Glueball-glueball scattering in a constituent gluon model

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Abstract. In this work we use a mapping technique to derive in the context of a constituent gluon model an effective Hamiltonian that involves explicit gluon degrees of freedom. We study glueballs with two gluons using the Fock-Tani formalism. In the present work we consider two possibilities for 0^{++}: (*i*) as a pure $s\bar{s}$ and calculate, in the context of a quark interchange picture, the cross-section; (*ii*) as a glueball where a new calculation for this cross-section is made, in the context of the constituent gluon model, with gluon interchange.

INTRODUCTION

The gluon self-coupling in QCD implies the existence of bound states of pure gauge fields known as glueballs. Numerous technical difficulties have so far been present in our understanding of their properties in experiments, largely because glueball states can mix strongly with nearby $q\bar{q}$ resonances. However recent experimental and lattice studies of 0^{++}, 2^{++} and 0^{-+} glueballs seem to be convergent.

In this work we use a mapping technique to derive in the context of a constituent gluon model an effective Hamiltonian that involves explicit gluon degrees of freedom. We study the glueball as a bound-state of two constituent gluons using the Fock-Tani formalism [1]. A glueball-glueball potential V_{gg}, scattering amplitude h_{fi} and a cross-section σ_{gg} can be obtained.

In the conventional quark model a 0^{++} state with $M_{0^{++}} = 1.73$ GeV is considered as $q\bar{q}$ bound state. The 0^{++} resonance is a isospin zero state so, in principal, it can be either represented as a $q\bar{q}$ bound state, a glueball, or a mixture. In particular there is growing evidence in the direction of large $s\bar{s}$ content with some mixture with the glue sector. It turns out that this resonance is an interesting system, in the theoretical point of view, where one can compare models. In the present work we consider two possibilities for 0^{++}: (*i*) as a pure $s\bar{s}$ and calculate, in the context of a quark interchange picture, the cross-section; (*ii*) as a glueball where a new calculation for this cross-section is made, in the context of the constituent gluon model, with gluon interchange.

On theoretical grounds, a simple potential model with massive constituent gluons, interacting by a two-body potential $V_{\rm aa}$, namely the model of Cornwall and Soni [2, 3] is used as an input in the Fock-Tani formalism for the microscopic model. This model has been studied recently in [4, 5] and the results are consistent with lattice and experiment.

A glueball-glueball potential can be obtained applying in a standard way the Fock-Tani transformed operators to the microscopic Hamiltonian

$$\mathscr{H}(\mu\nu;\sigma\rho) = T_{\rm aa}(\mu)a^{\dagger}_{\mu}a_{\mu} + \frac{1}{2}V_{\rm aa}(\mu\nu;\sigma\rho)a^{\dagger}_{\mu}a^{\dagger}_{\nu}a_{\rho}a_{\sigma}$$

where one obtains for the glueball-gluball potential V_{gg}

$$V_{gg} = \sum_{i=1}^{4} V_i(\alpha\gamma;\delta\beta)g^{\dagger}_{\alpha}g^{\dagger}_{\gamma}g_{\delta}g_{\beta} \tag{1}$$

and the $V_i(\alpha\gamma;\delta\beta)$ are given by

$$V_1 = 2V_{aa}(\mu\nu;\sigma\rho)\Phi^{\star\mu\tau}_{\alpha}\Phi^{\star\nu\xi}_{\gamma}\Phi^{\rho\xi}_{\delta}\Phi^{\sigma\tau}_{\beta} \;\; , \;\; V_2 = 2V_{aa}(\mu\nu;\sigma\rho)\Phi^{\star\mu\tau}_{\alpha}\Phi^{\star\nu\xi}_{\gamma}\Phi^{\rho\tau}_{\delta}\Phi^{\sigma\xi}_{\beta}$$
$$V_3 = V_{aa}(\mu\nu;\sigma\rho)\Phi^{\star\mu\nu}_{\alpha}\Phi^{\star\lambda\xi}_{\gamma}\Phi^{\sigma\lambda}_{\delta}\Phi^{\rho\xi}_{\beta} \;\; , \;\; V_4 = V_{aa}(\mu\nu;\sigma\rho)\Phi^{\star\mu\xi}_{\alpha}\Phi^{\star\nu\lambda}_{\gamma}\Phi^{\lambda\xi}_{\delta}\Phi^{\rho\sigma}_{\beta} \, .$$

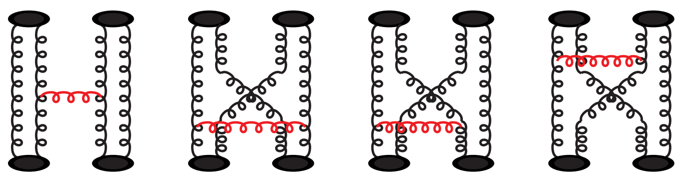

FIGURE 1. Diagrams representing the scattering amplitude h_{fi} for glueball-glueball interaction with constituent gluon interchange.

The next step is to obtain the scattering T-matrix from Eq. (1). Due to translational invariance, the T-matrix element is written as a momentum conservation delta-function, times a Born-order matrix element, h_{fi}:

$$T(\alpha\beta;\gamma\delta) = (\alpha\beta|V_{gg}|\gamma\delta) = \delta^{(3)}(\vec{P}_f - \vec{P}_i)\,h_{fi}$$

where $\vec{P}_f$ and $\vec{P}_i$ are the final and initial momenta of the two-glueball system. This result can be used in order to evaluate the glueball-glueball scattering cross-section

$$\sigma_{gg} = \frac{4\pi^5 s}{s-4M_G^2}\int_{-(s-4M_G^2)}^{0} dt\,|h_{fi}|^2 \tag{2}$$

where M_G is the glueball mass, s and t are the Mandelstam variables.

CONCLUSIONS

In this work we have extended the Fock-Tani formalism to a hadronic model in which the bound state is composed by bosons (gluons). The Cornwall-Soni constituent gluon model was used as the microscopic model. This model depends on several parameters: $\lambda = 3\alpha_s$; β, the string tension; k, a smearing constant and m, the constituent gluon

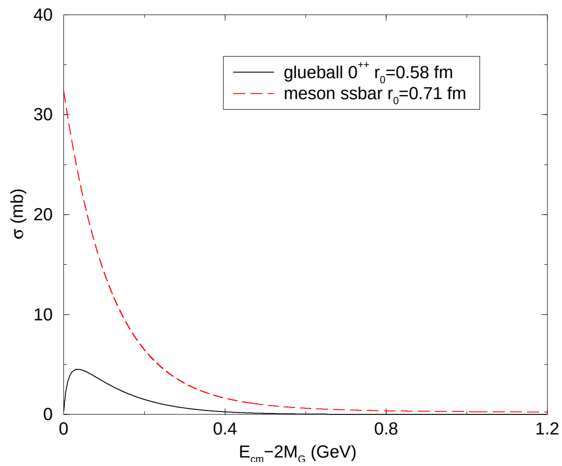

FIGURE 2. Cross-section comparison for 0^{++} with the following parameters $\beta = 0.1$, $\lambda = 1.8$, $k = 0.21$, gluon mass $m = 0.6$ GeV. The $s\bar{s}$ quark model parameters: $m_q = 0.55$ GeV, $\alpha_s = 0.6$.

mass. In the present we solve a mass equation in order to fix these parameters: $\beta = 0.1$, $\lambda = 1.8$, $k = 0.21$, gluon mass $m = 0.6$ GeV, for a radius $r_0 = 0.58$ fm. A comparison of the cross-sections reveals that a 0^{++} composed by $s\bar{s}$ quarks, with parameters $\alpha_s = 0.6$, $m_q = 0.55$ GeV and $r_0 = 0.71$ fm, adjusted to fit $\pi - \pi$ and $K - K$ scattering [1] implies in a much larger cross-section and different in shape than in the constituent gluon picture. This could represent a guiding-line criterion for distinguishing between pictures. The 0^{++} in our calculation is an extreme case, in which one has two models representing a pure state resonance. For other resonances, with a larger degree of mixture, this effect in the cross-section will not be clean, but we believe it will still be present validating this approach as a form of identifying glueballs in the meson spectrum.

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