# The velocities of light in modified QED vacua*

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QED vacua under the influence of external conditions (background fields, finite temperature, boundary conditions) can be considered as dispersive media whose complex behaviour can no longer be described in terms of a single universal vacuum velocity of light $c$. Beginning in the early 1950's (J.S. Toll), quantum field theoretic investigations have led to considerable insight into the relation between the vacuum structure and the propagation of light. Recent years have witnessed a significant growth of activity in this field of research. After a short overview, two characteristic situations are discussed: the propagation of light in a constant homogeneous magnetic field and in a Casimir vacuum. The latter appears to be particularly interesting because the Casimir vacuum has been found to exhibit modes of the propagation of light with phase and group velocities larger than $c$ in the low frequency domain $\omega \ll m$ where $m$ is the electron mass. The impact of this result on the front velocity of light in a Casimir vacuum is discussed by means of the Kramers-Kronig relation.

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## I. INTRODUCTION

Relativistic quantum field theory can be understood as having emerged, historically, from a combination of special relativity and quantum mechanics, but as its very fathers have always been very aware it is not a synthesis of these two theories. In standard relativistic quantum field theory, space-time is considered as a fixed arena in which the physical processes take place. The characteristics of the propagation of light are considered as *classical* input to the theory. This entails the view that there exists a single universal vacuum velocity of light $c$. However, starting in the early 1950's [1] research in quantum electrodynamics (QED) has revealed that higher conceptional sophistication in discussing the propagation of light in a vacuum is required. This more advanced insight derives from studies of QED vacua which have been modified by means of external conditions [background fields (electromagnetic, gravitational), finite temperature (heatbath), boundary conditions]. These modified vacua can be explored by studying the behaviour of particles immersed into the vacuum. One particular method, which is of special conceptional significance, consist in the investigation of the propagation of photons (light) in the vacuum modified by external conditions. The characteristics of the propagation of light then describe certain aspects of the vacuum structure. As a result, it has been found that modified QED vacua are complicated dispersive media which exhibit almost every phenomenon which is well known from ordinary condensed matter media in this respect. This is a *quantum* effect which is caused by the phenomenon of vacuum polarization. The propagation of light may depend on:

- the photon polarization,
- the direction of propagation,
- the photon frequency $\omega$ [(temporal) dispersion],
- the space-time location.

The description of these dependencies can conveniently be performed by means of a refractive index $n(\omega)$ for the appropriate modes of the propagation of light (we indicate the dependence on the photon frequency $\omega$ only; unless specified otherwise, $n(\omega)$ denotes the real part of the refractive index throughout the paper). Accordingly, as the refractive index of modified QED vacua is not a universal constant one needs to consider different notions of light velocities which have different physical significance (for a discussion of a number of notions of a propagation velocity see, e.g., [2,3]). In this article, we will concentrate our attention onto three velocities, the phase velocity

$$v_{\rm ph} \;=\; \frac{c}{n(\omega)} \quad , \tag{1}$$

the group velocity

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$$v_{\rm gr} \;=\; \frac{c}{n_{gr}(\omega)} \quad , \quad n_{gr}(\omega) \;=\; n(\omega) \;+\; \omega\frac{\partial n(\omega)}{\partial\omega} \quad , \tag{2}$$

and the front velocity

$$v_{\rm fr} \;=\; \frac{c}{n(\infty)} \quad , \tag{3}$$

which is the velocity related to the concept of the space-time structure (note that, in general, in modified QED vacua $n(\infty)$ has to be calculated and cannot be set equal to 1 just by default).

The behaviour of the different velocities can be discussed as soon as information about the refractive indices for the different modes propagating in the modified QED vacua is available. The task of quantum field theory consists in the calculation of these refractive indices. To this end one has to calculate the (renormalized) effective Maxwell action $\Gamma_{\rm eff}[\mathbf{E},\mathbf{B}]$ under the external conditions under consideration ($\mathbf{E}$, $\mathbf{B}$ are the field strengths of the test wave which are assumed to be sufficiently small). Within quantum field theory this is a well established procedure. The effective Maxwell action provides us with a convenient interface between quantum field theory and classical field theory. Once the effective Maxwell action is known, at least in some sensible approximation, a dispersion analysis can be performed by considering the effective equations of motion (effective Maxwell equations). This analysis proceeds along the same lines as the analysis for any other action would proceed, irrespective of its origin.

Within QED (as in many quantum field theoretic models), the effective (Maxwell) action can only explicitly be calculated, even at a given order of perturbation theory, if certain approximations are applied. The most commonly applied approximation is to consider the propagation of low frequency photons with $\omega \ll m$, i.e., of photons whose wavelength is much larger than the Compton wavelength of the electron ($m$ is the electron mass, $\hbar, c = 1$ throughout the paper unless inserted explicitly). Below we list the situations studied so far together with a qualitative overview of the results obtained for the low frequency refractive index $n(0) = n\,(\omega\ (\ll m))$. The references given in the column 'further literature' are not meant to represent a complete list. This, in particular, concerns the investigation of the propagation of light in electromagnetic background fields which has a rather long and rich history.

| | refractive index | first studied | further literature |
|---|---|---|---|
| – background fields | | | |
| – electromagnetic | $n(0) \geq 1$ | 1952 Toll [1] | [4–13] |
| – gravitational | $n(0) \gtreqless 1$ | 1980 Drummond/Hathrell [14] | [15,10,16,17,11,18–20] |
| – finite temperature | $n(0) > 1$ | 1983 Tarrach [21], corrected: 1990 Barton [22] | [23,24,12,13] |
| – Casimir vacuum (boundary conditions) | $n(0) \leq 1$ | 1990 Scharnhorst [25], Barton [22] | [26–28,24,12,20] |

The above results have all been obtained by considering 1-loop or 2-loop contributions to the effective Maxwell action only (which in the low frequency approximation yield the leading contribution in many cases). Very few results are available, even in 1-loop or 2-loop approximation, for arbitrary photon frequencies $\omega$. However, as one can see from Eq. (3), if one wants to discuss the front velocity of light, which is of greatest conceptional significance in discussing the space-time structure of the modified vacuum, one has to study the effective Maxwell action for arbitrarily large frequencies $\omega$ of the test photon exploring this vacuum. As interesting as any 1-loop or 2-loop calculation for arbitrary frequencies $\omega$ may be, the calculation of $n(\infty)$ is a truly nonperturbative task [28], p. 2041, which has not yet been solved in any of the situations listed above.

Latorre, Pascual and Tarrach [24] have proposed the following unified formula (sum rule) for the above listed results for the low frequency refractive index $n(0)$ which facilitates their qualitative understanding:

$$\bar{n}(0) \;=\; \frac{\sum_i n(0)}{\sum_i} \;=\; 1 \;+\kappa\rho \quad , \quad \kappa \;=\; \frac{44}{135}\,\frac{\alpha^2}{m^4} \quad . \tag{4}$$

Here, the sum (integral) $\sum_i$ extends over all directions of propagation and all polarizations, $\rho$ and $\alpha$ are the (vacuum) energy density and the fine structure constant, respectively. This sum rule has further been studied very recently in [12,13] within an elegant formalism which, so far, rigorously only applies to electromagnetic background fields, but can be extended to other situations.

## II. PROPAGATION OF LIGHT IN A CONSTANT HOMOGENEOUS MAGNETIC FIELD

In this section we want to illustrate the complicated behaviour of modified QED vacua by means of the example of a vacuum in the presence of a constant homogeneous magnetic background field (denoted by $\mathbf{B}_0$).

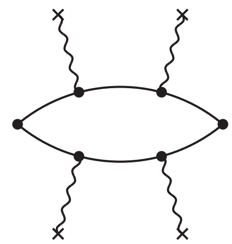

FIG. 1. Typical 1-loop diagram contributing to the effective Maxwell action in the presence of a magnetic background field (indicated by wavy lines)

For weak magnetic fields $\mathbf{B}_0$ and in the low frequency domain $\omega \ll m$, the following results have been obtained for the refractive index at the 1-loop level (cf. Fig. 1; $\Theta = \angle(\mathbf{B}_0, \mathbf{k})$, $\mathbf{k}$ is the wave vector of the test photon) [1,6,7].

$$n_{\|}(0) = 1 \ + \frac{8\alpha^2}{45} \frac{\mathbf{B}_0^2}{m^4} \sin^2\Theta \quad , \quad \mathbf{E} \parallel \mathbf{k} \times \mathbf{B}_0 \tag{5}$$

$$n_{\perp}(0) = 1 \ + \frac{14\alpha^2}{45} \frac{\mathbf{B}_0^2}{m^4} \sin^2\Theta \quad , \quad \mathbf{B} \parallel \mathbf{k} \times \mathbf{B}_0 \tag{6}$$

One immediately recognizes that, except for the case of propagation parallel to the magnetic field $\mathbf{B}_0$, the refractive indices for the two possible photon polarizations are different. Consequently, the QED vacuum in the presence of a magnetic (background) field is a birefringent medium! In the case of a constant homogeneous magnetic field, also (1-loop) results for the refractive index at arbitrary frequencies $\omega$ are available [1,4,5,7–9]. Figure 2 shows the behaviour of the two different refractive indices over a wide range of frequencies (we restrict our consideration to the real part of the refractive index). Figure 3 displays the polarization averaged refractive indices related to the phase and group velocities. No doubt, the graphs look quite similar to the graphs for the refractive index of some, say, condensed matter medium. It deserves to be emphasized that significant deviations from Eqs. (5), (6) occur for frequencies $\omega \gg m$ only. From Figs. 2, 3 one recognizes that for sufficiently large frequencies $\omega$ the phase and group velocities of light become larger than $c$, but the front velocity of light remains equal to $c$ (i.e., $n(\infty)$ is found to be equal to 1; incidentally, note that with increasing frequency the validity of QED perturbation theory is subject to scrutiny).

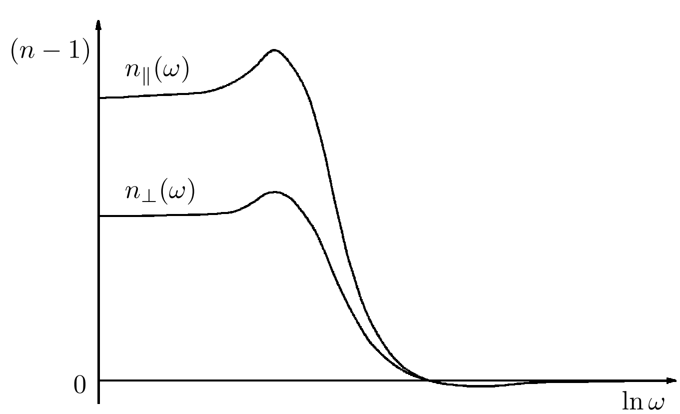


FIG. 2. Qualitative behaviour of the refractive index $n(\omega)$, the drawing is adapted from Fig. 3.5A of Ref. [1], p. 91

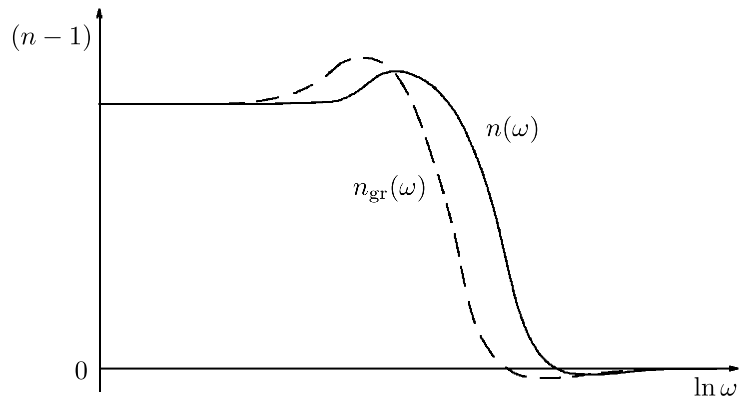


FIG. 3. Qualitative behaviour of the polarization averaged refractive indices $n(\omega)$, $n_{\rm gr}(\omega)$ (note that the arbitrary units applied to the axes are different here from those used in Fig. 2); the drawing is adapted from Fig. 2 of Ref. [4], p. 711; also see [5], Sec. 5.A

## III. THE CASIMIR VACUUM

The Casimir effect [29–31] is a macroscopic quantum effect which is considered to be a manifestation of the existence of (photon) vacuum fluctuations. In its best known form, it consists in the mutual attraction of two parallel uncharged conducting plates (mirrors) in vacuo separated by a distance $L$ (cf. Fig. 4) according to the law

$$p = -\frac{\pi^2}{240}\frac{1}{L^4} = -\frac{\partial E_{\rm vac}}{\partial L}\ , \quad E_{\rm vac} = -\frac{\pi^2}{720}\frac{1}{L^3}\ , \tag{7}$$

where $p$ is the force per unit area (Casimir pressure) and $E_{\rm vac}$ is the vacuum energy per unit area of the mirrors (the vacuum energy density $\rho$ is given by $E_{\rm vac}/L$).

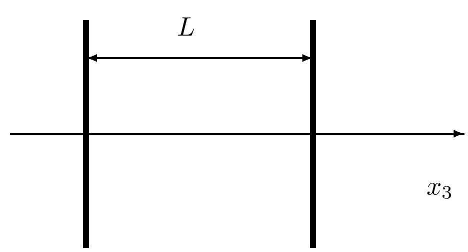


FIG. 4. The Casimir vacuum: Two uncharged parallel conducting plates enforcing at their surfaces the standard boundary conditions $\mathbf{E}_\parallel = 0 = \mathbf{B}_\perp$ on photon vacuum fluctuations

The effect has recently been verified quantitatively in the laboratory [32,33]. It is the result of the boundary conditions enforced on the photon vacuum fluctuations at the mirror surfaces[1]. The change in the vacuum structure effected by the parallel mirrors which manifests itself in terms of the Casimir effect also affects the propagation of light in the Casimir vacuum, as can be recognized from Eq. (4) (by the term 'Casimir vacuum' we denote the vacuum between the mirrors). In order to study this problem one has to calculate the effective Maxwell action for the given situation. This has been done in [25,22] in an approximation which considers plates distances $L$ much larger than the electron Compton wavelength $m^{-1}$ and low frequency test photons ($\omega \ll m$). The result can reliably be calculated within QED perturbation theory by taking into account the diagrams shown in Fig. 5.

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[1]As the Casimir effect is an infrared (i.e., long distance) phenomenon its physics is fairly independent of the fact that in reality the boundary conditions cease to apply beyond the cut-off frequency of the mirrors. This comment also applies to the discussion of the propagation of light in a Casimir vacuum including the problem of the front velocity of light in it.

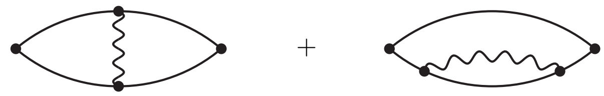

FIG. 5. 2-loop diagrams contributing in a nontrivial way to the effective Maxwell action in a Casimir vacuum

For the effective Maxwell action one finds the following result (between the mirrors).

$$\Gamma_{\text{eff}}[\mathbf{E},\mathbf{B}] \;=\; \frac{1}{2}\int \mathrm{d}^4x \left(\epsilon_{ij}E^iE^j \;-\; \mu^{-1}_{ij}B^iB^j\right) \tag{8}$$

$$\left\{ \begin{array}{rcl} \epsilon_{11} & = & \epsilon_{22} \\ & & \mu_{33} \end{array} \right\} = 1 \left\{ \begin{array}{c} + \\ - \end{array} \right\} \frac{\pi^2}{180}\,\frac{\alpha^2}{(mL)^4}\,\left[g(x_3) - \frac{11}{45}\right]$$

$$\left\{ \begin{array}{rcl} & & \epsilon_{33} \\ \mu_{11} & = & \mu_{22} \end{array} \right\} = 1 \left\{ \begin{array}{c} + \\ - \end{array} \right\} \frac{\pi^2}{180}\,\frac{\alpha^2}{(mL)^4}\,\left[g(x_3) + \frac{11}{45}\right]$$

Here, $g(x_3)$ is some explicitly known function which the diagonal elements of the permittivity and permeability tensors (all non-diagonal elements are zero) depend on but not the refractive index itself calculated in WKB approximation. For the low frequency refractive index one finds

$$n_{\|}(0) = 1 \tag{9}$$

for propagation of light parallel to the mirrors (as is expected from the residual Lorentz invariance with respect to boosts parallel to the mirrors) and

$$n_{\perp}(0) = 1 \;-\; \frac{11\pi^2}{(90)^2}\,\frac{\alpha^2}{(mL)^4} \tag{10}$$

for propagation perpendicular to the mirrors (in the space between them). In the approximation used the propagation of light is non-dispersive, consequently the phase and group velocities agree and are given by $c/n_{\perp}(0) > c$ (incidentally, analogous results have been found in the case of gravitational background fields). The change in these velocities related to the presence of the parallel mirrors is interesting as a matter of principle, but it is unmeasurably small in practice. Moreover, it cannot be detected by test waves whose Fourier components respect $\omega \ll m$ [26–28].

In qualitative respect, of most interest appears to be the question what can be said about the front velocity of light $c/n_{\perp}(\infty)$ in the Casimir vacuum. So far, no calculation of the refractive index $n(\omega)$ for arbitrary frequencies $\omega$ has been performed (an attempt for a complete 2-loop calculation made in [24] has turned out to be incorrect). Quite generally, the calculation of the infinite frequency limit of the refractive index is a nonperturbative task the technical tools we are presently lacking for[2]. However, in the present case there is an alternative to the explicit calculation. This alternative approach is based on structural arguments which are linked to basic physical principles [28]. The only nonperturbative relation at hand is the standard Kramers-Kronig dispersion relation for the refractive index which embodies the principle of local causality, i.e., the principle that no effect can precede its cause. The standard Kramer-Kronig relation reads

$$\mathrm{Re}\; n(\omega) \;=\; \mathrm{Re}\; n(\infty) \;+\; \frac{2}{\pi}\int_0^\infty \mathrm{d}\omega'\;\frac{\omega'\;\mathrm{Im}\; n(\omega')}{\omega'^2 - \omega^2} \tag{11}$$

and can immediately be specialized to the following form we are mainly interested in (for a more detailed discussion see [28]).

$$n(\infty) \;=\; n(0) \;-\; \frac{2}{\pi}\int_0^\infty \frac{\mathrm{d}\omega'}{\omega'}\;\mathrm{Im}\; n(\omega') \tag{12}$$

[2]The point is that for $\omega \ll m$ one can rely on a derivative expansion of the effective Maxwell action. For the Casimir vacuum, e.g., the 2-loop contribution to $n(0)$ yields the dominant term if $mL \gg 1$ applies. However, for $\omega \to \infty$ ($\omega \gg m$) the derivative expansion is no longer appropriate. Then, loops of arbitrary order are contributing in principle.

If one assumes that the Casimir vacuum behaves like a passive medium (Im $n_\perp(\omega) \geq 0$) from Eq. (12) one finds the inequality

$$1 \ > \ n_\perp(0) \ \geq \ n_\perp(\infty) \ \ . \tag{13}$$

Eq. (13) provides us with an upper bound on $n_\perp(\infty)$ irrespective of any nonperturbative contribution. Consequently, we may conclude (we denote this conclusion by the term 'alternative A'):

(A.) The front velocity of light $c/n_\perp(\infty)$ in a Casimir vacuum is larger than the front velocity of light $c$ in the unbounded space vacuum .

As unconventional as this conclusion may appear to be, it does not contradict the special theory of relativity. In this context, it is important to realize that the parallel mirrors (i.e., the Casimir vacuum) define a distinguished reference system. The equivalence of all inertial systems special relativity is based on no longer applies (this point, however, is not exclusively related to the Casimir vacuum, or other modified vacua, and can also be found in discussions of classical media, cf. [34], e.g.). Furthermore, the above conclusion does not entail any violation of the principle of local causality[3]. A well defined (but modified) light cone continues to exist. And finally, it is the etalon phenomenon itself which the space-time structure is defined by which is modified ("light does never move faster than light"). In other words, the light cone related to $c$ is no longer of any physical significance and, therefore, cannot be used to construct thought experiments exhibiting some causality violation.

The above conclusion (alternative A) with respect to the front velocity of light $c/n_\perp(\infty)$ in a Casimir vacuum relies on Eq. (13) which has been derived on the basis of a few assumptions. The alternative A can only be evaded if these assumptions were not valid. Consequently, mainly there are two (further) *logical* alternatives to it [28]:

B. The standard Kramers-Kronig relation fails to be valid in the present context.

C. Im $n_\perp(\omega) < 0$, at least for some range of frequencies $\omega$. In this case the Casimir vacuum would behave like an active medium amplifying the test wave. This seems to be in conflict with the principle of energy conservation unless a physical explanation removing this concern could be found (presently, for it even not a guess exists).

If one wished to avoid the alternative A one would have to face the logical alternatives B or C. It should, however, be emphasized that the latter alternatives would entail fairly unconventional physics.

## IV. CONCLUSIONS

QED vacua under the influence of external conditions are complicated dispersive media. The appearance of (phase and group) velocities larger than $c$ is a common phenomenon in these vacua. The front velocity of light $c/n(\infty)$ in such vacua cannot be calculated by means of presently available theoretical tools – this is a truly nonperturbative task. Regarding the special case of the Casimir vacuum, it can be said that the low frequency refractive index $n_\perp(0)$ for propagation perpendicular to two parallel mirrors (in the space between them) has reliably been calculated within QED perturbation theory and found to be smaller than 1. This result can be used indirectly to infer information about the related front velocity by relying on the standard Kramers-Kronig relation which embodies the principle of local causality (i.e., the fact that there can be no effect preceding its cause). If one then conventionally assumes that the Casimir vacuum behaves like a passive medium (Im $n_\perp(\omega) \geq 0$) one is led to conclude that the front velocity of light $c/n_\perp(\infty)$ in a Casimir vacuum is larger than the front velocity of light $c$ in the unbounded space vacuum[4]. It should finally be emphasized that this conclusion does not involve any serious conceptual dangers and does not, in particular, contradict the special theory of relativity.

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[3] This, however, is controversially being discussed. For a recent paper in this respect see [20].

[4] One should bear in mind, however, that this conclusion depends on a few assumptions which might fail to apply, although this would entail fairly unconventional and, therefore, equally interesting physics as pointed out at the end of Sec. III.

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