

## Research Article

# A New Physical Parameter Identification Method for Two-Axis On-Road Vehicles: Simulation and Experiment

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A new physical parameter identification method for two-axis on-road vehicle is presented. The modal parameters of vehicle are identified by using the State Variable Method. To make it possible to determine the matrices  $M$ ,  $C$ , and  $K$  of the vehicle, a known mass matrix  $\Delta M$  is designed to add into the vehicle in order to increase the number of equations ensuring that the number of equations is more than the one of unknowns. Therefore, the physical parameters of vehicle can be estimated by using the least square method. To validate the presented method, a numerical simulation example and an experiment example are given in this paper. The numerical simulation example shows that the largest of absolute value of percentage error is 1.493%. In the experiment example, a school bus is employed in study for the parameter identification. The simulation result from full-car model with the estimated physical parameters is compared with the test result. The agreement between the simulation and the test proves the effectiveness of the proposed estimation method.

## 1. Introduction

Physical parameters are essential for the dynamic modeling and analysis of road vehicles [1], but in practice they are very difficult to obtain. Therefore, methods to estimate these parameters are of great interest to researchers as well as to the industry [2–5]. Physical parameter identification is the second type of dynamical inverse problem. Generally speaking, its procedure could be divided into two steps: (1) modal parameter identification, which can identify modal parameters of vehicle, such as natural frequencies, damping ratios, and modal shapes; (2) physical parameter identification, which can identify physical parameters of vehicle, for example, the mass, damping coefficient, and stiffness parameters of vehicle.

During these decades, the researchers have proposed many different kinds of methods for modal parameter identification. These methods can be classified into two main categories: frequency domain and time domain methods. When natural frequencies are not close or damping ratios

are not high, the modal parameter can be identified by using frequency domain methods, such as peak picking method (P-P), polynomial fitting, maximum likelihood identification (MLI), and PolyMAX method. Otherwise, they are difficult to be identified by using frequency domain methods [5]. However, time domain methods, such as Ibrahim Time Domain (ITD), Eigensystem Realization Algorithm (ERA), Autoregressive Moving Average (ARMA), Empirical Mode Decomposition (EMD), Stochastic Subspace identification (SSI), can be suitable for modal parameter identification in all cases. Because modal frequencies of vehicle body are the 1–3 Hz range and the corresponding damping ratios are between 0.1 and 0.5 [5], time domain methods may be more appropriate to identify the modal parameter of vehicle.

In recent years, many time domain identification methods have been developed to identify the parameter of the on-road vehicle. Kumar and Shankar used global and substructure approaches in the time domain to identify the parameter of structures with nonlinearities [2]. The online parameter estimation method proposed by Rozyn and Zhang used

the equivalent suspension stiffness coefficient to represent suspension and wheel stations in order to simplify modeling [3]. Venture et al. presented a robotics approach, which is based on a multibody dynamic system that allows the automatic computation of the dynamic identification model, to estimate the dynamic parameters of a car [4]. Based on subspace identification method of modal parameters, Dong et al. used a new parameter identification method for estimating roll and pitch moments of inertia of the on-road vehicle [5]. Huh et al. designed a vehicle mass estimator for adaptive roll stability control [6]. Koulocheris et al. applied a parametric identification method to estimate structural parameters of commercial passenger vehicle [7]. All of these methods can only identify the part of physical parameters of vehicle. It is still difficult to identify all physical parameters of vehicle by using conventional parameter identification method.

This paper presents a new physical parameter identification method for two-axis on-road vehicle. The State Variable Method (SVM), which is presented by Zhang and Hayama to identify the modal parameter and physical parameter of structural system [8], is employed to identify the modal parameter of vehicle. In the parameter identification procedure, the presented method proposed that the modal parameter of original vehicle and vehicle which added a known mass are both identified simultaneously. The purpose of this procedure is to guarantee that the number of equations is more than the number of unknowns. Therefore, physical parameters of on-road vehicle can be calculated by using least square method. To verify the presented method, a numerical simulation example and an experiment example are given in this paper. A school bus is employed as an experiment example to study the physical parameter identification, based on the free-decay responses collected from drop test. Vehicle modal parameters of the school bus, including natural frequencies, damping ratios, and mode shapes, can be obtained from the measured free-decay responses through SVM. When the modal parameter of the school bus is obtained, furthermore, vehicle physical parameters, such as inertial parameters, stiffness, and damping, can be estimated by solving the inverse problem [9, 10]. The numerical simulation example and experiment example illustrate that the presented method is effective and accurate for physical parameter identification of a two-axis on-road vehicle.

## 2. Outline of the Method

**2.1. Notation.** The parameters of the 7-DOF vehicle model are as follows:

$m_s$ : sprung mass,

$I_{xx}$ : sprung mass inertia of the roll,

$I_{yy}$ : sprung mass inertia of the pitch,

$m_{uf}/m_{ur}$ : unsprung mass of the front/rear suspension of vehicle,

$c_{sf}/c_{sr}$ : damping coefficient of the front/rear suspension of shock absorber,

$k_{sf}/k_{sr}$ : spring stiffness of the front/rear suspension of vehicle,

$k_{tf}/k_{tr}$ : tire spring stiffness of the front/rear suspension of vehicle,

$t_{f1}/t_{r1}$ : half of shock absorber left-to-right distance at front/rear of vehicle,

$t_{f2}/t_{r2}$ : half of leaf spring left-to-right distance at front/rear of vehicle,

$a/b$ : distance from sprung mass CG to the front/rear axle,

$z_s$ : vertical displacement of sprung mass,

$\theta$ : pitch angle of vehicle body,

$\varphi$ : roll angle of vehicle body,

$z_{ui}$ : vertical displacement of unsprung mass ( $i = A, B, C, D$ ),

$z_{gi}$ : displacement of ground input ( $i = A, B, C, D$ ).

**2.2. Parameter Identification Method.** Figure 1 shows a 7-DOF vehicle model, and the dynamic equation of the vehicle system is

$$M\ddot{X} + C\dot{X} + KX = F, \quad (1)$$

where vectors  $X$ ,  $\dot{X}$ , and  $\ddot{X}$  are, respectively, the displacement, the velocity, and the acceleration of the vehicle,  $M$ ,  $C$ , and  $K$  are, respectively, the mass, damping, and stiffness matrices of the vehicle system, and vector  $F$  is the excitation of vehicle:

$$X = \{z_s \ \theta \ \varphi \ z_{uA} \ z_{uB} \ z_{uC} \ z_{uD}\}^T,$$

$$M = \text{diag} \{m_s \ I_{yy} \ I_{xx} \ m_{uf} \ m_{uf} \ m_{ur} \ m_{ur}\},$$

$$C = \begin{bmatrix} 2(c_{sf} + c_{sr}) & 2(bc_{sr} - ac_{sf}) & 0 & -c_{sf} & -c_{sf} & -c_{sr} & -c_{sr} \\ 2(bc_{sr} - ac_{sf}) & 2(a^2c_{sf} + b^2c_{sr}) & 0 & ac_{sf} & ac_{sf} & -bc_{sr} & -bc_{sr} \\ 0 & 0 & 2(t_{f1}^2c_{sf} + t_{r1}^2c_{sr}) & -t_{f1}c_{sf} & t_{f1}c_{sf} & -t_{r1}c_{sr} & t_{r1}c_{sr} \\ -c_{sf} & ac_{sf} & -t_{f1}c_{sf} & c_{sf} & 0 & 0 & 0 \\ -c_{sf} & ac_{sf} & t_{f1}c_{sf} & 0 & c_{sf} & 0 & 0 \\ -c_{sr} & -bc_{sr} & -t_{r1}c_{sr} & 0 & 0 & c_{sr} & 0 \\ -c_{sr} & -bc_{sr} & t_{r1}c_{sr} & 0 & 0 & 0 & c_{sr} \end{bmatrix},$$

$$K = \begin{bmatrix} 2(k_{sf} + k_{sr}) & 2(bk_{sr} - ak_{sf}) & 0 & -k_{sf} & -k_{sf} & -k_{sr} & -k_{sr} \\ 2(bk_{sr} - ak_{sf}) & 2(a^2k_{sf} + b^2k_{sr}) & 0 & ak_{sf} & ak_{sf} & -bk_{sr} & -bk_{sr} \\ 0 & 0 & 2(t_{f2}^2k_{sf} + t_{r2}^2k_{sr}) & -t_{f2}k_{sf} & t_{f2}k_{sf} & -t_{r2}k_{sr} & t_{r2}k_{cr} \\ -k_{sf} & ak_{sf} & -t_{f2}k_{sf} & k_{sf} + k_{tf} & 0 & 0 & 0 \\ -k_{sf} & ak_{sf} & t_{f2}k_{sf} & 0 & k_{sf} + k_{tf} & 0 & 0 \\ -k_{sr} & -bk_{sr} & -t_{r2}k_{sr} & 0 & 0 & k_{sr} + k_{tr} & 0 \\ -k_{sr} & -bk_{sr} & t_{r2}k_{sr} & 0 & 0 & 0 & k_{sr} + k_{tr} \end{bmatrix},$$

$$F = \{0 \ 0 \ 0 \ k_{tA}z_{gA} \ k_{tB}z_{gB} \ k_{tC}z_{gC} \ k_{tD}z_{gD}\}^T, \quad (2)$$

where the superscript “ $T$ ” denotes vector or matrix transpose. Consider the free vibration analysis of vehicle system; the excitation force vector  $F$  is zero vector. Introducing state vector  $Y = \{X \ \dot{X}\}^T$ , (1) can be rewritten in the state equation:

$$\dot{Y} = AY, \quad (3)$$

where

$$A = \begin{bmatrix} O & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}. \quad (4)$$

In the discrete time domain, the system can be described in the form of the difference equation:

$$Y(k+1) = A_1Y(k), \quad (5)$$

where

$$Y(k) = \{X^T(k) \ X^T(k+1)\}^T, \quad (6)$$

$$Y(k+1) = \{X^T(k+1) \ X^T(k+2)\}^T$$

are the discrete state vectors sampled at  $t = k\Delta T$ ,  $(k+1)\Delta T$ ,  $\Delta T$  is the sampling interval and

$$A_1 = \exp(A\Delta T) \quad (7)$$

is the transition matrix of the time discrete system.

In order to obtain the transition matrix  $A_1$ , the following finite difference equation can be constructed:

$$\bar{\Phi} = A_1\Phi + \tilde{\Phi}, \quad (8)$$

where  $\tilde{\Phi}$  is an error matrix and the state matrices  $\Phi$  and  $\bar{\Phi}$  are defined as

$$\Phi = [Y(1) \ Y(2) \ \cdots \ Y(N)], \quad (9)$$

$$\bar{\Phi} = [Y(2) \ Y(3) \ \cdots \ Y(N+1)],$$

where  $N$  is the column number of state matrix  $\Phi$ . The transition matrix  $A_1$  can be determined by using the least squares method, and it can be expressed as

$$A_1 = (\bar{\Phi}\Phi^T)(\Phi\Phi^T)^{-1}. \quad (10)$$

According to the relation between the  $Z$ -plane and the  $S$ -plane, the eigenvalues and eigenvectors of the matrix  $A$  can be obtained:

$$\lambda_i = \frac{\ln(z_i)}{\Delta T}, \quad (11)$$

$$\Phi_i = P_i, \quad (i = 1, 2, \dots, 7),$$

where  $\lambda_i$ ,  $\Phi_i$  are the eigenvalues and eigenvectors of the matrix  $A$  and  $z_i$ ,  $P_i$  are the eigenvalues and eigenvectors of the transition matrix  $A_1$ . Once the eigenvalues and eigenvectors of the matrix  $A$  are determined, the matrix  $A$  can be calculated through the following formula:

$$A = \begin{bmatrix} \Phi & \Phi^* \\ \Phi\Lambda & \Phi^*\Lambda^* \end{bmatrix} \begin{bmatrix} \Lambda & \\ & \Lambda^* \end{bmatrix} \begin{bmatrix} \Phi & \Phi^* \\ \Phi\Lambda & \Phi^*\Lambda^* \end{bmatrix}^{-1}, \quad (12)$$

where  $\Lambda^*$  and  $\Phi^*$  are, respectively, the conjugate matrices of  $\Lambda$  and  $\Phi$ :

$$\Lambda = \text{diag}[\lambda_1 \ \lambda_2 \ \cdots \ \lambda_7], \quad (13)$$

$$\Phi = [\Phi_1 \ \Phi_2 \ \cdots \ \Phi_7]. \quad (14)$$

The aforementioned method, which identifies the state matrix of the vehicle system, is called the State Variable Method. Although the state matrix  $A$  of the original vehicle system is identified, it is still difficult to determinate the mass matrix  $M$ , the damping coefficient matrix  $C$ , and the stiffness matrix  $K$  of the original vehicle by solving (3). This is because the number of equations is less than the number of unknowns in (3). To make it easy to identify the parameters of the original vehicle, a known mass matrix  $\Delta M$  is added into the original vehicle system. Then, the state matrix of vehicle with additional mass can be represented as

$$\bar{A} = \begin{bmatrix} O & I \\ -(M + \Delta M)^{-1}K & -(M + \Delta M)^{-1}C \end{bmatrix}. \quad (15)$$

The new state matrix  $\bar{A}$  can also be determined by using SVM. Now, the problem becomes that the number of equations is more than the number of unknowns when (3) and (15)

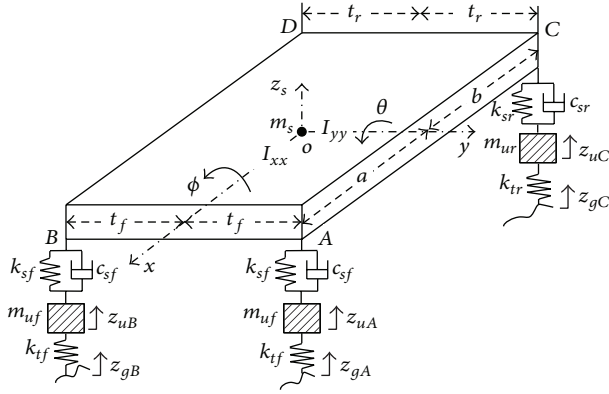


FIGURE 1: A 7-DOF vehicle model.

are combined. It can be easy to determinate the parameters of the original vehicle by using the least squares estimation method. Let the symbols  $A_{21}$  and  $\bar{A}_{21}$ , respectively, denote the submatrix of  $A$  and  $\bar{A}$ . The expressions of submatrices  $A_{21}$  and  $\bar{A}_{21}$  are, respectively,

$$A_{21} = -M^{-1}K, \quad (16)$$

$$\bar{A}_{21} = -(M + \Delta M)^{-1}K. \quad (17)$$

Solving (16) and (17), the mass matrix  $M$  can be obtained:

$$M = \Delta M \bar{A}_{21} (A_{21} - \bar{A}_{21})^{-1}. \quad (18)$$

When the mass matrix  $M$  is identified, the damping coefficient matrix  $C$  and stiffness matrix  $K$  of the vehicle system can be easily calculated by solving (3).

### 3. A Numerical Simulation Example

In order to validate the presented method, a numerical simulation example is given. The physical parameters of a 7-DOF vehicle model, which are used in the numerical simulation example, are listed in Table 1.

All the parameters in Table 1 are given in [6]. If the SVM method is chosen to identify the physical parameters of vehicle in the numerical simulation example, the free-decay responses of vehicle must be obtained first. To obtain the free-decay responses of vehicle, the half-sine bump is chosen as the road excitation of the vehicle. And the function of half-sine bump is expressed as follow:

$$z_g(t) = \begin{cases} 0.09 \sin(5\pi t) & 0.1 \leq t \leq 0.3 \\ 0 & t < 0.1, \text{ or } t > 0.3. \end{cases} \quad (19)$$

TABLE 1: The physical parameters of a 7-DOF vehicle model.

Notation	Description	Values	Units
$M_s$	Sprung mass	1380	kg
$I_{yy}$	Sprung mass inertia of the pitch	2440	kg·m <sup>2</sup>
$I_{xx}$	Sprung mass inertia of the roll	380	kg·m <sup>2</sup>
$m_{uf}$	Unsprung mass of the front	40.5	kg
$m_{ur}$	Unsprung mass of the rear	45.4	kg
$k_{sf}$	Suspension stiffness of the front	17000	N/m
$k_{sr}$	Suspension stiffness of the rear	22000	N/m
$k_{tf}$	Tire stiffness of the front	192000	N/m
$k_{tr}$	Tire stiffness of the rear	192000	N/m
$c_{sf}$	Suspension damping coefficient of the front	1500	N·s/m
$c_{sr}$	Suspension damping coefficient of the rear	1500	N·s/m
$a$	Distance from the sprung mass CG to the front axle	1.25	m
$b$	Distance from the sprung mass CG to the rear axle	1.51	m
$t_f$	Half width of the front axle	0.74	m
$t_r$	Half width of the rear axle	0.74	m

The simulations include the response simulation of original vehicle and the response simulation of vehicle with an additional known mass. If the incremental percentage of mass is less than 10%, the relative variety of modal parameters is less than 5%. Consider the real signal contains the noise, if incremental of mass is too small, it will cause the large error of physical parameter identification of vehicle. It is suggested that the incremental percentage of mass is about 10% in order to achieve better identification accuracy. In this manuscript, assume the additional mass matrix  $\Delta M$  being as following,

$$\Delta M = \begin{bmatrix} 110 & & & & & & \\ & 240 & & & & & \\ & & 40 & & & & \\ & & & 5 & & & \\ & & & & 5 & & \\ & & & & & 5 & \\ & & & & & & 5 \end{bmatrix}. \quad (20)$$

In case of the roll mode excitation, in which road excitation force of one side of vehicle is nonzero and the other side of vehicle is zero, seven modal modes of vibration of the vehicle can be stimulated. Therefore, all modal parameters of 7-DOF vehicle model can be identified through the SVM method in the roll mode excitation simulation. In this paper, the response simulations of vehicle in the roll mode excitation are only considered. Supposing the displacements of the ground input  $z_{gA}$  and  $z_{gC}$  both are the function of road excitation  $z_g$  and the displacements of the ground input  $z_{gB}$  and  $z_{gD}$  both are zero function. Let symbols  $M_{id}$ ,  $C_{id}$ , and  $K_{id}$

represent the mass matrix, the damping coefficient matrix, and the stiffness matrix of the original vehicle identified

by using the SVM method. The  $M_{id}$ ,  $C_{id}$ , and  $K_{id}$  are, respectively,

$$M_{id} = \begin{bmatrix} 1379.50 & -0.08 & 0 & 0.08 & 0.08 & 0.09 & 0.10 \\ -0.10 & 2439.19 & 0 & -0.10 & -0.10 & 0.15 & 0.15 \\ 0 & 0 & 379.77 & 0.06 & -0.06 & 0.07 & -0.08 \\ 0.14 & -0.15 & 0.09 & 39.99 & 0 & 0 & 0 \\ 0.14 & -0.15 & -0.09 & 0 & 39.99 & 0 & 0 \\ 0.15 & 0.19 & 0.09 & 0 & 0 & 44.82 & 0 \\ 0.15 & 0.19 & -0.09 & 0 & 0 & 0 & 44.82 \end{bmatrix}, \quad (21)$$

$$C_{id} = \begin{bmatrix} 5986.40 & 776.51 & 0.03 & -1494.27 & -1494.33 & -1493.99 & -1493.87 \\ 776.70 & 11501.65 & 0.01 & 1867.88 & 1867.87 & -2255.99 & -2255.98 \\ 0.02 & 0.04 & 3277.32 & -1105.40 & 1105.41 & -1105.33 & 1105.31 \\ -1478.32 & 1847.92 & -1093.58 & 1475.91 & -0.02 & -0.29 & 0.28 \\ -1478.31 & 1847.94 & 1093.59 & -0.01 & 1475.90 & 0.26 & -0.28 \\ -1477.81 & -2231.61 & -1093.28 & -0.31 & 0.27 & 1475.75 & -0.06 \\ -1477.80 & -2231.58 & 1093.29 & 0.28 & -0.31 & -0.08 & 1475.75 \end{bmatrix}, \quad (22)$$

$$K_{id} = \begin{bmatrix} 77835.9 & 23869.4 & 0.55 & -16655.4 & -16660.2 & -21590.9 & -21580.0 \\ 23872.4 & 153127.8 & 0.11 & 20815.7 & 20815.6 & -32601.4 & -32599.8 \\ 0.25 & 0.52 & 42613.4 & -12311.1 & 12311.6 & -15966.7 & 15964.0 \\ -16756.9 & 20946.6 & -12394.1 & 206074.1 & -0.22 & -8.40 & 11.46 \\ -16756.8 & 20946.9 & 12395.1 & 0.65 & 206073.9 & 9.36 & -7.95 \\ -21678.9 & -32736.5 & -16039.4 & -8.17 & 7.31 & 210959.3 & 1.22 \\ -21678.8 & -32736.1 & 16039.6 & 8.82 & -8.93 & -1.29 & 210959.6 \end{bmatrix}. \quad (23)$$

Comparing the expressions of  $M_{id}$ ,  $C_{id}$ , and  $K_{id}$  with the  $M$ ,  $C$ , and  $K$ , the physical parameters of the original vehicle can be calculated by using least square method. Table 2 gives the comparison of the parameters of identification with the ones of vehicle. It also shows that the largest of absolute value of percentage error is 1.493%; half of absolute values of percentage error are less than 1%. This numerical simulation example illustrates that the presented method is very effective for the physical parameter of vehicle identification.

## 4. An Experiment Example

**4.1. A School Bus Experiment.** A school bus, shown in Figure 2, is employed in the study of the parameter identification. There are eight accelerometers in total used in the tests. Four of them are mounted to the four corners of the chassis, and the rest four are mounted to the four wheel stations. Two of the mounted accelerometers are shown in Figure 3. The free-decay tests contain two groups of tests: (1) vehicle without loading and (2) vehicle with loading. Each group of tests includes bounce test, pitch test, and roll test of the vehicle. In the bounce test, four wheels of the school bus are

dropped simultaneously off the blocks with the same height in order to concentrate the excitation energy to stimulate the bounce mode. Similarly, in the pitch test, front or rear wheels of the school bus are dropped off the blocks to stimulate the pitch mode. In the roll test, left or right wheels of the school bus are dropped off the blocks to stimulate the roll mode. In the test with loading, the additional mass of the vehicle is 470 kg, the additional inertia in the pitching direction of the vehicle is 1057.5 kg·m<sup>2</sup>, and the additional inertia in the roll direction of the vehicle is 339.6 kg·m<sup>2</sup>. The acceleration responses of free-decay test are measured by acceleration transducers.

Figures 4–6 show the fast Fourier transformation (FFT) results of the vehicle drop tests. In the modal parameter estimation of the school bus, the SVM is employed to extract dynamic transition matrix from the free-decay test through the measurement of eight channels. The modal parameters of vehicle with/without loading, which are natural frequencies and damping ratios, are obtained by solving the eigenvalues problem of the transition matrix. Table 3 shows the estimated modal parameters of the test vehicle. Since the vehicle is asymmetric in the longitudinal direction,

TABLE 2: Comparison of the parameters of identification with the ones of vehicle.

Notation	Known	Identified	Percentage error (%)
$M_s$	1380	1379.50	-0.036
$I_{yy}$	2440	2439.20	-0.033
$I_{xx}$	380	379.77	-0.059
$m_{uf}$	40.5	39.99	-1.261
$m_{ur}$	45.4	44.82	-1.288
$k_{sf}$	17000	16600.17	-0.587
$k_{sr}$	22000	21824.96	-0.796
$k_{tf}$	192000	189173.82	-1.472
$k_{tr}$	192000	189134.48	-1.493
$c_{sf}$	1500	1493.31	-0.446
$c_{sr}$	1500	1492.88	-0.475
$a$	1.25	1.232	-1.436
$b$	1.51	1.500	-0.692
$t_f$	0.74	0.731	-1.223
$t_r$	0.74	0.733	-0.916



FIGURE 2: The drop test of the vehicle.



FIGURE 3: Accelerometer location of the chassis.

the body-dominant pitch mode is very difficult to be identified by FFT. In Table 3, we can find that the errors in the damped natural frequency estimated by FFT and the damped natural frequency estimated by the SVM are small.

In the physical parameter estimation, the total mass of the vehicle, the distance from sprung mass CG to the front/rear axle, and the half of leaf spring left-to-right distance at front/rear of vehicle can be determined by the weight and measure. And these physical parameter values are listed in Table 4. Other physical parameters can be determined by solving the state matrix  $A$  of the original vehicle system and the state matrix  $\bar{A}$  of the vehicle system after adding a known mass. Therefore, the physical parameters of the school bus

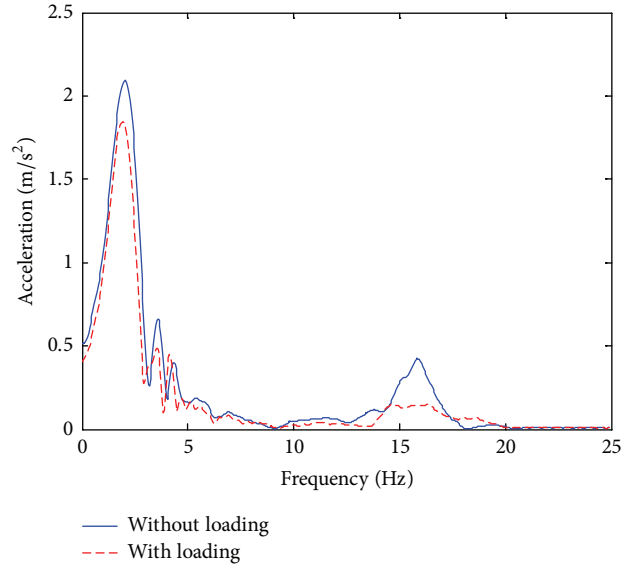


FIGURE 4: The FFT results of the vertical acceleration in the bounce test.

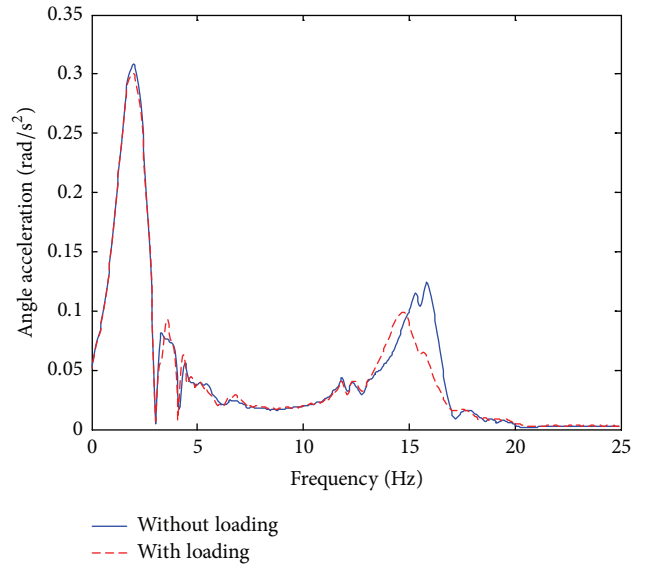


FIGURE 5: The FFT results of the pitch angular acceleration in the pitch test.

are estimated and listed in Table 5. The equivalent suspension damping coefficient contains the damping coefficient of leaf spring and shock absorber.

**4.2. Numerical Simulation.** To verify the result, the estimated physical parameters are applied to the 7-DOF full-car model to run simulation, and the result is compared with the tests. Figures 7–9 show the comparisons of the acceleration response in bounce test, the pitch angular acceleration in pitch test, and the roll angular acceleration in roll test. The results show that the simulation matches the tests well for the large amplitude oscillation at the beginning, while the

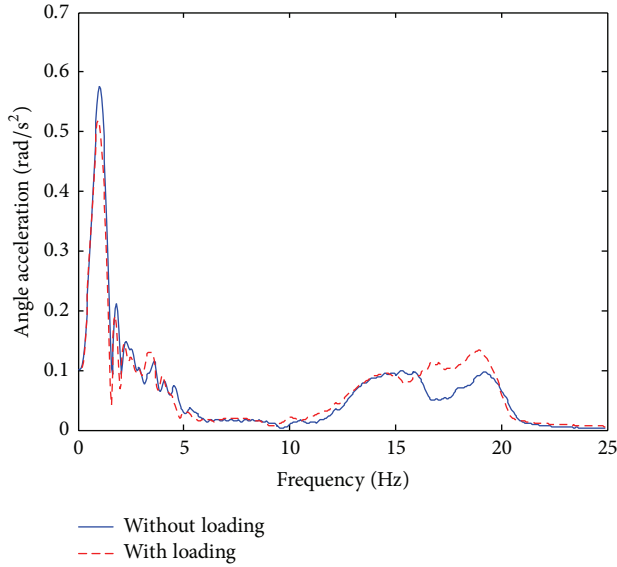


FIGURE 6: The FFT results of the roll angle acceleration in the roll test.

TABLE 3: The estimated vehicle modal parameters.

	FFT $f_d$ (Hz)	SVM $f_d$ (Hz)	Error $\varepsilon$ (%)	SVM $\omega_n$ (Hz)	SVM $\zeta$
Body-dominant bounce mode	2.018	1.9913	1.34%	2.0074	0.1264
Body-dominant pitch mode	—	2.8204	—	2.8995	0.1626
Body-dominant roll mode	1.06	1.0616	-0.15%	1.0707	0.1297
Wheel-dominant bounce mode	12.21	12.4157	-1.66%	12.3449	0.1066
Wheel-dominant pitch mode	12.66	12.4418	1.75%	12.4550	0.0984
Wheel-dominant roll mode	13.47	13.8558	-2.78%	14.0952	0.1835
Wheel-dominant articulation mode	13.57	13.9655	-2.83%	14.1808	0.1736

TABLE 4: The known physical parameters of the school bus.

Notation	Description	Values	Units
$M$	Total mass of the vehicle	5240	kg
$a$	Distance from sprung mass CG to the front axle	2.22	m
$b$	Distance from sprung mass CG to the rear axle	1.73	m
$t_{f2}$	Half of leaf spring left-to-right distance at front of vehicle	0.4	m
$t_{r2}$	Half of leaf spring left-to-right distance at rear of vehicle	0.5	m

agreement is not good for the small amplitude oscillation at the end. This is due to the nonlinearity of the leaf spring.

TABLE 5: Estimated physical parameters for the school bus.

Notation	Description	Estimate values	Units
$M_s$	Sprung mass	4320	kg
$I_{yy}$	Sprung mass inertia of the pitch	8800	kg·m <sup>2</sup>
$I_{xx}$	Sprung mass inertia of the roll	3450	kg·m <sup>2</sup>
$m_{uf}$	Unsprung mass of the front	130	kg
$m_{ur}$	Unsprung mass of the rear	330	kg
$k_{sf}$	Suspension stiffness of the front	155000	N/m
$k_{sr}$	Suspension stiffness of the rear	260000	N/m
$k_{tf}$	Tire stiffness of the front	880000	N/m
$k_{tr}$	Tire stiffness of the rear	1760000	N/m
$c_{sf}$	Equivalent suspension damping coefficient of the front	4000	N·s/m
$c_{sr}$	Equivalent suspension damping coefficient of the rear	5000	N·s/m
$t_{f1}/t_{r1}$	Equivalent half of the shock absorber left-to-right distance at front/rear of vehicle	0.64	m

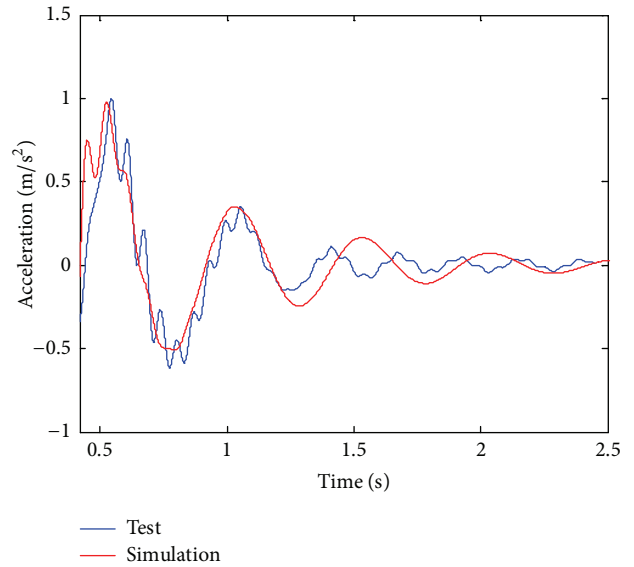


FIGURE 7: Comparison of the vertical acceleration of testing with the one of simulation in bounce test.

The dynamic equivalent stiffness of the leaf spring in small amplitude oscillation tends to increase due to its hysteretic characteristics, while the damping coefficient of the leaf spring increases as it deforms more [11]. When the deformation of leaf spring increases, its dry friction force increases. And this dry friction force in turn determines the damping coefficient of the leaf spring. Thus the nonlinear damping characteristics of the leaf spring can be summarized as it increases with the deformation of the leaf spring.

These nonlinear characteristics can explain the phenomenon in Figures 7, 8, and 9. Vehicle ride performance is mainly determined by the performance of leaf spring in large vibration amplitude.

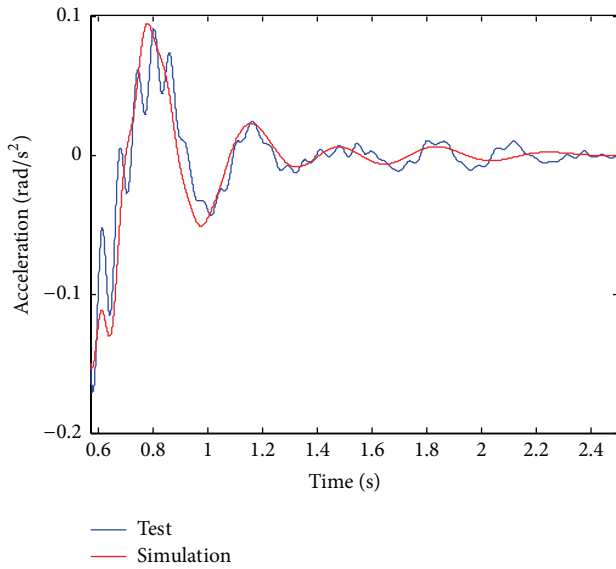


FIGURE 8: Comparison of the pitch angle acceleration of testing with the one of simulation in pitch test.

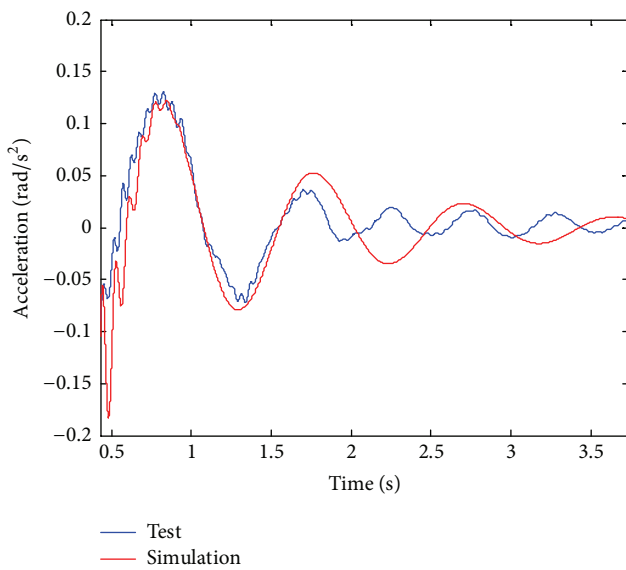


FIGURE 9: Comparison of the roll angle acceleration of testing with the one of simulation in roll test.

## 5. Conclusion

A new physical parameter identification method for two-axis vehicles has been presented. The modal parameter of vehicle is identified by using SVM. In order to easily identify the physical parameter of vehicle, the modal parameter of the vehicle which added a known mass is identified in the presented method. The main purpose of this procedure is to increase the number of equations and thus to ensure that the number of equations is more than the number of unknowns, when the mass matrix, the damping coefficient matrix, and the stiffness matrix of vehicle are calculated by using the expression of the state matrix of vehicle. Furthermore,

the physical parameters of vehicle can be estimated by using the least square method. A numerical simulation example and an experiment example are, respectively, given in this paper. A numerical simulation example shows that the presented method is very effective for the physical parameter identification for two-axis vehicle. A school bus is taken as an experimental example to illustrate the identification method. The estimated parameters are applied to the 7-DOF full-car model and its simulation result is then compared with the tests. The good agreement of the results validates the effectiveness of the proposed method in estimating vehicle physical parameters. Furthermore, the nonlinear characteristics of the leaf spring are discussed to explain the phenomenon observed from the tests.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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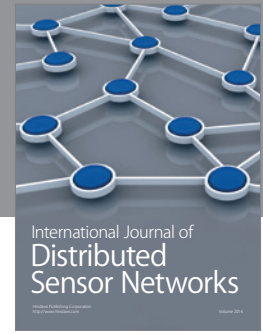
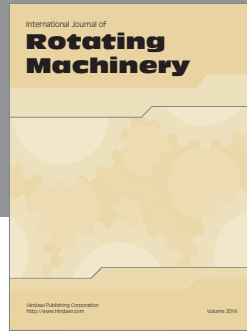
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