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Research Article

Chaotic Dynamic Analysis of Aquatic Phytoplankton System

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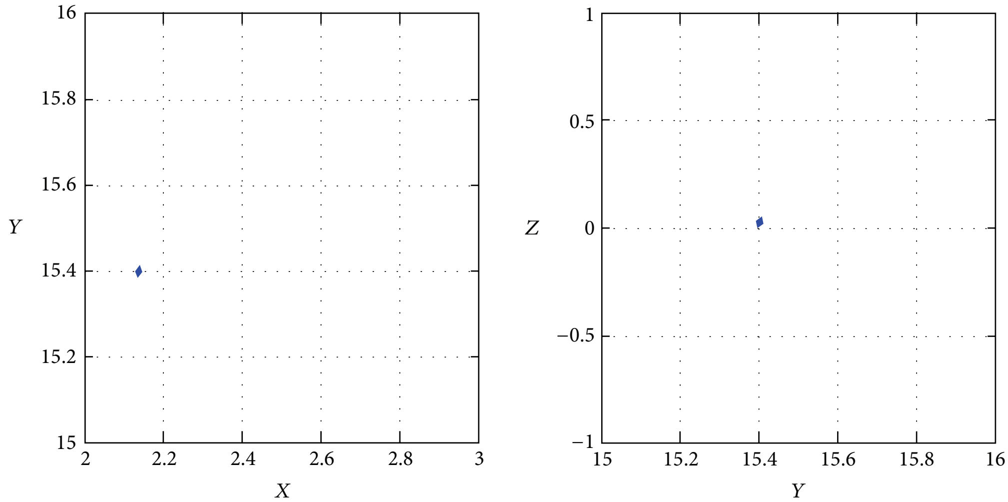
This study analyzed the effect of nonlinear dynamic parameter of phytoplankton toxin emission on the system. Many previous studies have indicated that the zooplankton, mollusks, and habitat factors generate nonlinear chaotic dynamic behavior, which is hardly controlled random behavior. Therefore, in order to understand in what parameter conditions the system has this nonlinear behavior, the linear and nonlinear behaviors resulting from different conditions are discussed. This study used numerical analysis of differential transformation method to analyze the phase of system and applied bifurcation diagrams, trajectory diagrams, Poincaré maps, and spectrograms to discuss and validate whether the system has chaos phenomenon.

1. Introduction

The phytoplankton is microscopic algae found in seas and fresh water, which plays a vital role in aquatic ecology because it is at the bottom of food chain. The phytoplankton grows by using water and CO_2 to absorb air. Scientists have indicated that the phytoplankton population is one of the factors causing global warming, so it is one of ecological items many ecologists want to recover actively. The effect of sharp increase and decrease in the quantity of phytoplankton always exists in the oceanic plankton ecology; it is the common characteristic. If the water has high nutrient content and conditions advantageous to growth, the algae grow rapidly or in large quantities. The low nutrient content and adverse conditions inevitably restrict their growth. If the phytoplankton contains high concentration pigments, it may darken or change the water color, known as red tide. The sudden occurrence and disappearance of this condition often influence the ecosystem significantly, and the adverse reaction of algae (HAB) harms human health, aquatic products, fishery, and tourism considerably. Therefore, this study discusses the effect of nonlinear dynamic parameter of phytoplankton toxin emission on the system.

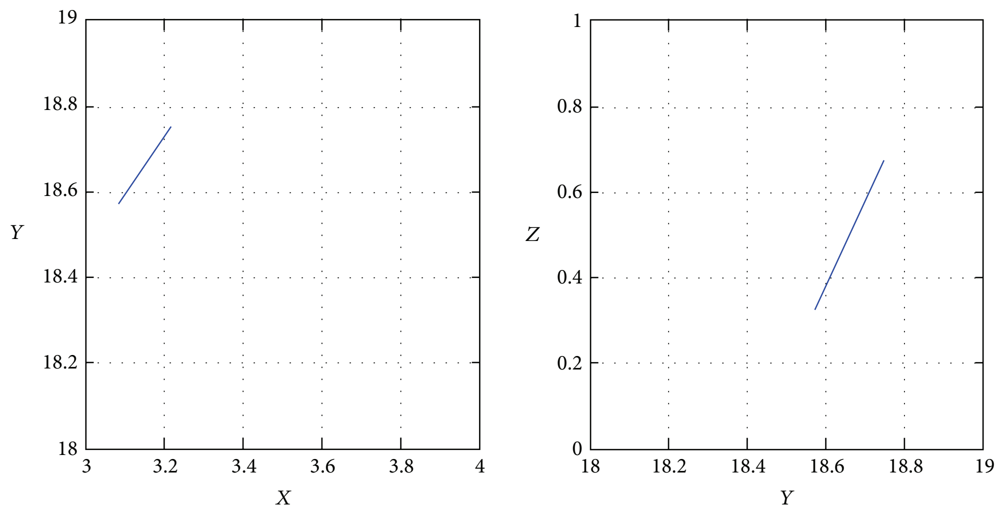
The HAB produced by algae is specific, but relevant control subjects are being researched and investigated by scientists. At present, it is confirmed that the toxin derived from the reproduction of phytoplankton is one of the major reasons influencing the ecosystem, and it plays an important role in the evolutionary process. When a specific phytoplankter releases toxin, the released toxin may influence other organisms due to cumulative effect, causing mass mortality. Such a high content of toxin caused mass mortality of fish and invertebrate in different places. There are many reasons for the release of toxin; one is that the phytoplankton produces toxin to avoid being eaten by zooplankton; it is a self-protection mechanism of organisms. The toxic substance released from phytoplankton reduces the food intake of animals, influencing the quantity of aquatic animals to some extent. Therefore, the amount, time, and frequency of toxin release are the most important parameters influencing the ecology. At present, some good evidences show that grazing herbivore in the initial stage of red tide plays a vital role. These observed data show that the TPP has a vital effect on the increase of zooplankton population. The TPP not only reduces the grazing pressure, but also controls the self-occurrence timing [1–3].

The predator-prey system of environmental heterogeneity and toxin production may influence the dynamics of phytoplankton significantly, but this system has complex characteristics; it is less noticed. Therefore, this study examined



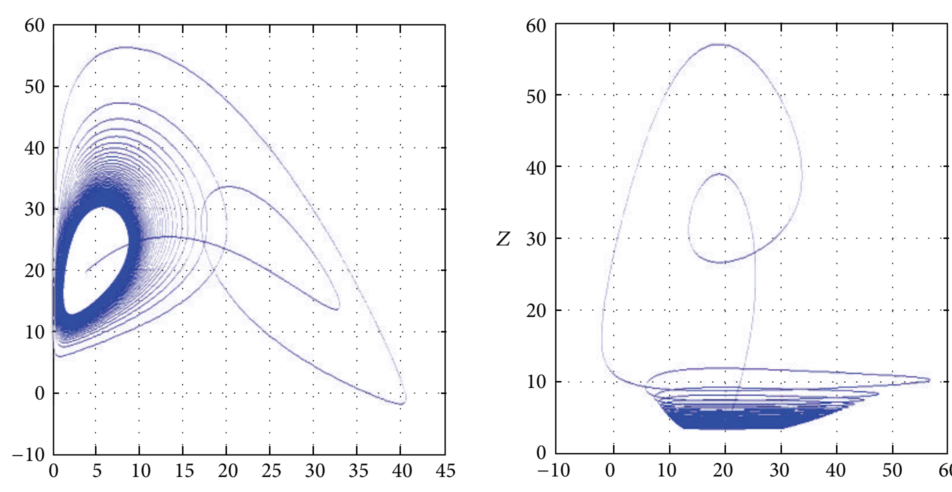
(A) (B)

(a)



(A) (B)

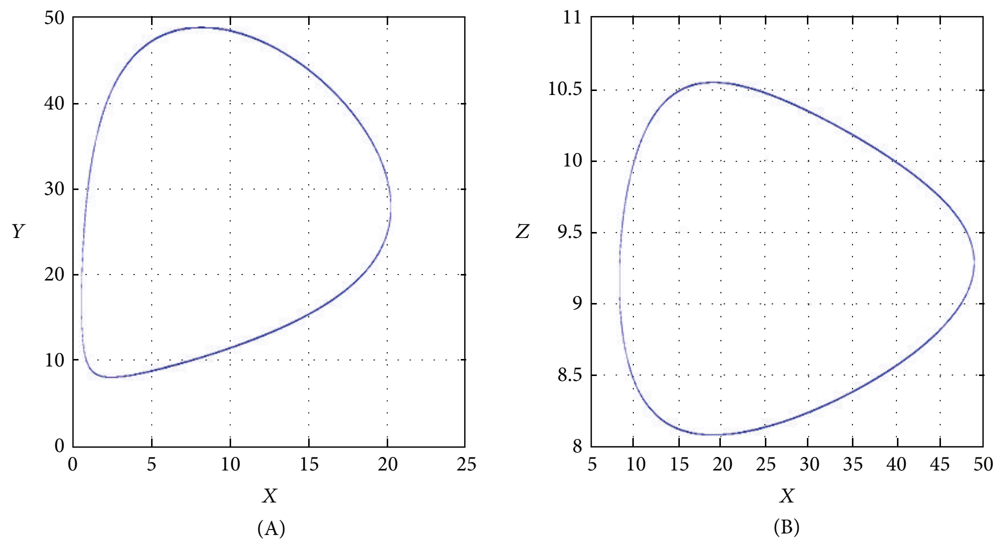
(b)



(A) (B)

(c)

Figure 1: Continued.



(d)

Figure 1: Trajectory diagram PD = (1) 6.0, (2) 6.98, (3) 6.99, (4) 8.0.

the defense mechanism of zooplankton or vegetarian in errantia. The diversity of open waters and habitat has too many influencing factors for the growth of organisms, and the growth conditions of phytoplankton in marine environment are influenced by a lot of uncertain factors; the rapid growth of HAB and innoxious precursor metabolites sometimes accelerate the production of toxin. Some experimental researches have shown that the chemical signals generated by the toxin can influence the species to directly cause interaction; hence, the communities in aquatic ecosystem are influenced significantly [4–6].

The TPP is released into the water. This kind of toxic chemicals reduces the grazing pressure of zooplankton; its effect on reducing the grazing pressure of zooplankton is well known. The toxicity may be one of mechanisms suppressing the zooplankton at high feeding rate. The laboratory research shows that the toxic substance has an important effect on the growth of zooplankton population, so it influences the interaction between phytoplankton and zooplankton significantly. In some regions with rich phytoplankton, the reduction of the species is adjusted and implemented gradually, as the dense phytoplankton or the toxic substance released from phytoplankton changes the balance of the ecoregion; this phenomenon is recognized and explained as exclusion principle. Some studies even show that reducing the quantity of toxin producing phytoplankton is effective on plankton bloom [7–11].

2. Theoretical Analysis and Research Method

2.1. Governing Equation of Ecological Model. The governing equation [12] of ecological model of phytoplankton is expressed as follows:

$$\begin{aligned}\frac{dx}{dt} &= \mathrm{pa1}\cdot x - b_1x^2 - \frac{\mathrm{PW}\cdot xy}{(x+\mathrm{PD})},\\ \frac{dy}{dt} &= \mathrm{pa2}\cdot y + \frac{w_1xy}{(x+D_1)} - \frac{w_2xy}{(y+D_2)},\\ \frac{dz}{dt} &= cz^2 - \frac{w_3z^2}{(y+D_3)},\end{aligned} \tag{1}$$

where x is the total quantity of phytoplankton or algae, y is the total quantity of zooplankton, and z is the total quantity of mollusks. There is a subtle mechanism among them; the total quantity of phytoplankton with toxic substance (x) is one of food sources of zooplankton and it influences the total quantity of zooplankton (y) if it is too large or too small. The zooplankton is one of food sources of mollusks; it influences the total quantity of mollusks (z) directly. The three species compose a food chain. There is already an equilibrium state in the food chain; this paper hopes to analyze the ecology.

In (1), pa1 is the growth rate of phytoplankton, pa2 is the mortality ratio of zooplankton, PD is the protection parameter of environment, PW or w_i $(i = 1 \sim 3)$ is the maximum value of different growth rate variations, D_3 is the loss of mollusks caused by the reduction of zooplankton, and D_1 is the protection parameter provided by the environment for phytoplankton. The amount of poisonous phytoplankton represents the increase and decrease in toxicity, meaning the toxin release period is closely related to the growth cycle of poisonous phytoplankton, and the variance in the mortality ratio and growth rate of zooplankton represents the effect of toxin release on the system. In addition, the protection for phytoplankton provided by the environment influences

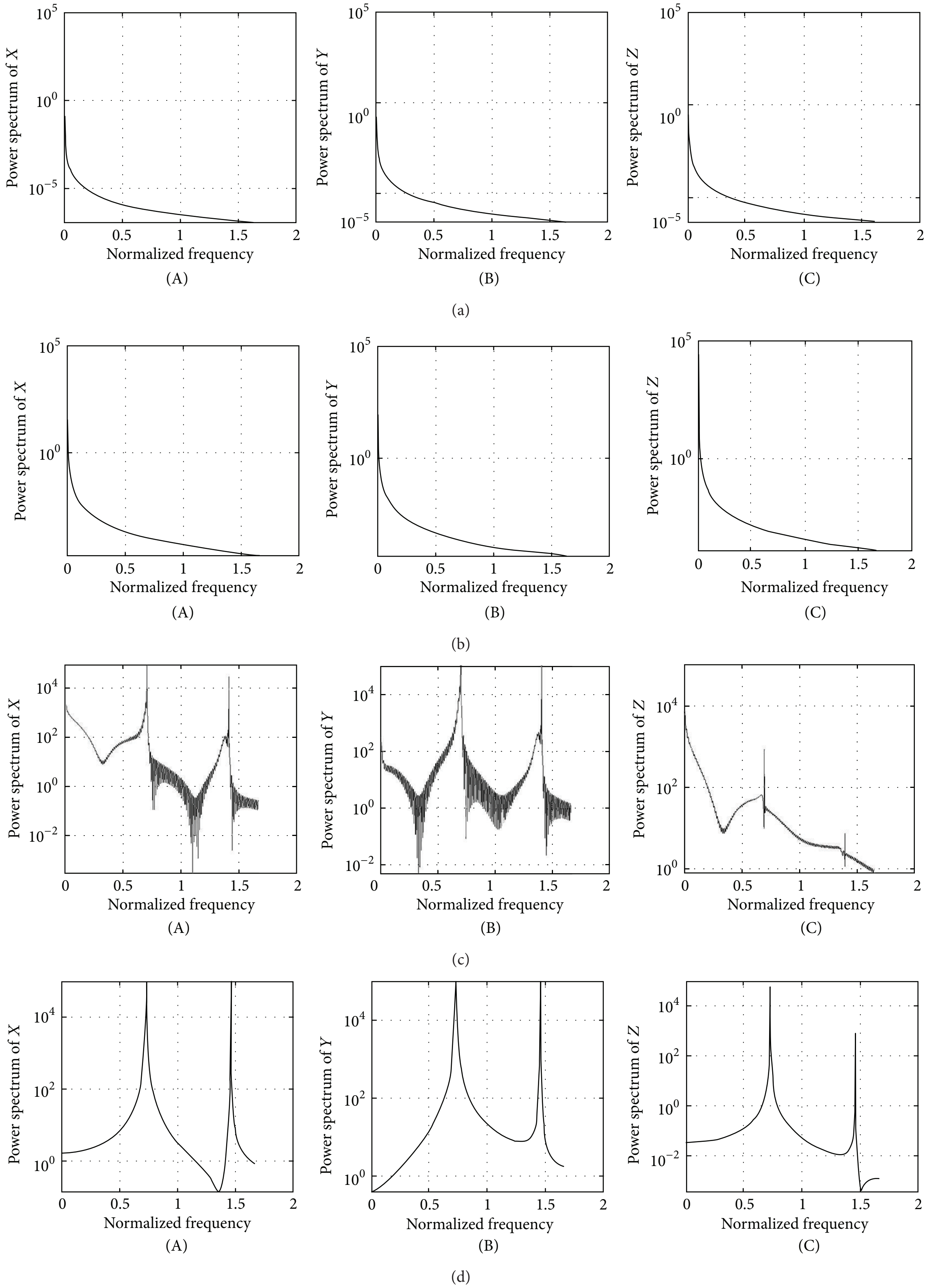


Figure 2: Spectrogram PD = (1) 6.0, (2) 6.98, (3) 6.99, (4) 8.0.

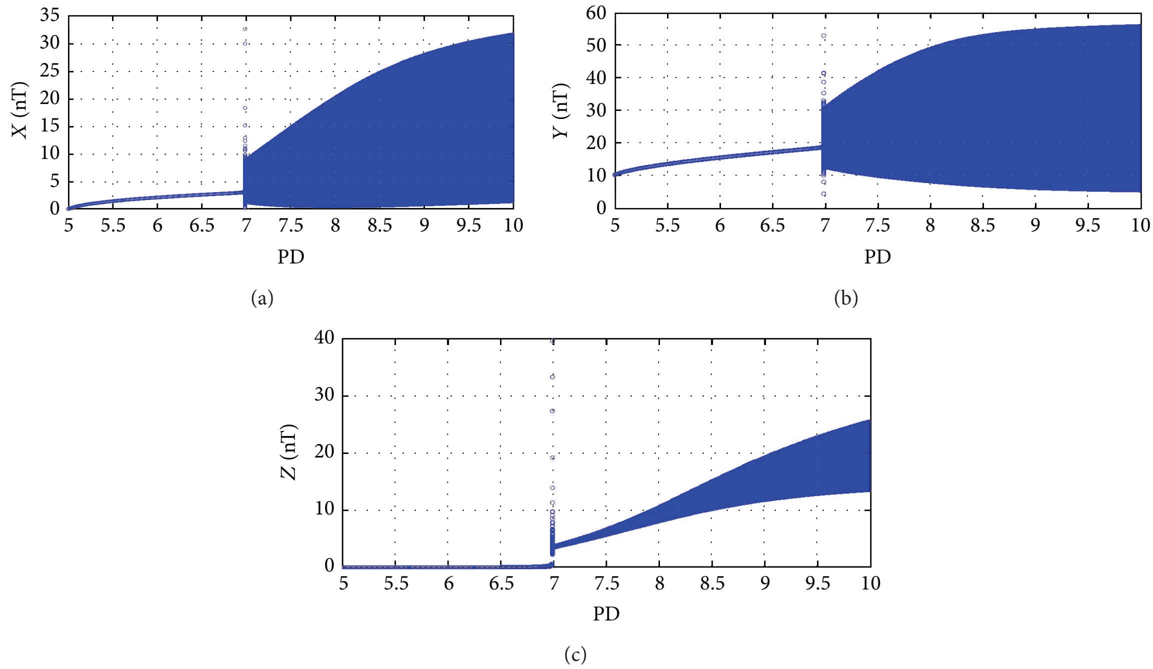


FIGURE 3: Bifurcation diagram (pa1 = 2, pa2 = 1, PW = 1).

the overall ecosystem. Therefore, in order to fully understand the actual effect of key parameters on the system, the PD will be analyzed.

2.2. Numerical Analysis of Differential Transformation Method

2.2.1. Differential Transformation Method. This paper uses differential transformation method [13–15] to solve (1); the basic principle is described below.

The differential transformation of function $f(t)$ is defined as

$$\hat{f}(k) = \frac{T^k}{k!}\left[\frac{\partial^k f(t)}{\partial t^k}\right]_{t=0}, \tag{2}$$

where $\hat{f}(k)$ is the transfer function in transform domain, $f(t)$ is the primitive function in time domain, T is the interval, and k is the transformation parameter. The inverse transform equation of $\hat{f}(k)$ is

$$f(t) = \sum_{k=0}^{\infty} \hat{f}(k)\left(\frac{t}{T}\right)^k. \tag{3}$$

The accuracy of the calculation by this method will be verified; namely, (4) and (5) are used for verification in the second time interval.

Consider

$$f_{i-1}(0) = f_i(0) + \sum_{k=1}^{\infty} \hat{f}_i(k)\left(\frac{-T_{i-1}}{T_i}\right)^k. \tag{4}$$

If $T_0 = T_1 = T_2 = \cdots$, the above equation is

$$f_{i-1}(0) = f_i(0) + \sum_{k=1}^{\infty} \hat{f}_i(k)(-1)^k. \tag{5}$$

2.2.2. Numerical Analysis of Differential Transformation Method. Equation (1) is transformed by differential transformation method and expressed as follows:

$$\begin{aligned}
\frac{k+1}{H}\overline{X}(k+1) &= \text{pa1}\cdot\overline{X}(k) - b_1\cdot\overline{X}(k)\otimes\overline{X}(k) - \frac{\text{PW}\cdot\overline{X}(k)\otimes\overline{Y}(k)}{\left(\overline{X}(k)+\text{PD}\right)}, \\
\frac{k+1}{H}\overline{Y}(k+1) &= \text{pa2}\cdot\overline{Y}(k) + \frac{W_1\cdot\overline{X}(k)\otimes\overline{Y}(k)}{\left(\overline{X}(k)+D_1\right)} - \frac{W_2\cdot\overline{X}(k)\otimes\overline{Y}(k)}{\left(\overline{Y}(k)+D_2\right)}, \\
\frac{k+1}{H}\overline{Z}(k+1) &= c\cdot\overline{Z}(k)\otimes\overline{Z}(k) - \frac{W_3\cdot\overline{Z}(k)\otimes\overline{Z}(k)}{\left(\overline{Y}(k)+D_3\right)}.
\end{aligned} \tag{6}$$

The x, y, and z values at each time point are calculated from the above equation and then the system behavior is analyzed by trajectory diagram, spectrogram, bifurcation diagram, and Poincaré maps.

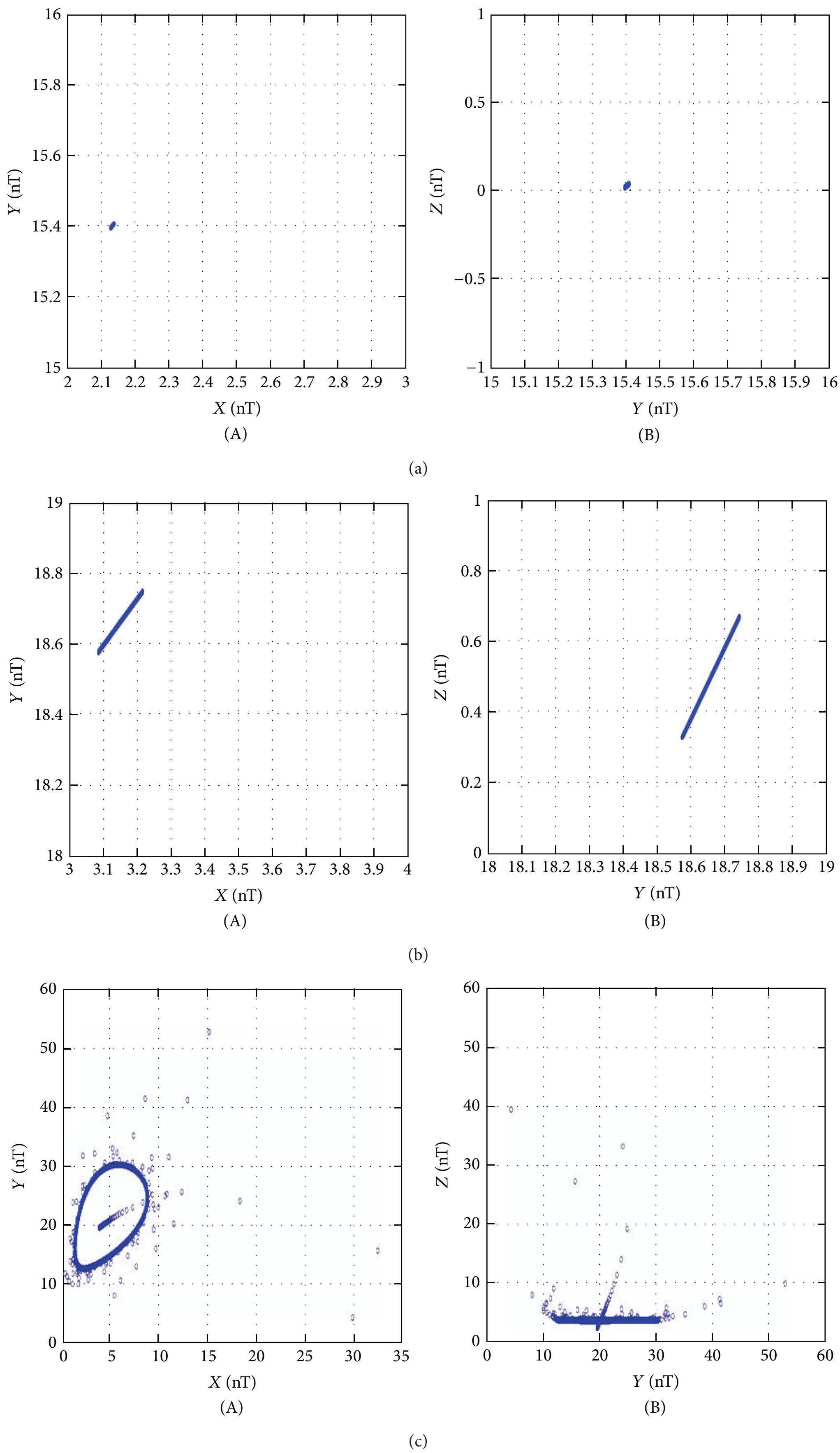
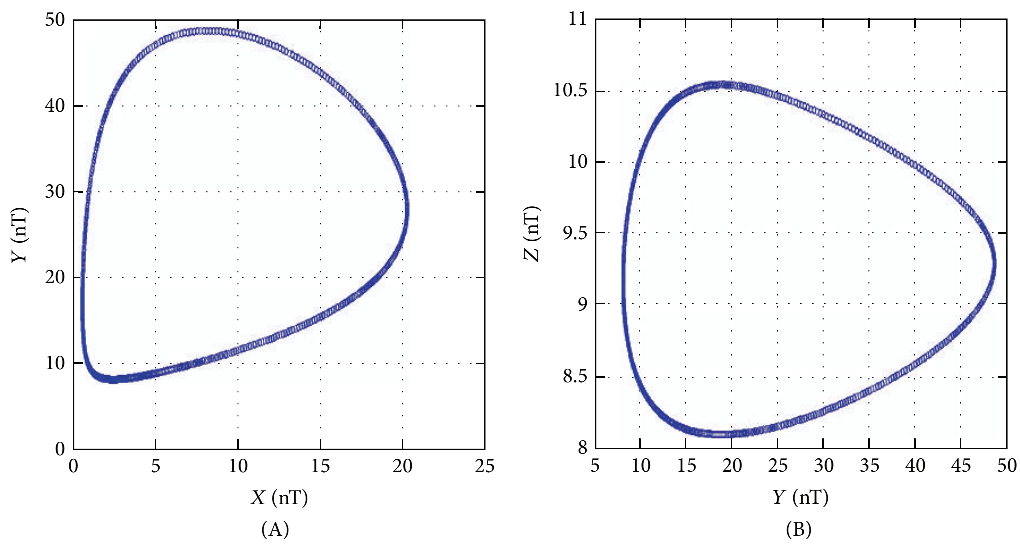


Figure 4: Continued.



(d)

FIGURE 4: Poincaré maps PD = (1) 6.0, (2) 6.98, (3) 6.99, (4) 8.0.

3. Numerical Result

The effect of PD on dynamic behavior of system is discussed; the data of numerical calculation are used to analyze the trajectory phase plane plot, spectrogram, bifurcation diagram, and Poincaré maps of system; and the details are described below.

3.1. Trajectory Diagram. The chaotic aquatic phytoplankton system has different types of periodic or aperiodic motion under the protection of environment. It is observed in Figures 1(a) and 1(b) that if PD = 6.0 and 6.98, the system is in periodic motion, and when PD = 6.99, the periodic motion changes to aperiodic motion; the state trajectory shows irregular pattern, as shown in Figure 1(c). As the PD increases to 8.0, the system shows stable motion behavior, as shown in Figure 1(d).

3.2. Spectral Analysis. The spectral analysis shows whether the signal of system is periodic motion when the chaotic aquatic phytoplankton system is under protection parameters provided by different environments. Figures 2(a) and 2(b) show when PD = 6.0 and 6.98, the system is T singly periodic motion; the spectrum clarifies the frequency. When PD = 6.99, the discrete spectrogram changes to continuous irregularity frequency values, so the state is nonlinear and aperiodic motion, as shown in Figure 2(c). When PD increases to 8.0, the spectrum of system clarifies discrete frequency values, as shown in Figure 2(d).

3.3. Bifurcation Diagram and Poincaré Maps. Figure 3 describes the general view of bifurcation behavior resulting from system variable parameter PD in chaotic dynamic ecosystem. As shown in Figure 3, the X, Y, and Z of chaotic dynamic ecosystem present regular T periodic motion when PD is in specific range ($5.0 \leqq \text{PD} < 6.99$). When PD = 6.0 and 6.98, the motion behavior is proved by Figures 4(a) and 4(b), displayed in single point, meaning that the system is T periodic motion. When PD increases to 6.99, the system shows nonperiodic motion; the Poincaré maps of PD = 6.99 (Figure 4(c)) show many irregular discrete points, providing the generation of chaos phenomenon. This chaos phenomenon transfers to quasiperiodic phenomenon when PD = 8.0, as shown in Figure 4(d); the Poincaré map shows a closed curve.

TABLE 1: Integration of dynamic behaviors at different values of parameter PD.

Parameter	Dynamic behavior
$5.0 \leqq \text{PD} < 6.99$	T
$6.99 \leqq \text{PD} < 8.0$	Chaos
$8.0 \leqq \text{PD} \leqq 10.0$	Quasi

4. Conclusion

This study analyzed the nonlinear motion behavior of chaotic aquatic phytoplankton and carried out numerical analysis according to system equation. Trajectory diagram, spectrogram, bifurcation diagram, and Poincaré maps were used to analyze different behaviors in different parameter conditions. The PD was used as bifurcation parameter to research the effect of different conditions on the overall system. The results showed that that range of $6.99 \leqq \text{PD} < 8.0$ results in chaos motion of system. Therefore, the protection parameters provided by the environment should be regulated timely to avoid the system being in the above conditions, as shown in Table 1.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] J. Chattopadhayay, R. R. Sarkar, and S. Mandal, "Toxin-producing plankton may act as a biological control for planktonic blooms—field study and mathematical modelling," *Journal of Theoretical Biology*, vol. 215, no. 3, pp. 333–344, 2002.

[2] J. Chattopadhyay and R. R. Sarkar, "Chaos to order: preliminary experiments with a population dynamics models of three trophic levels," *Ecological Modelling*, vol. 163, no. 1-2, pp. 45–50, 2003.

[3] S. Gakkhar and A. Singh, "A delay model for viral infection in toxin producing phytoplankton and zooplankton system," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 11, pp. 3607–3620, 2010.

[4] P. A. Dixon, M. J. Milicich, and G. Sugihara, "Episodic fluctuations in larval supply," *Science*, vol. 283, no. 5407, pp. 1528–1530, 1999.

[5] L. H. Erbe, H. I. Freedman, and V. Sree Hari Rao, "Three-species food-chain models with mutual interference and time delays," *Mathematical Biosciences*, vol. 80, no. 1, pp. 57–80, 1986.

[6] V. Rai and R. K. Upadhyay, "Evolving to the edge of chaos: chance or necessity?" *Chaos, Solitons and Fractals*, vol. 30, no. 5, pp. 1074–1087, 2006.

[7] F. C. Hansen, "Trophic interactions between zooplankton and *Phaeocystis* cf. globosa," *Helgoländer Meeresuntersuchungen*, vol. 49, no. 1–4, pp. 283–293, 1995.

[8] V. Kaitala, J. Ylikarjula, and M. Heino, "Non-unique population dynamics: basic patterns," *Ecological Modelling*, vol. 135, no. 2-3, pp. 127–134, 2000.

[9] V. Rai and R. K. Upadhyay, "Chaotic population dynamics and biology of the top-predator," *Chaos, Solitons & Fractals*, vol. 21, no. 5, pp. 1195–1204, 2004.

[10] P. Turchin and S. P. Ellner, "Living on the edge of chaos: Population dynamics of fennoscandian voles," *Ecology*, vol. 81, no. 11, pp. 3099–3116, 2000.

[11] R. K. Upadhyay, "Multiple attractors and crisis route to chaos in a model food-chain," *Chaos, Solitons & Fractals*, vol. 16, no. 5, pp. 737–747, 2003.

[12] R. K. Upadhyay and J. Chattopadhyay, "Chaos to order: role of toxin producing phytoplankton in aquatic systems," *Nonlinear Analysis: Modelling and Control*, vol. 10, no. 4, pp. 383–396, 2005.

[13] C. Wang, M. Jang, and Y. Yeh, "Bifurcation and nonlinear dynamic analysis of a flexible rotor supported by relative short gas journal bearings," *Chaos, Solitons and Fractals*, vol. 32, no. 2, pp. 566–582, 2007.

[14] C. Wang, "Theoretical and nonlinear behavior analysis of a flexible rotor supported by a relative short herringbone-grooved gas journal-bearing system," *Physica D: Nonlinear Phenomena*, vol. 237, no. 18, pp. 2282–2295, 2008.

[15] C. Wang, "Application of a hybrid method to the nonlinear dynamic analysis of a flexible rotor supported by a spherical gas-lubricated bearing system," *Nonlinear Analysis, Theory, Methods and Applications*, vol. 70, no. 5, pp. 2035–2053, 2009.