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Research Article

Adaptive Second-Order Synchronization of Two Heterogeneous Nonlinear Coupled Networks

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This paper investigates the second-order synchronization of two heterogeneous nonlinear coupled networks by introducing controller and adaptive laws. Based on Lyapunov stability properties and LaSalle invariance principle, it is proved that the position and the velocity of two heterogeneous nonlinear coupled networks are asymptotically stable. Finally, some numerical simulations are presented to verify the analytical results.

1. Introduction

In recent years, people have paid more attention to the synchronization problem of complex networks due to their broad applications, such as biology, physics, communication, computer, and the traffic [1–3].

Various models and algorithms about complex networks have been investigated based on different tasks or interests. To achieve the synchronization of complex networks, the adaptive strategy is one of the most interesting topics on the synchronization problem of complex networks. In [4], the authors introduced an adaptive synchronization scheme in complex networks which was linked through nonlinearly coupling. [5] considered the consensus problems of multiagent systems with second-order nonlinear dynamics by introducing the distributed control gains. In [6], the agents of second-order multiagent systems were governed by both position and velocity consensus protocol with time-varying velocity. In [7], the authors studied the group-consensus problem of second-order nonlinear multiagent systems. There have been many studies about the second-order networks [8–11]. However, due to the limit of outside influences and communication conditions, the dynamics of the coupling nodes can be different; so the heterogeneous networks models were proposed in [12–15]. In [16], the authors investigated the consensus problem of heterogeneous multiagent systems. In [17], the authors discussed the adaptive consensus of second-order multiagent systems with heterogeneous nonlinear dynamics and time-varying delays. In [18], the authors studied the finite-time consensus problem of heterogeneous multiagent systems consisting of first-order and second-order integrator agents. In [19], the authors investigated the containment control problem of heterogeneous multiagent systems. The recent papers focus on the synchronization of single network [20, 21]. However, the synchronization can also occur in two or more networks [22, 23], such as the inside doors and the outside doors of city subways. In [24], the authors investigated the synchronization between two coupled complex networks. In [25], the authors further solved the synchronization problem of two nonlinear coupled networks. In [26], the number of nodes, dynamics, and topological structures of the two complex networks were different. However, the second-order synchronization of two heterogeneous nonlinear coupled networks has not been investigated.

Motivated by this, in this paper, we focus on the problem of adaptive second-order synchronization of two heterogeneous nonlinear coupled networks. The main contributions of this paper are threefold: (1) the nonlinear intrinsic dynamics of each node is heterogeneous; (2) the synchronization occurs in the two heterogeneous nonlinear coupled networks; (3) controller and adaptive laws are introduced to solve the second-order synchronization of the two heterogeneous nonlinear coupled networks. Particularly, even if the topological structure is unknown, the two heterogeneous nonlinear networks can achieve synchronization by introducing the suitable controller and adaptive laws.

This paper is organized as follows. In Section 2, the second-order models of two heterogeneous nonlinear coupled networks are given. Moreover, some preliminaries are introduced to solve the adaptive synchronization. Section 3 presents the main results and the theoretical analysis of the second-order synchronization of two heterogeneous nonlinear coupled networks. Some numerical simulations of the theoretical results are given in Section 4. Finally, the conclusion is made in Section 5.

2. Preliminaries and Problem Statement

Consider the second-order models of two heterogeneous nonlinear coupled networks consisting of N identical nodes described by

$$\begin{aligned}
\dot{x}_i(t) &= v_i(t)\\
\dot{v}_i(t) &= f_i(t, x_i(t), v_i(t)) + \sum_{j\in\mathcal{N}_i} \mu_1 a_{ij}\left(x_j(t) - x_i(t)\right)\\
&\quad + \sum_{j\in\mathcal{N}_i} \mu_2 a_{ij}\left(v_j(t) - v_i(t)\right)\\
&\quad + \xi\left(f_i(t, q_i(t), p_i(t)) - g_i(t, x_i(t), v_i(t))\right) + u_i,
\end{aligned} \tag{1}$$

$$\begin{aligned}
\dot{q}_i(t) &= p_i(t)\\
\dot{p}_i(t) &= g_i(t, q_i(t), p_i(t)) + \sum_{j\in\mathcal{N}_i} \mu_3 b_{ij}\left(q_j(t) - q_i(t)\right)\\
&\quad + \sum_{j\in\mathcal{N}_i} \mu_4 b_{ij}\left(p_j(t) - p_i(t)\right)\\
&\quad + (1-\xi)\left(f_i(t, q_i(t), p_i(t)) - g_i(t, x_i(t), v_i(t))\right),
\end{aligned} \tag{2}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$, $q_i(t) = (q_{i1}(t), q_{i2}(t), \ldots, q_{in}(t))^T \in R^n$ $(i = 1, 2, \ldots, N)$ describe the position vectors of networks (1) and (2), respectively, and $v_i(t) = (v_{i1}(t), v_{i2}(t), \ldots, v_{in}(t))^T \in R^n$, $p_i(t) = (p_{i1}(t), p_{i2}(t), \ldots, p_{in}(t))^T \in R^n$ $(i = 1, 2, \ldots, N)$ are their velocity vectors, respectively. $f_i : R^n \to R^n$ and $g_i : R^n \to R^n$ are continuous functions. $\mu_i > 0$ $(i = 1, 2, 3, 4)$ are the position and velocity coupling strengths in two networks, respectively. $A = [a_{ij}] \in R^{n\times n}$ and $B = [b_{ij}] \in R^{n\times n}$ denote the coupling configurations of the two networks, respectively. If there exists communication channel between node i and node j, then $a_{ij} > 0$, $b_{ij} > 0$ $(i \neq j)$; otherwise, $a_{ij} = 0$, $b_{ij} = 0$ $(i \neq j)$, and the diagonal elements are defined as $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij}$, $b_{ii} = -\sum_{j=1, j\neq i}^{N} b_{ij}$. N_i is the neighbor set of node i. $0 \le \xi \le 1$ describe the nonlinear coupling parameter of both networks. u_i is the controller of network.

Define the position error and velocity error of the ith node as

$$\begin{aligned}
m_i(t) &= x_i(t) - q_i(t)\\
n_i(t) &= v_i(t) - p_i(t).
\end{aligned} \tag{3}$$

Differentiating $m_i(t)$ and $n_i(t)$, then

$$\begin{aligned}
\dot{m}_i(t) &= \dot{x}_i(t) - \dot{q}_i(t) = v_i(t) - p_i(t) = n_i(t)\\
\dot{n}_i(t) &= f_i(t, x_i(t), v_i(t)) + \sum_{j\in\mathcal{N}_i} \mu_1 a_{ij}\left(x_j(t) - x_i(t)\right)\\
&\quad + \sum_{j\in\mathcal{N}_i} \mu_2 a_{ij}\left(v_j(t) - v_i(t)\right) - g_i(t, q_i(t), p_i(t))\\
&\quad - \sum_{j\in\mathcal{N}_i} \mu_3 b_{ij}\left(q_j(t) - q_i(t)\right)\\
&\quad - \sum_{j\in\mathcal{N}_i} \mu_4 b_{ij}\left(p_j(t) - p_i(t)\right)\\
&\quad + (2\xi - 1)\left(f_i(t, q_i(t), p_i(t)) - g_i(t, x_i(t), v_i(t))\right)\\
&\quad + u_i.
\end{aligned} \tag{4}$$

Denoting $\tilde{a}_{ij} = a_{ij} - \hat{a}_{ij}$ and $\tilde{b}_{ij} = b_{ij} - \hat{b}_{ij}$, we can have

$$\begin{aligned}
\dot{\tilde{a}}_{ij} &= -\dot{\hat{a}}_{ij} = -(n_i + m_i)^T\\
&\quad \cdot \left(\mu_1\left(x_j(t) - x_i(t)\right) + \mu_2\left(v_j(t) - v_i(t)\right)\right),\\
\dot{\tilde{b}}_{ij} &= -\dot{\hat{b}}_{ij} = (n_i + m_i)^T\\
&\quad \cdot \left(\mu_3\left(q_j(t) - q_i(t)\right) + \mu_4\left(p_j(t) - p_i(t)\right)\right),\\
\dot{E}_i &= -2\left\|n_i\right\|^2 - 2\left\|m_i\right\|^2.
\end{aligned} \tag{5}$$

The controller is designed as

$$\begin{aligned}
u_i(t) &= -2\xi\left(f_i(t, q_i(t), p_i(t)) - g_i(t, x_i(t), v_i(t))\right)\\
&\quad + E_i\left(m_i(t) + n_i(t)\right)\\
&\quad - \sum_{j\in\mathcal{N}_i} \mu_1 \hat{a}_{ij}\left(x_j(t) - x_i(t)\right)\\
&\quad - \sum_{j\in\mathcal{N}_i} \mu_2 \hat{a}_{ij}\left(v_j(t) - v_i(t)\right)\\
&\quad + \sum_{j\in\mathcal{N}_i} \mu_3 \hat{b}_{ij}\left(q_j(t) - q_i(t)\right)\\
&\quad + \sum_{j\in\mathcal{N}_i} \mu_4 \hat{b}_{ij}\left(p_j(t) - p_i(t)\right).
\end{aligned} \tag{6}$$

Combining (6), system (4) can be rewritten as

$$\begin{aligned}
\dot{m}_i(t) &= n_i(t) \\
\dot{n}_i(t) &= f_i(t, x_i(t), v_i(t)) - f_i(t, q_i(t), p_i(t)) \\
&\quad + \sum_{j\in\mathcal{N}_i} \mu_1 \widetilde{a}_{ij}(x_j(t) - x_i(t)) \\
&\quad + \sum_{j\in\mathcal{N}_i} \mu_2 \widetilde{a}_{ij}(v_j(t) - v_i(t)) \\
&\quad + g_i(t, x_i(t), v_i(t)) - g_i(t, q_i(t), p_i(t)) \\
&\quad - \sum_{j\in\mathcal{N}_i} \mu_3 \widetilde{b}_{ij}(q_j(t) - q_i(t)) \\
&\quad - \sum_{j\in\mathcal{N}_i} \mu_4 \widetilde{b}_{ij}(p_j(t) - p_i(t)) \\
&\quad + E_i(m_i(t) + n_i(t)).
\end{aligned} \tag{7}$$

In the following, the necessary definition, assumption, and lemma will be presented for discussing the second-order synchronization of two heterogeneous nonlinear coupled networks.

Definition 1. Networks (1) and (2) are said to achieve second-order synchronization if

$$\begin{aligned}
\lim_{t\to\infty} \|x_i(t) - q_i(t)\| &= 0 \\
\lim_{t\to\infty} \|v_i(t) - p_i(t)\| &= 0;
\end{aligned} \tag{8}$$

that is,

$$\begin{aligned}
\lim_{t\to\infty} \|m_i(t)\| &= 0 \\
\lim_{t\to\infty} \|n_i(t)\| &= 0,
\end{aligned} \tag{9}$$

for $i = 1, 2, \ldots, N$.

Assumption 2. For every f_i of network (1) and g_i of network (2) ($\forall x, v, q, p \in R^n$), there exist the constants $\rho_1 > 0$, $\rho_2 > 0$ such that

$$\begin{aligned}
&\|f_i(t, x_i(t), v_i(t)) - f_i(t, q_i(t), p_i(t))\| \\
&\quad \le \rho_1(\|x_i(t) - q_i(t)\| + \|v_i(t) - p_i(t)\|), \\
&\|g_i(t, x_i(t), v_i(t)) - g_i(t, q_i(t), p_i(t))\| \\
&\quad \le \rho_2(\|x_i(t) - q_i(t)\| + \|v_i(t) - p_i(t)\|).
\end{aligned} \tag{10}$$

Lemma 3 (see [17]). *For any vectors $x, y \in R^n$ and positive definite matrix $G \in R^{n\times n}$, the following matrix inequality holds:*

$$2x^T y \le x^T G x + y^T G^{-1} y. \tag{11}$$

3. Main Results

In this section, we will investigate the second-order synchronization of two heterogeneous nonlinear coupled networks and provide the detailed analysis.

Theorem 4. *Consider networks (1) and (2) steered by (6) under Assumption 2, then the position and velocity of each node can asymptotically synchronize.*

Proof. Constructing the Lyapunov function,

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{12}$$

where

$$\begin{aligned}
V_1(t) &= \frac{1}{2}\sum_{i=1}^{N}(m_i(t) + n_i(t))^T(m_i(t) + n_i(t)), \\
V_2(t) &= \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\widetilde{b}_{ij}^2 + \frac{1}{2}\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\widetilde{a}_{ij}^2, \\
V_3(t) &= \frac{1}{2}\sum_{i=1}^{N}(E_i + k_i)^2,
\end{aligned} \tag{13}$$

and where k_i is a positive constant.

Differentiating $V_1(t)$, then

$$\begin{aligned}
\dot{V}_1(t) &= \sum_{i=1}^{N} m_i^T n_i + \sum_{i=1}^{N} m_i^T \dot{n}_i + \sum_{i=1}^{N} n_i^T n_i + \sum_{i=1}^{N} n_i^T \dot{n}_i \le \frac{1}{2} \\
&\quad \cdot \sum_{i=1}^{N} m_i^T m_i + \frac{3}{2}\sum_{i=1}^{N} n_i^T n_i + \sum_{i=1}^{N}(m_i + n_i)^T \dot{n}_i \le \frac{1}{2} \\
&\quad \cdot \sum_{i=1}^{N} m_i^T m_i + \frac{3}{2}\sum_{i=1}^{N} n_i^T n_i + \sum_{i=1}^{N}(m_i + n_i)^T \Bigg[\rho_1(m_i + n_i) \\
&\quad + \sum_{j\in\mathcal{N}_i}\mu_1\widetilde{a}_{ij}(x_j(t) - x_i(t)) \\
&\quad + \sum_{j\in\mathcal{N}_i}\mu_2\widetilde{a}_{ij}(v_j(t) - v_i(t)) + \rho_2(m_i + n_i) \\
&\quad - \sum_{j\in\mathcal{N}_i}\mu_3\widetilde{b}_{ij}(q_j(t) - q_i(t)) \\
&\quad - \sum_{j\in\mathcal{N}_i}\mu_4\widetilde{b}_{ij}(p_j(t) - p_i(t)) + E_i(m_i + n_i)\Bigg] = \frac{1}{2} \\
&\quad \cdot \sum_{i=1}^{N} m_i^T m_i + \frac{3}{2}\sum_{i=1}^{N} n_i^T n_i + \sum_{i=1}^{N}(m_i + n_i)^T(\rho_1 + \rho_2)(m_i \\
&\quad + n_i) + \sum_{i=1}^{N}(m_i + n_i)^T\Bigg[\sum_{j\in\mathcal{N}_i}\mu_1\widetilde{a}_{ij}(x_j(t) - x_i(t)) \\
&\quad + \sum_{j\in\mathcal{N}_i}\mu_2\widetilde{a}_{ij}(v_j(t) - v_i(t))\Bigg] + \sum_{i=1}^{N}(m_i + n_i)^T \\
&\quad \cdot \Bigg[-\sum_{j\in\mathcal{N}_i}\mu_3\widetilde{b}_{ij}(q_j(t) - q_i(t))
\end{aligned}$$

$$
\begin{aligned}
&-\sum_{j\in\mathcal{N}_i}\mu_4\tilde{b}_{ij}\left(p_j(t)-p_i(t)\right)\Bigg]+\sum_{i=1}^{N}(m_i+n_i)^T E_i\,(m_i\\
&+n_i)=\frac{1}{2}\sum_{i=1}^{N}m_i^T m_i+\frac{3}{2}\sum_{i=1}^{N}n_i^T n_i+\sum_{i=1}^{N}m_i^T\rho m_i\\
&+\sum_{i=1}^{N}m_i^T\rho n_i+\sum_{i=1}^{N}n_i^T\rho m_i+\sum_{i=1}^{N}n_i^T\rho n_i+\sum_{i=1}^{N}(m_i+n_i)^T\\
&\cdot\Bigg[\sum_{j\in\mathcal{N}_i}\mu_1\tilde{a}_{ij}\left(x_j(t)-x_i(t)\right)\\
&+\sum_{j\in\mathcal{N}_i}\mu_2\tilde{a}_{ij}\left(v_j(t)-v_i(t)\right)\Bigg]+\sum_{i=1}^{N}(m_i+n_i)^T\\
&\cdot\Bigg[-\sum_{j\in\mathcal{N}_i}\mu_3\tilde{b}_{ij}\left(q_j(t)-q_i(t)\right)\\
&-\sum_{j\in\mathcal{N}_i}\mu_4\tilde{b}_{ij}\left(p_j(t)-p_i(t)\right)\Bigg]+\sum_{i=1}^{N}(m_i+n_i)^T E_i\,(m_i\\
&+n_i)\le\frac{1}{2}\sum_{i=1}^{N}m_i^T m_i+\frac{3}{2}\sum_{i=1}^{N}n_i^T n_i+\rho\sum_{i=1}^{N}m_i^T m_i\\
&+\rho\sum_{i=1}^{N}n_i^T n_i+\rho\sum_{i=1}^{N}m_i^T m_i+\rho\sum_{i=1}^{N}n_i^T n_i+\sum_{i=1}^{N}(m_i+n_i)^T\\
&\cdot\Bigg[\sum_{j\in\mathcal{N}_i}\mu_1\tilde{a}_{ij}\left(x_j(t)-x_i(t)\right)\\
&+\sum_{j\in\mathcal{N}_i}\mu_2\tilde{a}_{ij}\left(v_j(t)-v_i(t)\right)\Bigg]+\sum_{i=1}^{N}(m_i+n_i)^T\\
&\cdot\Bigg[-\sum_{j\in\mathcal{N}_i}\mu_3\tilde{b}_{ij}\left(q_j(t)-q_i(t)\right)\\
&-\sum_{j\in\mathcal{N}_i}\mu_4\tilde{b}_{ij}\left(p_j(t)-p_i(t)\right)\Bigg]+\sum_{i=1}^{N}(m_i+n_i)^T E_i\,(m_i\\
&+n_i)=\left(\frac{1}{2}+2\rho\right)\sum_{i=1}^{N}m_i^T m_i+\left(\frac{3}{2}+2\rho\right)\sum_{i=1}^{N}n_i^T n_i\\
&+\sum_{i=1}^{N}(m_i+n_i)^T\Bigg[\sum_{j\in\mathcal{N}_i}\mu_1\tilde{a}_{ij}\left(x_j(t)-x_i(t)\right)\\
&+\sum_{j\in\mathcal{N}_i}\mu_2\tilde{a}_{ij}\left(v_j(t)-v_i(t)\right)\Bigg]+\sum_{i=1}^{N}(m_i+n_i)^T\\
&\cdot\Bigg[-\sum_{j\in\mathcal{N}_i}\mu_3\tilde{b}_{ij}\left(q_j(t)-q_i(t)\right)\\
&-\sum_{j\in\mathcal{N}_i}\mu_4\tilde{b}_{ij}\left(p_j(t)-p_i(t)\right)\Bigg]+\sum_{i=1}^{N}(m_i+n_i)^T E_i\,(m_i\\
&+n_i),
\end{aligned}
\tag{14}
$$

where $\rho=\rho_1+\rho_2$.

Differentiating $V_2(t)$, we get

$$
\begin{aligned}
\dot{V}_2(t)&=\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\tilde{b}_{ij}\dot{\tilde{b}}_{ij}+\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\tilde{a}_{ij}\dot{\tilde{a}}_{ij}\\
&=\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\tilde{b}_{ij}(n_i+m_i)^T\\
&\cdot\left[\mu_3\left(q_j(t)-q_i(t)\right)+\mu_4\left(p_j(t)-p_i(t)\right)\right]\\
&+\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i}\tilde{a}_{ij}(n_i+m_i)^T\\
&\cdot\left[-\mu_1\left(x_j(t)-x_i(t)\right)-\mu_2\left(v_j(t)-v_i(t)\right)\right].
\end{aligned}
\tag{15}
$$

Differentiating $V_3(t)$, then

$$
\begin{aligned}
\dot{V}_3(t)&=\sum_{i=1}^{N}(E_i+k_i)\dot{E}_i=\sum_{i=1}^{N}E_i\dot{E}_i+\sum_{i=1}^{N}k_i\dot{E}_i\\
&=\sum_{i=1}^{N}E_i\left(-2\|n_i\|^2-2\|m_i\|^2\right)\\
&\quad+\sum_{i=1}^{N}k_i\left(-2\|n_i\|^2-2\|m_i\|^2\right).
\end{aligned}
\tag{16}
$$

Combining $\dot{V}_1(t)$, $\dot{V}_2(t)$, and $\dot{V}_3(t)$, then we can have

$$
\begin{aligned}
\dot{V}(t)&\le\left(\frac{1}{2}+2\rho\right)\sum_{i=1}^{N}m_i^T m_i+\left(\frac{3}{2}+2\rho\right)\sum_{i=1}^{N}n_i^T n_i\\
&+\sum_{i=1}^{N}(m_i+n_i)^T E_i(m_i+n_i)\\
&+\sum_{i=1}^{N}E_i\left(-2\|n_i\|^2-2\|m_i\|^2\right)\\
&+\sum_{i=1}^{N}k_i\left(-2\|n_i\|^2-2\|m_i\|^2\right)\\
&=\left(\frac{1}{2}+2\rho\right)\sum_{i=1}^{N}m_i^T m_i+\left(\frac{3}{2}+2\rho\right)\sum_{i=1}^{N}n_i^T n_i\\
&+\sum_{i=1}^{N}m_i^T E_i n_i+\sum_{i=1}^{N}n_i^T E_i n_i+\sum_{i=1}^{N}m_i^T E_i m_i\\
&+\sum_{i=1}^{N}n_i^T E_i m_i+\sum_{i=1}^{N}E_i\left(-2\|n_i\|^2-2\|m_i\|^2\right)
\end{aligned}
$$

$$\begin{aligned}
&+\sum_{i=1}^{N}k_i\left(-2\left\|n_i\right\|^2-2\left\|m_i\right\|^2\right)\\
&\leq\left(\frac{1}{2}+2\rho\right)\sum_{i=1}^{N}m_i^Tm_i+\left(\frac{3}{2}+2\rho\right)\sum_{i=1}^{N}n_i^Tn_i\\
&\quad+2\sum_{i=1}^{N}m_i^TE_im_i+2\sum_{i=1}^{N}n_i^TE_in_i\\
&\quad+\sum_{i=1}^{N}E_i\left(-2\left\|n_i\right\|^2-2\left\|m_i\right\|^2\right)\\
&\quad+\sum_{i=1}^{N}k_i\left(-2\left\|n_i\right\|^2-2\left\|m_i\right\|^2\right)\\
&=\left(\frac{1}{2}+2\rho\right)\sum_{i=1}^{N}m_i^Tm_i+\left(\frac{3}{2}+2\rho\right)\sum_{i=1}^{N}n_i^Tn_i\\
&\quad-2k_i\sum_{i=1}^{N}\left\|m_i\right\|^2-2k_i\sum_{i=1}^{N}\left\|n_i\right\|^2\leq 0,
\end{aligned}\tag{17}$$

if $k_i \geq 3/4 + \rho$. Based on LaSalle invariance principle, we can know that, for any initial states, the error solution will tend to zero, which implies that networks (1) and (2) can asymptotically synchronize with controller (6). □

Remark 5. When the topology structure is unknown, the two heterogeneous nonlinear coupled networks also can be asymptotically synchronized by controller (6).

Corollary 6. *If the coupled networks (1) and (2) have the identical dynamics, the networks can asymptotically synchronize through the following controller:*

$$\begin{aligned}
u_i(t)=&-\sum_{j\in\mathcal{N}_i}\mu_1\widehat{a}_{ij}\left(x_j(t)-x_i(t)\right)\\
&-\sum_{j\in\mathcal{N}_i}\mu_2\widehat{a}_{ij}\left(v_j(t)-v_i(t)\right)\\
&+\sum_{j\in\mathcal{N}_i}\mu_3\widehat{b}_{ij}\left(q_j(t)-q_i(t)\right)\\
&+\sum_{j\in\mathcal{N}_i}\mu_4\widehat{b}_{ij}\left(p_j(t)-p_i(t)\right)\\
&+E_i\left(m_i(t)+n_i(t)\right),
\end{aligned}\tag{18}$$

where $\widehat{a}_{ij}, \widehat{b}_{ij}, E_i$ *are the same as Theorem 4.*

4. Simulations

In this section, several numerical simulations are given to illustrate the analytical results.

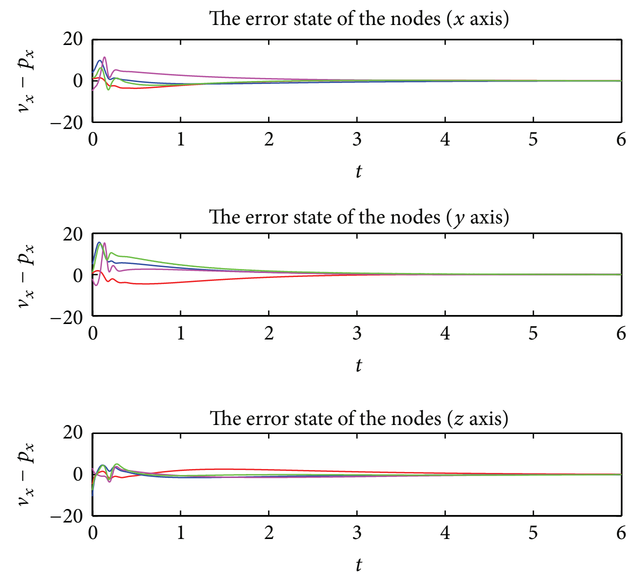

Figure 1: The error trajectories for velocity with controllers.

We choose Lorentz system of two different parameters as nonlinear dynamics of networks (1) and (2). The Lorentz system is described as the following:

$$\begin{aligned}
\dot{x}(t)&=v(t)\\
\dot{v}(t)&=\begin{cases}\alpha\left(v_y-v_x\right)\\ \beta v_x-v_xv_z-v_y\\ v_xv_y-\gamma v_y,\end{cases}
\end{aligned}\tag{19}$$

where α, β, and γ are the parameters. For network (1), let $\alpha = 10$, $\beta = 28$, and $\gamma = 8/3$; and for network (2), let $\alpha = 16$, $\beta = 4$, and $\gamma = 45.92$.

The coupling matrixes of networks (1) and (2) with four nodes, respectively, are described by the following matrixes:

$$\begin{aligned}
A&=\begin{bmatrix}-2&1&0&1\\1&-2&1&0\\0&1&-1&0\\1&0&0&-1\end{bmatrix},\\
B&=\begin{bmatrix}-2&1&0&1\\1&-2&1&0\\0&1&-2&1\\1&0&1&-2\end{bmatrix}.
\end{aligned}\tag{20}$$

With controller (6), it can be found that the velocity and position of networks (1) and (2) can synchronize to the synchronous state, described as Figures 1-2, respectively. However, if networks (1) and (2) without the controller are in the same conditions, we can find that velocity and position of the networks cannot achieve synchronization, when $\xi = 0.1$. The simulations are shown in Figures 3 and 4.

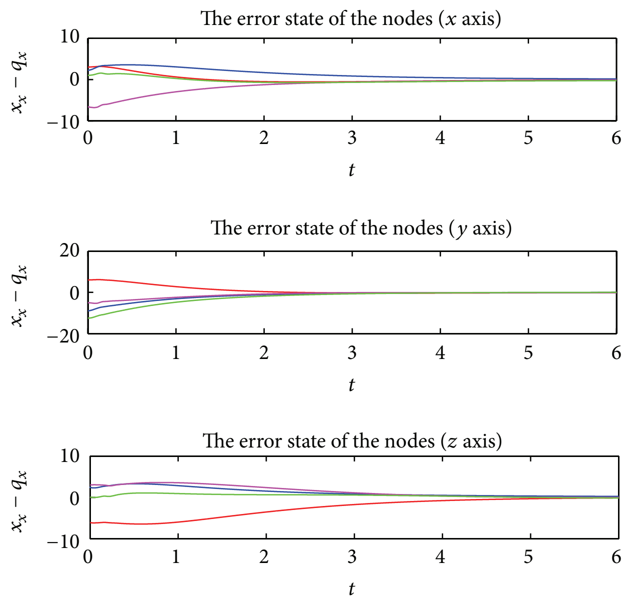

FIGURE 2: The error trajectories for position with controllers.

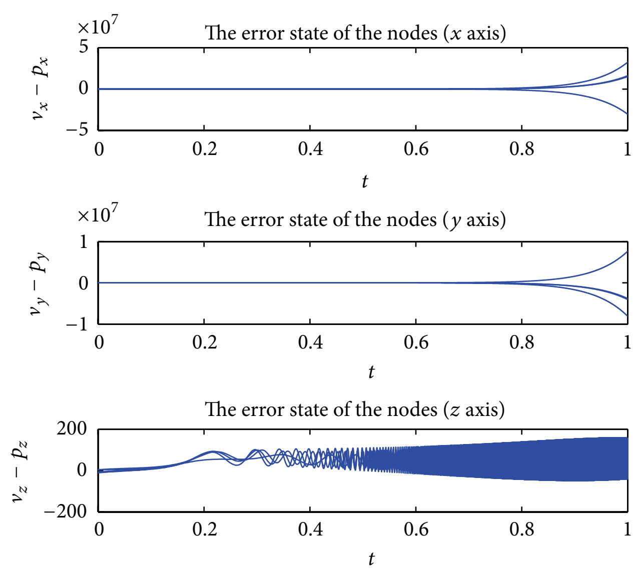

FIGURE 3: The error trajectories for velocity without controllers.

5. Conclusion

In this paper, we have considered the adaptive second-order synchronization of two heterogeneous nonlinear coupled networks. By constructing a valid Lyapunov function, we have proved that the networks can achieve asymptotically synchronization with the given controller and adaptive laws. Particularly, even if the topological structure is unknown, the networks also can be synchronized by the given controller and adaptive laws.

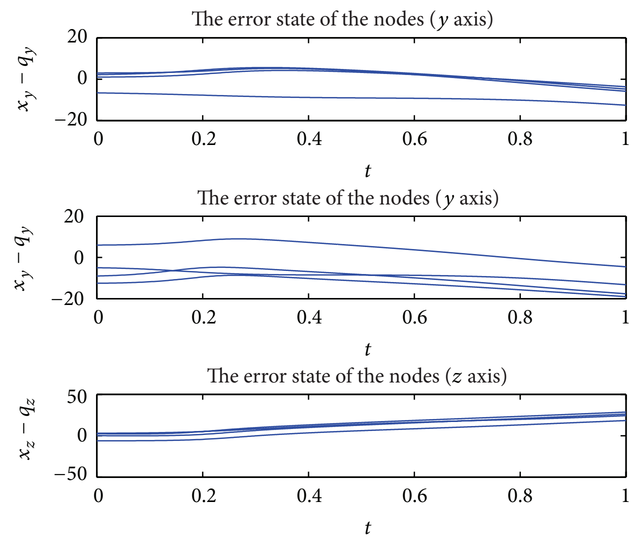

FIGURE 4: The error trajectories for position without controllers.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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