Research Article
Hybrid Charmonium and the $\rho - \pi$ Puzzle

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Using the method of QCD sum rules, we estimate the energy of the lowest hybrid charmonium state, and find it to be at the energy of the $\Psi'(2S)$ state, about 600 MeV above the $J/\Psi(1S)$ state. Since our solution is not consistent with a pure hybrid at this energy, we conclude that the $\Psi'(2S)$ state is probably an admixed $c\bar{c}$ and hybrid $c\bar{c}g$ state. From this conjecture, we find a possible explanation of the famous $\rho - \pi$ puzzle.

1. Introduction

A hybrid meson is a state composed of a quark and antiquark color octet, along with a valence gluon, giving a color zero particle. Hybrids are of great interest in studying the nature of QCD. The nonperturbative method of QCD sum rules [1, 2] has long been used to predict the energies of light quark hybrid mesons [3, 4] and hybrid baryons (see [5]). In the present work, we use this method to estimate the energy of the lowest charmonium hybrid. A major motivation for the present work is to understand the nature of the $\Psi'(2S)$ state, and to find a possible explanation of the long-standing $\rho - \pi$ puzzle.

The $\rho - \pi$ puzzle concerns the branching ratios for hadronic decays of the $\Psi'(2S)$ state compared to the $J/\Psi(1S)$ state. By taking ratios of hadronic decays to gamma decays for these heavy-quark states, the wave functions at the origin cancel, and by using the lowest-order diagrams, one obtains the ratios of branching rates for two charmonium states:

$$R = \frac{B\big(\Psi'(2S) \longrightarrow h\big)}{B\big(J/\Psi(1S) \longrightarrow h\big)} = \frac{B\big(\Psi'(2S) \rightarrow e^+e^-\big)}{B\big(J/\Psi(1S) \longrightarrow e^+e^-\big)} \simeq 0.13, \qquad (1.1)$$

the so-called 13% rule. For the $\Psi'(2S)$ state compared to the $J/\Psi(1S)$ state, however, the hadronic (e.g., $\rho - \pi$) decay ratio is more than an order of magnitude smaller than predicted [6]. This is the $\rho - \pi$ puzzle. There have been many, many theoretical attempts to explain this

puzzle: Chen and Braaten [7] review earlier work by Hou and Soni, Brodsky, Lepage and Tuan, Karl and Roberts, Chaichian and Tornqvist, Pinsky, Brodsky and Karliner, and Li, Bugg and Zou. All seem to agree that this is an unsolved puzzle. More recently, there has been an attempt to locate the source of the problem [8], with the suggestion that there is a cancellation of two processes in the $\Psi'(2S)$ decay. Our present work suggests that one can obtain such a cancellation by including valence gluonic structure.

In Section 2, we show that the energy of the lowest hybrid charmonium state with the quantum numbers $J^{PC} = 1^{--}$ is that of the $\Psi'(2S)$, but our solution is not consistent with a pure hybrid. Since the $c\bar{c}(2S)$ state is also expected to have that energy (about 600 MeV above that of the $J/\Psi(1S)$ state), we predict that the $\Psi'(2S)$ state is an admixture of $c\bar{c}(2S)$ and hybrid components. In Section 3, we show that this can provide a solution to the $\rho - \pi$ puzzle. In Section 4, we give our conclusions and compare our results to lattice gauge calculations.

2. Hybrid 1^{--} charmonium using QCD sum rules

We now will use the method of QCD sum rules to attempt to find the lowest hybrid charmonium state, assuming that such a pure hybrid charmonium meson with quantum numbers $J^{PC} = 1^{--}$ exists. First, let us review the method and the criteria for determining if one has obtained a satisfactory and accurate solution.

2.1. Method of QCD sum rules for a hybrid charmonium meson

The starting point of the method of QCD sum rules is the correlator, which for a hybrid meson is

$$\Pi^{\mu\nu}(x) = \left\langle T\left[J_H^\mu(x)J_H^\mu(0)\right]\right\rangle, \tag{2.1}$$

with the current $J_H^\mu(x)$ creating the hybrid state being studied. The QCD sum rule is obtained by evaluating $\Pi^{\mu\nu}$ in two ways. First, after a Fourier transform to momentum space, a dispersion relation gives the left-hand side (LHS) of the sum rule:

$$\Pi(q)_{\text{lhs}}^{\mu\nu} = \frac{\text{Im}\Pi^{\mu\nu}(M_A)}{\pi\left(M_A^2 - q^2\right)} + \int_{s_o}^\infty ds \frac{\text{Im}\Pi^{\mu\nu}(s)}{\pi\left(s - q^2\right)}, \tag{2.2}$$

where M_A is the mass of the state A (assuming zero width) and s_o is the start of the continuum—a parameter to be determined. The imaginary part of $\Pi(s)$, with the term for the state we are seeking shown as a pole (corresponding to a $\delta(s - M_A^2)$ term in ImΠ) and the higher-lying states produced by J_H^μ shown as the continuum, is illustrated in Figure 1.

Next, $\Pi^{\mu\nu}(q)$ is evaluated by an operator product expansion (OPE), giving the right-hand side (RHS) of the sum rule:

$$\Pi(q)_{\text{rhs}}^{\mu\nu} = \sum_k c_k(q)\langle 0|\mathcal{O}_k|0\rangle, \tag{2.3}$$

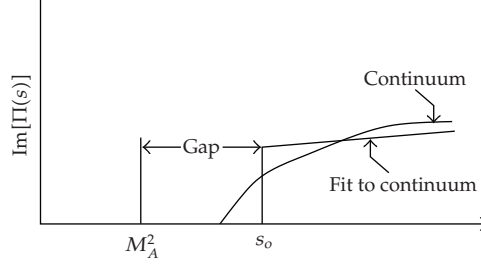with increasing k corresponding to increasing dimension of $\mathcal{O}_k$.

Figure 1: QCD sum rule study of a state A with mass M_A (no width).

After a Borel transform, $\mathcal{B}$, defined in Appendix B, in which the q variable is replaced by the Borel mass, M_B, the final QCD sum rule has the form

$$\frac{1}{\pi}e^{-M_{\text{HHM}}^2/M_B^2} + \mathcal{B}\int_{s_o}^{\infty}ds\frac{\text{Im}\left[\Pi(s)\right]}{\pi(s-q^2)} = \mathcal{B}\sum_k c_k(q)\langle 0|\mathcal{O}_k|0\rangle, \tag{2.4}$$

as we will show in what follows. Note that the Borel transform produces an exponential decrease with increasing values of s, as shown by the pole term in (2.4). This reduces the contribution of the continuum, where $s \geq M_B^2$.

This sum rule is used to estimate the heavy-hybrid mass, M_{HHM}. One of the main sources of error is the treatment of the continuum. In addition to the parameter s_o, one must parameterize the effective shape of the continuum. The criteria for a satisfactory solution are: (1) the contribution of the continuum should not be as large as the pole term in the LHS; (2) with an exact sum rule, the value of M_{HHM} is independent of the value of M_B (with the approximation of a fit to the continuum, there should be a minimum or maximum in the value of M_{HHM} versus M_B, and the value of M_{HHM} at this extremum should be approximately the value of M_B); (3) there should be a gap between the solution for M_{HHM}^2 and s_o, which reduces the contribution of the continuum to be smaller than the pole term due to the factor of e^{-s/M_B^2} after the Borel transform, as we will explain in what follows. If the value of s_o is much larger than the expected excited hybrid states, however, the solution is not physical.

2.2. Previous results for hybrids using QCD sum rules and lattice QCD methods

Since mesons with certain quantum numbers, such as 1^{-+}, cannot have a standard $q\bar{q}$ meson composition, there is a strong motivation for both experimental and theoretical searches for hybrid mesons, which can have such states. These states are called exotic or hermaphrodite mesons. Shortly after the introduction of QCD sum rules, they were used to attempt to find the masses and widths of exotics. The most accurate calculations [3, 4] predicted 1^{-+} hybrid light-quark mesons in the 1.3–1.7 GeV region, where such a state has been found [9]. The solutions satisfy the criteria for a good solution, as explained above. For example, for a solution that has a hybrid mass of 1.3 GeV, the value of s_o was 1.7 GeV, which is a reasonable separation of the lowest from the higher hybrid states. On the other hand, for these light meson hybrids, many terms in the OPE are needed, which significantly increases the uncertainty. This explains why previous calculations [3, 4] have found a rather wide range of values for the light-quark exotic hybrid.

For heavy-quark hybrids, the higher-order terms in the OPE are quite small, so the QCD sum rule method is more accurate. Since we are not trying to predict exotic hybrids, however, there is the serious complication of nonhybrid meson-hybrid meson mixing, which will be discussed in detail in what follows. This is an important aspect of our present work.

There have been many lattice QCD calculations of glueballs and light-quark hybrids [10]. The most recent calculations of light-quark hybrids find the lightest exotics to be about 2 GeV [11], quite a bit higher in energy than the QCD sum rule calculations. This probably is due to the fact that lattice QCD calculations for light quarks have some inconsistencies at the present time, while they are much more accurate for heavy-quark systems [12], as are QCD sum rules. Exotic charmonium states have been calculated using lattice QCD, and the 1^{-+} was found to be about 4.4 GeV [13], with the expectation that the 1^{--} hybrid charmonium state is at a similar energy. For the calculation of nonexotic hybrid mesons, such as 1^{--} hybrids, there are other difficulties for both methods, as we will discuss in what follows.

2.3. Heavy-hybrid meson correlator

For a hybrid meson with quantum numbers 1^{--} we use the standard current [3, 4]:

$$J_H^\mu = \overline{\Psi} \Gamma_\nu G^{\mu\nu} \Psi, \tag{2.5}$$

with $\Gamma_\nu = C\gamma_\nu$, where C is the charge conjugation operator, γ_ν is the usual Dirac matrix, and Ψ is the heavy-quark field. Carrying out a four-dimensional Fourier transform, the correlator in momentum space is

$$\Pi^{\mu\nu}(p) = \int \frac{d^4 p_1}{(2\pi)^4} \text{Tr}\left[S^{ab}\Gamma_\alpha S^{ba}\Gamma_\beta\right](p - p_1) \text{Tr}\left[G^{\mu\alpha}G^{\nu\beta}\right](p_1), \tag{2.6}$$

where S^{ab} is the quark propagator, with colors a and b. The color properties of $G^{\mu\alpha}$, the gluon color field, with μ, α Dirac indices, are given in Appendix A. Note that the traces are both fermion and color traces. Details of $\text{Tr}[S^{ab}\Gamma_\alpha S^{ba}\Gamma_\beta](p)$ and $\text{Tr}[G^{\mu\alpha}G^{\nu\beta}](p_1)$ are given in Appendix A.

It is important to recognize that the correlator used in the QCD sum rule method is similar to the correlator used in the lattice gauge approach, with the same objective of finding the mass of a heavy-hybrid meson.

As described in the preceding subsection, the correlator is evaluated in the method of QCD sum rules via an OPE and a dispersion relation.

2.4. OPE of the scalar correlator in momentum space

The QCD sum rule method uses an OPE in dimension (or inverse momentum) (2.3). For the hybrid meson, the lowest-order diagram [14] is shown in Figure 2.
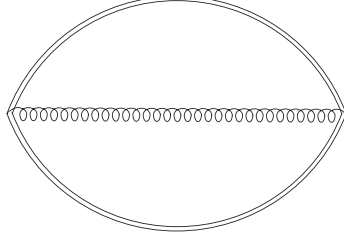
Figure 2: Lowest-order term in sum rule.

Using the standard quark and gluon propagators (see Appendix A for some details), we find the following formula for this lowest-dimensional process:

$$\Pi_1^{\mu\nu}(p) = -6 \int \frac{d^4 p_1}{(2\pi)^4} \left\{ g^{\mu\nu} \frac{[p_1 \cdot (p - p_1)]^2}{p_1^2} - \frac{p_1 \cdot (p - p_1)}{p_1^2} [p^\mu p_1^\nu + p^\nu (p - p_1)^\mu - 2p_1^\mu (p - p_1)^\nu] \right\}$$

$$\times \left\{ \frac{2}{3} \frac{M_Q^2}{(p - p_1)^2} + \left[\frac{9}{4}(p - p_1)^2 - \frac{41}{6}M_Q^2 - \frac{2}{3}M_Q^4 \frac{1}{(p - p_1)^2} \right] I_0(p - p_1) \right\}, \tag{2.7}$$

with

$$I_0(p) = \int_0^1 \frac{d\alpha}{\alpha(1 - \alpha)p^2 - M_Q^2}. \tag{2.8}$$

Extracting the scalar correlator Π^S, defined by $\Pi^{\mu\nu}(p) = (p_\mu p_\nu / p^2 - g^{\mu\nu})\Pi^V(p) + (p_\mu p_\nu / p^2)\Pi^S(p)$, and carrying out the p_1 integrals, one finds for $\Pi_1^S(p)$, the scalar term of $\Pi_1^{\mu\nu}(p)$,

$$\Pi_1^S(p) = -\frac{3}{(4\pi)^2} \int_0^1 \frac{d\alpha}{\alpha(1 - \alpha)}$$

$$\times \left\{ \left[\int_0^1 \frac{d\beta \, p^2 \beta}{p^2(1 - \beta) - M_Q^2/(\alpha - \alpha^2)} \right. \right.$$

$$\times \left[-\frac{4}{3} \frac{M_Q^6}{\alpha - \alpha^2} - \frac{155}{12} \frac{M_Q^6}{(\alpha - \alpha^2)^2} + \frac{319}{12} p^2 \frac{M_Q^4}{\alpha - \alpha^2} - \frac{4}{3} M_Q^4 p^2 - \frac{41}{3} M_Q^2 p^4 \right] \tag{2.9}$$

$$+ \frac{p^4}{2} \int_0^1 d\beta \, \frac{-1}{p^2(1 - \beta) - M_Q^2/(\alpha - \alpha^2)}$$

$$\left. \left. \times \left[\frac{55}{3} M_Q^2 \left(p^2 - \frac{M_Q^2}{\alpha - \alpha^2} \right) \left(-3\beta + \beta^2 - \frac{\beta^3}{3} \right) - \frac{8}{3} M_Q^4 \left(4\beta - 3\beta^2 + \frac{\beta^3}{3} \right) \right] \right\} \right\}$$
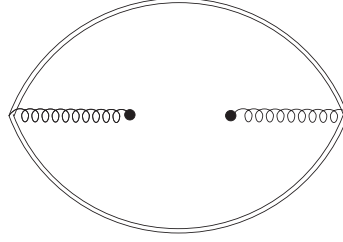
+ terms with three and four integrals.

Figure 3: Gluon condensate term in sum rule.

We find that the terms with three and four integrals are very small, and we do not include them in our calculation.

The second term in the OPE for the heavy-quark hybrid correlator includes the gluon condensate, illustrated in Figure 3.

For this process, the correlator $\Pi_2^{\mu\nu}(p)$ has the same form as (2.3), except that the gluon trace used to obtain $\Pi_1^{\mu\nu}(p)$ is replaced with the trace over the gluon condensate [1–4], which gives

$$\mathrm{Tr}\left[G^{\mu\alpha}G^{\nu\beta}\right](p_1) = (2\pi)^4\delta^4(p_1)\frac{12}{96}\langle G^2\rangle(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\beta}g^{\nu\alpha}). \tag{2.10}$$

The fermionic factor is the same as for Figure 2. From this, one finds

$$\Pi_2^{\mu\nu}(p)$$
$$= -\frac{3i}{2}\frac{\langle G^2\rangle}{(4\pi)^2}\left\{g^{\mu\nu}\left(-M_Q^2-\frac{p^2}{4}\right)(p^2-4M_Q^2)I_0+p^\mu p^\nu\left(\frac{11}{2}p^2-\frac{53}{3}M_Q^2+\frac{4}{3}\frac{M_Q^4}{p^2}\right)I_0+\frac{4}{3}\frac{M_Q^2}{p^2}\right\}, \tag{2.11}$$

giving for the scalar part of Figure 3

$$\Pi_2^S(p) = -\frac{3i}{2}\frac{\langle G^2\rangle}{(4\pi)^2}\left(\frac{11}{4}p^4 - \frac{59}{6}M_Q^2p^2 + \frac{14}{3}M_Q^4\right)I_0. \tag{2.12}$$

2.5. Borel transform of the correlator

To ensure convergence of the OPE of the correlator, one performs a Borel transform $\mathcal{B}$ [1, 2], defined in Appendix B. This is discussed in detail in the early papers on QCD sum rules [1–4]. For our problem, we have assumed that $\Pi^S(p) \simeq \Pi_1^S(p) + \Pi_2^S(p)$, with higher-order terms being very small. We shall see that even Π_2 is essentially negligible within the accuracy of the method, and the convergence after the Borel transform is established.

Using the equations in Appendix B, from (B.1), (B.2), and (B.3), where the quantities B_1 through B_8 are given, with $\mathcal{B}\Pi_1^S(p) \equiv \tilde{\Pi}_1^S(M_B)$, we find

$$\tilde{\Pi}_1^S(M_B)$$

$$= -\frac{1}{4(4\pi)^2}\left[-16M_Q^6 B_1 - 155M_Q^6 B_2 + 319B_3 - 16M_Q^4 B_4 - 164B_5 - 110(B_6 - M_Q^2 B_7) - 16M_Q^4 B_8\right]$$

$$= -\frac{1}{2(4\pi)^2}M_Q^4 \int_0^\infty d\delta\, e^{-2(M_Q^2/M_B^2)(1+\delta)}$$

$$\times \left\{\left[-310\frac{\delta}{1+\delta} + 638\delta - 656(1+\delta)\right]K_3\left(2\frac{M_Q^2}{M_B^2}(1+\delta)\right)\right.$$

$$+ \left[-1892\frac{\delta}{1+\delta} + 606\delta - 1968(1+\delta) - 32\left(4\delta - 3\frac{\delta^2}{1+\delta} + \frac{\delta^3}{3(1+\delta)^2}\right)\right]K_2\left(2\frac{M_Q^2}{M_B^2}(1+\delta)\right)$$

$$+ \left[-4778\frac{\delta}{1+\delta} + 9442\delta - 4920(1+\delta) - 128\left(4\delta - 3\frac{\delta^2}{1+\delta} + \frac{\delta^3}{3(1+\delta)^2}\right)\right]K_1\left(2\frac{M_Q^2}{M_B^2}(1+\delta)\right)$$

$$\left.+ \left[-1356\frac{\delta}{1+\delta} + 6284\delta - 3280(1+\delta) - 96\left(4\delta - 3\frac{\delta^2}{1+\delta} + \frac{\delta^3}{3(1+\delta)^2}\right)\right]K_0\left(2\frac{M_Q^2}{M_B^2}(1+\delta)\right)\right\}$$

+ multiple integral terms.

$$(2.13)$$

The multiple integral terms in (2.11) are small and are dropped, and δ is a variable of integration.

In a similar way, taking the Borel transform of $\Pi_2^S(p)$, with $\mathcal{B}\Pi_2^S(p) \equiv \tilde{\Pi}_2^S(M_B)$, we find

$$\tilde{\Pi}_2^S(M_B) = -i\frac{3}{2(4\pi)^2}M_Q^4 e^{-2(M_Q^2/M_B^2)}\left[11K_2\left(2\frac{M_Q^2}{M_B^2}\right) + \frac{14}{3}K_1\left(2\frac{M_Q^2}{M_B^2}\right) + 18K_0\left(2\frac{M_Q^2}{M_B^2}\right)\right].$$

$$(2.14)$$

2.6. QCD sum rule for hybrid charm meson

The method of QCD sum rules uses a dispersion relation for the correlator, which it equates to the correlator's operator product. Following the usual convention, we call the dispersion relation the LHS and the OPE the RHS: Π_{lhs} = dispersion relation, Π_{rhs} = OPE. Neglecting the width of the hybrid meson, the dispersion relation has the form of a pole and a continuum: $\int_0^\infty ds(\text{Im}\Pi(s)/(s - p^2))$. The dispersion relation is evaluated in Euclidean space, $p^2 \to -Q^2$, and the continuum is assumed to start at $s = s_o$. After the Borel transform, the form we use for the LHS is

$$\tilde{\Pi}_{\text{lhs}}(M_B^2) = Fe^{-M_{\text{HH}}^2/M_B^2} + (L_1 M_B^2 + L_2 M_B^4)e^{-s_o/M_B^2}, \qquad (2.15)$$

with F the numerator of the pole, and L_1 and L_2 constants used to fit the form of the continuum. We should use a standard method to eliminate F, and fit the sum rule
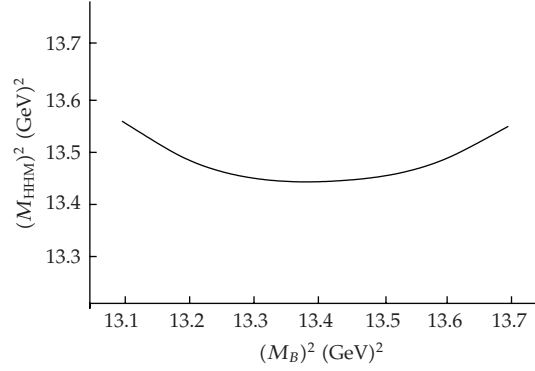
Figure 4: QCD sum rule for heavy-hybrid charm meson.

requirement that the solution should not be sensitive to M_B. In theory, the exact solution should be independent of M_B.

For convenience in carrying out the sum rule, we have fit the RHS of the correlator to a polynomial in the Borel mass, $\tilde{\Pi}_1(M_B) = a_1 M_B^2 + a_2 M_B^4 + a_3 M_B^6 + a_4 M_B^8$ and $\tilde{\Pi}_2(M_B) = b_1 M_B^2 + b_2 M_B^4 + b_3 M_B^6 + b_4 M_B^8$ with $a_1 = 122.29$, $a_2 = -143.88$, $a_3 = 45.79$, $a_4 = -0.346$, $b_1 = 616.0$, $b_2 = -279.1$, $b_3 = -226.56$, and $b_4 = 144.515$, with units GeV^6.

We find that for all values of M_B relevant to the sum rule, $\tilde{\Pi}_2$ is just about 1% of $\tilde{\Pi}_1$, and drop it, as the method is only valid to a few percent. The sum rule is obtained by taking the ratio of $\tilde{\Pi}$ to $\partial_{1/M_B^2}\tilde{\Pi}$. To do this, we use the relations

$$
\partial_{1/M_B^2} e^{-A/M_B^2} K_\nu\left(-\frac{A}{M_B^2}\right)\bigg|_{\nu>0} = -A e^{-A/M_B^2} K_\nu\left(-\frac{A}{M_B^2}\right)
$$
$$
+ \frac{A}{2} e^{-A/M_B^2}\left[K_{\nu-1}\left(-\frac{A}{M_B^2}\right) - K_{\nu+1}\left(-\frac{A}{M_B^2}\right)\right], \qquad (2.16)
$$
$$
\partial_{1/M_B^2} e^{-A/M_B^2} K_0\left(-\frac{A}{M_B^2}\right) = -A e^{-A/M_B^2}\left[K_0\left(-\frac{A}{M_B^2}\right) - K_1\left(-\frac{A}{M_B^2}\right)\right].
$$

Taking the ratio of $\partial_{1/M_B^2}\tilde{\Pi}_{\text{lhs}}(M_B) = \partial_{1/M_B^2}\tilde{\Pi}_{\text{rhs}}(M_B)$ to the equation $\tilde{\Pi}_{\text{lhs}}(M_B) = \tilde{\Pi}_{\text{rhs}}(M_B)$, one obtains the sum rule for the mass of the heavy charmonium hybrid meson:

$$
M_{\text{HH}}^2 = \left\{ e^{-s_o/M_B^2}\left[s_o(L_1 M_B^2 + L_2 M_B^4) + L_1 M_B^4 + 2L_2 M_B^6 \right] + \partial_{1/M_B^2}\tilde{\Pi}_1^S \right\}
$$
$$
\times \left\{ e^{-s_o/M_B^2}\left[(L_1 M_B^2 + L_2 M_B^4) - \tilde{\Pi}_1^S \right] \right\}^{-1}. \qquad (2.17)
$$

The result of the QCD sum rule fit is shown in **Figure 4**, with $s_o = 60.0\,\text{GeV}^2$, $L_1 = -99.0\,\text{GeV}^4$, and $L_2 = 6.06\,\text{GeV}^2$.

The parameters L_1 and L_2 have magnitudes expected for the fit to the continunmm. Except for the large value of s_o, the solution satisfies the criteria for a successful QCD sum rule, as **Figure 4** shows. The solution for the mass is a few percent of the value of M_B in the

region of stability. The mass predicted for the hybrid is

$$M_{\text{HHM}} \simeq 3.66 \, \text{GeV},\tag{2.18}$$

within a 10% accuracy of the sum rule method, while the experimental mass of the $\Psi'(2S)$ state [15] is

$$M(\Psi'(2S)) = 3.68 \, \text{GeV}.\tag{2.19}$$

The large value of s_o, however, predicts that the excited hybrids are at a very high energy, as found in lattice gauge calculations, and that the $\Psi'(2S)$ cannot be a pure hybrid. Therefore, we expect that the physical $\Psi'(2S)$ is an admixture with a charm meson and a hybrid charm component. A second, orthogonal mixed state will be in the continuum. As we now show, this can provide a solution to the $\rho - \pi$ puzzle.

3. Hybrid-normal charmonium and a possible solution to the $\rho - \pi$ puzzle

In our treatment of hadronic decays of the $\Psi'(2S)$ state, we use the Sigma/Glueball model, which was motivated by the BES analysis of glueball decay [16] and the study of scalar mesons and scalar glueballs using QCD sum rules [17, 18]. We briefly review this model in what follows.

3.1. The Sigma/Glueball model

In energy regions where there are both scalar mesons and scalar glueballs, it is expected that 0^{++} states will be an admixture of mesons and glueballs. For this reason, when using QCD sum rules to find such states, one must use currents that are a linear combination of glueball and meson currents. A scalar glueball current can have the form

$$J^G(x) = \alpha_s G^2,\tag{3.1}$$

while a scalar meson current has the form

$$J^m(x) = \frac{1}{2}\left(\overline{u}(x)u(x) - \overline{d}(x)d(x)\right).\tag{3.2}$$

We use for our 0^{++} current [17] (with M_o needed for correct dimensions)

$$J_{0^{++}} = \beta M_o J_m + \left(1 - |\beta|\right) J_G.\tag{3.3}$$

The QCD sum rule calculation makes use of the correlator $\Pi(x) = \langle T[J_{0^{++}} J_{0^{++}}]\rangle$. The cross term between J^m and J^G is evaluated by using the scalar glueball-meson coupling theorem [19, 20]:

$$\int dx \langle T[J^G(x) J^m(0)]\rangle \simeq -\frac{32}{9}\langle \overline{q}q\rangle,\tag{3.4}$$

with $\langle \overline{q}q\rangle \equiv$ the quark condensate, which is illustrated in Figure 5.
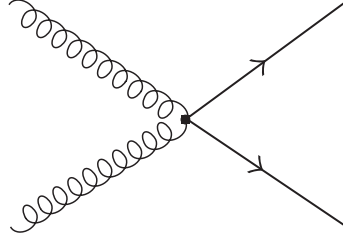
Figure 5: Scalar glueball-meson coupling theorem.

The results of the QCD sum rule calculations [17, 18] are that there are three solutions:

80% scalar glueball at 1500 MeV → the f_0 (1500);

80% scalar meson at 1350 MeV → the f_0 (1370);

Light Scalar Glueball 400–600 MeV → the Sigma/Glueball.

The Sigma/Glueball model follows from the existence of the σ, a scalar $\pi - \pi$ resonance with a broad width and the same mass as the scalar glueball found in the sum rule calculations, and makes use of the glueball-meson coupling shown in Figure 5. It has been used for the study of the Roper resonance decay into a nucleon and a σ [5, 21], and the prediction of σ production in proton-proton high-energy collisions [22], and other applications. The relevance for our present work is that just as it is possible to determine the mixing parameter for a state consisting of a scalar meson and a scalar glueball, it should be possible to determine the $c\bar{c}$ and hybrid $c\bar{c}g$ admixture for charmonium systems.

3.2. Hybrid mixing model and the $\rho - \pi$ puzzle

The solution to the $\rho - \pi$ puzzle by the mixing of hybrid and normal meson components of the $\Psi(2S)$ state can be understood from Figures 6 and 7, which illustrate the decay of a $c\bar{c}$ and a $c\bar{c}g$ state into two hadrons. The $c\bar{c}$ decay involves the matrix element $\langle\pi\rho|O|\Psi'(c\bar{c},2S)\rangle$.

The corresponding hybrid decay involves the matrix element $\langle\pi\rho|O'|\Psi'(c\bar{c}g,2S)\rangle$, with the diagram shown in Figure 7.

Assuming that the 2 s state is a $c\bar{c}$-$c\bar{c}g$ admixture,

$$|\Psi'(2S)\rangle = b|\Psi'(c\bar{c},2S)\rangle + \sqrt{1-b^2}|\Psi'(c\bar{c}g,2S)\rangle, \tag{3.5}$$

and recognizing that the O and O' matrix elements are approximately equal, we see that the solution to the $\rho - \pi$ puzzle requires

$$b + \sqrt{1-b^2} \simeq 0.1. \tag{3.6}$$

The solution of the $\rho - \pi$ puzzle would be given if

$$b \simeq -.7. \tag{3.7}$$

In other words, if $b \simeq -.7$, we have found a solution of the $\rho - \pi$ puzzle. The evaluation of b is rather complicated, and the testing of this conjecture will be carried out in future work.
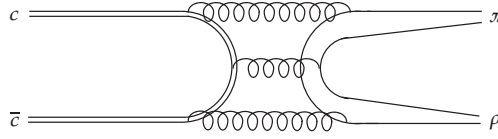
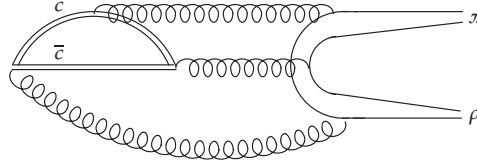**Figure 6:** Lowest-order PQCD diagram for a $c\bar{c}$ decay into two hadrons.

Figure 7: Decay of a $c\bar{c}g$ state into two hadrons.

4. Conclusion

Using the method of QCD sum rules, we have shown that the $\Psi'(2S)$ state cannot be a pure charmonium hybrid. We have found that the energy of the lowest $J^{PC} = 1^{--}$ hybrid charmonium state is approximately the same as the $\Psi'(2S)$ state, about 600 MeV above the $J/\Psi(1S)$ state, but that the QCD sum rule solution is not consistent with a pure hybrid. The standard model prediction for $c\bar{c}(2S)$ is at approximately the same energy. Therefore, we expect that the physical $\Psi'(2S)$ state is an admixture of a $c\bar{c}(2S)$ and a $c\bar{c}(8)g(8)(2S)$. Using this picture, we find a possible solution to the famous $\rho - \pi$ puzzle.

There have been many lattice calculations of exotic hybrid mesons. There is experimental evidence for an exotic light-quark 1^{-+} meson (see [9] for a discussion) at 1.4 to 1.6 GeV, which is consistent with QCD sum rule calculations [3, 4], while lattice calculations find the lowest 1^{-+} hybrid at 1.9 to 2.1 GeV. The lowest energy 1^{-+} charmonium hybrid found in lattice calculations is at 4.4 GeV [13], about 800 MeV above our 1^{--} hybrid charmonium solution. This is consistent with our large value of s_o. For the 1^{--} state, we have shown that one must use a mixed $c\bar{c}$ and $c\bar{c}g$ current, which we shall use in future work. This use of a mixed current to define the correlator has not been done in lattice QCD caluclations, but with the QCD sum rule method, it can be done in a rather straight-forward calculation.

It is interesting that the energy difference between the $\Upsilon(2S)$ and the $\Upsilon(1S)$ is also approximately 600 MeV. If this is the energy of a $\Upsilon(nS)$ hybrid, this could provide a solution to the puzzling σ decays of $\Upsilon(nS)$ states that have recently been observed [23]. Investigation of this system is a topic of future research.

Appendices

A. Heavy-quark hybrid correlator in momentum space

The current to create a heavy-quark hybrid meson with $J^{PC} = 1^{--}$ is

$$J_H^\mu = \overline{\Psi}\Gamma_\nu G^{\mu\nu}\Psi, \tag{A.1}$$

where Ψ is the heavy-quark field, $\Gamma_\nu = C\gamma_\nu$, γ_ν is the usual Dirac matrix, C is the charge conjugation operator, and the gluon color field is

$$G^{\mu\nu} = \sum_{a=1}^{8} \frac{\lambda_a}{2} G_a^{\mu\nu}, \tag{A.2}$$

with λ_a the SU(3) generator ($\text{Tr}[\lambda_a\lambda_b] = 2\delta_{ab}$). From this, one finds for the correlator of a heavy-quark hybrid meson

$$\Pi^{\mu\nu}(p) = \int d^4x\, e^{ip\cdot x} \langle T[J_H^\mu(x) J_H^\mu(0)] \rangle$$
$$= \int \frac{d^4p_1}{(2\pi)^4} \text{Tr}[S^{ab}\Gamma_\alpha S^{ba}\Gamma_\beta](p-p_1)\text{Tr}[G^{\mu\alpha}G^{\nu\beta}](p_1), \tag{A.3}$$

where S^{ab} is the quark propagator, a standard Dirac propagator for a fermion with mass M_Q, with colors a and b. Using $C = i\gamma^2\gamma^0$, $\text{Tr}[\gamma_\alpha\gamma_\beta] = 4g_{\alpha\beta}$, and $\text{Tr}[\gamma_\lambda\gamma_\alpha\gamma_\delta\gamma_\beta] = 4(g_{\lambda\alpha}g_{\delta\beta} - g_{\lambda\delta}g_{\alpha\beta} + g_{\lambda\beta}g_{\delta\alpha})$:

$$\text{Tr}[S^{ab}\Gamma_\alpha S^{ba}\Gamma_\beta](p)$$
$$= (-24i)\int \frac{d^Dk}{(2\pi)^D} \left[-M_Q^2 g_{\alpha\beta} + (p_\lambda k_\delta - k_\lambda k_\delta)(g_{\lambda\alpha}g_{\delta\beta} - g_{\lambda\delta}g_{\alpha\beta} + g_{\lambda\beta}g_{\delta\alpha})\right] \frac{1}{(k^2 - M_Q^2)\left[(p-k)^2 - M_Q^2\right]}. \tag{A.4}$$

To use dimensional regularization, we define the quantity $D = 4 - \epsilon$, and let $\epsilon \to 0$ to complete the integrals. One can then show

$$\int \frac{d^Dk}{(2\pi)^D} \frac{1}{(k^2 - M_Q^2)\left[(p-k)^2 - M_Q^2\right]} = \frac{I_0}{(8\pi)^2}, \tag{A.5}$$

with

$$I_0 = \int_0^1 \frac{d\alpha}{(\alpha - \alpha^2)p^2 - M_Q^2}. \tag{A.6}$$

The first term in the OPE, shown in Figure 2, has a standard gluon propagator. For the gluon trace [3, 4] one finds

$$\text{Tr}[G^{\mu\alpha}G^{\nu\beta}](p_1) = -4\pi^2 i\left(g_{\alpha\beta}\frac{p_1^\mu p_1^\nu}{p_1^2} + g_{\mu\nu}\frac{p_1^\alpha p_1^\beta}{p_1^2} - g_{\mu\beta}\frac{p_1^\alpha p_1^\nu}{p_1^2} - g_{\alpha\nu}\frac{p_1^\mu p_1^\beta}{p_1^2}\right). \tag{A.7}$$

The correlator for the process shown in Figure 2 is found to be

$$\Pi_1^{\mu\nu}(p) = -6\int \frac{d^4 p_1}{(2\pi)^4}\left\{g^{\mu\nu}\frac{[p_1\cdot(p-p_1)]^2}{p_1^2} - \frac{p_1\cdot(p-p_1)}{p_1^2}[p^\mu p_1^\nu + p^\nu(p-p_1)^\mu - 2p_1^\mu(p-p_1)^\nu]\right\}$$

$$\times\left\{\frac{2}{3}\frac{M_Q^2}{(p-p_1)^2} + \left[\frac{9}{4}(p-p_1)^2 - \frac{41}{6}M_Q^2 - \frac{2}{3}M_Q^4\frac{1}{(p-p_1)^2}\right]I_0(p-p_1)\right\}.$$

$$(A.8)$$

The next term in the OPE for this heavy-quark system, where quark condensates are negligible, is the gluon condensate term, shown in Figure 3. The trace over the quark propagators is the same as in (A.4). The gluon field trace for this term is

$$\langle\text{Tr}[G^{\mu\alpha}G^{\nu\beta}](p_1)\rangle = (2\pi)^4\delta^4(p_1)\frac{12}{96}\langle G^2\rangle(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\beta}g^{\nu\alpha}).$$

$$(A.9)$$

From this, one finds $\Pi_2^{\mu\nu}$, given in (2.11).

B. Borel transforms

A key method that enables one to use the operator expansion to get accurate sum rules is the use of the Borel transform [1, 2], $\mathcal{B}$, defined by

$$\mathcal{B} = \lim_{q^2, n\to\infty}\frac{1}{(n-1)!}(q^2)^n\left(-\frac{d}{dq^2}\right)^n\bigg|_{q^2/n=M_B^2}.$$

$$(B.1)$$

Two key equations which we need are (with K_ν the modified Bessel functions)

$$\mathcal{B}\left(\frac{1}{m^2-p^2}\right)^k = \frac{e^{-m^2/M_B^2}}{(k-1)!(M_B^2)^{k-1}},$$

$$\int_0^\infty x^{\nu-1}e^{a/x-bx} = 2\left(\frac{a}{b}\right)^{\nu/2}K_\nu(2\sqrt{ab}).$$

$$(B.2)$$

Transforms used in the body of the paper are

$$\mathcal{B}I_0 = \int_0^1\frac{d\alpha}{(\alpha-\alpha^2)}\mathcal{B}\left(p^2 - \frac{M_Q^2}{(\alpha-\alpha^2)}\right)^{-1} = 2e^{-2M_Q^2/M_B^2}K_0\left(\frac{2M_Q^2}{M_B^2}\right),$$

$$B_1 = \mathcal{B}\int_0^1\frac{d\alpha}{(\alpha-\alpha^2)^2}p^2\int_0^1\frac{d\beta\,\beta}{(1-\beta)[p^2 - M_Q^2/(\alpha-\alpha^2)(1-\beta)]}$$

$$= 2M_Q^2\int_0^\infty d\delta\,\frac{\delta}{1+\delta}e^{-2(M_Q^2/M_B^2)(1+\delta)}\left[2K_2\left(2\frac{M_Q^2}{M_B^2}(1+\delta)\right) + 8K_1\left(2\frac{M_Q^2}{M_B^2}(1+\delta)\right)\right.$$

$$\left. + 6K_0\left(2\frac{M_Q^2}{M_B^2}(1+\delta)\right)\right],$$

$$B_2 = \mathcal{B} \int_0^1 \frac{d\alpha}{(\alpha - \alpha^2)^3} p^2 \int_0^1 \frac{d\beta\,\beta}{(1 - \beta)\left[p^2 - M_Q^2/(\alpha - \alpha^2)(1 - \beta)\right]}$$

$$= 2M_Q^2 \int_0^\infty d\delta\, \frac{\delta}{1 + \delta} e^{-2(M_Q^2/M_B^2)(1+\delta)} \left[2K_3\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right) + 12K_2\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right)\right.$$

$$\left. + 30K_1\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right) + 20K_0\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right)\right],$$

$$B_3 = \mathcal{B} \int_0^1 \frac{d\alpha}{(\alpha - \alpha^2)^3} p^4 \int_0^1 \frac{d\beta\,\beta}{(1 - \beta)\left[p^2 - M_Q^2/(\alpha - \alpha^2)(1 - \beta)\right]}$$

$$= 2M_Q^4 \int_0^\infty d\delta\, \delta e^{-2(M_Q^2/M_B^2)(1+\delta)} \left[2K_3\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right) + 12K_2\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right)\right.$$

$$\left. + 30K_1\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right) + 20K_0\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right)\right],$$

$$B_4 = \mathcal{B} \int_0^1 \frac{d\alpha}{(\alpha - \alpha^2)} p^4 \int_0^1 \frac{d\beta\,\beta}{(1 - \beta)\left[p^2 - M_Q^2/(\alpha - \alpha^2)(1 - \beta)\right]}$$

$$= 2M_Q^4 \int_0^\infty d\delta\, \delta e^{-2(M_Q^2/M_B^2)(1+\delta)} \left[2K_2\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right)\right],$$

$$B_5 = \mathcal{B} \int_0^1 \frac{d\alpha}{(\alpha - \alpha^2)} p^6 \int_0^1 \frac{d\beta\,\beta}{(1 - \beta)\left[p^2 - M_Q^2/(\alpha - \alpha^2)(1 - \beta)\right]}$$

$$= 2M_Q^6 \int_0^\infty d\delta\, \delta(1 + \delta) e^{-2(M_Q^2/M_B^2)(1+\delta)} \left[2K_3\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right) + 12K_2\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right)\right.$$

$$\left. + 30K_1\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right) + 20K_0\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right)\right],$$

$$B_6 = \mathcal{B} \int_0^1 \frac{d\alpha}{(\alpha - \alpha^2)} p^6 \int_0^1 \frac{d\beta(-3\beta + \beta^2 - \beta^3/3)}{(1 - \beta)\left[p^2 - M_Q^2/(\alpha - \alpha^2)(1 - \beta)\right]}$$

$$= 2M_Q^6 \int_0^\infty d\delta\left(-3\delta + \frac{\delta^2}{1 + \delta} - \frac{\delta^3}{3(1 + \delta)^2}\right) e^{-2(M_Q^2/M_B^2)(1+\delta)}$$

$$\times \left[2K_3\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right) + 12K_2\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right)\right.$$

$$\left. + 30K_1\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right) + 20K_0\left(2\frac{M_Q^2}{M_B^2}(1 + \delta)\right)\right],$$

$$B_7 = \mathcal{B} \int_0^1 \frac{d\alpha}{(\alpha - \alpha^2)^2} p^4 \int_0^1 \frac{d\beta(-3\beta + \beta^2 - \beta^3/3)}{(1 - \beta)\left[p^2 - M_Q^2/(\alpha - \alpha^2)(1 - \beta)\right]} = \frac{1}{M_Q^2} B_6,$$

$$B_8 = \mathcal{B} \int_0^1 \frac{d\alpha}{(\alpha - \alpha^2)} p^4 \int_0^1 \frac{d\beta(4\beta - 3\beta^2 + \beta^3/3)}{(1-\beta)\left[p^2 - M_Q^2/(\alpha - \alpha^2)(1-\beta)\right]}$$

$$= 2M_Q^4 \int_0^\infty d\delta \left(4\delta - \frac{3\delta^2}{1+\delta} + \frac{\delta^3}{3(1+\delta)^2}\right) e^{-2(M_Q^2/M_B^2)(1+\delta)}$$

$$\times \left[2K_2\left(2\frac{M_Q^2}{M_B^2}(1+\delta)\right) + 8K_1\left(2\frac{M_Q^2}{M_B^2}(1+\delta)\right) + 6K_0\left(2\frac{M_Q^2}{M_B^2}(1+\delta)\right)\right].$$
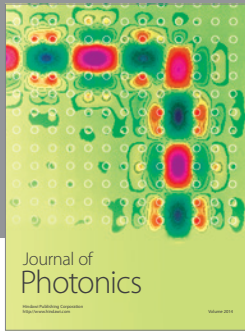
$$\text{(B.3)}$$

Acknowledgments

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