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Research Article

On Level p Siegel Cusp Forms of Degree Two

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We give a simple formula for the Fourier coefficients of some degree-two Siegel cusp form with level p.

1. Introduction

In the previous paper [1], the second and the third authors introduced a simple construction of a Siegel cusp form of degree 2. This construction has an advantage because the Fourier coefficients are explicitly computable. After this work was completed, Kikuta and Mizuno proved that the p-adic limit of a sequence of the aforementioned cusp forms becomes a Siegel cusp form of degree 2 with level p.

In this paper, we give an explicit description of the Fourier expansion of such a form. This result shows that the cusp form becomes a *nonzero* cusp form of weight 2 on $\Gamma_0^2(p)$ if $p > 7$ and $p \equiv 3 \pmod 4$.

2. Siegel Modular Forms of Degree 2

We start by recalling the basic facts of Siegel modular forms.

The *Siegel upper half-space* of degree 2 is defined by

$$\mathbb{H}_2 := \{ Z = X + iY \in \mathrm{Sym}_2(\mathbb{C}) \mid Y > 0 \ (\text{positive-definite}) \}. \tag{2.1}$$

Then the degree 2 Siegel modular group $\Gamma^2 := \mathrm{Sp}_2(\mathbb{Z})$ acts on $\mathbb{H}_2$ discontinuously. For a congruence subgroup $\Gamma' \subset \Gamma^2$, we denote by $M_k(\Gamma')$ (resp., $S_k(\Gamma')$) the corresponding space of Siegel modular forms (resp., cusp forms) of weight k.

We will be mainly concerned with the Siegel modular group Γ^2 and the congruence subgroup

$$\Gamma_0^2(N) := \left\{ \begin{pmatrix} AB \\ CD \end{pmatrix} \in \Gamma^2 \mid C \equiv O \pmod{N} \right\}. \tag{2.2}$$

In both cases, $F \in M_k(\Gamma')$ has a Fourier expansion of the form

$$F(Z) = \sum_{0 \leq T \in \Lambda_2} a(T;F) \exp[2\pi i \operatorname{tr}(TZ)], \tag{2.3}$$

where

$$\Lambda_2 := \left\{ T = \left(t_{ij}\right) \in \mathrm{Sym}_2(\mathbb{Q}) \mid t_{11}, t_{22}, 2t_{12} \in \mathbb{Z} \right\}, \tag{2.4}$$

and $a(T;F)$ is the Fourier coefficient of F at T.

3. Siegel Cusp Form of Degree 2

In the previous paper [1], we constructed a cusp form $f_k \in S_k(\Gamma^2)$ whose Fourier coefficients are explicitly computable. We review the result.

First, we recall the definition of Cohen's function. Cohen defined an arithmetical function $H(r,N)$ $(r, N \in \mathbb{Z}_{\geq 0})$ in [2]. In the case that r and N satisfy $(-1)^r N = D \cdot f^2$ where D is a fundamental discriminant and $f \in \mathbb{N}$, the function is given by

$$H(r,N) = L\left(1-r, \chi_D\right) \sum_{0<d \mid f} \mu(d) \chi_D(d) d^{r-1} \sigma_{2r-1}\left(\frac{f}{d}\right). \tag{3.1}$$

Here, $L(s,\chi)$ is the Dirichlet L-function with character χ, and μ is the Möbius function. For the precise definition of $H(r,N)$, see [2, page 272].

Secondly, we introduce Krieg's function $G(s,N)$ $(s, N \in \mathbb{Z}_{\geq 0})$ associated with the Gaussian field $\mathbb{Q}(i)$. Let χ_{-4} be the Kronecker character associated with $\mathbb{Q}(i)$. Krieg's function $G(s,N) = G_{\mathbb{Q}(i)}(s,N)$ over $\mathbb{Q}(i)$ is defined by

$$G(s,N) := \begin{cases} \dfrac{1}{1+\left|\chi_{-4}(N)\right|} \left(\sigma_{s,\chi_{-4}}(N) - \widetilde{\sigma}_{s,\chi_{-4}}(N)\right), & \text{if } N > 0, \\ -\dfrac{B_{s+1,\chi_{-4}}}{2(s+1)}, & \text{if } N = 0, \end{cases} \tag{3.2}$$

where $B_{m,\chi}$ is the generalized Bernoulli number with character χ,

$$\sigma_{s,\chi_{-4}}(N) := \sum_{0<d \mid N} \chi_{-4}(d) d^s, \qquad \widetilde{\sigma}_{s,\chi_{-4}}(N) := \sum_{0<d \mid N} \chi_{-4}\left(\frac{N}{d}\right) d^s. \tag{3.3}$$

This function was introduced by Krieg [3] to describe the Fourier coefficients of Hermitian Eisenstein series of degree 2.

The following theorem is one of the main results in [1].

Theorem 3.1. *There exists a Siegel cusp form $f_k \in S_k(\Gamma^2)$ whose Fourier coefficients $a(T; f_k)$ are given as follows:*

$$a(T; f_k) = \sum_{0<d|\varepsilon(T)} d^{k-1} \alpha_k\left(\frac{4\det(T)}{d^2}\right), \tag{3.4}$$

where

$$\alpha_k(N) := H(k-1, N) - \frac{B_{2k-2}}{B_{k-1,\chi_{-4}}} \sum_{\substack{s\in\mathbb{Z} \\ s^2\leq N}} G\left(k-2, N-s^2\right),$$
$$\varepsilon(T) := \max\left\{l \in \mathbb{N} \mid l^{-1}T \in \Lambda_2\right\}. \tag{3.5}$$

Here, B_m is the mth Bernoulli number.

Remark 3.2. The above result shows that the cusp form f_k is a form in the Maass space (cf. [1]).

4. p-Adic Siegel Modular Forms

The cusp form f_k introduced in Theorem 3.1 was constructed by the difference between the Siegel Eisenstein series E_k and the restriction of the Hermitian Eisenstein series $E_{k,\mathbb{Q}(i)}$:

$$f_k = c_k \cdot \left(E_k - E_{k,\mathbb{Q}(i)}|_{\mathbb{H}_2}\right), \tag{4.1}$$

for some $c_k \in \mathbb{Q}$. The p-adic properties of the Eisenstein series E_k and $E_{k,\mathbb{Q}(i)}$ are studied by the second author (cf. [4, 5]). After our work [1] was completed, Kikuta and Mizuno studied p-adic properties of our form f_k. The following statement is a special case in [6].

Theorem 4.1. *Let p be a prime number satisfying $p \equiv 3 \pmod 4$, and $\{k_m\}$ is the sequence defined by*

$$k_m = k_m(p) := 2 + (p-1)p^{m-1}. \tag{4.2}$$

Then there exists the p-adic limit

$$f_p^* := \lim_{m\to\infty} f_{k_m}, \tag{4.3}$$

and f_p^* represents a cusp form of weight 2 with level p, that is,

$$f_p^* \in S_2\left(\Gamma_0^2(p)\right). \tag{4.4}$$

Remark 4.2. (1) The p-adic convergence of modular forms is interpreted as the convergence of the Fourier coefficients.

(2) Kikuta and Mizuno studied a similar problem under more general situation. They noted that if we take the sequence $\{k_m\}$ with $k_m = k + (p-1)p^{m-1}$, $k \in \mathbb{N}$ $(k > 4)$, then $\lim_{m\to\infty} f_{k_m}$ is no longer a cusp form [6, Theorem 1.7].

(3) The cuspidality of f_p^* essentially results from the fact that there are no nontrivial modular forms of weight 2 on the full modular group Γ^2.

5. Main Result

In this section, we give an explicit formula for the Fourier coefficients of f_p^*.

To describe $a(T; f_p^*)$, we will introduce two functions H_p^* and G_p^*.

First, for $N \in \mathbb{N}$ with $N \equiv 0$ or $3 \pmod 4$, we write N as $N = -D \cdot f^2$ where D is a fundamental discriminant and $f \in \mathbb{N}$. Then, we define

$$H_p^*(N) := -(1 - \chi_D(p))B_{1,\chi_D} \sum_{\substack{0<d\mid f \\ (d,p)=1}} \mu(d)\chi_D(d)\sigma_1^*\left(\frac{f}{d}\right), \tag{5.1}$$

where

$$\sigma_1^*(m) = \sum_{\substack{0<d\mid m \\ (d,p)=1}} d. \tag{5.2}$$

Secondly, for $N \in \mathbb{Z}_{\geq 0}$, we define

$$G_p^*(N) := \begin{cases} \dfrac{1-(-1)^{\mathrm{ord}_p(N)}}{1+|\chi_{-4}(N)|}\, \sigma_{0,\chi_{-4}}^*(N), & \text{if } N > 0, \\ \dfrac{1}{2}, & \text{if } N = 0, \end{cases} \tag{5.3}$$

where

$$\sigma_{0,\chi_{-4}}^*(N) = \sum_{\substack{0<d\mid N \\ (d,p)=1}} \chi_{-4}(d). \tag{5.4}$$

Remark 5.1. From the definition, the following holds:

$$G_p^*(N) = 0 \quad \text{if } p \nmid N. \tag{5.5}$$

The main theorem of this paper can be stated as follows.

Theorem 5.2. *Let p be a prime number satisfying $p \equiv 3 \pmod 4$. Then the Fourier coefficients $a(T; f_p^*)$ of $f_p^* \in S_2(\Gamma_0^2(p))$ are given by*

$$a\left(T; f_p^*\right) = \sum_{\substack{0<d\mid \varepsilon(T) \\ (d,p)=1}} d\alpha_p^*\left(\frac{4\det(T)}{d^2}\right), \tag{5.6}$$

where

$$\alpha_p^*(N) := H_p^*(N) - \frac{p-1}{6} \sum_{\substack{s \in \mathbb{Z} \\ s^2 \le N}} G_p^*\left(N - s^2\right). \tag{5.7}$$

Here, H_p^* and G_p^* are the functions defined in (5.1) *and* (5.3), *respectively.*

From Theorems 3.1 and 4.1, the proof of Theorem 5.2 is reduced to show that

$$\lim_{m \to \infty} \alpha_{k_m}(N) = \alpha_p^*(N). \tag{5.8}$$

We proceed the proof of (5.8) step by step.

Lemma 5.3. *Consider the following:*

$$\lim_{m \to \infty} H(k_m - 1, N) = H_p^*(N). \tag{5.9}$$

Proof. Under the description $N = -D \cdot f^2$, we can write $H(k_m - 1, N)$ as

$$H(k_m - 1, N) = -\frac{B_{k_m - 1, \chi_D}}{k_m - 1} \sum_{0 < d \mid f} \mu(d) \chi_D(d) d^{k_m - 2} \sigma_{2k_m - 3}\left(\frac{f}{d}\right), \tag{5.10}$$

(cf. (3.1)).

Using Kummer's congruence, we obtain

$$\lim_{m \to \infty} \frac{B_{k_m - 1, \chi_D}}{k_m - 1} = (1 - \chi_D(p)) B_{1, \chi_D}. \tag{5.11}$$

On the other hand, we have

$$\lim_{m \to \infty} \sum_{0 < d \mid f} \mu(d) \chi_D(d) d^{k_m - 2} \sigma_{2k_m - 3}\left(\frac{f}{d}\right) = \sum_{\substack{0 < d \mid f \\ (d,p)=1}} \mu(d) \chi_D(d) \sigma_1^*\left(\frac{f}{d}\right), \tag{5.12}$$

because

$$\begin{gathered} \lim_{m \to \infty} d^{k_m - 2} = \begin{cases} 1, & \text{if } p \nmid d, \\ 0, & \text{if } p \mid d, \end{cases} \\ \lim_{m \to \infty} \sigma_{2k_m - 3}(l) = \lim_{m \to \infty} \sum_{0 < d \mid l} d^{1 + 2(p-1)p^{m-1}} = \sum_{\substack{0 < d \mid l \\ (d,p)=1}} d = \sigma_1^*(l), \quad (l \in \mathbb{N}). \end{gathered} \tag{5.13}$$

This proves (5.9). □

Lemma 5.4. *Consider the following:*

$$\lim_{m\to\infty}\frac{B_{2k_m-2}}{B_{k_m-1,\chi_{-4}}}\sum_{\substack{s\in\mathbb{Z}\\ s^2\le N}} G\left(k_m-2, N-s^2\right) = \frac{p-1}{6}\sum_{\substack{s\in\mathbb{Z}\\ s^2\le N}} G_p^*\left(N-s^2\right). \tag{5.14}$$

Proof. First, we calculate the factor of Bernoulli numbers. Again by Kummer's congruence, we obtain

$$\begin{aligned}\lim_{m\to\infty}\frac{B_{2k_m-2}}{B_{k_m-1,\chi_{-4}}} &= 2\lim_{m\to\infty}\frac{B_{2k_m-2}}{2k_m-2}\cdot\frac{k_m-1}{B_{k_m-1,\chi_{-4}}}\\ &= 2\cdot(1-p)\cdot\frac{B_2}{2}\cdot\frac{1}{(1-\chi_{-4}(p))B_{1,\chi_{-4}}}\\ &= \frac{p-1}{6}.\end{aligned} \tag{5.15}$$

Here, we used the facts that $\chi_{-4}(p) = -1$ and $B_{1,\chi_{-4}} = -1/2$.

Next we calculate

$$\lim_{m\to\infty}\sum_{\substack{s\in\mathbb{Z}\\ s^2\le N}} G\left(k_m-2, N-s^2\right). \tag{5.16}$$

If $N' := N - s^2 > 0$, then

$$G(k_m-2, N') = \frac{1}{1+|\chi_{-4}(N')|}\left(\sigma_{k_m-2,\chi_{-4}}(N') - \widetilde{\sigma}_{k_m-2,\chi_{-4}}(N')\right), \tag{5.17}$$

(cf. (3.2)). Therefore, we need to calculate

$$\lim_{m\to\infty}\sigma_{k_m-2,\chi_{-4}}(N'), \qquad \lim_{m\to\infty}\widetilde{\sigma}_{k_m-2,\chi_{-4}}(N'). \tag{5.18}$$

We have

$$\lim_{m\to\infty}\sigma_{k_m-2,\chi_{-4}}(N') = \lim_{m\to\infty}\sum_{0<d\mid N'}\chi_{-4}(d)d^{(p-1)p^{m-1}} = \sum_{\substack{0<d\mid N'\\ (d,p)=1}}\chi_{-4}(d). \tag{5.19}$$

To calculate $\lim_{m\to\infty}\widetilde{\sigma}_{k_m-2,\chi_{-4}}(N')$, we write N' as $N' = p^e\cdot N''$, $(p, N'') = 1$, namely, $e = \mathrm{ord}_p(N')$. Then we have

$$\begin{aligned}\lim_{m\to\infty}\widetilde{\sigma}_{k_m-2,\chi_{-4}}(N') &= \lim_{m\to\infty}\sum_{0<d\mid N'}\chi_{-4}\left(\frac{N'}{d}\right)d^{(p-1)p^{m-1}}\\ &= \sum_{0<d\mid N''}\chi_{-4}(p^e\cdot N'')\\ &= (\chi_{-4}(p))^e\sum_{\substack{0<d\mid N'\\ (d,p)=1}}\sum\chi_{-4}(d).\end{aligned} \tag{5.20}$$

Combining these formulas, we obtain

$$\begin{aligned}\lim_{m\to\infty}\left(\sigma_{k_m-2,\chi_{-4}}(N')-\tilde{\sigma}_{k_m-2,\chi_{-4}}(N')\right)&=\left(1-\left(\chi_{-4}(p)\right)^{\mathrm{ord}_p(N')}\right)\sum_{\substack{0<d\mid N'\\(d,p)=1}}\chi_{-4}(d)\\&=\left(1-(-1)^{\mathrm{ord}_p(N')}\right)\sigma^{*}_{0,\chi_{-4}}(N').\end{aligned}\tag{5.21}$$

If $N'=N-s^2=0$, then

$$G(k_m-2,0)=-\frac{B_{k_m-1,\chi_{-4}}}{2(k_m-1)}.\tag{5.22}$$

Thus, we get

$$\lim_{m\to\infty}G(k_m-2,0)=-(1-\chi_{-4}(p))\frac{B_{1,\chi_{-4}}}{2}=\frac{1}{2}.\tag{5.23}$$

Consequently,

$$\lim_{m\to\infty}\sum_{\substack{s\in\mathbb{Z}\\s^2\le N}}G\left(k_m-2,N-s^2\right)=\sum_{\substack{s\in\mathbb{Z}\\s^2\le N}}G^{*}_p\left(N-s^2\right).\tag{5.24}$$

The identity (5.14) immediately follows due to these formulas. □

The proof of Theorem 5.2 is completed by combining Lemmas 5.3 and 5.4.

An advantage of the formula (5.6) is that we can prove the nonvanishing property for the cusp form f^{*}_p for $p>7$.

Corollary 5.5. *Assume that $p\equiv 3\pmod 4$. If $p>7$, then f^{*}_p does not vanish identically.*

Proof. We calculate the Fourier coefficient $a(T;f^{*}_p)$ at $T=\left(\begin{smallmatrix}1&0\\0&1\end{smallmatrix}\right)$. From the theorem, we have

$$\begin{aligned}a\left(\begin{pmatrix}1&0\\0&1\end{pmatrix};f^{*}_p\right)&=\alpha^{*}_p(4)\\&=H^{*}_p(4)-\frac{p-1}{6}\left(G^{*}_p(4)+2G^{*}_p(3)+2G^{*}_p(0)\right).\end{aligned}\tag{5.25}$$

The assumption $p\equiv 3\pmod 4$ implies that

$$H^{*}_p(4)=-(1-\chi_{-4}(p))B_{1,\chi_{-4}}=1.\tag{5.26}$$

On the other hand, $G^{*}_p(3)=G^{*}_p(4)=0$ (because $p\nmid 3,4$) and $G^{*}_p(0)=1/2$. Hence,

$$\frac{p-1}{6}\left(G^{*}_p(4)+2G^{*}_p(3)+2G^{*}_p(0)\right)=\frac{p-1}{6}.\tag{5.27}$$

Consequently, we obtain

$$a\left(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}; f_p^*\right) = \alpha_p^*(4) = 1 - \frac{p-1}{6} = \frac{7-p}{6} < 0 \tag{5.28}$$

if $p > 7$. □

Remark 5.6. We have $f_3^* = f_7^* = 0$. These identities are consistent with the fact that $\dim S_2(\Gamma_0^2(3)) = \dim S_2(\Gamma_0^2(7)) = 0$ (see [7]).

6. Numerical Examples

In this section, we present numerical examples concerning our Siegel cusp forms. To begin with, we recall the theta series associated with quadratic forms.

Let $S = S^{(2m)}$ be a half-integral, positive-definite symmetric matrix of rank $2m$.

We associate the theta series

$$\vartheta(S,Z) = \sum_{X\in M_{2m,2}(\mathbb{Z})} \exp\left[2\pi i \operatorname{tr}\left({}^t XSXZ\right)\right], \quad Z \in \mathbb{H}_2. \tag{6.1}$$

If we take a symmetric $S = S^{(2m)} > 0$ with level p, then

$$\vartheta(S,Z) \in M_m\left(\Gamma_0^2(p)\right). \tag{6.2}$$

In some cases, we can construct cusp forms by taking a linear combination of theta series.

The Case $p = 11$. Set

$$Q_1^{(11)} = \begin{pmatrix} 1 & 0 & \frac{1}{2} & 0\\ 0 & 1 & 0 & \frac{1}{2}\\ \frac{1}{2} & 0 & 3 & 0\\ 0 & \frac{1}{2} & 0 & 3 \end{pmatrix}, \quad Q_2^{(11)} = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 1 & 0 & \frac{1}{2}\\ \frac{1}{2} & 0 & 4 & 2\\ \frac{1}{2} & \frac{1}{2} & 2 & 4 \end{pmatrix}, \quad Q_3^{(11)} = \begin{pmatrix} 2 & 1 & \frac{1}{2} & \frac{1}{2}\\ 1 & 2 & 0 & \frac{1}{2}\\ \frac{1}{2} & 0 & 2 & 1\\ \frac{1}{2} & \frac{1}{2} & 1 & 2 \end{pmatrix}, \tag{6.3}$$

and $\vartheta_i = \vartheta(Q_i^{(11)}, Z)$. It is known that $\dim S_2(\Gamma_0^2(11)) = 1$ (cf. [7]). We can take a nonzero element of $S_2(\Gamma_0^2(11))$ as

$$C_2(11) = 3\vartheta_1 - 2\vartheta_2 - \vartheta_3 \tag{6.4}$$

(Yoshida's cusp form cf. [8]).

Table 1 gives a first few examples for the Fourier coefficient of f_{11}^* and $C_2(11)$.

Table 1

T	$\begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & 1/2 \\ 1/2 & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 3 & 1/2 \\ 1/2 & 1 \end{pmatrix}$
$a(T; f_{11}^*)$	$2/3$	$-2/3$	0	0	$-2/3$
$a(T; C_2(11))$	-24	24	0	0	24
T	$\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & 1/2 \\ 1/2 & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$	$\begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix}$
$a(T; f_{11}^*)$	$-2/3$	$2/3$	$2/3$	0	$4/3$
$a(T; C_2(11))$	24	-24	-24	0	-48

Table 2

N	3	4	7	8	11	12	15	16	19	20	23	24	27	28	31
$\alpha_{11}^*(N)$	$2/3$	$-2/3$	0	0	$-2/3$	$-2/3$	$2/3$	$4/3$	0	$2/3$	$-2/3$	0	$-2/3$	0	$-2/3$
N	32	35	36	39	40	43	44	47	48	51	52	55	56	59	60
$\alpha_{11}^*(N)$	0	0	0	0	0	0	$2/3$	0	0	0	0	$2/3$	$4/3$	$-2/3$	-2
N	63	64	67	68	71	72	75	76	79	80	83	84	87	88	91
$\alpha_{11}^*(N)$	0	$-4/3$	2	0	$2/3$	0	$4/3$	0	0	$-4/3$	0	0	0	0	$-8/3$
N	92	95	96	99	100										
$\alpha_{11}^*(N)$	2	0	0	$4/3$	0										

The relation between f_{11}^* and $C_2(11)$ is

$$f_{11}^* = -\frac{1}{36} C_2(11). \tag{6.5}$$

Further examples of the Fourier coefficients of f_{11}^* can be obtained from Table 2.

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