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# Research Article

# **Design of Adaptive Switching Controller for Robotic Manipulators with Disturbance**

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Two adaptive switching control strategies are proposed for the trajectory tracking problem of robotic manipulator in this paper. The first scheme is designed for the supremum of the bounded disturbance for robot manipulator being known; while the supremum is not known, the second scheme is proposed. Each proposed scheme consists of an adaptive switching law and a PD controller. Based on the Lyapunov stability theorem, it is shown that two new schemes can guarantee tracking performance of the robotic manipulator and be adapted to the alternating unknown loads. Simulations for two-link robotic manipulator are carried out and show that the two schemes can avoid the overlarge input torque, and the feasibility and validity of the proposed control schemes are proved.

#### 1. Introduction

With the increasing number of robotic manipulators used in industry, it has become an important research field for the control of manipulators with unknown or changing dynamics. The excellent performance of tracking can be considered as one of the most important requirements of robotic manipulator because of its highly nonlinear, multivariable, strongly coupling, and time-varying systems. Many schemes were presented in the past years [1-4]. Generally speaking, in the process of operating robotic manipulators, there are many uncertainties and disturbances, such as the nonlinear friction and variational payloads. Those uncertainties lead to the instability of the robot system and deteriorate the system performance further. As a matter of fact, it is difficult to find a precise dynamic model for a robot manipulator, so it is not realistic to control the robotic manipulators with these modelbased algorithms [1, 5] relying heavily on the accuracy of the model in the modern automatic industry production line.

For compensation of those uncertainties stemming from inaccurate estimation of inertial parameters of the load mass changes and so on, many methods are proposed to overcome these uncertainties, such as adaptive control [6], iterative

learning control [7], neural network control [8], fuzzy logic control [9, 10], and sliding mode control [11].

The adaptive control is very popular to cope with the parameter uncertainty of robotic system [12-15]. A lot of schemes integrating advantages of the adaptive control and other methods have been proposed. Chen and Papavassilopoulos [16] developed a law which is a combination of the variable structure control (VSC) law and switching- $\sigma$ adaptive law to enhance the control of robotic dynamics with unknown parameters and bounded disturbances. However, subject to the nature of VSC, discontinuities caused by chatter in the control signal still exist. In order to reduce amplitudes of these undesired oscillations, second-order sliding mode control algorithm has been developed for particular control systems [17]. Hamerlain et al. [18] proposed a robust control law for decreasing the chatter, based on a generalised sliding mode control that switches on the derivative of control instead of the control input itself.

Iterative learning control requires that the reference trajectory is periodic with known period [19, 20]. The key idea is to use the information obtained in the preceding trial to improve the performance in the current one. Liuzzo and Tomei [21] designed an adaptive learning PD controller

which learned the input reference signals by identifying their Fourier coefficients and developing in Fourier series expansion the input reference signals of every joint, but no discussion in the case of output feedback. Tayebi and Islam [22] proposed some adaptive iterative learning control (ILC) schemes based upon the use of a proportionalderivative (PD) feedback structure, for which an iterative term was added to cope with the unknown parameters and disturbances. The presented controllers only require that the PD and learning gains are the positive definiteness condition and the bounds of the robot parameters are not needed. Ouyang et al. [23] proposed an adaptive switching learning PD controller (ASL-PD) with a combination of the feedback PD control law with a gain switching technique and the feedforward learning control law with the input torque profile. The scheme is integrated into the iterative learning procedure and the trajectory tracking converges very fast. Though the performance of all the iterative learning control for robotic manipulator is very well, this is still limited to the same initial conditions for each iteration.

Neural networks (NNs) and fuzzy systems are two typical tools to parameterize the nonlinear systems with unknown nonlinearities [24–29]. It has been proved that the fuzzy logic systems (FLSs) and neural networks (NNs) can approximate arbitrary nonlinear continuous functions to a given accuracy on a closed set [30, 31]. Mulero-Martinez [32] proposed a new Gaussian radial basis function (GRBF) static neurocontroller, which is a two-stage controller acting in a supervisory fashion by means of a switching logic and allowing arbitration between a neural network (NN) and a robust proportionalderivative controller. This structure is intended to reduce the effects of the curse of dimensionality in multidimensional systems by fully exploiting the mechanical properties of the robot manipulator. Yu et al. [33] presented a robust adaptive neural switching controller which can relax the hypothesis that the bounds of external disturbance and approximation errors of neural networks are known. RBF neural networks have been adopted to approximate unknown functions of robotic manipulators; meanwhile an  $H_{\infty}$  controller was designed to enhance system robustness and stabilization due to the existence of the compound disturbance which consisted of approximation errors of the neural networks and external disturbance. Lam and Leung [34] proposed a fuzzy controller involving a fuzzy combination of local fuzzy and global switching state-feedback controllers. By using fuzzy logic techniques, the undesirable chattering effect introduced by the global switching state-feedback controller can be eliminated. Liu et al. [35] proposed an adaptive fuzzy optimal controller developed for a class of nonlinear discrete-time systems in strict-feedback form. The fuzzy approximation can be used to handle unknown functions.

In fact, from an implementation point of view, neural networks (NNs) and fuzzy systems are much more complicated. It is well known that the linear PD control is one of the most simple and effective control methods, which has been widely used in the field of industrial robots [36–39]. However, it is shown from the field of application that a very large initial output requirement for the driving mechanism is a drawback to further the application of the linear PD control. Actually,

driving mechanism generally cannot provide the larger initial torque for the linear PD control. Moreover, the maximum torque from robot manipulator is limited, which is restricted to further improving the performance of systems by adjusting the coefficient of PD control. As a result, many schemes for nonlinear PD control were brought up [40, 41]; but for most of them, there were only parameters for PD, which meant that the coefficient of proportional and differential was still larger, and the output of torque was still overlarge.

Practically, robots often must pick up or lay down some objects and the load for manipulator is not constant. Therefore, parameter jumping exists in this system. So it is difficult for the traditional adaptive control to solve the above problem. It is well known that a system with a jumping parameter can be viewed as a switched system whose subsystems differ from each other only by parameters [42–44]. Robotic manipulator can be modeled as switching systems which are used to model many physical or manmade systems displaying switching features. There are a few of works combining the adaptive control with the switched system in order to deal with the above problem [45–48].

The purpose of this paper is to provide an efficient solution. In this paper, two adaptive switching controllers with PD parameters for a serial *n*-joint robotic manipulator are discussed. The first is designed for the supremum of bounded disturbance for robot manipulator being known; the other is contrary; the supremum of bounded disturbance for robot manipulator is not known. The main contributions of this paper are threefold:

- (1) According to whether the supremum of bounded disturbance for robot manipulator is known, two difficult disturbance compensation algorithms are designed. Those strategies are all composed of a nonlinear PD and compensated controller. The portion of nonlinear PD can avoid the overlarge output of initial torque, and the adaptive controller including a regression matrix can compensate the dynamic uncertainty of robot manipulator.
- (2) Advantages of methods are as follows: when the initial error is bigger, the nonlinear PD feedback plays a main role, which can avoid the overlarge output of initial torque; when the error is smaller, the adaptive controller plays a main role, which can obtain the good dynamic performance.
- (3) Based on switched common Lyapunov function method, the adaptive updated laws and the switching signals have been developed to guarantee that the resulting closed-loop system is asymptotically Lyapunov stable and the position of manipulator's joint can follow any given bounded desired output signal. Finally, a simulation example of robotic manipulator is given to illustrate the proposed methods.

The organization of this paper is as follows. In Section 2, the switched robot model and some properties are given. Two different adaptive switching controllers with PD parameters are designed separately in Section 3. In Section 4, results of simulation are shown. In the end, conclusions are given.

## 2. Problem Statement and Preliminaries

Considering an *n*-link robotic manipulator (Lewis et al. [49])

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + \omega, \tag{1}$$

where  $q \in R^n$ ,  $\dot{q} \in R^n$ , and  $\ddot{q} \in R^n$  are the vector of joint angles, velocity, and acceleration, respectively.  $\tau \in R^n$  is the torque input vector,  $\omega \in R^n$  is the disturbance input and errors,  $D(q) \in R^{n \times n}$  is the symmetric positive definite inertial matrix,  $C(q, \dot{q}) \in R^n$  is vector of centripetal and Coriolis torques, and  $G(q) \in R^n$  stands for the vector gravitational forces.

Some properties and assumptions of robot (1) are listed; they will be useful in stability analysis (Lewis et al. [49]; Barambones and Etxebarria [50]; Ge et al. [51]).

*Property 1.* D(q),  $C(q,\dot{q})$ , and G(q) of dynamic model (1) can be linearly parameterized as

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = W(q,\dot{q},\ddot{q})\theta, \qquad (2)$$

where  $W(q, \dot{q}, \ddot{q}) \in \mathbb{R}^{n \times m}$  is the nonlinear regression matrix on the vector of joints and  $\theta$  is the unknown constant vector on the load of robotic manipulator.

Assumption 1.  $q_d(t), \dot{q}_d(t), \ddot{q}_d(t) \in \mathbb{R}^n$  are the desired vector of joint position, joint speed, and joint acceleration, which are bounded.

Assumption 2. The disturbance input and errors  $\omega$  satisfy

$$\|\omega\| \le d_1 + d_2 \|e\| + d_3 \|\dot{e}\|,$$
 (3)

where  $d_1$ ,  $d_2$ , and  $d_3$  are all positive constants and  $e = q - q_d$  is the tracking error.

**Lemma 3** (Barbalat's lemma [52]). If  $f, \dot{f} \in L_{\infty}$  and  $f \in L_p$  for some  $p \in [1, \infty)$ , then  $f(t) \to 0$  as  $t \to \infty$ .

Considering the payload variation, the following switched model of robotic manipulator for subsystems has been used:

$$D_{\sigma}(q)\ddot{q} + C_{\sigma}(q,\dot{q})\dot{q} + G_{\sigma}(q) = W(q,\dot{q},\ddot{q})\theta_{\sigma}$$

$$= \tau + \omega,$$
(4)

where  $\sigma(t):[0,+\infty)\to \Lambda=\{1,2,\ldots,N\}$  is the switching signal dominated by the load.  $W(q,\dot{q},\ddot{q})$  is the nonlinear regressor matrix. Our object is to design an adaptive switching controller to obtain the stability of robotic system, and the tracking error e converges to zero asymptotically whether the supremum of bounded disturbance for robot manipulator is known or not.

Remark 4. For model (4), the conventional adaptive control is not effective, because the uncertain parameter must be constant. But when (4) is considered as a switch system, designing an adaptive controller for each subsystem will be an efficient scheme for the different loads.

# 3. Adaptive Switching Controller Design

The following adaptive switching controllers will be classified as two sections: the first is designed under the condition of knowing the supremum of bounded disturbance for robot manipulator, and the other is done without knowing it.

3.1. Case I: The Supremum of Bounded Disturbance for Robot Manipulator Is Known. This section introduces the adaptive switching controller applied to robotic manipulator. Our purpose is to design a robustly stable controller to ensure the system stability and improve the robot tracking performance in the case of the variational payload for robotic manipulator. For the dynamic model of robot (4), the proposed controller is as follows:

$$\tau = -K_{\nu}\dot{e} - K_{\rho}e + W(q, \dot{q}, \ddot{q})\hat{\theta}_{\sigma} + u, \tag{5}$$

$$u = [u_1, u_2, \dots, u_n]^T,$$

$$u_i = -(d_1 + d_2 ||e|| + d_3 ||\dot{e}||) \operatorname{sgn}(e_i),$$
(6)

where  $K_p$  and  $K_v$  are the proportional and derivative gain matrices, respectively, which are positive definite matrices.  $\hat{\theta}_i$  is the estimation of  $\theta_i$ . Only when the ith subsystem is active will  $\hat{\theta}_i$  work on it. The presented adaptive law is

$$\dot{\tilde{\theta}}_{i}^{T} = -e^{T}W(q, \dot{q}, \ddot{q})\Gamma_{i}^{-1}, \quad i = \sigma, 
\dot{\tilde{\theta}}_{i}^{T} = 0, \quad i \neq \sigma,$$
(7)

where  $\tilde{\theta}_{\sigma} = \hat{\theta}_{\sigma} - \theta_{\sigma}$ .

*Remark 5.* The adaptive laws (7) make the estimation parameters change when the corresponding subsystem is active, which can avoid the coupling of different estimation vectors.

**Theorem 6.** For robotic system (4) and the adaptive switching controller (5), (7) can guarantee the global convergence of the tracking error. That is,  $\lim_{t\to\infty} e(t) = 0$ .

*Proof.* Combining (4) with (5), (9) can be obtained:

$$-K_{\nu}\dot{e} - K_{\rho}e + W(q, \dot{q}, \ddot{q})\hat{\theta}_{\sigma} + u + \omega = W(q, \dot{q}, \ddot{q})\theta_{\sigma}, \quad (8)$$

$$K_{\nu}\dot{e} = -K_{p}e + W\left(q, \dot{q}, \ddot{q}\right)\widetilde{\theta}_{\sigma} + u + \omega. \tag{9}$$

Choose a Lyapunov function candidate:

$$V(e,H) = \frac{1}{2} \left[ e^T K_{\nu} e + \operatorname{tr} \left( H^T \Gamma H \right) \right], \tag{10}$$

where  $H^T = [\tilde{\theta}_1, \dots, \tilde{\theta}_N]$  and  $\Gamma = \text{diag}(\Gamma_1, \dots, \Gamma_N)$  with  $\Gamma_i > 0$ ,  $i = 1, 2, \dots, N$ .

Taking the time of derivative of V(e, H), using (7) and (9), we have

$$\dot{V}(e,H) = e^{T} K_{\nu} \dot{e} + \dot{\tilde{\theta}}_{\sigma}^{T} \Gamma_{\sigma} \tilde{\theta}_{\sigma}$$

$$= e^{T} \left( -K_{p} e + W \left( q, \dot{q}, \ddot{q} \right) \tilde{\theta}_{\sigma} + u + \omega \right)$$

$$+ \dot{\tilde{\theta}}_{\sigma}^{T} \Gamma_{\sigma} \tilde{\theta}_{\sigma}$$

$$= -e^{T} K_{p} e + e^{T} u + e^{T} \omega$$

$$+ \left( e^{T} W \left( q, \dot{q}, \ddot{q} \right) + \dot{\tilde{\theta}}_{\sigma}^{T} \Gamma \right) \tilde{\theta}_{\sigma}$$

$$= -e^{T} K_{p} e + e^{T} u + e^{T} \omega.$$

$$(11)$$

$$= -e^{T} K_{p} e + e^{T} u + e^{T} \omega.$$

Because of Assumption 2, we have

$$e^{T}u = \sum_{i=1}^{n} e_{i} \left[ -\left(d_{1} + d_{2} \|e\| + d_{3} \|\dot{e}\|\right) \operatorname{sgn}\left(e_{i}\right) \right]$$

$$= \sum_{i=1}^{n} \left( -d_{1} - d_{2} \|e\| - d_{3} \|\dot{e}\|\right) |e_{i}| \qquad (12)$$

$$\leq \sum_{i=1}^{n} \left( -\|\omega\| \cdot |e_{i}|\right) = -\|\omega\| \cdot \|e\|.$$

Because  $e^T \omega \le ||e^T|| \cdot ||\omega||, e^T u + e^T \omega \le 0$ 

So: 
$$\dot{V}(e, H) \le -e^T K_p e < 0. \quad \forall e \ne 0.$$
 (13)

It means that  $\dot{V}(e, H)$  is a nonincreasing function over time t. Hence,  $\forall t \geq 0$ ,  $V(e(t), H(t)) \leq V(e(0), H(0))$ , which implies that V(e, H) is bound for the signals e and H. Integrating both sides of (13) over  $[0, +\infty)$  leads to

$$\int_{0}^{\infty} e^{T} e \, dt \le \frac{1}{\lambda_{\min}} \int_{0}^{\infty} e^{T} K_{p} e \, dt$$

$$= \frac{1}{\lambda_{\min}} \left( V(0) - V(\infty) \right) \le \frac{1}{\lambda_{\min}} V(0),$$
(14)

where  $\lambda_{\min}$  is the minimum eigenvalue of  $K_p$ . Equation (14) implies  $e \in L_2$ . It can be seen that q,  $\dot{q}$ , and  $\ddot{q}$  are all bounded from Assumption 1 and e is bounded too. According to Property 1 and Assumptions 1-2, the boundedness of  $W(q, \dot{q}, \ddot{q})$  can be ensured. Therefore, from (9),  $\dot{e} \in L_{\infty}$ . Using Lemma 3 (Barbalat's lemma), we have  $\lim_{t\to\infty} e(t) = 0$ . So far, the proof has been completed.

3.2. Case 2: The Supremum of Bounded Disturbance for Robot Manipulator Is Not Known. In this section, a sufficient condition is proposed to ensure the system is stable without knowing the supremum of bounded disturbance.

**Theorem 7.** For robotic system (4), the following control laws (15), (16), (17), and (18) can guarantee system (4) to obtain global asymptotic stability:

$$\tau = -K_{\nu}\dot{e} - K_{p}e + W\left(q,\dot{q},\ddot{q}\right)\widehat{\theta}_{\sigma} + u, \tag{15}$$

$$u = -\frac{\left(\hat{d}f\right)^2}{df \|e\| + \varepsilon^2}e,\tag{16}$$

$$\dot{\overline{\theta}}_{i}^{T} = -e^{T}W(q, \dot{q}, \ddot{q})\Gamma_{i}^{-1}, \quad i = \sigma, 
\dot{\overline{\theta}}_{i}^{T} = 0, \quad i \neq \sigma, \quad (\widetilde{\theta}_{\sigma} = \widehat{\theta}_{\sigma} - \theta_{\sigma}),$$
(17)

$$\dot{\hat{d}} = \gamma_1 f \|e\|, \quad \hat{d}(0) = 0,$$

$$\dot{\varepsilon} = -\gamma_2 \varepsilon,$$
(18)

where  $K_p$  and  $K_v$  are the proportional and derivative gain matrices, respectively, which are positive definite matrices;  $d = d_1 + d_2 + d_3$ ,  $\tilde{d} = d - \hat{d}$ ,  $f = \max(1, \|e\|, \|\dot{e}\|)$ , and  $\hat{d}$  is the estimated value of d;  $\gamma_1$  and  $\gamma_2$  are all the positive constant value

Proof. Choosing a Lyapunov function candidate

$$V(e, H) = \frac{1}{2} \left[ e^{T} K_{\nu} e + \operatorname{tr} \left( H^{T} \Gamma H \right) \right]$$

$$+ \frac{1}{2} \left( \gamma_{1}^{-1} \widetilde{d}^{2} + \gamma_{2}^{-1} \varepsilon^{2} \right),$$
(19)

where  $H^T = [\tilde{\theta}_1, \dots, \tilde{\theta}_N]$  and  $\Gamma = \text{diag}(\Gamma_1, \dots, \Gamma_N)$  with  $\Gamma_i > 0$ ,  $i = 1, 2, \dots, N$ .

Similar to Theorem 6, combining (4) with (5)

$$-K_{\nu}\dot{e} - K_{p}e + W(q, \dot{q}, \ddot{q})\hat{\theta}_{\sigma} + u + \omega = W(q, \dot{q}, \ddot{q})\theta_{\sigma}, \quad (20)$$

$$K_{\nu}\dot{e} = -K_{p}e + W(q, \dot{q}, \ddot{q})\widetilde{\theta}_{\sigma} + u + \omega. \tag{21}$$

Taking the time of derivative of V(e, H), using (17) and (21), we have

$$\dot{V}(e,H) = e^{T}K_{\nu}\dot{e} + \dot{\tilde{\theta}}_{\sigma}^{T}\Gamma_{\sigma}\tilde{\theta}_{\sigma} + \gamma_{1}^{-1}\tilde{d}\dot{\tilde{d}} + \gamma_{2}^{-1}\varepsilon\dot{\varepsilon}$$

$$= e^{T}\left(-K_{p}e + W\left(q,\dot{q},\ddot{q}\right)\tilde{\theta}_{\sigma} + u + \omega\right)$$

$$+ \dot{\tilde{\theta}}_{\sigma}^{T}\Gamma_{\sigma}\tilde{\theta}_{\sigma} + \gamma_{1}^{-1}\tilde{d}\dot{\tilde{d}} + \gamma_{2}^{-1}\varepsilon\dot{\varepsilon}$$

$$= -e^{T}K_{p}e + e^{T}u + e^{T}\omega$$

$$+ \left(e^{T}W\left(q,\dot{q},\ddot{q}\right) + \dot{\tilde{\theta}}_{\sigma}^{T}\Gamma\right)\tilde{\theta}_{\sigma} + \gamma_{1}^{-1}\tilde{d}\dot{\tilde{d}}$$

$$+ \gamma_{2}^{-1}\varepsilon\dot{\varepsilon}$$

$$= -e^{T}K_{p}e + e^{T}u + e^{T}\omega + \gamma_{1}^{-1}\tilde{d}\dot{\tilde{d}} + \gamma_{2}^{-1}\varepsilon\dot{\varepsilon}.$$
(22)

According to (16), we have

$$\dot{V}(e,H) = -e^{T}K_{p}e - e^{T}\frac{\left(\hat{d}f\right)^{2}}{\hat{d}f \|e\| + \varepsilon^{2}}e + e^{T}\omega + \gamma_{1}^{-1}\tilde{d}\dot{\tilde{d}}$$

$$+ \gamma_{2}^{-1}\varepsilon\dot{\varepsilon},$$
(23)

because

$$e^{T}\omega \leq \|e\| \cdot \|\omega\|,$$

$$\|\omega\| \leq d_{1} + d_{2} \|e\| + d_{3} \|\dot{e}\| \leq df,$$

$$e^{T}e = \|e\|^{2},$$

$$\dot{\tilde{d}} = -\dot{\hat{d}} = -\gamma_{1} f \|e\|.$$

$$(24)$$

There is

$$\begin{split} \dot{V}\left(e,H\right) &\leq -e^{T}K_{p}e - \frac{\left(\widehat{d}f\right)^{2}}{\widehat{d}f\left\|e\right\| + \varepsilon^{2}}\left\|e\right\|^{2} + df\left\|e\right\| \\ &+ \gamma_{1}^{-1}\widetilde{d}\dot{\overline{d}} + \gamma_{2}^{-1}\varepsilon\dot{\varepsilon} \\ &= -e^{T}K_{p}e - \frac{\left(\widehat{d}f\right)^{2}}{\widehat{d}f\left\|e\right\| + \varepsilon^{2}}\left\|e\right\|^{2} + df\left\|e\right\| \\ &- \widetilde{d}f\left\|e\right\| - \varepsilon^{2} \\ &= -e^{T}K_{p}e - \frac{\left(\widehat{d}f\right)^{2}}{\widehat{d}f\left\|e\right\| + \varepsilon^{2}}\left\|e\right\|^{2} + \widehat{d}f\left\|e\right\| - \varepsilon^{2} \\ &= -e^{T}K_{p}e + \frac{-\left(\widehat{d}f\right)^{2}\left\|e\right\|^{2} + \left(\widehat{d}f\right)^{2}\left\|e\right\|^{2} - \varepsilon^{4}}{\widehat{d}f\left\|e\right\| + \varepsilon^{2}} \\ &= -e^{T}K_{p}e - \frac{\varepsilon^{4}}{\widehat{d}f\left\|e\right\| + \varepsilon^{2}}. \end{split}$$

Because of  $\hat{d} > 0$  (it can be seen from the definition of  $\hat{d}$ ),

So: 
$$\dot{V}(e, H) \le -e^T K_p e < 0 \quad \forall e \ne 0.$$
 (26)

Similarly, from the proof of Theorem 6, we have  $\lim_{t\to\infty} e(t) = 0$ . The proof has been completed.

*Remark 8.* From the process of controller design, it can be seen that it is easier to obtain the parameters of proportional and differential. And controller can be working on the state of arbitrary switching.

#### 4. Simulation

In this section, the above proposed adaptive switching strategies are employed to control the robotic manipulator to illustrate the feasibility and effectiveness. Simulations are carried out for a two-DOF planar manipulator whose load is persistently changing. The model of robotic dynamics is

$$\begin{bmatrix}
D_{11}(q_{2}) & D_{21}(q_{2}) \\
D_{12}(q_{2}) & D_{22}(q_{2})
\end{bmatrix} \begin{bmatrix} \ddot{q}_{1} \\ \ddot{q}_{2} \end{bmatrix} \\
+ \begin{bmatrix} -C_{12}(q_{2}) \dot{q}_{2} & -C_{12}q_{2}(\dot{q}_{1} + \dot{q}_{2}) \\
C_{12}(q_{2}) \dot{q}_{1} & 0
\end{bmatrix} \begin{bmatrix} \dot{q}_{1} \\ \dot{q}_{2} \end{bmatrix}$$

$$+ \begin{bmatrix} G_{1}(q_{1}, q_{2}) g \\ G_{2}(q_{1}, q_{2}) g \end{bmatrix} + \begin{bmatrix} \omega_{1} \\ \omega_{2} \end{bmatrix} = \begin{bmatrix} \tau_{1} \\ \tau_{2} \end{bmatrix},$$
(27)

where

$$D_{11}(q_{2}) = (m_{1} + m_{2}) r_{1}^{2} + m_{2}r_{2}^{2} + 2m_{2}r_{1}r_{2}\cos q_{2},$$

$$D_{12}(q_{2}) = m_{2}r_{2}^{2} + m_{2}r_{1}r_{2}\cos q_{2},$$

$$D_{22}(q_{2}) = m_{2}r_{2}^{2},$$

$$C_{12}(q_{2}) = m_{2}r_{1}r_{2}\sin q_{2},$$

$$G_{1}(q_{1}, q_{2}) = (m_{1} + m_{2}) r_{1}\cos q_{2}$$

$$+ m_{2}r_{2}\cos (q_{1} + q_{2}),$$

$$G_{2}(q_{1}, q_{2}) = m_{2}r_{2}\cos (q_{1} + q_{2}).$$

$$(28)$$

The linkage is composed of two rigid beams with actuators mounted at the joints. The load can be considered as a part of the second link. Parameters of dynamic model (1) are as follows.

The length and mass of robot are  $r_1 = 1$  and  $r_2 = 0.8$  and  $m_1 = 0.5$ ,  $m_2 = 0.5$  ( $\sigma = 1$ ), and  $m_2 = 1$  ( $\sigma = 2$ ).

The given reference trajectory and initial state of system are

$$q_{d_1} = \sin(2\pi t),$$

$$q_{d_2} = \sin(2\pi t),$$

$$\begin{bmatrix} q_1 \\ \dot{q}_1 \\ q_2 \\ \dot{a}_2 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \\ 0 \end{bmatrix}.$$
(29)

Control parameters are

$$K_p = \text{diag}(50, 50),$$
  
 $K_d = \text{diag}(180, 180),$ 
  
 $\Gamma = \text{diag}(5, 5).$ 
(30)

*Case 1.* It is switching adaptive control when the supremum of bounded disturbance for robot manipulator is known.

Choose disturbance input and errors as

$$\omega = d_1 + d_2 \|e\| + d_3 \|\dot{e}\|, \quad d_1 = 2, \ d_2 = 3, \ d_3 = 6.$$
 (31)

Simulation results are shown in Figures 1–4. The switching signal is shown in Figure 1. Figure 2 denotes the tracking error performance of two links. The bounded control input for the two links of robotic manipulator is given in Figures 3 and 4. Comparing with the literature [31], we can see that the torque for two links, which is the control input of robotic system, is smaller apparently. From the above, it can be inferred that the proposed control scheme has provided better control performance. As is claimed in Theorem 6, Figure 2 shows the tracking errors converge to zero.

*Case 2.* It is switching adaptive control when the supremum of bounded disturbance for robot manipulator is not known.

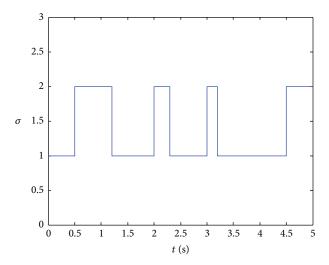


FIGURE 1: The switching signal.

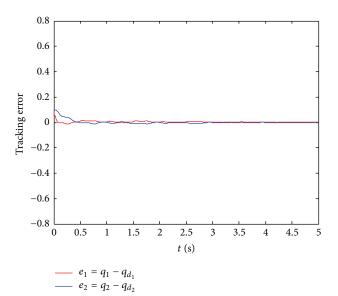


FIGURE 2: Tracking error for Case 1.

Choose  $\gamma_1 = 20$  and  $\gamma_2 = 20$ .

In Case 2, the switch signal is still arbitrary as Figure 1, and simulation results are shown in Figures 5-8. Figure 5 denotes the tracking error performance of two links which converge to zero. It is shown that Theorem 7 in this paper can guarantee that the system output follows the given bounded desired output signal and the tracking error performance is well achieved. In order to show the advantage of the adaptive switching controller, the tracking error using the PID controller is obtained in Figure 6. Comparing Figure 5 with Figure 6, we can see that the proposed controller is superior to the PID controller apparently in terms of convergence speed and tracking accuracy. No overlarge control inputs for the two links of robotic manipulator are given in Figures 7-8, which are suitable for the requirement of the engineering. From the above analysis, it illustrates that the proposed control scheme can ensure the robotic system stability.

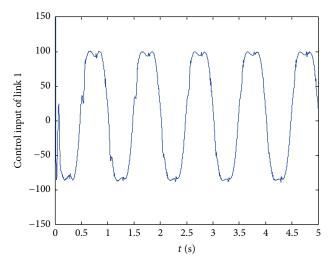


FIGURE 3: Torque for link 1 (Case 1).

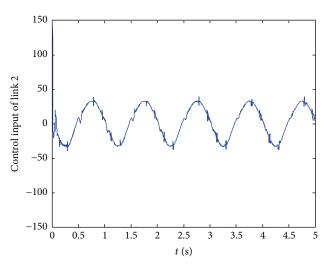


FIGURE 4: Torque for link 2 (Case 1).

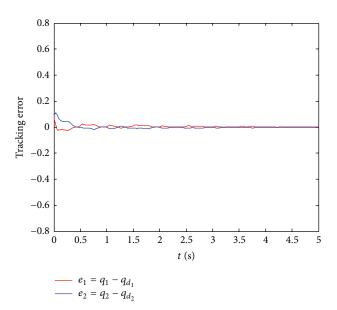


FIGURE 5: Tracking error for Case 2.

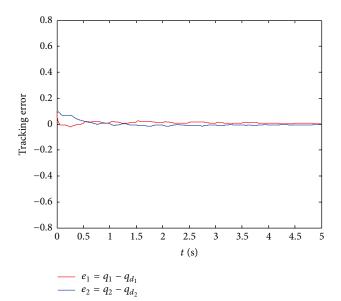


FIGURE 6: Tracking error (PID).

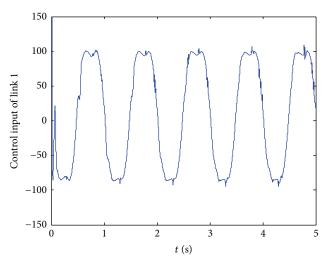


FIGURE 7: Torque for link 1 (Case 2).

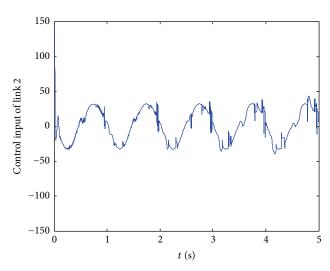


FIGURE 8: Torque for link 2 (Case 2).

#### 5. Conclusion

In this paper, two adaptive switching control schemes have been investigated for robotic manipulator with changing loads. The first scheme is designed for the supremum of bounded disturbance for robot manipulator being known, and the other is contrary. When the corresponding subsystem is activated, the proposed adaptive update law works. Based on Lyapunov stability theorem, it is shown that the proposed control scheme can guarantee the tracking performance of robotic manipulator system. Simulations show that the satisfactory tracking performance can be obtained and the adaptive switching controller is simple to realize for engineering applications. In our further work, we will try to extend the proposed results to the case of force tracking for the end effector of robotic manipulators.

#### **Competing Interests**

The authors declare no conflict of interests.

### **Authors' Contributions**

Shumin Fei managed the overall progress and gave some useful advises on this project. Zhen Yang carried out the theoretical analysis and numerical calculations for the results. Both authors have read and approved the final paper.

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