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Research Article

Dynamic Behaviors of a Discrete Periodic Predator-Prey-Mutualist System

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A nonautonomous discrete predator-prey-mutualist system is proposed and studied in this paper. Sufficient conditions which ensure the permanence and existence of a unique globally stable periodic solution are obtained. We also investigate the extinction property of the predator species; our results indicate that if the cooperative effect between the prey and mutualist species is large enough, then the predator species will be driven to extinction due to the lack of enough food. Two examples together with numerical simulations show the feasibility of the main results.

1. Introduction

As was pointed out by Berryman [1], the dynamic relationship between predator and prey has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. Recently, predator-prey models have been studied widely [2–7]. It brings to our attention that all the works of [2–7] are dealing with the relationship between two species, while, in the real world, the relationship among species is very complicated and it needs to consider the three-species models. Many scholars [8–13] studied the dynamic behaviors of the three-species models.

Moreover, mutualism is one of the most important relationships in the theory of ecology. Mutualism is a symbiotic association between any two species and the interaction between the two species is beneficial to both of the species [14]. Already, many scholars [15–21] studied the dynamic behaviors of cooperative models. It brings to our attention that although predator-prey and mutualism can be recognized as major issues in both applied mathematics and theoretical ecology, few scholars have considered predator-prey system with cooperation in three species. But this phenomenon really exists in nature. For example, while aphids are preyed by natural enemies, they are protected by some natural friends like ants; there ants eat the honeydew that aphids excrete and help to overcome the resource scarcity of offspring [22, 23].

In 2009, Rai and Krawcewicz [24] proposed the following predator-prey-mutualist system:

$$\begin{aligned}
\frac{dx}{dt} &= \alpha x\left(1-\frac{x}{K}\right)-\frac{\beta xz}{1+my},\\
\frac{dy}{dt} &= \gamma y\left(1-\frac{y}{lx+L_0}\right),\\
\frac{dz}{dt} &= z\left(-s+\frac{c\beta x}{1+my}\right),
\end{aligned} \tag{1}$$

where $x(t)$, $y(t)$, and $z(t)$ denote the densities of prey, mutualist, and predator population at any time t, respectively; they applied the equivariant degree method to study Hopf bifurcations phenomenon of the system.

Recently, Yang et al. [25] argued that, due to seasonal effects of weather, temperature, food supply, mating habits, and so forth, a more appropriate system should be a nonautonomous one, and they proposed and studied the following system:

$$\dot{x} = x\left(a_1(t) - b_1(t)x - \frac{c_1(t)z}{d_1(t) + d_2(t)y}\right),$$
$$\dot{y} = y\left(a_2(t) - \frac{y}{d_3(t) + d_4(t)x}\right), \tag{2}$$
$$\dot{z} = z\left(-a_3(t) + \frac{k_1(t)c_1(t)x}{d_1(t) + d_2(t)y} - b_2(t)z\right).$$

By using the Brouwer fixed pointed theorem and constructing a suitable Lyapunov function, the authors obtained a set of sufficient conditions for the existence of a globally asymptotically stable periodic solution in system (2). It is well known that the discrete time models are more appropriate than the continuous ones when the size of the population is rarely small or the population has nonoverlapping generations. It has been found that the dynamic behaviors of the discrete system are rather complex and contain more rich dynamics than the continuous ones. To the best of the authors knowledge, still no scholar proposes and studies the discrete predator-prey-mutualist system; this motivated us to study the following system:

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left\{a_1(n) - b_1(n)x_1(n) - \frac{c_1(n)x_3(n)}{d_1(n) + d_2(n)x_2(n)}\right\}, \\ x_2(n+1) &= x_2(n)\exp\left\{a_2(n) - \frac{x_2(n)}{d_3(n) + d_4(n)x_1(n)}\right\}, \\ x_3(n+1) &= x_3(n)\exp\left\{-a_3(n) + \frac{k_1(n)c_1(n)x_1(n)}{d_1(n) + d_2(n)x_2(n)} - b_2(n)x_3(n)\right\}, \end{aligned} \tag{3}$$

where $x_1(n)$, $x_2(n)$, and $x_3(n)$ are the population sizes of the prey, mutualist, and predator at nth generation, respectively, $a_1(n)$ and $a_2(n)$ are the intrinsic growth rate of prey and mutualist at nth generation, $a_3(n)$ is the death rate of the predator at nth generation, $k_1(n)$ is called the conversion rate at nth generation, which denotes the fraction of the prey biomass being converted to predator biomass, and $c_1(n)$ is the capture rate of the prey at nth generation. The sequences of $d_4(n)$, $d_2(n)$ are the mutualism sequences. We mention here that, in system (3), we consider the density restriction term of predator species ($b_2(n)z$); such a consideration is needed since the density of any species is restricted by the environment [10]. Here, we assume that $a_i(n)$ $(i = 1,2,3)$, $b_j(n), c_j(n)$ $(j = 1,2)$ $k_1(n)$, and $d_r(n)$ $(r = 1,2,3,4)$ are all bounded nonnegative sequences. $a_i(n)$ $(i = 1,2,3), b_j(n)$ $(j = 1,2)$ are strictly positive sequences. Note that

$$\begin{aligned} x_1(n) &= x_1(0)\exp\sum_{k=1}^{n-1}\left[a_1(k) - b_1(k)x_1(k) - \frac{c_1(k)x_3(k)}{d_1(k) + d_2(k)x_2(k)}\right], \\ x_2(n) &= x_2(0)\exp\sum_{k=1}^{n-1}\left[a_2(k) - \frac{x_2(k)}{d_3(k) + d_4(k)x_1(k)}\right], \\ x_3(n) &= x_3(0)\exp\sum_{k=1}^{n-1}\left[-a_3(k) + \frac{k_1(k)c_1(k)x_1(k)}{d_1(k) + d_2(k)x_2(k)} - b_2(k)x_3(k)\right]. \end{aligned} \tag{4}$$

From the point of view of biology, in the sequence, we assume that $x_1(0) > 0$, $x_2(0) > 0$, $x_3(0) > 0$, and then from (4), we know that the solutions of system (3) are positive. We use the following notations for any bounded sequence $x(n)$:

$$x^u = \sup_{n\in N} x(n), \qquad x^l = \inf_{n\in N} x(n). \tag{5}$$

We arrange the rest of the paper as follows. In Section 2, we establish a permanence result for (3). In Section 3, the sufficient conditions about the uniqueness and global attractivity of the periodic solution of (3) are obtained. In Section 4, the sufficient conditions about the extinction of predator species and the stability of prey-mutualist species are obtained. Finally, two suitable examples are given to illustrate that the conditions of the main theorem are feasible. We end this paper by a brief discussion.

2. Permanence

Theorem 1. *Assume the inequalities* $k_1^u c_1^u B_1/d_1^l - a_3^l > 0$, *and every positive solution* $(x_1(n), x_2(n), x_3(n))$ *of system (3) satisfies*

$$\limsup_{n\to\infty} x_1(n) \le B_1, \quad \limsup_{n\to\infty} x_2(n) \le B_2, \quad \limsup_{n\to\infty} x_3(n) \le B_3, \tag{6}$$

where

$$\begin{aligned} B_1 &= \frac{\exp\{a_1^u - 1\}}{b_1^l}, \\ B_2 &= \left(d_3^u + d_4^u B_1\right)\exp\{a_2^u - 1\}, \\ B_3 &= \frac{1}{b_2^l}\exp\left\{-a_3^l + \frac{k_1^u c_1^u B_1}{d_1^l} - 1\right\}. \end{aligned} \tag{7}$$

Proof. Since $x_1(0) > 0$, $x_2(0) > 0$, and $x_3(0) > 0$, then $x_1(n) > 0$, $x_2(n) > 0$, and $x_3(n) > 0$, for $n \geq 0$. We only need to prove that

$$\limsup_{n\to\infty} x_1(n) \leq B_1. \tag{8}$$

Since similar result can be shown for $x_2(n)$ and $x_3(n)$, then (6) follows obviously. We first assume that there exists $l_0 \in N$ such that $x_1(l_0+1) \geq x_1(l_0)$. Then

$$a_1(l_0) - b_1(l_0)x_1(l_0) - \frac{c_1(l_0)x_3(l_0)}{d_1(l_0)+d_2(l_0)x_2(l_0)} \geq 0. \tag{9}$$

Hence,

$$x_1(l_0) \leq \frac{a_1(l_0)}{b_1(l_0)} \leq \frac{a_1^u}{b_1^l}. \tag{10}$$

It follows that

$$\begin{aligned} x_1(l_0+1) &= x_1(l_0)\exp\Bigg\{a_1(l_0) - b_1(l_0)x_1(l_0) \\ &\quad - \frac{c_1(l_0)x_3(l_0)}{d_1(l_0)+d_2(l_0)x_2(l_0)}\Bigg\} \leq x_1(l_0)\exp\{a_1(l_0) \\ &\quad - b_1(l_0)x_1(l_0)\} \leq x_1(l_0)\exp\{a_1^u - b_1^l x_1(l_0)\} \\ &\leq \frac{\exp\{a_1^u - 1\}}{b_1^l} = B_1; \end{aligned} \tag{11}$$

here we used

$$\max_{x\in R} x\exp(a-bx) = \frac{\exp(a-1)}{b}, \quad \text{for } a,b>0. \tag{12}$$

We claim that

$$x_1(n) \leq B_1, \quad n \geq l_0. \tag{13}$$

By way of contradiction, assume that there exists $p_0 > l_0$ such that $x_1(p_0) > B_1$. Then $p_0 \geq l_0+2$. Let $\widetilde{p_0} \geq l_0+2$ be the smallest integer such that $x_1(\widetilde{p_0}) > B_1$. Then $x_1(\widetilde{p_0}) > B_1 \geq x_1(\widetilde{p_0}-1)$, which implies $x_1(\widetilde{p_0}) > x_1(\widetilde{p_0}-1)$. The above argument produces that $x_1(\widetilde{p_0}) \leq B_1$, a contradiction. This proves the claim. Now, we assume that $x(n+1) < x(n)$ for all $n \in N$. In particular, $\lim_{n\to\infty} x(n)$ exists, denoted by $\overline{x}_1$. We claim that $\overline{x}_1 \leq a_1^u/b_1^l$. By way of contradiction, assume that $\overline{x}_1 > a_1^u/b_1^l$. Taking limit in the first equation in system (3) gives

$$\begin{aligned} &\lim_{n\to\infty}\left(a_1(n) - b_1(n)x_1(n) - \frac{c_1(n)x_3(n)}{d_1(n)+d_2(n)x_2(n)}\right) \\ &\quad = 0, \end{aligned} \tag{14}$$

which is a contradiction since

$$\begin{aligned} &a_1(n) - b_1(n)x_1(n) - \frac{c_1(n)x_3(n)}{d_1(n)+d_2(n)x_2(n)} \\ &\quad \leq a_1(n) - b_1(n)x_1(n) \leq a_1^u - b_1^l\overline{x}_1 < 0 \\ &\qquad \text{for } n \in N. \end{aligned} \tag{15}$$

This proves the claim. Note that

$$\begin{gathered} \exp(x-1) > x \quad (x>0). \\ \frac{\exp(a_1^u-1)}{b_1^l} = \frac{a_1^u}{b_1^l}\frac{\exp(a_1^u-1)}{a_1^u} > \frac{a_1^u}{b_1^l}. \end{gathered} \tag{16}$$

It follows that (8) holds. This completes the proof of the main result. □

Theorem 2. *Assume the inequalities*

$$\begin{aligned} &\frac{k_1^l c_1^l D_1}{d_1^u + d_2^u B_2} - a_3^u > 0, \\ &a_1^l - \frac{c_1^u B_3}{d_1^l + d_2^l D_2} > 0, \end{aligned} \tag{H_1}$$

where B_2 and B_3 are the same as in Theorem 1. Then

$$\begin{aligned} &\liminf_{n\to\infty} x_1(n) \geq D_1, \\ &\liminf_{n\to\infty} x_2(n) \geq D_2, \\ &\liminf_{n\to\infty} x_3(n) \geq D_3, \end{aligned} \tag{17}$$

where

$$\begin{aligned} D_1 &= \frac{a_1^l\left(d_1^l + d_2^l D_2\right) - c_1^u B_3}{b_1^u\left(d_1^l + d_2^l D_2\right)} \\ &\quad \cdot \exp\left\{a_1^l - b_1^u B_1 - \frac{c_1^u B_3}{d_1^l + d_2^l D_2}\right\}, \\ D_2 &= a_2^l d_3^l \exp\left\{a_2^l - \frac{B_2}{d_3^l}\right\}, \\ D_3 &= \frac{k_1^l c_1^l D_1 - a_3^u\left(d_1^u + d_2^u B_2\right)}{\left(d_1^u + d_2^u B_2\right) b_2^u} \\ &\quad \cdot \exp\left\{-a_3^u + \frac{k_1^l c_1^l D_1}{d_1^u + d_2^u B_2} - b_2^u B_3\right\}. \end{aligned} \tag{18}$$

Proof. We first show that

$$\liminf_{n\to\infty} x_2(n) \geq D_2. \tag{19}$$

For any $\varepsilon > 0$, there exists $n^* \in N$ such that

$$\begin{aligned} &x_1(n) \leq B_1 + \varepsilon, \\ &x_2(n) \leq B_2 + \varepsilon, \\ &x_3(n) \leq B_3 + \varepsilon, \\ &\qquad \text{for } n \geq n^*. \end{aligned} \tag{20}$$

First, we assume that there exists $l_0 \geq n^*$ such that $x_2(l_0+1) \leq x_2(l_0)$. Note that, for $n \geq l_0$,

$$\begin{aligned} x_2(n+1) &= x_2(n)\exp\left\{a_2(n) - \frac{x_2(n)}{d_3(n)+d_4(n)x_1(n)}\right\} \\ &\geq x_2(n)\exp\left\{a_2(n) - \frac{x_2(n)}{d_3(n)}\right\} \\ &\geq x_2(n)\exp\left\{a_2^l - \frac{x_2(n)}{d_3^l}\right\}. \end{aligned} \tag{21}$$

In particular, with $n = l_0$, we get

$$a_2^l - \frac{x_2(l_0)}{d_3^l} \leq 0, \tag{22}$$

which implies that $x_2(l_0) \geq a_2^l d_3^l$. Then

$$\begin{aligned} x_2(l_0+1) &= x_2(l_0)\exp\left\{a_2(l_0) - \frac{x_2(l_0)}{d_3(l_0)+d_4(l_0)x_1(l_0)}\right\} \\ &\geq x_2(l_0)\exp\left\{a_2^l - \frac{B_2+\varepsilon}{d_3^l}\right\} \\ &\geq a_2^l d_3^l \exp\left\{a_2^l - \frac{B_2+\varepsilon}{d_3^l}\right\}. \end{aligned} \tag{23}$$

Let

$$x_{2\varepsilon} \overset{\text{def}}{=} a_2^l d_3^l \exp\left\{a_2^l - \frac{B_2+\varepsilon}{d_3^l}\right\}. \tag{24}$$

We claim that

$$x_2(n) \geq x_{2\varepsilon} \quad \text{for } n \geq l_0. \tag{25}$$

By way of contradiction, assume that there exists $p_0 > l_0$ such that $x_2(p_0) < x_{2\epsilon}$. Then $p_0 \geq l_0 + 2$. Let $\widetilde{p_0} \geq l_0 + 2$ be the smallest integer such that $x_2(\widetilde{p_0}) < x_{2\epsilon}$. Then $x_2(\widetilde{p_0} - 1) \geq x_{2\epsilon} > x_2(\widetilde{p_0})$, and clearly $x_2(\widetilde{p_0} - 1) > x_2(\widetilde{p_0})$. The above argument produces that $x_2(\widetilde{p_0}) \geq x_{2\epsilon}$ for all large n. Then $\lim_{n\to\infty} x_2(n)$ exists, denoted by $\underline{x_2}$. We claim that $\underline{x_2} \geq a_2^l d_3^l$. By way of contradiction, assume that $\underline{x_2} < a_2^l d_3^l$. Taking limit in the second equation in system (3) gives

$$\lim_{n\to\infty}\left(a_2(n) - \frac{x_2(n)}{d_3(n)+d_4(n)x_1(n)}\right) = 0, \tag{26}$$

which is a contradiction since

$$\begin{aligned} \lim_{n\to\infty}\left(a_2(n) - \frac{x_2(n)}{d_3(n)+d_4(n)x_1(n)}\right) &\geq a_2^l - \frac{\underline{x_2}}{d_3^l} \\ &> 0. \end{aligned} \tag{27}$$

This proves the claim. Note that

$$\begin{aligned} \frac{B_2}{d_3^l} &= \frac{d_3^u \exp\{a_2^u - 1\} + d_4^u \exp\{a_1^u + a_2^u - 2\}/b_1^l}{d_3^l} \\ &\geq \exp\{a_2^u - 1\} + \frac{d_4^u \exp\{a_1^u + a_2^u - 2\}}{b_1^l d_3^l} \\ &\geq a_2^u + \frac{d_4^u \exp\{a_1^u + a_2^u - 2\}}{b_1^l d_3^l}. \end{aligned} \tag{28}$$

Clearly, $a_2^l - B_2/d_3^l < 0$, so $D_2 < a_2^l d_3^l$. We can easily see that (19) holds. The proof of the other two inequalities is similar to the above analysis and we omit the detail here. This completes the proof of the main result. □

As a direct corollary of Theorems 1 and 2, from the definition of permanence, we have the following.

Theorem 3. *Assume that* (H_1) *holds. Then system (3) is permanent.*

It should be noticed that, from the inequality $k_1^l c_1^l D_1/(d_1^u + d_2^u B_2) - a_3^u > 0$ *and from the proofs of Theorems 1 and 2, one knows that where* (H_1) *holds, the set* $[D_1, B_1] \times [D_2, B_2] \times [D_3, B_3]$ *is an invariant set of system (3).*

3. Existence and Stability of a Periodic Solution

Due to seasonal effects of weather, temperature, food supply, mating habits, contact with predators, and other resources or physical environmental quantities, we can assume temporal to be cyclic or periodic [26–28]. In this section, we consider system (3) with $a_i(n)$ $(i = 1,2,3)$, $b_j(n), c_j(n)$ $(j = 1,2)$, $k_1(n)$, and $d_r(n)$ $(r = 1,2,3,4)$ being periodic with a common period. More precisely, we assume that there exists a positive integer w such that, for $n \in N$,

$$\begin{aligned} &0 < a_i(n+w) = a_i(n), \\ &0 < b_j(n+w) = b_j(n), \\ &0 < c_j(n+w) = c_j(n), \\ &0 < d_k(n+w) = d_k(n), \\ &0 < k_1(n+w) = k_1(n). \end{aligned} \tag{29}$$

Let $B_i, D_i, i = 1,2,3$, be the same as in Theorems 1 and 2. Our first result concerns the existence of a periodic solution.

Theorem 4. *Assume that* (H_1) *holds; then system (3) has* w*-periodic solution, denoted by* $(\overline{x_1}(n), \overline{x_2}(n), \overline{x_3}(n))$.

Proof. As noted at the end of the last section that $[D_1, B_1] \times [D_2, B_2] \times [D_3, B_3]$ is an invariant set of system (3), thus we can define a mapping F on $[D_1, B_1] \times [D_2, B_2] \times [D_3, B_3]$ by

$$\begin{aligned} &F(\overline{x_1}(0), \overline{x_2}(0), \overline{x_3}(0)) = (\overline{x_1}(w), \overline{x_2}(w), \overline{x_3}(w)) \\ &\text{for } (\overline{x_1}(0), \overline{x_2}(0), \overline{x_3}(0)) \in [D_1, B_1] \times [D_2, B_2] \times [D_3, B_3]. \end{aligned} \tag{30}$$

Obviously, F depends continuously on $(\overline{x_1}(0), \overline{x_2}(0), \overline{x_3}(0))$. Thus, F is continuous and maps the compact set $[D_1, B_1] \times [D_2, B_2] \times [D_3, B_3]$ into itself. Therefore, F has a fixed point $(\overline{x_1}(n), \overline{x_2}(n), \overline{x_3}(n))$. It is easy to see that the solution $(\overline{x_1}(n), \overline{x_2}(n), \overline{x_3}(n))$ is w-periodic solution of system (3). This completes the proof. □

Now, under some additional conditions, we study the global stability of the periodic solution obtained in Theorem 4.

Theorem 5. *Assume that (29) and (H_1) hold, and*

$$\begin{aligned}
\lambda_1 &= \max\left\{\left|1 - b_1^u B_1\right|, \left|1 - b_1^l D_1\right|\right\} + \frac{c_1^u B_3 d_2^u B_2}{\left(d_1^l + d_2^l D_2\right)^2} \\
&\quad + \frac{c_1^u B_3}{d_1^l + d_2^l D_2} < 1, \\
\lambda_2 &= \max\left\{\left|1 - \frac{B_2}{d_3^l + d_4^l D_1}\right|, \left|1 - \frac{D_2}{d_3^u + d_4^u B_1}\right|\right\} \\
&\quad + \frac{d_4^u B_1 B_2}{\left(d_3^l + d_4^l D_1\right)^2} < 1, \\
\lambda_3 &= \max\left\{\left|1 - b_2^u B_3\right|, \left|1 - b_2^l D_3\right|\right\} + \frac{c_1^u k_1^u B_1}{d_1^l + d_2^l D_2} \\
&\quad + \frac{c_1^u k_1^u B_1 B_2 d_2^u}{\left(d_1^l + d_2^l D_2\right)^2} < 1.
\end{aligned} \tag{31}$$

Then for every solution $(\overline{x_1}(n), \overline{x_2}(n), \overline{x_3}(n))$ of system (3), one has

$$\begin{aligned}
\lim_{n\to\infty} \left(x_1(n) - \overline{x_1}(n)\right) &= 0, \\
\lim_{n\to\infty} \left(x_2(n) - \overline{x_2}(n)\right) &= 0, \\
\lim_{n\to\infty} \left(x_3(n) - \overline{x_3}(n)\right) &= 0,
\end{aligned} \tag{32}$$

where $(\overline{x_1}(n), \overline{x_2}(n), \overline{x_3}(n))$ is w-periodic solution obtained in Theorem 4.

Proof. Let

$$\begin{aligned}
x_1(n) &= \overline{x}_1(n) \exp(u(n)), \\
x_2(n) &= \overline{x}_2(n) \exp(v(n)), \\
x_3(n) &= \overline{x}_{31}(n) \exp(z(n)).
\end{aligned} \tag{33}$$

Then system (3) is equivalent to

$$\begin{aligned}
u(n+1) &= u(n) + b_1(n)\overline{x}_1(n)\left[1 - \exp\{u(n)\}\right] \\
&\quad + c_1(n)\overline{x}_3(n)\frac{\left[1 - \exp\{z(n)\}\right]}{d_1(n) + d_2(n)x_2(n)} \\
&\quad - \frac{c_1(n)\overline{x}_3(n)\overline{x}_2(n)d_2(n)\left[1 - \exp\{v(n)\}\right]}{\left(d_1(n) + d_2(n)\overline{x}_2(n)\right)\left(d_1(n) + d_2(n)x_2(n)\right)}, \\
v(n+1) &= v(n) + \frac{\overline{x}_2(n)\left[1 - \exp\{v(n)\}\right]}{d_3(n) + d_4(n)x_1(n)} \\
&\quad - \frac{\overline{x}_2(n)d_4(n)\overline{x}_1(n)\left[1 - \exp\{u(n)\}\right]}{\left(d_3(n) + d_4(n)\overline{x}_1(n)\right)\left(d_3(n) + d_4(n)x_1(n)\right)}, \\
z(n+1) &= z(n) \\
&\quad - \frac{k_1(n)c_1(n)\overline{x}_1(n)\left[1 - \exp\{u(n)\}\right]}{d_1(n) + d_2(n)x_2(n)} \\
&\quad + b_2(n)\overline{x}_3(n)\left[1 - \exp\{z(n)\}\right] \\
&\quad + \frac{k_1(n)c_1(n)\overline{x}_1(n)\overline{x}_2(n)d_2(n)\left[1 - \exp\{v(n)\}\right]}{\left(d_1(n) + d_2(n)x_2(n)\right)\left(d_1(n) + d_2\overline{x}_2(n)\right)}.
\end{aligned} \tag{34}$$

By using the mean-value theorem, it follows that

$$\begin{aligned}
u(n+1) &= u(n)\left[1 - b_1(n)\overline{x}_1(n)\exp\{\theta_1(n)u(n)\}\right] \\
&\quad - \frac{c_1(n)\overline{x}_3(n)z(n)\exp\{\theta_3(n)z(n)\}}{d_1(n) + d_2(n)x_2(n)} \\
&\quad + \frac{c_1(n)d_2(n)\overline{x}_2(n)\overline{x}_3(n)\exp\{\theta_2(n)v(n)\}v(n)}{\left(d_1(n) + d_2(n)x_2(n)\right)\left(d_1(n) + d_2(n)\overline{x}_2(n)\right)} \\
v(n+1) &= v(n)\left[1 - \frac{\overline{x}_2(n)\exp\{\theta_2(n)v(n)\}}{d_3(n) + d_4(n)x_1(n)}\right] \\
&\quad + \frac{d_4(n)\overline{x}_1(n)\overline{x}_2(n)u(n)\exp\{\theta_1(n)u(n)\}}{\left(d_3(n) + d_4(n)x_1(n)\right)\left(d_3(n) + d_4(n)\overline{x}_1(n)\right)}, \\
z(n+1) &= z(n)\left[1 - b_2(n)\overline{x}_3(n)\exp\{\theta_3(n)z(n)\}\right] \\
&\quad + \frac{c_1(n)k_1(n)\overline{x}_1(n)u(n)\exp\{\theta_1(n)u(n)\}}{d_1(n) + d_2(n)x_2(n)} \\
&\quad - \frac{c_1(n)k_1(n)d_2(n)\overline{x}_1(n)\overline{x}_2(n)\exp\{\theta_2(n)v(n)\}v(n)}{\left(d_1(n) + d_2(n)x_2(n)\right)\left(d_1(n) + d_2(n)\overline{x}_2(n)\right)},
\end{aligned} \tag{35}$$

where $(\theta_1(n), \theta_2(n), \theta_3(n)) \in [0, 1]$. To complete the proof, it suffices to show that

$$\lim_{n\to\infty} u(n) = \lim_{n\to\infty} v(n) = \lim_{n\to\infty} z(n) = 0. \tag{36}$$

In view of (31), we can choose $\varepsilon > 0$ small enough such that

$$\begin{aligned}
\lambda_1^\epsilon &= \max\left\{\left|1 - b_1^u(B_1 + \varepsilon)\right|, \left|1 - b_1^l(D_1 - \varepsilon)\right|\right\} \\
&\quad + \frac{c_1^u(B_3 + \varepsilon)d_2^u(B_2 + \varepsilon)}{\left(d_1^l + d_2^l(D_2 - \varepsilon)\right)^2} + \frac{c_1^u(B_3 + \varepsilon)}{d_1^l + d_2^l(D_2 - \varepsilon)} < 1 \\
\lambda_2^\epsilon &= \max\left\{\left|1 - \frac{(B_2 + \varepsilon)}{d_3^l + d_4^l(D_1 - \varepsilon)}\right|,\right.
\end{aligned}$$

$$\left|1-\frac{(D_2-\varepsilon)}{d_3^u+d_4^u(B_1+\varepsilon)}\right|\Bigg\}+\frac{d_4^u(B_1+\varepsilon)(B_2+\varepsilon)}{\left(d_3^l+d_4^l(D_1-\varepsilon)\right)^2}<1$$

$$\begin{aligned}\lambda_3^\varepsilon=\max&\left\{\left|1-b_2^u(B_3+\varepsilon)\right|,\left|1-b_2^l(D_3-\varepsilon)\right|\right\}\\&+\frac{c_1^u k_1^u(B_1+\varepsilon)}{d_1^l+d_2^l(D_2-\varepsilon)}+\frac{c_1^u k_1^u(B_1+\varepsilon)(B_2+\varepsilon)d_2^u}{\left(d_1^l+d_2^l(D_2-\varepsilon)\right)^2}\\&<1.\end{aligned}\tag{37}$$

According to Theorems 1 and 2, there exists $n_0\in N$ such that

$$\begin{aligned}&D_1-\varepsilon\le\overline{x}_1(n)\le B_1+\varepsilon,\\&D_2-\varepsilon\le\overline{x}_2(n)\le B_2+\varepsilon,\\&D_3-\varepsilon\le\overline{x}_3(n)\le B_3+\varepsilon.\\&D_1-\varepsilon\le x_1(n)\le B_1+\varepsilon,\\&D_2-\varepsilon\le x_2(n)\le B_2+\varepsilon,\\&D_3-\varepsilon\le x_3(n)\le B_3+\varepsilon\end{aligned}\tag{38}$$

for $n\ge n_0$.

Notice that $\theta_1(n)\in[0,1]$ implies that $\overline{x}_1(n)\exp\{\theta_1(n)u(n)\}$ lies between $\overline{x}_1(n)$ and $x_1(n)$. Similarly, $\overline{x}_2(n)\exp\{\theta_2(n)v(n)\}$ lies between $\overline{x}_2(n)$ and $x_2(n)$, and $\overline{x}_3(n)\exp\{\theta_3(n)z(n)\}$ lies between $\overline{x}_3(n)$ and $x_3(n)$. From (35), we get

$$\begin{aligned}|u(n+1)|\le{}&|u(n)|\max\left\{\left|1-b_1^u(B_1+\varepsilon)\right|,\right.\\&\left.\left|1-b_1^l(D_1-\varepsilon)\right|\right\}+|v(n)|\frac{c_1^u(B_3+\varepsilon)d_2^u(B_2+\varepsilon)}{\left(d_1^l+d_2^l(D_2-\varepsilon)\right)^2}\\&+|z(n)|\frac{c_1^u(B_3+\varepsilon)}{d_1^l+d_2^l(D_2-\varepsilon)},\\|v(n+1)|\le{}&|v(n)|\max\left\{\left|1-\frac{B_2+\varepsilon}{d_3^l+d_4^l(D_1-\varepsilon)}\right|,\right.\\&\left.\left|1-\frac{D_2-\varepsilon}{d_3^u+d_4^u(B_1+\varepsilon)}\right|\right\}\\&+|u(n)|\frac{d_4^u(B_1+\varepsilon)(B_2+\varepsilon)}{\left(d_3^l+d_4^l(D_1-\varepsilon)\right)^2},\\|z(n+1)|\le{}&|z(n)|\max\left\{\left|1-b_2^l(D_3-\varepsilon)\right|,\right.\\&\left.\left|1-b_2^u(B_3+\varepsilon)\right|\right\}+|u(n)|\frac{c_1^u k_1^u(B_1+\varepsilon)}{d_1^l+d_2^l(D_2-\varepsilon)}\\&+|v(n)|\frac{c_1^u k_1^u d_2^u(B_1+\varepsilon)(B_2+\varepsilon)}{\left(d_1^l+d_2^l(D_2-\varepsilon)\right)^2}\end{aligned}\tag{39}$$

for $n\ge n_0$. Let $\lambda=\max\{\lambda_1^\varepsilon,\lambda_2^\varepsilon,\lambda_3^\varepsilon\}$. Then $\lambda<1$. In view of (39), we get

$$\begin{aligned}&\max\{|u(n+1)|,|v(n+1)|,|z(n+1)|\}\\&\quad\le\lambda\max\{|u(n)|,|v(n)|,|z(n)|\},\quad n\ge n_0.\end{aligned}\tag{40}$$

This implies

$$\begin{aligned}&\max\{|u(n)|,|v(n)|,|z(n)|\}\\&\quad\le\lambda^{n-n_0}\max\left\{\left|u(n_0)\right|,\left|v(n_0)\right|,\left|z(n_0)\right|\right\},\quad n\ge n_0.\end{aligned}\tag{41}$$

Therefore (36) holds and the proof is complete. □

4. Extinction of Predator Species and Stability of Prey-Mutualist Species

In this section, we also consider system (3) with $a_i(n)$ $(i=1,2,3)$, $b_j(n),c_j(n)$ $(j=1,2)$ $k_1(n)$, and $d_r(n)$ $(r=1,2,3,4)$ being periodic with a common period $w>0$. By developing the analysis technique of [29], we show that, under some suitable assumption, the predator will be driven to extinction while prey-mutualist will be globally attractive to a certain solution of a logistic equation.

We consider a discrete logistic equation

$$x(n+1)=x(n)\exp\left(a_1(n)-b_1(n)x(n)\right),\quad n\in N.\tag{42}$$

Theorem 6. *For any positive solution x^* of (42), one has*

$$m_1\le\liminf_{n\to\infty}x^*\le\limsup_{n\to\infty}x^*\le B_1,\tag{43}$$

where $m_1=(a_1^l/b_1^u)\exp\{a_1^l-b_1^uB_1\}$ and B_1 is defined by Theorem 1. Furthermore, there exists w-periodic solution for (42).

The proof of the above claim follows that of Theorems 1 and 2 with slight modification and we omit the detail here.

Theorem 7. *Assume that the inequality*

$$\frac{b_1^u\exp\left(a_1^u-1\right)}{b_1^l}<2\tag{H_2}$$

holds. Let $x^*(n)$ be a periodic solution of (42). Then, for every positive solution $x(n)$ of (42), one has

$$\lim_{n\to\infty}\left(x(n)-x^*(n)\right)=0.\tag{44}$$

Proof. Let

$$x(n)=x^*(n)\exp\{p(n)\}.\tag{45}$$

Then system (42) is equivalent to

$$\begin{aligned}p(n+1)&=p(n)+b_1(n)\left(x^*(n)-x(n)\right)\\&=p(n)-b_1(n)x^*(n)\left[\exp p(n)-1\right].\end{aligned}\tag{46}$$

By using the mean-value theorem, it follows that

$$p(n+1) = p(n)\left[1 - b_1(n)x^*(n)\exp\{\theta_4(n)p(n)\}\right], \quad (47)$$

where $\theta_4(n) \in [0,1]$. To complete the proof, it suffices to show that

$$\lim_{n\to\infty} p(n) = 0; \quad (48)$$

we first assume that

$$\lambda^* = \max\left\{\left|1 - b_1^u B_1\right|, \left|1 - b_1^l m_1\right|\right\} < 1; \quad (49)$$

then we can choose positive constant $\varepsilon > 0$ small enough such that

$$\lambda_\varepsilon^* = \max\left\{\left|1 - b_1^u(B_1+\varepsilon)\right|, \left|1 - b_1^l(m_1-\varepsilon)\right|\right\} < 1. \quad (50)$$

According to Theorem 6, there exists $n^* \in N$ such that

$$\begin{aligned} &m_1 - \varepsilon \le x(n), \\ &x^*(n) \le B_1 + \varepsilon, \\ &\qquad\qquad n \ge n^*. \end{aligned} \quad (51)$$

Notice that $\theta_4(n) \in [0,1]$ implies that $x^*(n)\exp\{\theta_4(n)p(n)\}$ lies between $x^*(n)$ and $x(n)$. From (47), we get

$$\begin{aligned} &\left|p(n+1)\right| \\ &\quad \le \left|p(n)\right| \max\left\{\left|1 - b_1^u(B_1+\varepsilon)\right|, \left|1 - b_1^l(m_1-\varepsilon)\right|\right\} \\ &\quad = \left|p(n)\right| \lambda_\varepsilon^*. \end{aligned} \quad (52)$$

This implies that

$$\left|p(n)\right| \le (\lambda_\varepsilon^*)^{n-n^*}\left|p(n^*)\right| \quad n \ge n^*. \quad (53)$$

Since $\lambda_\varepsilon^* < 1$ and ε is arbitrarily small, we obtain $\lim_{n\to\infty} p(n) = 0$, and it means that (48) holds when $\lambda^* < 1$.

Note that

$$1 - b_1^u B_1 \le 1 - b_1^l m_1 < 1; \quad (54)$$

thus, $\lambda^* < 1$ is equivalent to

$$1 - b_1^u B_1 > -1, \quad (55)$$

or

$$b_1^u B_1 = \frac{b_1^u}{b_1^l}\exp\{a_1^u - 1\} < 2. \quad (56)$$

Now, we can conclude that (48) is satisfied as (H_2) holds, and so

$$\lim_{n\to\infty}\left(x(n) - x^*(n)\right) = 0. \quad (57)$$

□

Theorem 8. *Assume that the inequality*

$$\frac{k_1^u c_1^u B_1}{d_1^l + d_2^l D_2} - a_3^l < 0 \quad (H_3)$$

holds, where D_2 and B_1 are defined by Theorems 1 and 2. Let $(x_1(n), x_2(n), x_3(n))$ be any positive solution of system (3); then $x_3(n) \to 0$ as $n \to +\infty$.

Proof. From (H_3) we can choose positive constant $\varepsilon > 0$ small enough such that inequality

$$\frac{k_1^u c_1^u(B_1+\varepsilon)}{d_1^l + d_2^l(D_2-\varepsilon)} - a_3^l < 0 \quad (58)$$

holds. Thus, there exists $\delta_2 > 0$,

$$\frac{k_1^u c_1^u(B_1+\varepsilon)}{d_1^l + d_2^l(D_2-\varepsilon)} - a_3^l < -\delta_2 < 0. \quad (59)$$

Let $(x_1(n), x_2(n), x_3(n))$ be any positive solution of system (3). For any $q \in N$, according to the equation of system (3), we obtain

$$\begin{aligned} \ln\frac{x_3(q+1)}{x_3(q)} &= -a_3(q) + \frac{k_1(q)c_1(q)x_1(q)}{d_1(q)+d_2(q)x_2(q)} \\ &\quad - b_2(q)x_3(q) \\ &\le -a_3(q) + \frac{k_1(q)c_1(q)x_1(q)}{d_1(q)+d_2(q)x_2(q)} \\ &\le -a_3^l + \frac{k_1^u c_1^u(B_1+\varepsilon)}{d_1^l + d_2^l(D_2-\varepsilon)} \le -\delta_2 < 0. \end{aligned} \quad (60)$$

Summating both sides of the above inequations from 0 to $n-1$, we obtain

$$\ln\frac{x_3(n)}{x_3(0)} < -\delta_2 n, \quad (61)$$

and then

$$x_3(n) < x_3(0)\exp(-\delta_2 n). \quad (62)$$

The above inequality shows that $x_3(n) \to 0$ exponentially as $n \to +\infty$. This completes the proof of Theorem 8. □

Theorem 9. *Assume (H_2), (H_3), and $B_2/(d_3^l + d_4^l D_1) < 2$ hold; also*

$$\begin{aligned} a_1^l - \frac{c_1^u B_3}{d_1^l + d_2^l D_2} &> 0, \\ \frac{k_1^u c_1^u B_1}{d_1^l} - a_3^l &> 0. \end{aligned} \quad (63)$$

Then for any positive solution $(x_1(n), x_2(n), x_3(n))$ of system (3), one has

$$\begin{aligned} \lim_{n\to\infty}\left(x_1(n) - x_1^*(n)\right) &= 0, \\ \lim_{n\to\infty}\left(x_2(n) - x_2^*(n)\right) &= 0. \end{aligned} \quad (64)$$

$x_1^*(n)$ is any positive solution of system (42) and $x_2^*(n)$ is any positive solution of the second equation of system (3).

Proof. Since (H_3) holds, it follows from Theorem 8 that

$$\lim_{n\to\infty} x_3(n) = 0. \tag{65}$$

To prove $\lim_{n\to\infty}(x_1(n) - x_1^*(n)) = 0$, let

$$x_1(n) = x_1^*(n)\exp(u(n)); \tag{66}$$

then from the first equation of system (3) and (66),

$$\begin{aligned} u(n+1) &= u(n) - b_1(n)x_1^*(n)\left(\exp(u(n)-1)\right) \\ &\quad - \frac{c_1(n)x_3(n)}{d_1(n)+d_2(n)x_2(n)}. \end{aligned} \tag{67}$$

Using the mean-value theorem, one has

$$\begin{aligned} \exp(u(n)-1) &= \exp\left(\zeta_1(n)u(n)\right)u(n), \\ &\qquad \zeta_1(n) \in (0,1). \end{aligned} \tag{68}$$

Then the first equation of system (3) is equivalent to

$$\begin{aligned} u(n+1) &= u(n)\left(1 - b_1(n)x_1^*(n)\exp\left(\zeta_1(n)u(n)\right)\right) \\ &\quad - \frac{c_1(n)x_3(n)}{d_1(n)+d_2(n)x_2(n)}. \end{aligned} \tag{69}$$

To complete the proof, it suffices to show that

$$\lim_{n\to\infty} u(n) = 0. \tag{70}$$

We first assume that

$$\lambda = \max\left\{\left|1 - b_1^u B_1\right|, \left|1 - b_1^l D_1\right|\right\} < 1, \tag{71}$$

and then we can choose positive constant $\varepsilon > 0$ small enough such that

$$\lambda_\varepsilon = \max\left\{\left|1 - b_1^u(B_1+\varepsilon)\right|, \left|1 - b_1^l(D_1-\varepsilon)\right|\right\} < 1. \tag{72}$$

For the above ε, according to Theorems 1, 2, and 8, there exists an integer $n_1 \in N$ such that

$$\begin{aligned} &D_1 - \varepsilon \le x_1(n) \le B_1 + \varepsilon, \\ &m_1 - \varepsilon \le x_1^*(n) \le B_1 + \varepsilon, \\ &x_3(n) \le \varepsilon, \\ &\qquad\qquad n \ge n_1. \end{aligned} \tag{73}$$

Noting that $m_1 \ge D_1$, then

$$\begin{aligned} &D_1 - \varepsilon \le x_1(n), \\ &x_1^*(n) \le B_1 + \varepsilon, \\ &x_3(n) \le \varepsilon, \\ &\qquad\qquad n \ge n_1. \end{aligned} \tag{74}$$

It follows from (74) that

$$\frac{c_1(n)}{d_1(n)+d_2(n)x_2(n)} \le \frac{c_1^u}{d_1^l + d_2^l(D_2-\varepsilon)} \doteq M_\varepsilon, \qquad n \ge n_1. \tag{75}$$

Noting that $\zeta_1(n) \in (0,1)$, it implies that $x_1^*(n)\exp(\zeta_1(n)u(n))$ lies between $x_1^*(n)$ and $x_1(n)$. From (69), (72)–(75), we get

$$\begin{aligned} &|u(n+1)| \\ &\quad \le |u(n)|\max\left\{\left|1 - b_1^u(B_1+\varepsilon)\right|, \left|1 - b_1^l(D_1-\varepsilon)\right|\right\} \\ &\qquad + \frac{c_1^u\varepsilon}{d_1^l + d_2^l(D_2-\varepsilon)} = |u(n)|\lambda_\varepsilon + M_\varepsilon\varepsilon, \\ &\qquad\qquad n \ge n_1. \end{aligned} \tag{76}$$

This implies that

$$|u(n)| \le \lambda_\varepsilon^{n-n_1}\left|u(n_1)\right| + \frac{1-\lambda_\varepsilon^{n-n_1}}{1-\lambda_\varepsilon}M_\varepsilon\varepsilon, \quad n \ge n_1. \tag{77}$$

Since $\lambda_\varepsilon < 1$ and ε is arbitrarily small, we obtain $\lim_{n\to\infty} u(n) = 0$, and it means that (70) holds when $\lambda < 1$.

Note that

$$1 - b_1^u B_1 \le 1 - b_1^l D_1 < 1; \tag{78}$$

thus, $\lambda < 1$ is equivalent to

$$1 - b_1^u B_1 > -1, \tag{79}$$

or

$$b_1^u B_1 = \frac{b_1^u}{b_1^l}\exp\left\{a_1^u - 1\right\} < 2. \tag{80}$$

Now, we can conclude that (70) is satisfied as (H_2) holds, and so

$$\lim_{n\to\infty}\left(x_1(n) - x_1^*(n)\right) = 0. \tag{81}$$

Next, we prove

$$\lim_{n\to\infty}\left(x_2(n) - x_2^*(n)\right) = 0. \tag{82}$$

Let

$$x_2(n) = x_2^*(n)\exp(v(n)). \tag{83}$$

If $k_1^u c_1^u B_1/d_1^l - a_3^l > 0$ and $a_1^l - c_1^u B_3/(d_1^l + d_2^l D_2) > 0$ hold, from Theorems 1 and 2, we know that $x_1(n), x_2(n)$ are bounded eventually. From the second inequality of (39),

$$\begin{aligned} |v(n+1)| &\le |v(n)|\max\left\{\left|1 - \frac{B_2+\varepsilon}{d_3^l + d_4^l(D_1-\varepsilon)}\right|,\right. \\ &\quad \left.\left|1 - \frac{D_2-\varepsilon}{d_3^u + d_4^u(B_1+\varepsilon)}\right|\right\} \\ &\quad + |u(n)|\frac{d_4^u(B_1+\varepsilon)(B_2+\varepsilon)}{\left(d_3^l + d_4^l(D_1-\varepsilon)\right)^2} \\ &\qquad\qquad n > n_0. \end{aligned} \tag{84}$$

We first assume that

$$\rho = \max\left\{\left|1-\frac{B_2}{d_3^l+d_4^l D_1}\right|,\left|1-\frac{D_2}{d_3^u+d_4^u B_1}\right|\right\}<1. \quad (85)$$

It follows from (70) that $\lim_{n\to\infty} u(n)=0$.

For any positive constant $\varepsilon>0$, there exists integer $n_2\ge \max(n_0,n_1)$ such that

$$\rho_\varepsilon = \max\left\{\left|1-\frac{(B_2+\varepsilon)}{d_3^l+d_4^l(D_1-\varepsilon)}\right|,\right.$$
$$\left.\left|1-\frac{(D_2-\varepsilon)}{d_3^u+d_4^u(B_1+\varepsilon)}\right|\right\}<1, \quad n\ge n_2 \quad (86)$$
$$|u(n)|<\varepsilon, \quad n\ge n_2.$$

Let

$$\frac{d_4^u(B_1+\varepsilon)(B_2+\varepsilon)}{\left(d_3^l+d_4^l(D_1-\varepsilon)\right)^2}\doteq A_\varepsilon. \quad (87)$$

From (84)–(87) we can conclude that

$$|v(n+1)|\le \rho_\varepsilon|v(n)|+\varepsilon A_\varepsilon, \quad n\ge n_2. \quad (88)$$

This implies that

$$|v(n)|\le \rho_\varepsilon^{n-n_2}|v(n_2)|+\varepsilon A_\varepsilon\frac{1-\rho_\varepsilon^{n-n_2}}{1-\rho_\varepsilon}, \quad n\ge n_2. \quad (89)$$

Since $\rho_\varepsilon<1$ and ε is arbitrarily small, we obtain $\lim_{n\to\infty}u(n)=0$. Note that

$$1-\frac{B_2}{d_3^l+d_4^l D_1}\le 1-\frac{D_2}{d_3^u+d_4^u B_1}<1; \quad (90)$$

thus $\rho_\varepsilon<1$ is equivalent to

$$1-\frac{B_2}{d_3^l+d_4^l D_1}>-1, \quad (91)$$

or

$$\frac{B_2}{d_3^l+d_4^l D_1}<2. \quad (92)$$

Now, we can conclude that $\lim_{n\to\infty}v(n)=0$. And so

$$\lim_{n\to\infty}\left(x_2(n)-x_2^*(n)\right)=0. \quad (93)$$

We can conclude that

$$\lim_{n\to\infty}\left(x_1(n)-x_1^*(n)\right)=0,$$
$$\lim_{n\to\infty}\left(x_2(n)-x_2^*(n)\right)=0, \quad (94)$$
$$n\ge n_2$$

This completes the proof of Theorem 9. □

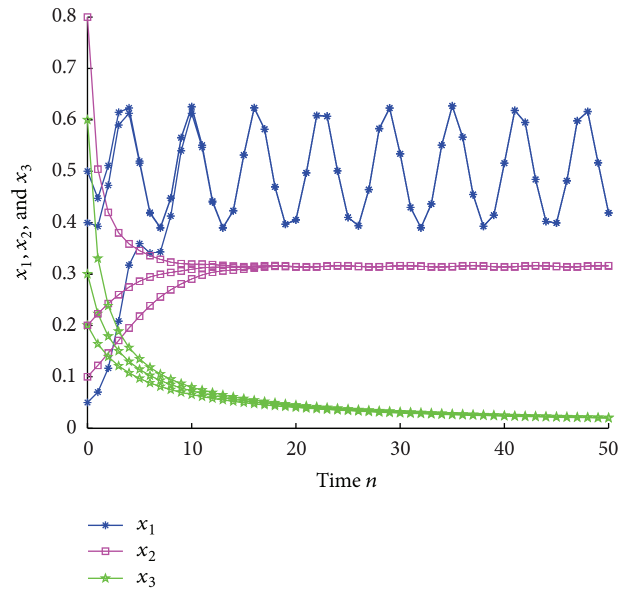

Figure 1: Dynamic behavior of the solution $(x_1(n),x_2(n),x_3(n))$ to system (95) with the initial conditions $(x_1(0),x_2(0),x_3(0))=(0.05,0.1,0.2)$, $(0.5,0.8,0.3)$ and $(0.4,0.2,0.6)$, respectively.

5. Examples and Numeric Simulations

In this section, we will give two examples to show the feasibility of our results.

Example 1. Consider the following system:

$$x_1(n+1)=x_1(n)\cdot\exp\left\{(0.5-0.2\cos(n))-x_1(n)-\frac{0.02x_3(n)}{1+x_2(n)}\right\},$$
$$x_2(n+1)=x_2(n)\exp\left\{0.3-\frac{x_2(n)}{1+0.1x_1(n)}\right\}, \quad (95)$$
$$x_3(n+1)=x_3(n)\cdot\exp\left\{-0.0001+\frac{0.01x_1(n)}{1+x_2(n)}-x_3(n)\right\}.$$

One could easily see that $k_1^l c_1^l D_1/(d_1^u+d_2^u B_2)-a_3^u\approx 0.0011>0$ and $a_1^l-c_1^u B_3/(d_1^l+d_2^l D_2)\approx 0.2940>0$, and then condition (H_1) is satisfied. According to Theorem 1, system (3) is permanent. Numerical simulation (see Figure 1) indicates the permanence of system (95).

Figure 1 shows the dynamic behaviors of system (95), which strongly supports our results.

Example 2. Consider the following system:

$$x_1(n+1)=x_1(n)\cdot\exp\left\{(0.7-0.3\cos(n))-2x_1(n)-\frac{0.02x_3(n)}{1+x_2(n)}\right\},$$

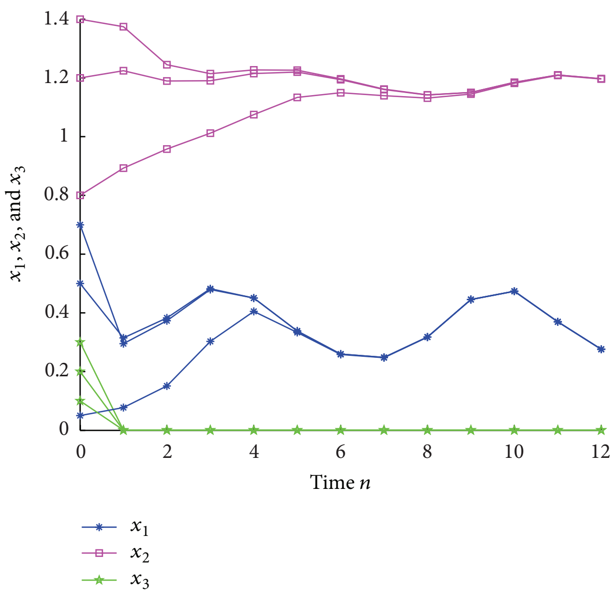

FIGURE 2: Dynamic behavior of the solution $(x_1(n), x_2(n), x_3(n))$ to system (96) with the initial conditions $(x_1(0), x_2(0), x_3(0)) = (0.05, 0.8, 0.1), (0.5, 1.2, 0.2)$ and $(0.7, 1.4, 0.3)$, respectively.

$$\begin{aligned} x_2(n+1) &= x_2(n) \\ &\cdot \exp\left\{(0.5 - 0.1\sin(n)) - \frac{x_2(n)}{2 + x_1(n)}\right\}, \\ x_3(n+1) &= x_3(n) \\ &\cdot \exp\left\{-0.004 + \frac{0.01 x_1(n)}{1 + x_2(n)} - 50 x_3(n)\right\}. \end{aligned} \tag{96}$$

We could easily see that $(b_1^u/b_1^l)\exp(a_1^u - 1) = 1 < 2$, $k_1^u c_1^u B_1/(d_1^l + d_2^l D_2) - a_3^l \approx -0.000703 < 0$, $B_2/(d_3^l + d_4^l D_1) \approx 0.7943 < 2$, $a_1^l - c_1^u B_3/(d_1^l + d_2^l D_2) \approx 0.3999 > 0$, and $k_1^u c_1^u B_1/d_1^l - a_3^l = 0.001 > 0$. Clearly, conditions of Theorem 9 are satisfied, and so $\lim_{n\to\infty}(x_1(n) - x_1^*(n)) = 0$, $\lim_{n\to\infty}(x_2(n) - x_2^*(n)) = 0$, and $\lim_{n\to\infty} x_3(n) = 0$.

Figure 2 shows the dynamic behaviors of system (96), which strongly supports our results.

6. Discussion

It is well known that prey-mutualist system can decrease predation risk; mutualism plays an important role in the dynamic behaviors of predator-prey populations. For system (3), we showed that the predator-prey-mutualist system will be coexistent in a globally stable state under some suitable conditions. We argued that it is an important topic to study the extinction of the species [29–32], since, with the development of modern society, more and more species are driven to extinction; this motivated us to study the extinction of the predator species. In Section 4, our results indicate that if the death rate of the predator species x_3 is big enough or the cooperate effect between species x_1 and x_2 is very strong, the predator species will be driven to extinction due to the fewer chances of meeting prey species. This can also be seen from (H_3); x_3 will be driven to extinction when d_2 becomes bigger.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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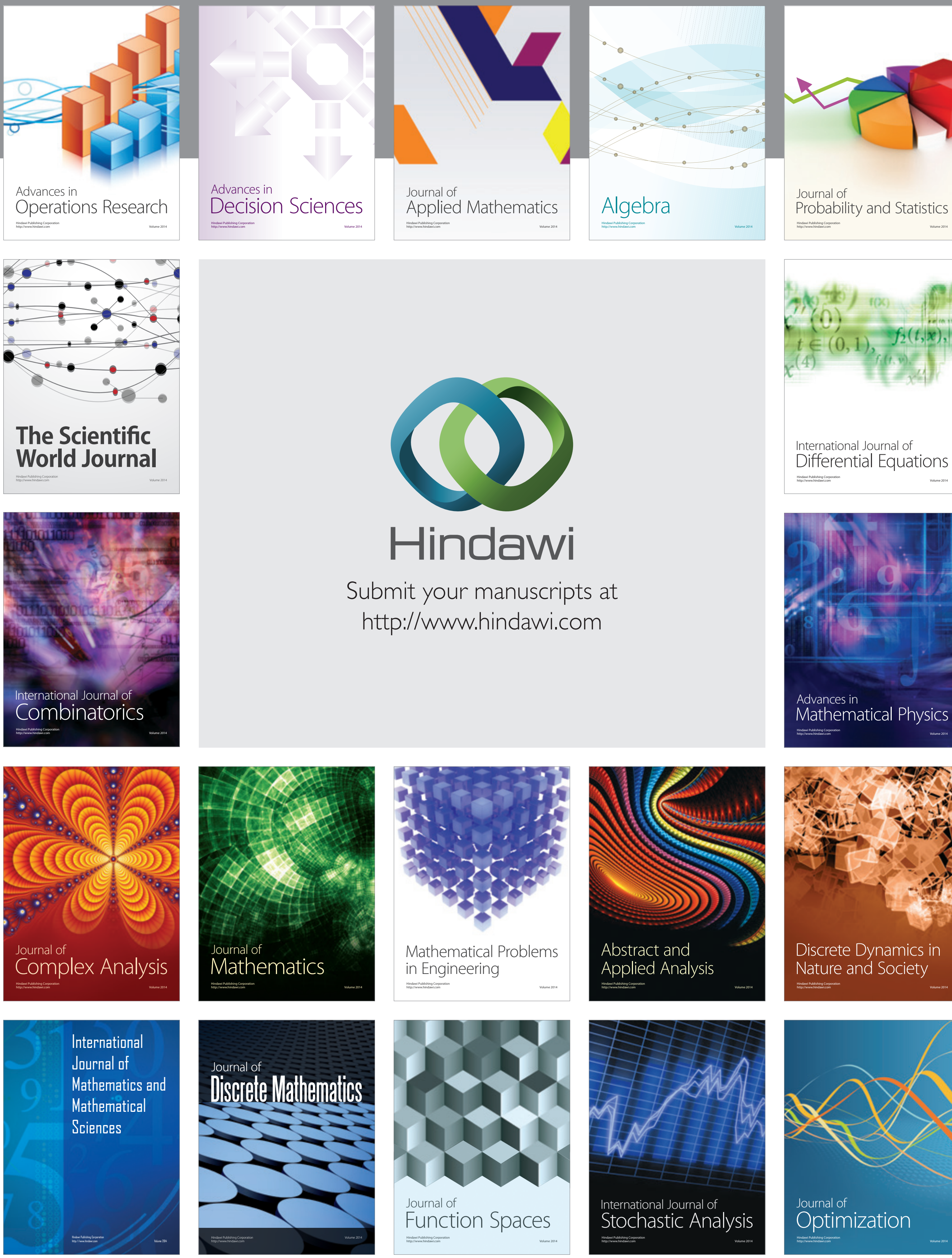

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