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Research Article

Iterative Approximation of the Minimal and Maximal Positive Solutions for Multipoint Fractional Boundary Value Problem on an Unbounded Domain

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By employing the monotone iterative method, this paper not only establishes the existence of the minimal and maximal positive solutions for multipoint fractional boundary value problem on an unbounded domain, but also develops two computable explicit monotone iterative sequences for approximating the two positive solutions. An example is given for the illustration of the main result.

1. Introduction

The fractional calculus has been recognized as an effective modeling methodology for describing hereditary properties of various materials and processes widely. For a lot of applications, we refer the reader to the books [1–5]. For some new development on the topic, see [6–17] and the references therein.

Recently, there has been a significant development on boundary value problems for fractional differential equations on *infinite intervals*; see papers [18–26], in which authors are devoted to investigating the existence of solutions and positive solutions by employing some fixed point theorems, Leray-Schauder nonlinear alternative theorem, or fixed point index theory.

By using Schauder's fixed point theorem combined with the diagonalization method, Arara et al. [18] studied the existence of the bounded solution of the following problem on infinite intervals:

$$\begin{gathered} {}^{C}D^{\alpha}_{0+}\, y(t) = f(t, y(t)), \quad 1 < \alpha \le 2, \\ y(0) = y_0, \quad y \text{ is bounded on } J, \end{gathered} \tag{1}$$

where $t \in J = [0, +\infty)$, $f \in C(J \times \mathbb{R}, \mathbb{R})$, $y_0 \in \mathbb{R}$, and ${}^{C}D^{\alpha}_{0+}$ is the Caputo fractional derivative of order α.

In [19], Zhao and Ge investigated the existence of positive solutions for the following fractional boundary value problem by employing the Leray-Schauder nonlinear alternative theorem:

$$\begin{gathered} D^{\alpha}_{0+}u(t) + f(t, u(t)) = 0, \quad 1 < \alpha \le 2, \\ u(0) = 0, \quad \lim_{t \to +\infty} D^{\alpha-1}_{0+} = \beta u(\xi), \end{gathered} \tag{2}$$

where $t \in J = [0, +\infty)$, $f \in C(J \times \mathbb{R}, [0, +\infty))$, $0 \le \xi$, $\eta < \infty$, and D^{α}_{0+} is the standard Riemann-Liouville fractional derivative.

Liang and Zhang [20] were concerned with the following nonlinear fractional differential equations with multipoint fractional boundary conditions on an unbounded domain:

$$\begin{gathered} D^{\alpha}u(t) + a(t) f(t, u(t)) = 0, \quad 0 < t < \infty, \\ u(0) = u'(0) = 0, \\ D^{\alpha-1}u(+\infty) = \sum_{i=1}^{m-2} \beta_i u(\xi_i), \end{gathered} \tag{3}$$

where $J = [0, +\infty)$, $2 < \alpha \le 3$, D^{α} denotes the Riemann-Liouville fractional derivative, $0 < \xi_1 < \xi_2 < \cdots < \xi_{m-2} < +\infty$, and $\beta_i > 0$, $i = 1, 2, \ldots, m-2$, satisfy

$0 < \sum_{i=1}^{m-2} \beta_i \xi_i^{\alpha-1} < \Gamma(\alpha)$. By using the fixed point index theory, authors gave sufficient conditions for the existence of multiple positive solutions to the above multi-point fractional boundary value problem.

However, very interesting and important question is "If we know the existence of the solution, how can we find it?" This question motivates us to reconsider problem (3). In this paper, we not only establish the existence of two positive solutions for problem (3), but also develop two computable explicit monotone iterative sequences for approximating the minimal and maximal positive solutions of (3), which is indeed an important and useful contribution to the existing literature on the topic. In addition, to start our work, we employ the monotone iterative method, which is different from the ones used in [18–26]. Let us state that this method was widely used for nonlinear problem; see, for instance, [27–38].

2. Preliminaries and Several Lemmas

In this section, we present some useful definitions and related theorems.

Definition 1 (see [2]). The Riemann-Liouville fractional derivative of order δ for a continuous function f is defined by

$$D^{\delta} f(t) = \frac{1}{\Gamma(n-\delta)} \left(\frac{d}{dt}\right)^{n} \times \int_0^t (t-s)^{n-\delta-1} f(s)\, ds, \quad n = [\delta] + 1, \tag{4}$$

provided the right-hand side is pointwise defined on $(0, \infty)$ and $[\delta]$ is the integer part of δ.

Definition 2 (see [2]). The Riemann-Liouville fractional integral of order δ for a function f is defined as

$$I^{\delta} f(t) = \frac{1}{\Gamma(\delta)} \int_0^t (t-s)^{\delta-1} f(s)\, ds, \quad \delta > 0, \tag{5}$$

provided that such integral exists.

Lemma 3 (see [20]). *Let $h \in C([0, +\infty))$. For $2 < \alpha < 3$, the fractional boundary value problem*

$$\begin{gathered} D^{\alpha} u(t) + h(t) = 0, \\ u(0) = u'(0) = 0, \\ D^{\alpha-1} u(+\infty) = \sum_{i=1}^{m-2} \beta_i u(\xi_i) \end{gathered} \tag{6}$$

has a unique solution

$$u(t) = \int_0^{+\infty} G(t,s) h(s)\, ds, \tag{7}$$

where

$$G(t,s) = G^{*}(t,s) + G^{**}(t,s), \tag{8}$$

with

$$\begin{gathered} G^{*}(t,s) = \frac{1}{\Gamma(\alpha)} \begin{cases} t^{\alpha-1} - (t-s)^{\alpha-1}, & 0 \le s \le t < +\infty, \\ t^{\alpha-1}, & 0 \le t \le s < +\infty, \end{cases} \\ G^{**}(t,s) = \frac{\sum_{i=1}^{m-2} \beta_i t^{\alpha-1}}{\Gamma(\alpha) - \sum_{i=1}^{m-2} \beta_i \xi_i^{\alpha-1}} G^{*}(\xi_i, s). \end{gathered} \tag{9}$$

Lemma 4 (see [20]). *For $(s,t) \in [0, +\infty) \times [0, +\infty)$, then Green's function $G(t,s)$ has the following properties:*

(1)

$$0 \le G(t,s) \le L t^{\alpha-1}, \tag{10}$$

(2)

$$\frac{G(t,s)}{1 + t^{\alpha-1}} \le L, \tag{11}$$

where

$$L = \frac{1}{\Gamma(\alpha)} + \frac{\sum_{i=1}^{m-2} \beta_i \xi_{m-2}^{\alpha-1}}{\Gamma(\alpha)\left(\Gamma(\alpha) - \sum_{i=1}^{m-2} \beta_i \xi_i^{\alpha-1}\right)}. \tag{12}$$

For the forthcoming analysis, we will use a Banach space:

$$X = \left\{ u \in C(J, \mathbb{R}) : \sup_{t \in J} \frac{|u(t)|}{1 + t^{\alpha-1}} < +\infty \right\}, \tag{13}$$

equipped with the norm

$$\|u\|_X = \sup_{t \in J} \frac{|u(t)|}{1 + t^{\alpha-1}}. \tag{14}$$

Define a cone $P \subset X$ by

$$P = \{u \in X : u(t) \ge 0, t \in J\} \tag{15}$$

and an operator $Q : X \to X$ as follows:

$$Qu(t) = \int_0^{\infty} G(t,s) a(s) f(s, u(s))\, ds. \tag{16}$$

Observe that multi-point fractional boundary value problem (3) has a solution if and only if the integral operator Q has a fixed point.

3. Main Results

In this section, we shall construct two explicit monotone iterative sequences which converge to the minimal and maximal positive solutions of (3).

Theorem 5. *Assume that the following conditions hold:*

(H_1) *$f \in C(J \times J, J)$, $f(t, 0) \not\equiv 0$ on any subinterval of J, and when u is bounded, $f(t, (1 + t^{\alpha-1})u)$ is bounded on J;*

(H_2) *$a : J \to J$ does not identically vanish on any subinterval of J and $0 < \int_0^{+\infty} a(t) dt < \infty$;*

(H_3) *$f(t,\cdot)$ is nondecreasing for any $t \in J$, and there exists a constant $b > 0$, such that $f(t,(1+t^{\alpha-1})u) \le b/L\int_0^{\infty} a(t)dt$ for $(t,u) \in J \times [0,b]$.*

Then the multi-point fractional boundary value problem (3) has the minimal and maximal positive solutions v^*, u^* in $(0, bt^{\alpha-1}]$, which can be obtained by the following two explicit monotone iterative sequences:

$$\begin{aligned} v_{n+1} &= \int_0^{+\infty} G(t,s)\,a(s)\,f(s,v_n(s))\,ds \\ &\quad \text{with initial value } v_0(t) = 0, \\ u_{n+1} &= \int_0^{+\infty} G(t,s)\,a(s)\,f(s,u_n(s))\,ds \\ &\quad \text{with initial value } u_0(t) = bt^{\alpha-1}. \end{aligned} \tag{17}$$

Moreover,

$$\begin{aligned} &v_0 \le v_1 \le \cdots \le v_n \cdots \le v^* \le \cdots \\ &\le u^* \cdots \le u_n \le \cdots \le u_1 \le u_0. \end{aligned} \tag{18}$$

Proof. By a similar process used in [20], it is easy to show that $Q: P \to P$ is completely continuous.

Now denote $B = \{u \in P, \|u\|_X \le b\}$; then we have $Q(B) \subset B$. In fact, let $u \in B$; then by (H_3) and (12), we have

$$\begin{aligned} \|Qu\|_X &= \sup_{t\in J} \int_0^{+\infty} \frac{G(t,s)}{1+t^{\alpha-1}} \left|a(s)\,f(s,u(s))\right| ds \\ &\le L\int_0^{+\infty} a(s)\,ds \cdot \frac{b}{L\int_0^{+\infty} a(s)\,ds} = b. \end{aligned} \tag{19}$$

That is, $Q(B) \subset B$.

Denote that $v_0(t) = 0$, $v_1 = Q0 = Qv_0$, and $v_2 = Q^2 0 = Qv_1$, for all $t \in J$. Since $v_0(t) = 0 \in B$ and $Q : B \to B$, then $v_1 \in Q(B) \subset B$ and $v_2 \in Q(B) \subset B$. So, we have

$$v_1(t) = (Q0)(t) \ge 0 = v_0(t), \quad \forall t \in J. \tag{20}$$

By condition (H_3), for $u, v \in B$ and $u \ge v$, we have

$$\begin{aligned} Qu(t) &= \int_0^{\infty} G(t,s)\,a(s)\,f(s,u(s))\,ds \\ &\ge \int_0^{\infty} G(t,s)\,a(s)\,f(s,v(s))\,ds = Qv(t). \end{aligned} \tag{21}$$

This proves that Q is a nondecreasing operator.

So, we have

$$v_2(t) = (Qv_1)(t) \ge (Qv_0)(t) = v_1(t), \quad \forall t \in J. \tag{22}$$

By the induction, define $v_{n+1} = Qv_n$, $n = 0,1,2,\ldots$. Then the sequence $\{v_n\}_{n=1}^{\infty} \subset Q(B) \subset B$ and satisfies the following relation:

$$v_{n+1}(t) \ge v_n(t), \quad \forall t \in J,\ n = 0,1,2,\ldots. \tag{23}$$

In view of the complete continuity of the operator Q and $v_{n+1} = Qv_n$, then $\{v_n\}_{n=1}^{\infty}$ is relative compact. That is, $\{v_n\}_{n=1}^{\infty}$ has a convergent subsequence $\{v_{n_k}\}_{k=1}^{\infty}$ and there exists a $v^* \in B$ such that $v_{n_k} \to v^*$ as $k \to \infty$. This, together with (23), holds $\lim_{n\to\infty} v_n = v^*$.

Since Q is continuous and $v_{n+1} = Qv_n$, then we have $Qv^* = v^*$. That is, v^* is a fixed point of the operator Q.

Denote that $u_0(t) = bt^{\alpha-1}$, $u_1 = Qu_0$, and $u_2 = Q^2u_0 = Qu_1$, for all $t \in J$. Since $u_0(t) \in B$ and $Q : B \to B$, then $u_1 \in Q(B) \subset B$ and $u_2 \in Q(B) \subset B$. By (H_3), we have

$$\begin{aligned} u_1(t) &= \int_0^{+\infty} G(t,s)\,a(s)\,f(s,u_0(s))\,ds \\ &\le \int_0^{+\infty} Lt^{\alpha-1} a(s)\,ds \cdot \frac{b}{\int_0^{+\infty} La(s)\,ds} \\ &= bt^{\alpha-1} = u_0(t), \quad \forall t \in J. \end{aligned} \tag{24}$$

Since Q is nondecreasing, then we have

$$u_2(t) = (Qu_1)(t) \le (Qu_0)(t) = u_1(t), \quad \forall t \in J. \tag{25}$$

By the induction, define $u_{n+1} = Qu_n$, $n = 0,1,2,\ldots$. Then the sequence $\{u_n\}_{n=1}^{\infty} \subset Q(B) \subset B$ and satisfies the following relation:

$$u_{n+1}(t) \le u_n(t), \quad \forall t \in J,\ n = 0,1,2,\ldots. \tag{26}$$

With an analysis exactly parallel to the proving process of $\lim_{n\to\infty} v_n = v^*$, we have that there exists a $u^* \in B$ such that $\lim_{n\to\infty} u_n = u^*$.

Since Q is continuous and $u_{n+1} = Qu_n$, we have $Qu^* = u^*$. That is, u^* is a fixed point of the operator Q.

Now, we are in a position to show that u^* and v^* are the maximal and minimal positive solutions of (3) in $(0, bt^{\alpha-1}]$.

Let $w \in [0, bt^{\alpha-1}]$ be any solution of (3). That is $Qw = w$. Noting that Q is nondecreasing and $v_0(t) = 0 \le w(t) \le bt^{\alpha-1} = u_0(t)$, then we have $v_1(t) = Qv_0(t) \le w(t) \le Qu_0(t) = u_1(t)$, for all $t \in J$.

Similarly, we can obtain

$$v_n(t) \le w(t) \le u_n(t), \quad \forall t \in J,\ n = 0,1,2,\ldots. \tag{27}$$

Since $u^* = \lim_{n\to\infty} u_n$ and $v^* = \lim_{n\to\infty} v_n$, it follows from (23)~(27) that

$$\begin{aligned} &v_0 \le v_1 \le \cdots \le v_n \cdots \le v^* \\ &\le w \le u^* \cdots \le u_n \le \cdots \le u_1 \le u_0. \end{aligned} \tag{28}$$

Since $f(t,0) \not\equiv 0$, for all $t \in J$, then 0 is not a solution of problem (3). Thus, by (28), we know that u^* and v^* are the maximal and minimal positive solutions of (3) in $(0, bt^{\alpha-1}]$, which can be obtained by the corresponding iterative sequences in (17).

This completes the proof. □

4. Example

Example 1. Take $\alpha = 5/2$, $\beta_1 = 3/10$, $\beta_2 = 1/5$, $\xi_1 = 1/4$, and $\xi_2 = 1$. Consider the following boundary value problem:

$$\begin{gathered} D^{5/2}u(t) + e^{-t} f(t, u(t)) = 0, \quad t \in (0, +\infty) \\ u(0) = u'(0) = 0, \\ D^{3/2}u(+\infty) = \frac{3}{10}u\left(\frac{1}{4}\right) + \frac{1}{5}u(1), \end{gathered} \tag{29}$$

where $a(t) = e^{-t}$ and

$$f(t,u) = \begin{cases} \dfrac{1}{100(1+t^4)} + \dfrac{1}{10}\left(\dfrac{u}{1+t^{3/2}}\right)^5, & 0 \le u \le 1, \\ \dfrac{1}{100(1+t^4)} + \dfrac{1}{10}\left(\dfrac{1}{1+t^{3/2}}\right)^5, & u > 1. \end{cases} \tag{30}$$

Now, we show that $f(t, (1+t^{\alpha-1})u)$ is bounded on J when u is bounded. Since

$$f\left(t, \left(1+t^{3/2}\right)u\right) = \begin{cases} \dfrac{1}{100(1+t^4)} + \dfrac{1}{10}u^5, & 0 \le u \le 1, \\ \dfrac{1}{100(1+t^4)} + \dfrac{1}{10}\left(\dfrac{1}{1+t^{3/2}}\right)^5, & u > 1. \end{cases} \tag{31}$$

Then we have $f(t, (1+t^{3/2})u) \le 11/100$. So condition (H_1) holds.

In view of $\int_0^{+\infty} a(t)dt = \int_0^{+\infty} e^{-t}dt = 1$, condition (H_2) holds.

By a simple computation, we have that $\Gamma(\alpha) = \Gamma(5/2) = 3\sqrt{\pi}/4$ and $L = (1/\Gamma(\alpha)) + (\sum_{i=1}^{m-2}\beta_i\xi_{m-2}^{\alpha-1}/\Gamma(\alpha)(\Gamma(\alpha) - \sum_{i=1}^{m-2}\beta_i\xi_i^{\alpha-1})) \approx 1.096741$. Taking $b = 1$, it follows that

$$\begin{aligned} f\left(t, \left(1+t^{3/2}\right)u\right) &\le 0.11 < \frac{1}{1.0968} \\ &\le \frac{b}{L\int_0^{\infty} a(s)\,ds}, \\ &\text{for } (t,u) \in J \times [0,1]. \end{aligned} \tag{32}$$

Hence, condition (H_3) holds. Thus all conditions of Theorem 5 are satisfied. Therefore, the fractional boundary value problem (29) has the minimal and maximal positive solutions in $(0, t^{3/2}]$, which can be obtained by two explicit monotone iterative sequences.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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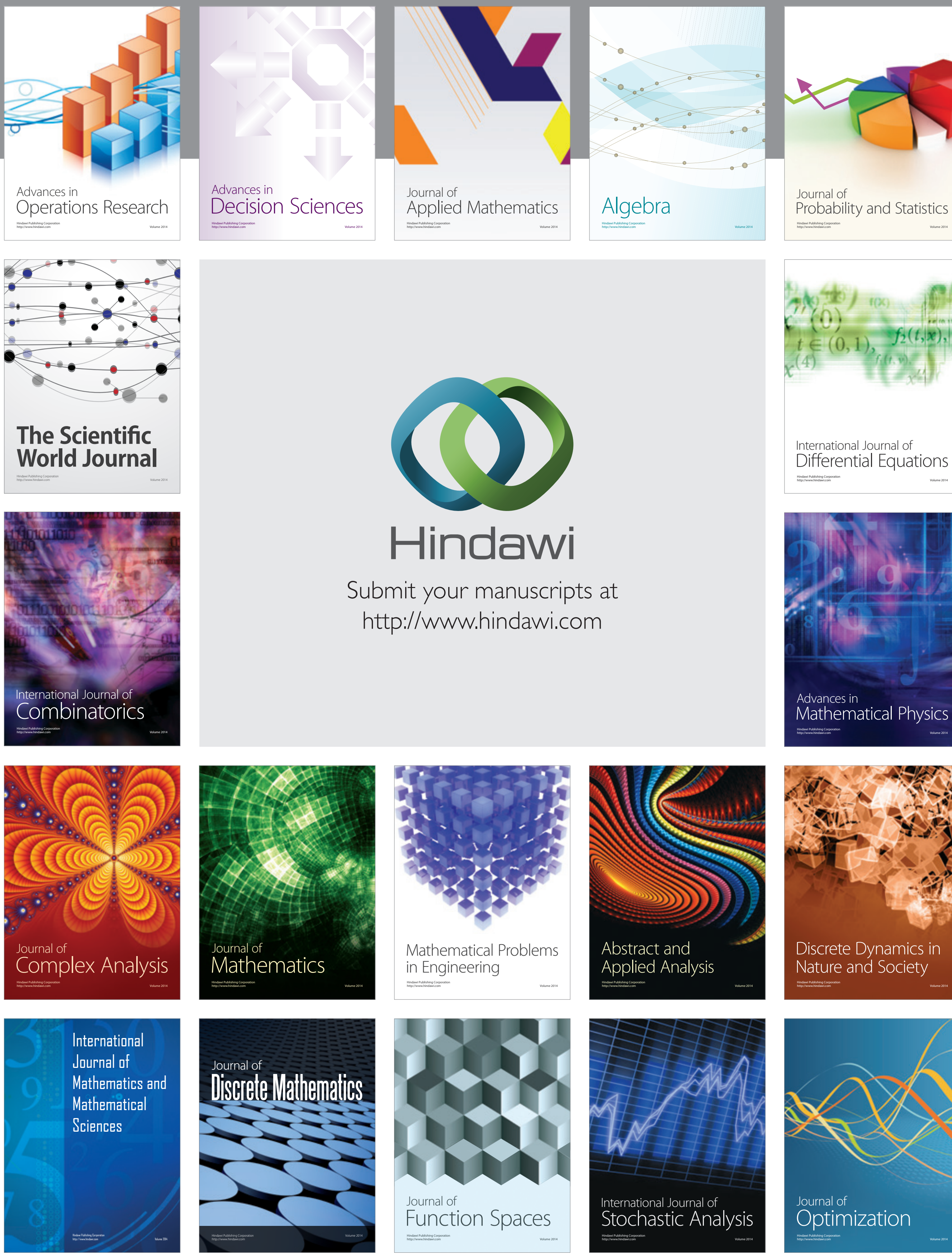
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