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Research Article

Power Series Solution for Solving Nonlinear Burgers-Type Equations

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Power series solution method has been traditionally used to solve ordinary and partial linear differential equations. However, despite their usefulness the application of this method has been limited to this particular kind of equations. In this work we use the method of power series to solve nonlinear partial differential equations. The method is applied to solve three versions of nonlinear time-dependent Burgers-type differential equations in order to demonstrate its scope and applicability.

1. Introduction

Power series solution (PSS) method is an old method that has been limited to solve linear differential equations, both ordinary differential equations (ODE) [1, 2] and partial differential equations (PDE) [3, 4]. Linear PDE have traditionally been solved using the separation of variables method because it permits obtaining a coupled system of ODE easier to solve with the PSS method. Some examples of these are the Legendre polynomials and the spherical harmonics used in Laplace's equations in spherical coordinates or in Bessel's equations in cylindrical coordinates [3, 4]. It is known that in nonlinear PDE (NLPDE) this procedure is not possible.

In this work we compare the spectral method (SM) with the PSS method solving three versions of nonlinear time-dependent Burgers-type equations [5] because we know that the SM is the more accurate numerical method. The SM with collocation points (SMCP) is a numerical technique applied to solve linear and nonlinear differential equations with high accurate approximations to the solution [6]. This has been used to solve PDE using polynomial interpolation function with an orthogonal basis such as Fourier, Chebyshev, or Legendre functions [7]. The SM has also been very successful to solve any kind of DE problems, including integro-differential problems [8], with Newman boundary values [9], and nonlinear PDE [10].

We use the symbolic computation package Matlab to obtain the algebraic operations for the truncated series approximation. This program helps to do easier the tedious algebraic operations.

2. Power Series Solution Method

We know that almost the totality of the NLPDE does not have a solution with an analytic expression, that is, a solution in a closed form of known functions. Our goal is to construct a solution using a power series, taking advantage of the capacity of power series to represent any function with algebraic series developing the idea to construct an approximate solution [11–17]. It also has the possibility to approximate a solution, inclusive if an analytic form does not exist, in a similar way like the Taylor's series approximate the functions. The existence of the PSS does not guarantee per se that the represented function has an exact approximation in distant points relative to the central value. However, considering that the PSS needs to satisfy the NLPDE, with initial values condition (IVC) or with boundary values conditions (BVC), therefore we can construct a well posed problem to obtain an accurate solution, constrained with all these limiting conditions [18]. Furthermore, the polynomial of the PSS is a smoothed function and this can guarantee the existence of a solution [18].

The PSS method represents a general solution with a series of unknown coefficients. When the PSS polynomial is substituted in the PDE we obtain a recurrence relation for the expansion coefficients. These coefficients should be expressed in function of the coefficients result from IVC or BVC. In this way, we obtain a system of equations depending on these initial value based coefficients. In order to obtain and solve a consistent algebraic system of equations, we also need the same number of coefficients and equations [11]. All these conditions, in the beginning, provide a guarantee that the PDE is a well posed problem; that is, existence, uniqueness, and smoothness of the solution are well defined [18].

Finally, the PSS method is a proposal to find a semianalytic solution as an asymptotic approximation (in space and time) of a finite series with minimal error in the expansion of terms of the series. From numerical analysis when a power series $\sum_k a_k x^k$ converges on an interval $(-c, c)$ to a function f, the radius of convergence is c. In our work, the radius of convergence is defined by each interval where our error was estimated as we see below.

3. Numerical Results

First, we consider the nonlinear time-dependent one-dimensional generalized Burgers-Huxley equation [5]:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} - \nu U^\delta \frac{\partial U}{\partial x} + \eta U\left(1 - U^\delta\right)\left(U^\delta - \gamma\right) \quad \text{with } (x,t) \in [A,B] \times [0,t] \tag{1}$$

with the initial condition

$$U(x, t = 0) = \frac{\gamma}{2} + \frac{\gamma}{2}\tanh\left[x\gamma\frac{-\nu\delta + \delta\sqrt{\nu^2 + 4\eta(1+\delta)}}{4(\delta+1)}\right]^{1/\delta}, \tag{2}$$

where γ, ν, η, and δ are real parameters. With $\delta = 1$, this equation admits a travelling wave solution. Then $U(x,t)$ reads

$$U(x,t) = \phi(z), \tag{3}$$

with $z = kx - wt$, where k and w represent the wave number and frequency of the travelling wave, respectively, working as unknown variables. Introducing (3) in (1) we obtain

$$-w\frac{d\phi}{dz} = k^2\frac{d^2\phi}{dz^2} - k\eta\phi\frac{d\phi}{dz} + \eta\phi(1-\phi)(\phi-\gamma). \tag{4}$$

The ansatz for (4) will be a PSS as

$$\phi(z) = \sum_{n=0}^{\infty} a_n z^n. \tag{5}$$

The respective derivatives and nonlinear terms in (4) result in

$$\begin{aligned}
\frac{d\phi}{dz} &= \sum_{k=0}^{\infty}(k+1)a_{k+1}z^k, \\
\frac{d^2\phi}{dz^2} &= \sum_{k=0}^{\infty}(k+1)(k+2)a_{k+2}z^k, \\
\phi\frac{d\phi}{dz} &= \sum_{k=0}^{\infty}\left(\sum_{i=0}^{k} k a_k a_{k+1-i}\right)z^k, \\
\phi^2 &= \sum_{k=0}^{\infty}\left(\sum_{i=0}^{k} a_k a_{k-i}\right)z^k, \\
\phi^3 &= \sum_{k=0}^{\infty}\left[\sum_{m=0}^{k}\left(\sum_{i=0}^{m} a_m a_{m-i}\right)a_{k-m}\right]z^k.
\end{aligned} \tag{6}$$

Substituting the series of (6) in (4), we obtain the recurrence relation

$$\begin{aligned}
a_{k+2} = {} & \frac{1}{k^2(k+1)(k+2)} \\
& \cdot \left[-k\nu\sum_{i=0}^{k} k a_k a_{k+1-i} - w(k+1)a_{k+1}\right. \\
& - \eta(\gamma+1)\sum_{i=0}^{k} a_k a_{k-i} + \gamma\eta a_k \\
& \left. + \eta\sum_{m=0}^{k}\left(\sum_{i=0}^{m} a_m a_{m-i}\right)a_{k-m}\right].
\end{aligned} \tag{7}$$

Solving with Matlab until degree $n = 3$ of PSS from (4), we obtain the following values for the coefficients:

$$\begin{aligned}
a_2 = {} & -\left(a_1 w + a_0^2\eta - a_0^3\eta - a_0\eta\gamma + a_0^2\eta\gamma\right. \\
& \left. - a_0 a_1 k\nu\right)\left(2k^2\right), \\
a_3 = {} & \left(a_1 w^2 + a_1^2 k^3\gamma + a_0^2\eta w - a_0^3\eta w\right. \\
& - a_0\eta\gamma w + a_0^2 a_1 k^2\nu^2 - 2a_0 a_1\eta k^2 \\
& + a_1\eta\gamma k^2 - a_0^3\eta k\nu + a_0^4\eta k\nu + a_0^2\eta\gamma w \\
& + 3a_0^2 a_1\eta k^2 - 2a_0 a_1 k\nu w - 2a_0 a_1\eta\gamma k^2 \\
& \left. + a_0^2\eta\gamma\nu - a_0^3\eta\gamma k\nu\right)\left(6k^4\right) \\
& \vdots
\end{aligned} \tag{8}$$

We will use the initial conditions to obtain the unknown coefficients (8). From the initial condition (2), we express

$U(x,0)$ as a polynomial series applying Taylor's theorem. Then

$$U(x,0) = \frac{\gamma}{2} + \frac{\gamma}{2}\left(x\gamma\frac{-\nu+\sqrt{\nu^2+8\eta}}{8} - \frac{1}{3}\left(x\gamma\frac{-\nu+\sqrt{\nu^2+8\eta}}{8}\right)^3 + \frac{2}{15}\left(x\gamma\frac{-\nu+\sqrt{\nu^2+8\eta}}{8}\right)^5 - \cdots \right). \quad (9)$$

Matching the coefficients of this polynomial with the coefficients (8) of our ansatz, we obtain the next values: $a_0 = \gamma/2$, $a_1 = \gamma/2$, $a_2 = 0$, $a_3 = -\gamma/6$, $a_4 = 0$, $a_5 = \gamma/15$, $a_6 = 0$, $a_7 = -17\gamma/630$, and so forth.

With a_2 and a_3 values matched to their respective coefficients in (8), we obtain an algebraic system of 2 equations with two variables. Solving this one, we obtain the value of the unknown variables k and w:

$$k = \gamma\frac{-\nu+\sqrt{\nu^2+8\eta}}{8}, \quad w = \frac{\eta\gamma}{2} - \frac{\eta\gamma^2}{4} - \frac{\gamma^2\nu^2}{16} + \frac{\gamma^2\nu\sqrt{\nu^2+8\eta}}{8}. \quad (10)$$

Then, the complete solution as PSS for the NLDE (1) reads

$$U(x,t) = \frac{\gamma}{2} + \frac{\gamma}{2}\left((kx-wt) - \frac{1}{3}(kx-wt)^3 + \frac{2}{15}(kx-wt)^5 - \frac{17}{315}(kx-wt)^7 + \cdots \right). \quad (11)$$

As it usually does when an approximate solution with PSS is obtained, a test of accuracy of the approximation must be performed. In this way, we calculate the absolute difference between exact and approximated solution defined as $E(x,t) = |U(x,t) - U'(x,t)|$, where U is the exact solution obtained from [5], and U' is the calculated solution (11), at the point (x,t), until the power degree $n = 19$, respectively. We compute the error, with the parameters values $\gamma = 0.001$, $\nu = 1$, and $\eta = 1$, within the intervals $x = [-2000, 2000]$ and $t = [0, 1000]$. This result is shown in Figure 1. This parameter set was selected because it is the same one used in [5] to do a comparison. The convergence of the power series, $\phi(z) = \sum_{n=0}^{\infty} a_n z^n$, depends on z and also the coefficient a_n, and then it is possible to adjust these ones to solve the NLDE and to find a solution that approximates its behavior to any distance and time, at less in the interval where we calculate the solution.

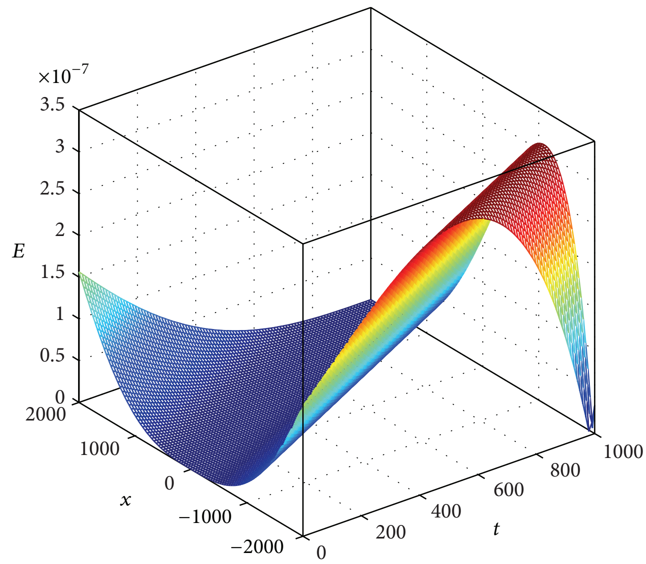

Figure 1: Absolute error of Burgers-Huxley equation, (1), with $\delta = 1$, $\gamma = 0.001$, $\nu = 1$, and $\eta = 1$.

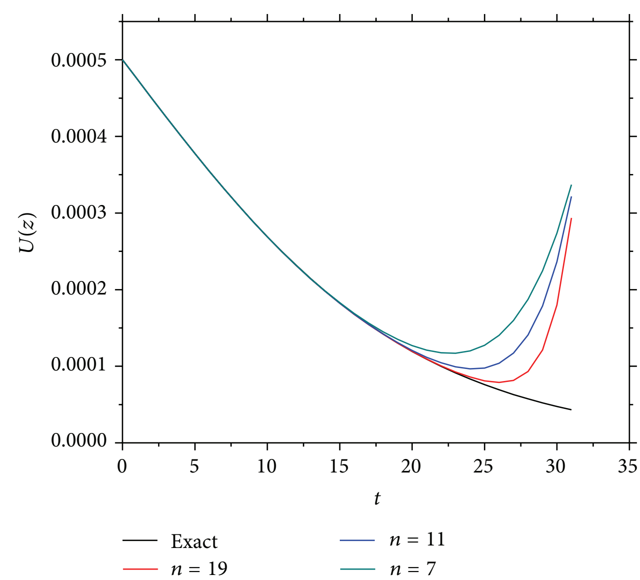

Figure 2: Trajectories of the approximate solution of the Burgers-Huxley equation, (1), with $\delta = 1$, $\gamma = 0.001$, $\nu = 1$, and $\eta = 1$, for $x = 0.9$.

We also compare the approximated solution with the spatial part fixed in $x = 0.9$, with different power degree polynomial ($n = 7, 11, 19$), relative to the exact result. This comparison is shown in Figure 2. From this figure we note that the improvement is better when the power degree increases. In a similar way, we solved (1) with $\delta = 2$ and $\delta = 3$. For $\delta = 2$ and $\gamma = \nu = \eta = 0.001$ in the intervals $x = [-100, 100]$ and $t = [0, 1000]$ we calculate the error shown in Figure 3. Finally, in Figure 4 we showed the absolute error of (1) for $\delta = 3$ with $\gamma = \nu = \eta = 0.001$ in the intervals $x = [-300, 300]$ and $t = [0, 10000]$.

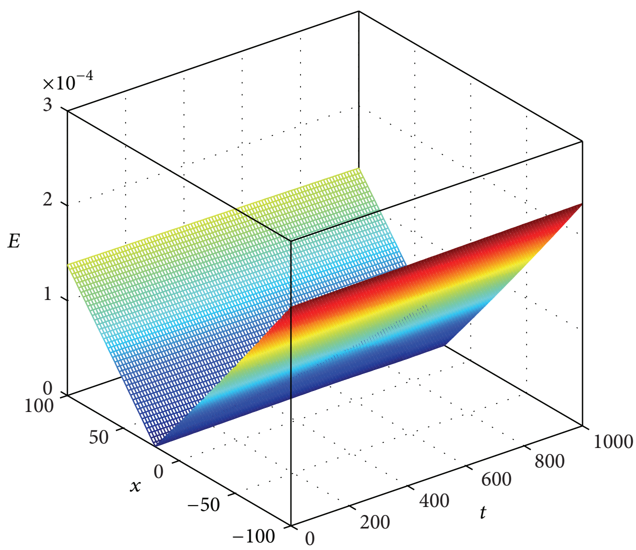

FIGURE 3: Absolute error of Burgers-Huxley equation, (1), with $\delta = 2$ and $\gamma = \nu = \eta = 0.001$.

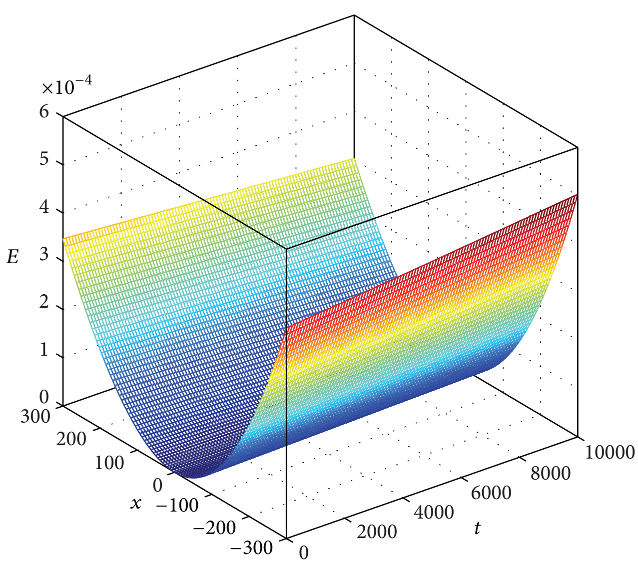

FIGURE 4: Absolute error of Burgers-Huxley equation, (1), with $\delta = 3$ and $\gamma = \nu = \eta = 0.001$.

For our second example, let us consider the nonlinear time-dependent one-dimensional Burgers-type equation [5]:

$$\frac{\partial U}{\partial t} + \nu U \frac{\partial U}{\partial x} - \mu \frac{\partial^2 U}{\partial x^2} = 0 \quad \text{with} \quad (x,t) \in [A,B] \times [0,t], \tag{12}$$

where c, ν, and μ are real parameters, with the initial condition $U(x, t = 0) = c/\nu - (c/\nu)\tanh[(c/2\mu)x]$. As in the first case, (12) admits a travelling wave solution. Performing the transformation

$$U(x,t) = \phi(z), \tag{13}$$

where $z = kx - wt$ and k and w are unknown parameters, and replacing (13) in (12) then

$$-w\frac{d\phi}{dz} - k^2\mu\frac{d^2\phi}{dz^2} + k\nu\phi\frac{d\phi}{dz} = 0. \tag{14}$$

We use the following PSS ansatz for (14):

$$\phi(z) = \sum_{n=0}^{\infty} a_n z^n, \tag{15}$$

then

$$\begin{aligned} \frac{d\phi}{dz} &= \sum_{n=1}^{\infty} n a_n z^{n-1}, \\ \frac{d^2\phi}{dz^2} &= \sum_{n=2}^{\infty} n(n-1) a_n z^{n-2}, \\ \phi\frac{d\phi}{dz} &= \sum_{n=0}^{\infty}\left(\sum_{i=0}^{k} n a_n a_{n+1-i}\right) z^n; \end{aligned} \tag{16}$$

or moving conveniently the sum index n to k, then

$$\begin{aligned} \frac{d\phi}{dz} &= \sum_{k=0}^{\infty} (k+1) a_{k+1} z^k, \\ \frac{d^2\phi}{dz^2} &= \sum_{k=0}^{\infty} (k+1)(k+2) a_{k+2} z^k, \\ \phi\frac{d\phi}{dz} &= \sum_{k=0}^{\infty}\left(\sum_{i=0}^{k} k a_k a_{k+1-i}\right) z^k. \end{aligned} \tag{17}$$

Substituting the above series into (14), we have

$$\begin{aligned} a_{k+2} = {} & \frac{1}{k^2\mu(k+1)(k+2)} \\ & \cdot \left[k\nu \sum_{i=0}^{k} k a_k a_{k+1-i} - w(k+1) a_{k+1}\right]. \end{aligned} \tag{18}$$

This recurrence relation is used in the PSS until the degree $n = 5$. Using Matlab to solve the recurrence algebraic system, we obtain

$$a_2 = -\left(a_1 w - a_0 a_1 k \nu\right)\left(2k^2\mu\right),$$

$$a_3 = \frac{\left(a_0^2 a_1 k^2 \nu^2 - 2a_0 a_1 k \nu w + \mu a_1^2 k^3 \nu + a_1 w^2\right)}{\left(6k^4 * \mu^2\right)},$$

$$a_4 = -\left(-a_0^3 a_1 k^3 \nu^3 + 3a_0^2 a_1 k^2 \nu^2 w - 4\mu a_0 a_1^2 k^4 \nu^2 - 3a_0 a_1 k \nu w^2 + 4\mu a_1^2 k^3 \nu w + a_1 w^3\right) \cdot \left(24k^6 \mu^3\right)^{-1},$$
$$a_5 = \left(a_0^4 a_1 k_4 \nu^4 - 4a_0^3 a_1 k^3 \nu^3 w + 11a_0^2 a_1^2 k^5 \mu \nu^3 + 6a_0^2 a_1 k^2 \nu^2 w^2 - 22a_0 a_1^2 k^4 \mu \nu^2 w - 4a_0 a_1 k \nu w^3 + 4a_1^3 k^6 \mu^2 \nu^2 + 11a_1^2 k^3 \mu \nu w^2 + a_1 w^4\right) \cdot \left(120k^8 \mu^4\right)^{-1},$$
$$\vdots \qquad (19)$$

Expanding the initial condition of (12) as a polynomial through the Taylor series,

$$U(x,0) = \frac{c}{\nu} - \frac{c}{\nu}\left(\frac{c}{2\mu}x - \frac{1}{3}\left(\frac{c}{2\mu}x\right)^3 + \frac{2}{15}\left(\frac{c}{2\mu}x\right)^5 - \frac{17}{315}\left(\frac{c}{2\mu}x\right)^7 + \cdots\right). \qquad (20)$$

Matching the coefficients of (20) with the coefficients of solution (19), we arrive to

$$a_0 = \frac{c}{\nu}, \quad a_1 = -\frac{c}{\nu}, \quad a_2 = 0, \quad a_3 = \frac{c}{3\nu}, \quad a_4 = 0, \quad a_5 = -\frac{2c}{15\nu}, \quad a_6 = 0, \quad a_7 = \frac{17c}{315\nu}, \text{ and so forth.} \qquad (21)$$

The a_2 and a_3 matched values are used to get a system of 2 equations with 2 variables, w and k. Solving this one, we obtain the value of the unknown variables w and k:

$$w = \frac{c^2}{2\mu}, \qquad k = \frac{c}{2\mu}, \qquad (22)$$

and the complete solution in (15) reads

$$U(x,t) = \frac{c}{\nu} - \frac{c}{\nu}\left(\left(\frac{c}{2\mu}x - \frac{c^2}{2\mu}t\right) - \frac{1}{3}\left(\frac{c}{2\mu}x - \frac{c^2}{2\mu}t\right)^3 + \frac{2}{15}\left(\frac{c}{2\mu}x - \frac{c^2}{2\mu}t\right)^5 - \frac{17}{315}\left(\frac{c}{2\mu}x - \frac{c^2}{2\mu}t\right)^7 + \cdots\right). \qquad (23)$$

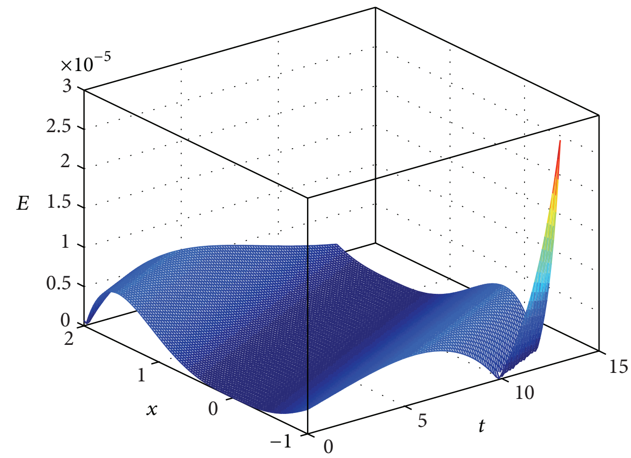

FIGURE 5: Absolute error of Burgers-type equation, (12), with $\nu = 10$, $\mu = 0.1$, and $c = 0.1$.

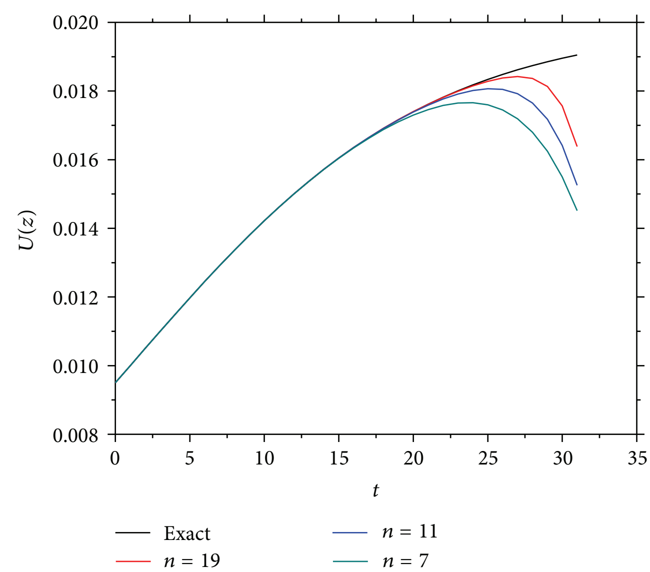

FIGURE 6: Trajectories of the approximate solution of the Burgers-type equation, (12), with $\nu = 10$, $\mu = 0.1$, and $c = 0.1$, for $x = 0.9$.

As in the previous example, we calculate the error $E(x,t)$ with the parameters values $\nu = 10$, $\mu = 0.1$, and $c = 0.1$ within the intervals $x = [-1, 2]$ and $t = [0, 13]$ until the power degree $n = 19$. This result is shown in Figure 5. In Figure 6 we make the comparison for a fixed $x = 0.9$ in the time interval $t = [0, 30]$ with several power degree polynomials ($n = 7, 11, 19$).

For the third example, we consider the nonlinear time-dependent one-dimensional generalized Burgers-Fisher-type equation [5]:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial x^2} - \nu U^\delta \frac{\partial U}{\partial x} + \gamma U\left(1 - U^\delta\right) \quad \text{with} \ (x,t) \in [A,B] \times [0,t] \qquad (24)$$

with the initial condition

$$U(x,t=0)=\frac{1}{2}-\frac{1}{2}\tanh\left[\frac{\nu}{4}x\right], \tag{25}$$

where ν, γ, and δ are real numbers. When $\delta=1$, this equation admits a travelling wave solution

$$U(x,t)=\phi(z), \tag{26}$$

with $z=kx-wt$. This one is replaced in (24)

$$-w\frac{d\phi}{dz}=k^2\frac{d^2\phi}{dz^2}-k\nu\phi\frac{d\phi}{dz}+\gamma\phi(1-\phi), \tag{27}$$

and we attempt a PSS

$$\phi(z)=\sum_{n=0}^{\infty}a_nz^n, \tag{28}$$

and so the terms of (27) can be written as

$$\begin{aligned}\frac{d\phi}{dz}&=\sum_{k=0}^{\infty}(k+1)a_{k+1}z^k,\\ \frac{d^2\phi}{dz^2}&=\sum_{k=0}^{\infty}(k+1)(k+2)a_{k+2}z^k,\\ \phi\frac{d\phi}{dz}&=\sum_{k=0}^{\infty}\left(\sum_{i=0}^{k}ka_ka_{k+1-i}\right)z^k,\\ \phi^2&=\sum_{k=0}^{\infty}\left(\sum_{i=0}^{k}a_ka_{k-i}\right)z^k.\end{aligned} \tag{29}$$

Substituting (29) into (27), we obtain the recurrence relation

$$\begin{aligned}a_{k+2}=&\frac{1}{k^2(k+1)(k+2)}\\ &\cdot\left[k\nu\sum_{i=0}^{k}ka_ka_{k+1-i}\sum_{i=0}^{k}a_ka_{k-i}\right.\\ &\left.-w(k+1)a_{k+1}-\gamma a_k\right].\end{aligned} \tag{30}$$

This recurrence relation is solved with Matlab until the degree $n=4$ and the coefficients result

$$\begin{aligned}a_2=&-\left(a_0\gamma+a_1w-a_0^2\gamma-a_0a_1k\nu\right)\left(2k^2\right),\\ a_3=&\left(\gamma a_0^3k\nu+a_0^2a_1k^2\nu^2-\gamma a_0^2k\nu-\gamma a_0^2w\right.\\ &+2\gamma a_0a_1k^2-2a_0a_1k\nu w+\gamma a_0w\\ &\left.+a_1^2k^3\nu-\gamma a_1k^2+a_1w^2\right)\cdot\left(6k^4\right)^{-1},\\ a_4=&\left(a_0^4\gamma k^2\nu^2+a_0^3a_1k^3\nu^3+2a_0^3\gamma^2k^2\right.\\ &-a_0^3\gamma k^2\nu^2-2a_0^3\gamma k\nu w+7a_0^2a_1\gamma k^3\nu\\ &-3a_0^2a_1k^2\nu^2w-3a_0^2\gamma^2k^2+2a_0^2\gamma k\nu w\\ &+a_0^2\gamma w^2+4a_0a_1^2k^4\nu^2-5a_0a_1\gamma k^3\nu\\ &-4a_0a_1\gamma k^2w+3a_0a_1k\nu w^2+a_0\gamma^2k^2\\ &-a_0\gamma w^2+2a_12\gamma k^4-4a_1^2k^3\nu w\\ &\left.+2a_1\gamma k^2w-a_1w^3\right)\left(2k^6\right),\\ &\vdots\end{aligned} \tag{31}$$

The initial condition (25) is expressed like a polynomial series applying Taylor's formula

$$\begin{aligned}U(x,0)=&\frac{1}{2}-\frac{1}{2}\left(\frac{\nu}{4}x-\frac{1}{3}\left(\frac{\nu}{4}x\right)^3\right.\\ &\left.+\frac{2}{15}\left(\frac{\nu}{4}x\right)^5-\frac{17}{315}\left(\frac{\nu}{4}x\right)^7+\cdots\right).\end{aligned} \tag{32}$$

Matching the coefficients of this polynomial with those of polynomial from solution (31), we obtain the value of the coefficients: $a_0=1/2$, $a_1=-1/2$, $a_2=0$, $a_3=1/6$, $a_4=0$, $a_5=-2/30$, $a_6=0$, $a_7=17/630$, and so forth. With a_2 and a_3 values matched to their respective coefficients of (31), we obtain a system of 2 equations with 2 variables to solve and to obtain the values of the unknown parameters k and w:

$$k=\frac{\nu}{4},\qquad w=\frac{\nu^2}{8}+\frac{\gamma}{2}. \tag{33}$$

Therefore from the solution (28) reads

$$\begin{aligned}&U(x,t)\\ &=\frac{1}{2}-\frac{1}{2}\left(\left(\frac{\nu}{4}x-\left(\frac{\nu^2}{8}+\frac{\gamma}{2}\right)t\right)-\frac{1}{3}\left(\frac{\nu}{4}x-\left(\frac{\nu^2}{8}+\frac{\gamma}{2}\right)t\right)^3\right.\\ &\qquad\left.+\frac{2}{15}\left(\frac{\nu}{4}x-\left(\frac{\nu^2}{8}+\frac{\gamma}{2}\right)t\right)^5-\cdots\right).\end{aligned} \tag{34}$$

We computed the error with (34) and the exact solution presented in [5] with the parameters values $\gamma=0.01$ and

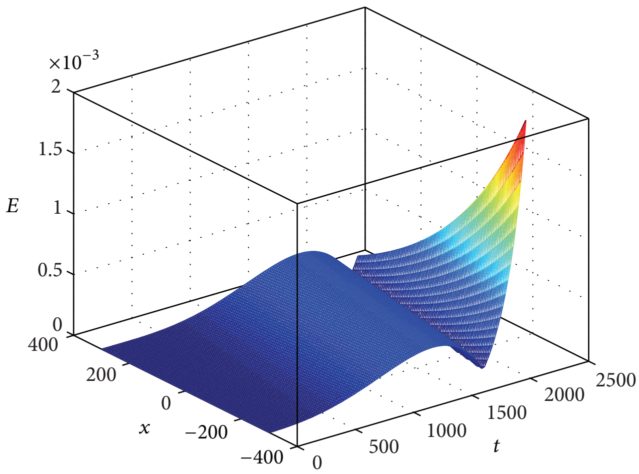

FIGURE 7: Absolute error of generalized Burgers-Fisher-type equation, (24), with $\delta = 1$, $\gamma = 0.01$, and $\nu = 0.01$.

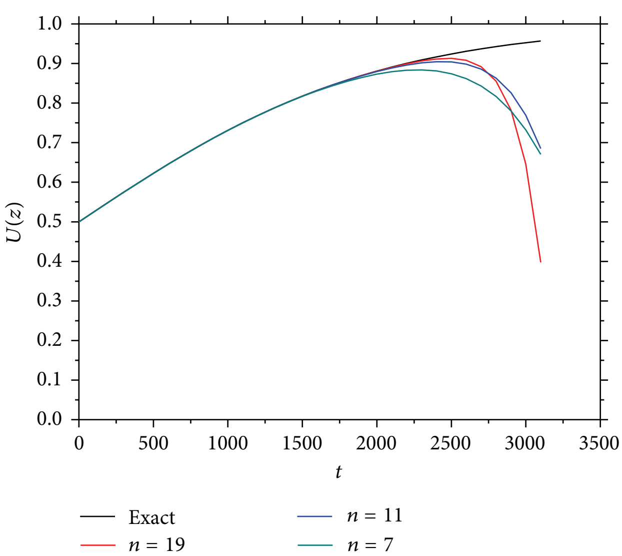

FIGURE 8: Trajectories of the approximate solution of generalized Burgers-Fisher-type equation, (24), with $\delta = 1$, $\gamma = 0.01$, and $\nu = 0.01$, for $x = 0.9$.

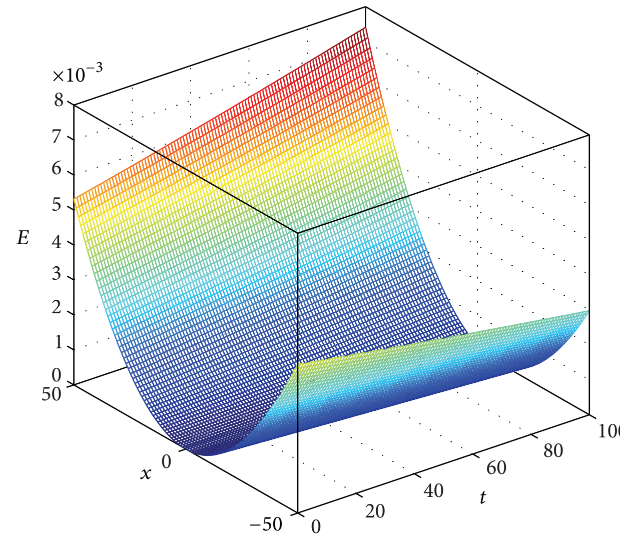

FIGURE 9: Absolute error of generalized Burgers-Fisher-type equation, (24), with $\delta = 2$, $\gamma = 0.001$, and $\nu = 0.25$.

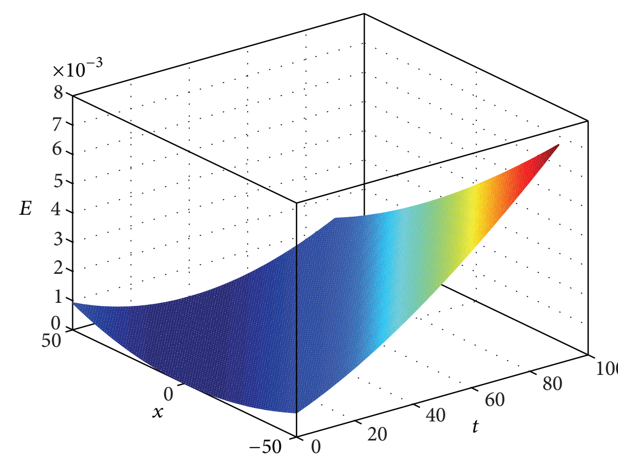

FIGURE 10: Absolute error of generalized Burgers-Fisher-type equation, (24), with $\delta = 3$, $\gamma = 0.001$, and $\nu = 1$.

$\nu = 0.01$ within the intervals $x = [-300, 300]$ and $t = [0, 2200]$ until the power degree $n = 19$. This error is shown in Figure 7. Figure 8 shows the trajectories of the approximate solution of problem (24) where $\delta = 1$, for $x = 0.9$, and $\gamma = 0.01$ and $\nu = 0.01$, in the time interval $t = [0, 3000]$, with the power degree polynomial ($n = 7, 11, 19$). In Figure 9, the error is shown with $\delta = 2$, and $\gamma = 0.001$ and $\nu = 0.25$, in the intervals $x = [-50, 50]$ and $t = [0, 100]$. In Figure 10, the computed error is shown for $\delta = 3$, with $\gamma = 0.001$ and $\nu = 1$, in the intervals $x = [-50, 50]$ and $t = [0, 100]$.

We remark that Figures 2, 6, and 8 are shown simple trajectories of the approximate solution due to the fact that they are the solution of this NLDE in the time interval with the axis x fixed (to compare with [5]) where the behavior is smoother. These are nonlinear equations where the error grows in an exponential way, at difference of the behavior of linear equations. We show that by adding more terms to the ansatz series, the error is reduced and the solution is extended to long times and an increased space in comparison with other methods such as the spectral method that is one of the most accurate numerical methods. The PSS method is employed to more complex trajectories as shown in [13], where the method is used to solve chaotic NLDE.

4. Discussion and Conclusions

In this work we have shown that it is possible to solve nonlinear differential equations with the power series solution method. This method is implemented as a general approximate solution for each nonlinear PDE or ODE, in a similar way to the solution of a Linear DE. We transform a nonlinear DE problem into an algebraic system of equations.

Therefore, with this method, it is possible to obtain a well posed problem when the system of DE is consistent. It means that the system has the same number of variables as equations, and consequently the number of algebraic variables (coefficients of expansion series) obtained with this method is the same as the number of equations.

We had shown that the PSS method is a semianalytic technique that leads, in an easier and exact way, to the solution of difficult differential equations with an approximated closed form expression. This is especially useful to solve nonlinear equations and leaves open the possibility to describe in an exact and convergent way the behavior of chaotic dynamical systems [13, 15]. Then, the solution can be approximated until the necessary degree into the power series. The convergence of the PSS method depends on the number of terms used in the power series. Once we know it, we can determine the domain of the space and time where the solution is valid. Three examples of the nonlinear time-dependent one-dimensional Burgers-Huxley, Burgers-type, and Burgers-Fisher-type equations were solved within this capable method. The errors between the exact analytic solution from [5] and our PSS are compared in each case. We obtained a better approximation in larger space and time intervals ($\sim 10^{-3}$–10^{-7}) with PSS of order $n = 19$ but not with the accuracy of [5] in a small space and time interval of the solution.

The advantages of the PSS method over the traditional methods are the same ones that are presented in the work with respect to the spectral method that solves any kind of NLDE. It means that it can solve NLDE with higher accuracy in an interval of time and space than the other traditional methods, adding more terms to the series.

In summary, we have shown that the PSS method is a technique which can be used to solve this kind of nonlinear differential equations [11]. In this work, the main goal was to solve the differential partial equation nonlinear Burgers type with dimension 1 (spatial) + 1 (time), doing a transformation of this one to a single nonlinear equation. We compared the PPS solution with that coming from the spectral method. We believe that it is possible to extend the solution to 2 or 3 spatial dimensions plus the time dimension. This possibility will be considered in future works. The procedure provides a much greater accuracy adding more terms into the series. The PSS method opens the possibility to analyze other characteristics of the NLPDE, obtaining better semianalytic approximations that involve less computational efforts.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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