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Research Article

H_∞ Control for Stochastic Systems with Markovian Switching and Time-Varying Delay via Sliding Mode Design

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This paper addresses the problem of H_∞ control for a class of uncertain stochastic systems with Markovian switching and time-varying delays. The system under consideration is subject to time-varying norm-bounded parameter uncertainties and an unknown nonlinear function in the state. An integral sliding surface corresponding to every mode is first constructed, and the given sliding mode controller concerning the transition rates of modes can deal with the effect of Markovian switching. The synthesized sliding mode control law ensures the reachability of the sliding surface for corresponding subsystems and the global stochastic stability of the sliding mode dynamics. A simulation example is presented to illustrate the proposed method.

1. Introduction

Many practical dynamics, for example, manufacturing systems, chemical process systems, computer controlled systems and communication systems, solar receiver control, and power systems, experience abrupt random changes in their structure. These changes are usually caused by random failure or repairs of the components, changing in subsystems interconnections, sudden environmental changes, and so forth. Such systems can be modeled as hybrid systems. One special class of hybrid systems is named Markovian jump systems (MJSs), whose system mode is governed by a Markov process. During the past decades, due to their potential applications in manufacturing systems and communications, considerable attention has been paid to the problems of stability and stabilization [1–5], H_∞ control and filtering [6–8], optimal tracking problem [9], and so on. The robust stability and stabilization of uncertain stochastic systems with time-varying delays are investigated by using the linear matrix inequality approach (LMI) [10]. Some stability criteria are obtained for a class of bilinear continuous time-delay uncertain systems with Markovian jump parameter [11]. In [12], a robust H_∞ controller is designed for uncertain systems with state delay. Huang and Mao [13] investigated the stabilization of stochastic linear systems by delayed state feedback controller. In [14], the method of Lyapunov functional is employed to study p-moment stability of nonlinear stochastic systems with impulsive and Markovian switching.

The SMC for stochastic systems has received an increasing attention. K. Chang and W. Chang [15] developd an SMC method to guarantee the robust state covariance assignment for perturbed stochastic multivariable systems via variable structure control. In [16], robust observer design for Ito stochastic time-delay systems has been studied via sliding mode control and the sufficient conditions for the asymptotic stability (in probability) of the sliding motion are derived. Niu et al. [17] paid some efforts to coping with the connection among the designed sliding surface corresponding to every mode for MJSs. However, there are little results reported on the SMC of stochastic systems with Markovian switching. The existence of uncertainties, time-varying delays, Markovian switching, and bilinear perturbations will make the problem more complex and challenging.

This paper studies the robust H_∞ control problem for uncertain stochastic systems with time-varying delays and Markovian switching. The systems under consideration may contain time-varying parameter uncertainties and bilinear stochastic perturbations, nonlinearities, and external disturbance. An integral-type sliding surface is constructed. Due to the existence of Markovian switching, in the design of sliding surface, a set of specified matrices are given to establish the connections among sliding surfaces corresponding to every mode. SMC law is synthesized to guarantee that the trajectories can be driven onto the specified sliding surfaces for each mode in finite time and the dynamics along the sliding surface for each mode is stochastically stable.

2. Preliminaries

Let (Ω, F, P) be a probability space with Ω the sample space, F the σ-algebra of subsets of the sample space, and P the probability measure. $\|\cdot\|$ and $\|\cdot\|_1$ denote the Euclidean norm and 1-norm of a vector or its induced matrix norm, respectively. If $a \in R^n$, we have that $\|a\| < \|a\|_1$. $\|\cdot\|_2$ stands for the usual $L_2[0,\infty]$ norm. For a real matrix M, $M > 0$ means that M is symmetric positive definite. I is the identity matrix with compatible dimension.

The Markov process $\{r_t,\ t \geq 0\}$ represents the switching between the different modes taking values in a finite state space $S = \{1, 2, \dots, N\}$ with generator $\pi = (\pi_{ij})_{N\times N}$ given by

$$\Pr\{r_{t+\Delta} = j \mid r_t = i\} = \begin{cases} \pi_{ij}\Delta + \mathrm{o}(\Delta), & \text{if } i \neq j \\ 1 + \pi_{ii}\Delta + \mathrm{o}(\Delta), & \text{if } i = j, \end{cases} \quad (1)$$

where π_{ij} is the transition rate from mode i to j and satisfies the following relations:

$$\pi_{ij} \geq 0, \qquad \pi_{ii} = -\sum_{j\neq i} \pi_{ij}, \quad (2)$$

and $\mathrm{o}(\Delta)$ is such that $\lim_{\Delta\to 0}\mathrm{o}(\Delta)/\Delta = 0$.

Consider the following stochastic system with Markovian switching and time-varying delay:

$$\begin{aligned} dx(t) = {} & \Big[\big(A(r_t) + \widetilde{A}(r_t)\big)x(t) \\ & + \big(A_d(r_t) + \widetilde{A}_d(r_t)\big)x(t-\tau(t)) \\ & + B(r_t)\big(u(t) + f(t, x(t), r_t)\big) + B_v(r_t)v(t)\Big]dt \\ & + D(r_t)\Big[\big(C(r_t) + \widetilde{C}(r_t)\big)x(t) \\ & + \big(C_d(r_t) + \widetilde{C}_d(r_t)\big)x(t-\tau(t))\Big]d\omega(t), \\ z(t) = {} & \big(E(r_t) + \widetilde{E}(r_t)\big)x(t) + F(r_t)v(t), \\ x(t) = {} & \varphi(t), \quad t \in [-d, 0], \end{aligned} \quad (3)$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control input, $z(t) \in R^p$ is the controlled output, $v(t) \in R^q$ is the exogenous disturbance input belonging to $L_2[0,\infty] \cap L_\infty[0,\infty]$, and $\omega(t)$ is a one-dimensional Brownian motion defined on the probability space (Ω, F, P). $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $B_v(r_t)$, $C(r_t)$, $C_d(r_t)$, $D(r_t)$, $E(r_t)$, and $F(r_t)$ are known real constant matrices with appropriate dimensions. In general, it is assumed that the pair $(A(r_t), B(r_t))$ is controllable and the input matrix $B(r_t)$ has full column rank. $\tau(t)$ is the time-varying delay satisfying

$$0 < \tau(t) \leq d < \infty, \quad \dot{\tau}(t) \leq h < 1, \quad (4)$$

where d and h are known real constant scalars, and $\varphi(t)$ is a continuous vector-valued initial function. $\widetilde{A}(r_t)$, $\widetilde{A}_d(r_t)$, $\widetilde{C}(r_t)$, $\widetilde{C}_d(r_t)$, and $\widetilde{E}(r_t)$ are parameter uncertainties, and unknown function $f(t, x(t), r_t)$ is an unknown nonlinear uncertainties.

For each $r_t = i \in S$, for the sake of convenience, denote $A(r_t) = A_i$ and $\widetilde{A}(r_t) = \widetilde{A}_i(t)$. Other matrices are defined as above. Then system (3) becomes

$$\begin{aligned} dx(t) = {} & \Big[\big(A_i + \widetilde{A}_i(t)\big)x(t) + \big(A_{di} + \widetilde{A}_{di}(t)\big)x(t-\tau(t)) \\ & + B_i\big(u(t) + f(t, x(t), i)\big), + B_{vi}v(t)\Big]dt \\ & + D_i\Big[\big(C_i + \widetilde{C}_i(t)\big)x(t), \\ & + \big(C_{di} + \widetilde{C}_{di}(t)\big)x(t-\tau(t))\Big]d\omega(t), \end{aligned} \quad (5)$$

$$z(t) = \big(E_i + \widetilde{E}_i(t)\big)x(t) + F_i v(t), \quad (6)$$

$$x(t) = \varphi(t), \quad t \in [-d, 0]. \quad (7)$$

For $r_t = i$, the admissible uncertainties are assumed to be norm bounded and can be described as

$$\begin{aligned} \big[\widetilde{A}_i(t)\ \ \widetilde{A}_{di}(t)\ \ \widetilde{E}_i(t)\big] &= N_{1i}F_{1i}(t)\big[H_{ai}\ \ H_{adi}\ \ H_{ei}\big], \\ \big[\widetilde{C}_i(t)\ \ \widetilde{C}_{di}(t)\big] &= N_{2i}F_{2i}(t)\big[H_{ci}\ \ H_{cdi}\big], \end{aligned} \quad (8)$$

$$\|f(t, x(t), i)\| \leq \eta_i \|x(t)\|, \quad (9)$$

where $\eta_i > 0$ is a constant scalar, H_{ai}, H_{adi}, H_{ci}, H_{cdi}, H_{ei}, N_{1i}, and N_{2i} are known real constant matrices, and $F_{1i}(t)$ and $F_{2i}(t)$ are unknown matrix functions satisfying

$$F_{1i}^T(t)F_{1i}(t) \leq I, \quad F_{2i}^T(t)F_{2i}(t) \leq I, \quad \forall t. \quad (10)$$

In the sequel, some concepts and lemmas about the stability of stochastic systems are used.

Definition 1. System (5)–(7) with $u(t) = 0$ is stochastically stable for $v(t) = 0$, if there exists a constant matrix $G > 0$ such that the following inequality holds for any pair of initial conditions (x_0, r_0)

$$E\left[\int_0^\infty x^T(t)x(t)\,dt \mid x_0,\ \sigma(0) = r_0\right] \leq x_0^T G x_0. \quad (11)$$

Definition 2. For given scalar $\gamma > 0$, the system (5)–(7) is said to stochastically stabilizable and satisfy $\|T_{zv}(s)\|_\infty \leq \gamma$ if the closed-loop system is stochastically stable for $v(t) = 0$ and for all mismatched uncertainties,

$$\|z(t)\|_{E_2} < \gamma\|v(t)\|_{E_2}, \quad (12)$$

holds, where $\|z(t)\|_{E_2} = \left(E\{\int_0^t |z(t)|^2 dt\}\right)^{1/2}$.

Definition 3. Let $C^{2,1}(R^n \times [t_0 - r, \infty] \times S; R^+)$ denote the family of all nonnegative functions $V(x,t,i)$ that are continuously twice differentiable in x and once differentiable with respect to t. For each $V(x,t,i) \in C^{2,1}(R^n \times [t_0 - r, \infty] \times S; R^+)$, define an infinitesimal operator LV from $R^n \times [t_0 - r, \infty] \times S$ to R^+ as follows:

$$\begin{aligned} LV(x,t,i) &= LV(x,t,i) \\ &= \lim_{\Delta \to 0} \frac{1}{\Delta} \left[E\left\{ V\left(x(t+\Delta), r_{t+\Delta}, t+\Delta\right) \mid \right.\right. \\ &\qquad \left.\left. x(t) = x, r_t = i, t \right\} - V(x,i,t) \right]. \end{aligned} \tag{13}$$

The following matrix inequalities will be essential for the proofs in Section 3.

Lemma 4 (see [18]). *Let Q, H, F, and G be real matrices of appropriate dimensions with $F^T F \le I$, then, for any real matrix $Q = Q^T$, there exists scalar $\varepsilon > 0$, and one has the following:*

$$\begin{gathered} Q + HF(t)G + G^T F^T(t) H^T < 0, \\ \forall F(t) \;\; s.t. \; F^T(t) F(t) \le I, \end{gathered} \tag{14}$$

if and only if there exists some scalar $\varepsilon > 0$ such that

$$Q + \varepsilon H H^T + \varepsilon^{-1} G^T G < 0. \tag{15}$$

Lemma 5 (see [18]). *For any real vectors a, b and matrix $X > 0$ of compatible dimensions*

$$a^T b + b^T a \le a^T X a + b^T X^{-1} b. \tag{16}$$

3. Main Results

This section constructs an SMC law $u(t)$ for system (5)–(7) such that the resultant closed-loop system is stochastically stable despite uncertainties, exogenous disturbance, time delay, and Markovian switching.

3.1. Integral-Type Sliding Surface. As the first step of SMC design, the integral-type sliding surface is constructed as follows:

$$\begin{aligned} s(x(t),i) &= B_i^T X_i x(t) \\ &\quad - \int_0^t B_i^T X_i (A_i + B_i K_i) x(\tau)\, d\tau, \end{aligned} \tag{17}$$

for each $r_t = i \in S$. In (17), the matrix $K_i \in R^{m\times n}$ is chosen such that the matrix $A_i + B_i K_i$ is Hurwitz. In particular, the matrix $X_i \in R^{n\times n}$ is to be designed so that $B_i^T X_i B_i$ is nonsingular and $B_i^T X_i D_i = 0$. Under the assumption that B_i is full rank, it can be easily shown that the nonsingularity of $B_i^T X_i B_i$ can be guaranteed if X_i is symmetric positive definite, that is, $X_i > 0$. In addition, the condition $B_i^T X_i D_i = 0$ will be incorporated in Theorem 6.

The solution $x(t)$ of the stochastic system (5)–(7) is expressed as follows:

$$\begin{aligned} x(t) &= \varphi(0) \\ &\quad + \int_0^t \left[\left(A_i + \widetilde{A}_i(s)\right) x(s) + \left(A_{di} + \widetilde{A}_{di}(s)\right) x(s - \tau(s)) \right. \\ &\qquad \left. + B_i \left(u(s) + f(s, x(s), i)\right) + B_{vi} v(s) \right] ds \\ &\quad + \int_0^t D_i \left[\left(C_i + \widetilde{C}_i(s)\right) x(s) \right. \\ &\qquad \left. + \left(C_{di} + \widetilde{C}_{di}(s)\right) x(s - \tau(s)) \right] d\omega(s). \end{aligned} \tag{18}$$

Under the condition that $B_i^T X_i D_i = 0$, it can be obtained from (17) and (18) that

$$\begin{aligned} s&(x(t),i) \\ &= B_i^T X_i \varphi(0) \\ &\quad + \int_0^t B_i^T X_i \left[\left(A_i + \widetilde{A}_i(s)\right) x(s) \right. \\ &\qquad + \left(A_{di} + \widetilde{A}_{di}(s)\right) x(s - \tau(s)) \\ &\qquad \left. + B_i \left(u(s) + f(s, x(s), i)\right) + B_{vi} v(s) \right] ds. \end{aligned} \tag{19}$$

According to the sliding mode theory, we have $s(x(t), i) = 0$ and $\dot{s}(x(t), i) = 0$. The equivalent control law in the sliding mode can be obtained by solving $\dot{s}(x(t), i) = 0$ as

$$\begin{aligned} u_{\text{eq}}(t) &= K_i x(t) - \left(B_i^T X_i B_i\right)^{-1} B_i^T X_i \\ &\quad \times \left[\widetilde{A}_i(t) x(t) + \left(A_{di} + \widetilde{A}_{di}(t)\right) x(t - \tau(t)) \right. \\ &\qquad \left. + B_{vi} v(t) \right] - f(t, x(t), i). \end{aligned} \tag{20}$$

Substituting (20) into (5), the sliding mode dynamics in $s(x(t), i) = 0$ can be written as

$$\begin{aligned} dx(t) &= \left\{ \left[A_i + B_i K_i + \widetilde{A}_i(t) - B_i \left(B_i^T X_i B_i\right)^{-1} B_i^T X_i \widetilde{A}_i(t) \right] x(t) \right. \\ &\quad + \left[A_{di} + \widetilde{A}_{di}(t) - B_i \left(B_i^T X_i B_i\right)^{-1} \right. \\ &\qquad \left. \times B_i^T X_i \left(A_{di} + \widetilde{A}_{di}(t)\right) \right] x(t - \tau(t)) \\ &\quad \left. + \left[B_{vi} v(t) - B_i \left(B_i^T X_i B_i\right)^{-1} B_i^T X_i B_{vi} v(t) \right] \right\} dt \\ &\quad + D_i \left[\left(C_i + \widetilde{C}_i(t)\right) x(t) \right. \\ &\qquad \left. + \left(C_{di} + \widetilde{C}_{di}(t)\right) x(t - \tau(t)) \right] d\omega(t). \end{aligned} \tag{21}$$

Now, the stochastic stability of the sliding mode dynamics described by (21) will be analyzed.

3.2. Stochastic Stability of Sliding Surface

Theorem 6. *Consider the stochastic delay system (5)–(7) with assumptions (4), (8)–(10), and the sliding mode surface (17). For a given scalar $\gamma > 0$, if there exist matrices $X_i > 0$ and $Q > 0$, scalars $\varepsilon_{1i} > 0$, $\varepsilon_{2i} > 0$, $\varepsilon_{3i} > 0$, and $\varepsilon_{4i} > 0$ ($i = 1, 2, \ldots, N$) satisfy the following LMIs:*

$$\begin{bmatrix}
\Pi_{1i} & * & * & * & * & * & * & * & * & * & * & * & * & * & * \\
\Pi_{5i}^T & \Pi_{2i} & * & * & * & * & * & * & * & * & * & * & * & * & * \\
B_{vi}^T X_i & 0 & \Pi_{3i} & * & * & * & * & * & * & * & * & * & * & * & * \\
X_i D_i C_i & X_i D_i C_{di} & 0 & -X_i & * & * & * & * & * & * & * & * & * & * & * \\
N_{2i}^T D_i^T X_i D_i C_i & N_{2i}^T D_i^T X_i D_i C_{di} & 0 & 0 & \Pi_{4i} & * & * & * & * & * & * & * & * & * & * \\
E_i & 0 & F_i & 0 & 0 & -I & * & * & * & * & * & * & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & -X_i & * & * & * & * & * & * & * & * \\
0 & X_i A_{di}^T & 0 & 0 & 0 & 0 & 0 & -X_i & * & * & * & * & * & * & * \\
\sqrt{2} B_i^T X_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -B_i^T X_i B_i & * & * & * & * & * & * \\
B_i^T X_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -B_i^T X_i B_i & * & * & * & * & * \\
N_{1i}^T X_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{1i} I & * & * & * & * \\
N_{1i}^T X_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2i} I & * & * & * \\
0 & 0 & 0 & 0 & 0 & N_{1i}^T X_i & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{4i} I & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & N_{1i}^T X_i & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{4i} I & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & N_{1i}^T X_i & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{4i} I
\end{bmatrix}, \tag{22}$$

$$B_i^T X_i D_i = 0, \tag{23}$$

with

$$\begin{aligned}
\Pi_{1i} &= X_i \left(A_i + B_i K_i\right) + \left(A_i + B_i K_i\right)^T X_i \\
&\quad + Q + \varepsilon_{1i} H_{ai}^T H_{ai} + \varepsilon_{3i} H_{ci}^T H_{ci} + \varepsilon_{4i} H_{ai}^T H_{ai} \\
&\quad + \varepsilon_{4i} H_{ei}^T H_{ei} + \sum_{j=1}^{N} \pi_{ij} X_j, \\
\Pi_{2i} &= -(1-h) Q + \varepsilon_{2i} H_{adi}^T H_{adi} + \varepsilon_{3i} H_{ci}^T H_{ci} + \varepsilon_{4i} H_{adi}^T H_{adi}, \\
\Pi_{3i} &= B_{vi}^T X_i B_{vi} - \gamma^2 I, \\
\Pi_{4i} &= -\varepsilon_{3i} I + N_{2i}^T D_i^T X_i D_i N_{2i}, \\
\Pi_{5i} &= X_i A_{di} + \varepsilon_{3i} H_{ci}^T H_{cdi},
\end{aligned} \tag{24}$$

and then the sliding motion (21) is stochastically stable and satisfies H_∞ performance for all $v(t) \in L_2[0, \infty)$.

Proof. Consider the following Lyapunov function:

$$V(x(t), i) = x^T(t) X_i x(t) + \int_{t-\tau(t)}^{t} x^T(s) Q x(s)\, ds. \tag{25}$$

By Definition 2, the stochastic stability for systems (21) is firstly established with $v(t) = 0$. From Definition 3, the infinitesimal operator $LV(x(t), i)$ for (21) with $v(t) = 0$ is obtained for each $i \in S$

$$\begin{aligned}
&LV(x(t), i) \\
&= 2x^T(t) X_i \Big[A_i + B_i K_i + \widetilde{A}_i(t) \\
&\qquad - B_i \left(B_i^T X_i B_i\right)^{-1} B_i^T X_i \widetilde{A}_i(t)\Big] x(t) + 2x^T(t) X_i \\
&\quad \times \Big[A_{di} + \widetilde{A}_{di}(t) \\
&\qquad - B_i \left(B_i^T X_i B_i\right)^{-1} B_i^T X_i \left(A_{di} + \widetilde{A}_{di}(t)\right)\Big] x(t-\tau(t)) \\
&\quad + \left[\left(C_i + \widetilde{C}_i(t)\right) x(t) + \left(C_{di} + \widetilde{C}_{di}(t)\right) x(t-\tau(t))\right]^T \\
&\quad \times D_i^T X_i D_i \\
&\quad \times \left[\left(C_i + \widetilde{C}_i(t)\right) x(t) + \left(C_{di} + \widetilde{C}_{di}(t)\right) x(t-\tau(t))\right]
\end{aligned}$$

$$+\sum_{j=1}^{N}\pi_{ij}x^T(t)X_jx(t)$$
$$+x^T(t)Qx(t)-(1-\dot{\tau}(t))x^T(t-\tau(t))Qx(t-\tau(t)). \tag{26}$$

Using Lemma 5, one has

$$\begin{aligned}
&2x^T(t)X_i\widetilde{A}_i(t)x(t)\\
&\quad\le \varepsilon_{1i}^{-1}x^T(t)X_iN_{1i}N_{1i}^TX_ix(t)\\
&\qquad+\varepsilon_{1i}x^T(t)H_{ai}^TH_{ai}x(t),\\
&2x^T(t)X_i\widetilde{A}_{di}(t)x(t-\tau(t))\\
&\quad\le \varepsilon_{2i}^{-1}x^T(t)X_iN_{1i}N_{1i}^TX_ix(t)\\
&\qquad+\varepsilon_{2i}x^T(t-\tau(t))H_{adi}^TH_{adi}x(t-\tau(t)),\\
&-2x^T(t)X_iB_i\left(B_i^TX_iB_i\right)^{-1}B_i^TX_i\widetilde{A}_i(t)x(t)\\
&\quad\le x^T(t)X_iB_i\left(B_i^TX_iB_i\right)^{-1}B_i^TX_ix(t)\\
&\qquad+x^T(t)\widetilde{A}_i^T(t)X_i\widetilde{A}_i(t)x(t),\\
&-2x^T(t)X_iB_i\left(B_i^TX_iB_i\right)^{-1}B_i^TX_i\left(A_{di}+\widetilde{A}_{di}(t)\right)x(t-\tau(t))\\
&\quad\le x^T(t)X_iB_i\left(B_i^TX_iB_i\right)^{-1}B_i^TX_ix(t)\\
&\qquad+x^T(t-\tau(t)\left(A_{di}+\widetilde{A}_{di}(t)\right)^T\\
&\qquad\times X_i\left(A_{di}+\widetilde{A}_{di}(t)\right)x(t-\tau(t)).
\end{aligned} \tag{27}$$

From (22), we can obtain $\varepsilon_{3i}I-N_{2i}^TD_i^TX_iD_iN_{2i}>0$. According to assumption (8), it has

$$\begin{aligned}
&\left[D_i\overline{C}_i+D_iN_{2i}F_{2i}(t)\overline{H}_{ci}\right]^TX_i\left[D_i\overline{C}_i+D_iN_{2i}F_{2i}(t)\overline{H}_{ci}\right]\\
&\quad\le \overline{C}_i^TD_i^TX_iD_i\overline{C}_i+\overline{C}_i^TD_i^TX_iD_iN_{2i}\\
&\quad\times\left(\varepsilon_{3i}I-N_{2i}^TD_i^TX_iD_iN_{2i}\right)^{-1}N_{2i}^TD_i^TX_iD_i\overline{C}_i\\
&\quad+\varepsilon_{3i}\overline{H}_{ci}^T\overline{H}_{ci},
\end{aligned} \tag{28}$$

where $\overline{C}_i=[C_i\ C_{di}]$, $\overline{H}_{ci}=[H_{ci}\ H_{cdi}]$. Define $\Phi=[x^T(t),x^T(t-\tau(t))]^T$. Then it follows from (26)–(28) that

$$LV(x(t),i)\le\Phi^T\Gamma_i\Phi, \tag{29}$$

in which

$$\begin{aligned}
\Gamma_i=&\begin{bmatrix}\Xi_{1i} & X_iA_{di}\\ A_{di}^TX_i & \Xi_{2i}\end{bmatrix}+\overline{C}_i^TD_i^TX_iD_i\overline{C}_i\\
&+\overline{C}_i^TD_i^TX_iD_iN_{2i}\left(\varepsilon_{3i}I-N_{2i}^TD_i^TX_iD_iN_{2i}\right)^{-1}\\
&\times N_{2i}^TD_i^TX_iD_i\overline{C}_i+\varepsilon_{3i}\overline{H}_{ci}^T\overline{H}_{ci},
\end{aligned} \tag{30}$$

with

$$\begin{aligned}
\Xi_{1i}=&X_i(A_i+B_iK_i)+(A_i+B_iK_i)^TX_i+Q+\varepsilon_{1i}H_{ai}^TH_{ai}\\
&+\varepsilon_{1i}^{-1}X_iN_{1i}N_{1i}^TX_i+\varepsilon_{2i}^{-1}X_iN_{1i}N_{1i}^TX_i+\widetilde{A}_i^T(t)X_i\widetilde{A}_i(t)\\
&+2X_iB_i\left(B_i^TX_iB_i\right)^{-1}B_i^TX_i,\\
\Xi_{2i}=&-(1-h)Q+\varepsilon_{2i}H_{adi}^TH_{adi}\\
&+\left(A_{di}+\widetilde{A}_{di}(t)\right)^TX_i\left(A_{di}+\widetilde{A}_{di}(t)\right).
\end{aligned} \tag{31}$$

So, if $\Gamma_i<0$, for any $\Phi\neq0$, we have

$$LV(x(t),i)<0. \tag{32}$$

According to Definition 1, the sliding mode dynamics (21) is stochastically stable.

By Schur's complement, $\Gamma_i<0$ is equivalent to the following LMI:

$$\begin{bmatrix}
\Xi_{1i}^* & * & * & * & * & * & * & * & *\\
\Pi_{5i}^T & \Xi_{2i}^* & * & * & * & * & * & * & *\\
X_iD_iC_i & X_iD_iC_{di} & -X_i & * & * & * & * & * & *\\
E_{2i}^TD_i^TX_iD_iC_i & E_{2i}^TD_i^TX_iD_iC_{di} & 0 & \Pi_{3i} & * & * & * & * & *\\
0 & 0 & 0 & 0 & -X_i & * & * & * & *\\
0 & X_iA_{di} & 0 & 0 & 0 & -X_i & * & * & *\\
\sqrt{2}B_i^TX_i & 0 & 0 & 0 & 0 & 0 & -B_i^TX_iB_i & * & *\\
N_{1i}^TX_i & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{1i}I & *\\
N_{1i}^TX_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2i}I
\end{bmatrix}+GF(t)H+H^TF^T(t)G^T<0, \tag{33}$$

where

$$\Xi_{1i}^{*} = X_i(A_i + B_iK_i) + (A_i + B_iK_i)^T X_i + Q + \varepsilon_{1i}H_{ai}^T H_{ai} + \varepsilon_{3i}H_{ci}^T H_{ci} + \sum_{j=1}^{N}\pi_{ij}X_j,$$

$$\Xi_{2i}^{*} = -(1-h)Q + \varepsilon_{2i}H_{adi}^T H_{adi} + \varepsilon_{3i}H_{cdi}^T H_{cdi},$$

$$G = \begin{bmatrix} H_{ai} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & H_{adi} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$F(t) = \begin{bmatrix} F_{1i}^T(t) & 0 \\ 0 & F_{2i}^T(t) \end{bmatrix},$$

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & N_{1i}^T X_i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{1i}^T X_i & 0 & 0 & 0 \end{bmatrix}. \tag{34}$$

According to Lemma 4, there exists positive scalar ε_{4i} such that the following LMI holds:

$$\begin{bmatrix} \Xi_{3i} & * & * & * & * & * & * & * & * & * & * \\ \Pi_{5i}^T & \Xi_{4i} & * & * & * & * & * & * & * & * & * \\ X_iD_iC_i & X_iD_iC_{di} & -X_i & * & * & * & * & * & * & * & * \\ E_{2i}^T D_i^T X_i D_i C_i & E_{2i}^T D_i^T X_i D_i C_{di} & 0 & \Pi_{3i} & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -X_i & * & * & * & * & * & * \\ 0 & X_iA_{di} & 0 & 0 & 0 & -X_i & * & * & * & * & * \\ \sqrt{2}B_i^T X_i & 0 & 0 & 0 & 0 & 0 & -B_i^T X_i B_i & * & * & * & * \\ N_{1i}^T X_i & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{1i}I & * & * & * \\ N_{1i}^T X_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_{2i}I & * & * \\ 0 & 0 & 0 & 0 & N_{1i}^T X_i & 0 & 0 & 0 & 0 & -\varepsilon_{4i}I & * \\ 0 & 0 & 0 & 0 & 0 & N_{1i}^T X_i & 0 & 0 & 0 & 0 & -\varepsilon_{4i}I \end{bmatrix} < 0, \tag{35}$$

with

$$\Xi_{3i} = X_i(A_i + B_iK_i) + (A_i + B_iK_i)^T X_i + Q + \varepsilon_{1i}H_{ai}^T H_{ai} + \varepsilon_{3i}H_{ci}^T H_{ci} + \varepsilon_{4i}H_{ai}^T H_{ai} + \sum_{j=1}^{N}\pi_{ij}X_j,$$

$$\Xi_{4i} = -(1-h)Q + \varepsilon_{2i}H_{adi}^T H_{adi} + \varepsilon_{3i}H_{cdi}^T H_{cdi} + \varepsilon_{4i}H_{adi}^T H_{adi}. \tag{36}$$

Moreover, by Schur's complement lemma, the above inequality is implied by (22).

Then, we will prove that the stochastic system (21) with (6) satisfies

$$\|z(t)\|_{E_2} < \gamma\|v(t)\|_2, \tag{37}$$

for all nonzero $v(t) \in L_2[0,\infty)$.

Choose the same Lyapunov function as (25). And the generator $LV(x(t),i)$ with $v(t) \neq 0$ can be calculated as follows:

$$\begin{aligned} &LV(x(t),i) \\ &= 2x^T(t)X_i\Big[A_i + B_iK_i + \widetilde{A}_i(t) \\ &\qquad - B_i\big(B_i^T X_i B_i\big)^{-1}B_i^T X_i\widetilde{A}_i(t)\Big]x(t) \\ &\quad + 2x^T(t)X_i\Big[A_{di} + \widetilde{A}_{di}(t) \\ &\qquad - B_i\big(B_i^T X_i B_i\big)^{-1}B_i^T X_i\big(A_{di} + \widetilde{A}_{di}(t)\big)\Big] \\ &\quad \times x(t-\tau(t)) + 2x^T(t)X_i \\ &\quad \times \Big[I - B_i\big(B_i^T X_i B_i\big)^{-1}B_i^T X_i\Big]B_{vi}v(t) \\ &\quad + \Big[\big(C_i + \widetilde{C}_i(t)\big)x(t) + \big(C_{di} + \widetilde{C}_{di}(t)\big)x(t-\tau(t))\Big]^T \\ &\quad \times D_i^T X_i D_i \\ &\quad \times \Big[\big(C_i + \widetilde{C}_i(t)\big)x(t) + \big(C_{di} + \widetilde{C}_{di}(t)\big)x(t-\tau(t))\Big] \\ &\quad + \sum_{j=1}^{N}\pi_{ij}x^T(t)X_jx(t) \\ &\quad + x^T(t)Qx(t) - (1-\dot{\tau}(t))x^T(t-\tau(t))Qx(t-\tau(t)). \end{aligned} \tag{38}$$

Let

$$J(t) \triangleq E\left\{\int_{t_0}^{t}\left[z^T(s)z(s) - \gamma^2 v^T(s)v(s)\right]ds\right\}. \tag{39}$$

So, under zero initial condition, we have $V(x(t_0), i) = 0$ for $x(t_0) = 0$. From Dynkin's formula, one has

$$E\{V(x(t), t)\} = E\left\{\int_{t_0}^{t} LV(x(s), s)\, ds\right\}. \quad (40)$$

Then, similar to the (26)–(29), we can obtain

$$\begin{aligned} J(t) &= E\left\{\int_{t_0}^{t}\left[z^T(s) z(s) - \gamma^2 v^T(s) v(s) + LV(x(s), s)\right] ds\right\} \\ &\quad - E\{V(x(t), t)\} \\ &\leq E\left\{\int_{0}^{t}\left[z^T(s) z(s) - \gamma^2 v^T(s) v(s) + LV(x(s), s)\right] ds\right\} \\ &= E\left\{\int_{0}^{t}\left[x^T(s) \;\; x^T(s-\tau(s)) \;\; v^T(s)\right]\right. \\ &\qquad \left.\times \Omega_i \left[x^T(s) \;\; x^T(s-\tau(s)) \;\; v^T(s)\right]^T ds\right\}, \end{aligned} \quad (41)$$

where

$$\Omega_i = \begin{bmatrix} \Theta_{1i} & \Theta_{3i} & X_i B_{vi} + \left(E_i + \widetilde{E}_i\right)^T F_i \\ \Theta_{3i}^T & \Theta_{2i} & 0 \\ B_{vi}^T X_i + F_i^T\left(E_i + \widetilde{E}_i\right) & 0 & B_{vi}^T X_i B_{vi} + F_i^T F_i - \gamma^2 I \end{bmatrix}, \quad (42)$$

with

$$\begin{aligned} \Theta_{1i} &= \Xi_{1i} + X_i B_i \left(B_i^T X_i B_i\right)^{-1} B_i^T X_i \\ &\quad + \left(E_i + \widetilde{E}_i\right)^T \left(E_i + \widetilde{E}_i\right) + C_i^T D_i^T X_i D_i C_i \\ &\quad + C_i^T D_i^T X_i D_i N_{2i} \\ &\quad \times \left(\varepsilon_{3i} I - N_{2i}^T D_i^T X_i D_i N_{2i}\right)^{-1} N_{2i}^T D_i^T X_i D_i C_i, \\ \Theta_{2i} &= \Xi_{2i} + C_{di}^T D_i^T X_i D_i C_{di} + C_{di}^T D_i^T X_i D_i N_{2i} \\ &\quad \times \left(\varepsilon_{3i} I - N_{2i}^T D_i^T X_i D_i N_{2i}\right)^{-1} N_{2i}^T D_i^T X_i D_i C_{di}, \\ \Theta_{3i} &= X_i A_{di} + C_i^T D_i^T X_i D_i N_{2i} \\ &\quad \times \left(\varepsilon_{3i} I - N_{2i}^T D_i^T X_i D_i N_{2i}\right)^{-1} \\ &\quad \times N_{2i}^T D_i^T X_i D_i C_{di}, \end{aligned} \quad (43)$$

and Ξ_{1i} and Ξ_{2i} are defined as in (30).

It is observed that $\Omega_i < 0$ from (22) and Schur's complement. Hence, by (39), inequality $\|z(t)\|_{E_2} < \gamma \|v(t)\|_2$ holds for all nonzero $v(t) \in L_2[0, \infty)$. The proof is complete. □

Remark 7. Notice that the condition in Theorem 6 is not a convex set due to the matrix equality constraints in (23). A simple algorithm is given to solve the feasibility problem. The equality condition $B_i^T X_i D_i = 0$ with X_i satisfying inequality (22) can be equivalently converted to $\mathrm{tr}[(B_i^T X_i D_i)^T B_i^T X_i D_i] = 0$.

Consider the following matrix inequality for $\alpha > 0$:

$$\left(B_i^T X_i D_i\right)^T B_i^T X_i D_i \leq \alpha I, \quad \text{for } i \in S. \quad (44)$$

By Schur's complement, one has

$$\begin{bmatrix} -\alpha I & B_i^T X_i D_i \\ D_i^T X_i B_i & -I \end{bmatrix} \leq 0, \quad \text{for } i \in S. \quad (45)$$

Now, the problem is changed to a problem of finding a global solution of the following minimization problem:

$$\min \alpha, \;\text{ subject to }\; (22) \;\text{ and }\; (45). \quad (46)$$

It can be seen that if the global infimum α of (46) equals to zero, the corresponding solutions will satisfy the LMIs (22) and (23). So, the sliding control problem is solvable.

3.3. SMC Law Synthesis. Now, we will synthesize a SMC law to ensure that the trajectories of system (5)–(7) can be driven to reach and keep the predefined surface $s(x(t), i) = 0$ from the initial time.

Theorem 8. *For the uncertain stochastic delay system* (5)–(7) *with Markovian jumping and assumptions* (4) *and* (8)–(10), *the sliding surface is designed as* (17) *where* X_i, $(i \in S)$ *are the solutions of LMI* (22)-(23). *Then the trajectories of system can be driven to the sliding surface* $s(x(t), i) = 0$ *in finite time (with probability one) by employing the following SMC law:*

$$\begin{aligned} u_i(t) &= K_i x(t) - \mu_i s(x(t), i) \\ &\quad - \left(B_i^T X_i B_i\right)^{-1} B_i^T X_i A_{di} x(t-\tau(t)) \\ &\quad - \frac{1}{2}\sum_{j=1}^{N} \pi_{ij} \left(B_j^T X_j B_j\right)^{-1} s(x(t), i) \\ &\quad - \rho_i(t) \operatorname{sgn}(s(x(t), i)), \end{aligned} \quad (47)$$

where

$$\begin{aligned} \rho_i(t) &= \eta_i \|x(t)\| + \left\|\left(B_i^T X_i B_i\right)^{-1} B_i^T X_i N_{1i}\right\| \\ &\quad \times \left(\left\|H_{ai} x(t)\right\| + \left\|H_{adi} x(t-\tau(t))\right\|\right) \\ &\quad + \left\|\left(B_i^T X_i B_i\right)^{-1} B_i^T X_i B_{vi}\right\| \|v(t)\|, \end{aligned} \quad (48)$$

with μ_i *are a small positive constant for each* $i \in S$.

Proof. By (19) and (47), and by observing that $B_i^T X_i D_i = 0$, it follows that

$$\begin{aligned}\dot{s}(x(t),i) &= B_i^T X_i \widetilde{A}_i(t)x(t) + B_i^T X_i \widetilde{A}_{di}(t)x(t-\tau(t)) \\ &\quad - \mu_i B_i^T X_i B_i s(x(t),i) + B_i^T X_i B_{vi} v(t) \\ &\quad - \frac{1}{2}\sum_{j=1}^{N}\pi_{ij} B_i^T X_i B_i \left(B_j^T X_j B_j\right)^{-1} s(x(t),i) \\ &\quad - \rho_i(t) B_i^T X_i B_i \operatorname{sgn}(s(x(t),i)) \\ &\quad + B_i^T X_i B_i f(t,x(t),i).\end{aligned} \tag{49}$$

Choose the following Lyapunov function candidate:

$$V(t) = \frac{1}{2}s^T(t)\left(B_i^T X_i B_i\right)^{-1} s(t). \tag{50}$$

Utilizing expression (49) yields

$$\begin{aligned}\dot{V}(t) &= s^T(x(t),i)\left(B_i^T X_i B_i\right)^{-1}\dot{s}(x(t),i) \\ &\quad + \frac{1}{2}\sum_{j=1}^{N}\pi_{ij} s^T(x(t),i)\left(B_j^T X_j B_j\right)^{-1} s(x(t),i) \\ &= s^T(x(t),i)\left(B_i^T X_i B_i\right)^{-1} B_i^T X_i \widetilde{A}_i(t)x(t) \\ &\quad + s^T(x(t),i)\left(B_i^T X_i B_i\right)^{-1} B_i^T X_i \widetilde{A}_{di}(t)x(t-\tau(t)) \\ &\quad - \mu_i s^T(x(t),i)s(x(t),i) \\ &\quad - \rho_i(t)s^T(x(t),i)\operatorname{sgn}(s(x(t),i)) \\ &\quad + s^T(x(t),i) f(t,x(t),i) \\ &\quad + s^T(x(t),i)\left(B_i^T X_i B_i\right)^{-1} B_i^T X_i B_{vi} v(t) \\ &\le \|s(x(t),i)\| \left\|\left(B_i^T X_i B_i\right)^{-1} B_i^T X_i N_{1i}\right\| \\ &\quad \times \left(\|H_{ai}x(t)\| + \|H_{adi}x(t-\tau(t))\|\right) \\ &\quad - \rho_i(t)\|s(x(t),i)\|_1 + \|s(x(t),i)\| \\ &\quad \times \left\|\left(B_i^T X_i B_i\right)^{-1} B_i^T X_i B_{vi}\right\| \|v(t)\| - \mu_i \|s(x(t),i)\|^2 \\ &\quad + \eta_i \|s(x(t),i)\| \|x(t)\|.\end{aligned} \tag{51}$$

Noting that $\|s(x(t),i)\|_1 \ge \|s(x(t),i)\|$ and substituting (48) into (51), one has

$$\dot{V}(t) \le -\mu_i \|s(x(t),i)\|^2 < 0, \quad \text{for } \|s(x(t),i)\| \ne 0. \tag{52}$$

This means that the trajectories of system (5)–(7) can be driven to remain in the sliding surface from the initial time. □

Remark 9. The use of sign function $\operatorname{sgn}(s(x(t),i))$ may cause chattering behavior, which is undesired in practical engineering background [19]. To avoid the shortcomings, the function $\tanh(s(x(t),i))$ is introduced to approximate function $\operatorname{sgn}(s(x(t),i))$ [20].

From SMC (47)-(48), it is seen that the bound η_i of $f(t,x(t),i)$ is needed to synthesize the SMC law. It is well known that a practical engineering system may lose the exact knowledge of the bound due to the noise pollution and measurement error. Then, an adaptive SMC law is further presented for the case when the bound η_i is unknown.

Theorem 10. *For the uncertain stochastic delay systems* (5)–(7) *with Markovian jumping and assumptions* (4) *and* (8)–(10)*, the sliding surface is designed as* (17) *where X_i, $(i \in S)$ are the solutions of LMI* (22)-(23)*. Then the trajectories of system can be driven to the sliding surface $s(x(t),i) = 0$ in finite time (with probability one) by employing the following SMC law:*

$$\begin{aligned}u_i(t) &= K_i x(t) - \mu_i s(x(t),i) \\ &\quad - \left(B_i^T X_i B_i\right)^{-1} B_i^T X_i A_{di} x(t-\tau(t)) \\ &\quad - \frac{1}{2}\sum_{j=1}^{N}\pi_{ij}\left(B_j^T X_j B_j\right)^{-1} s(x(t),i) \\ &\quad - \widehat{\rho}_i(t)\operatorname{sgn}(s(x(t),i)),\end{aligned} \tag{53}$$

where

$$\begin{aligned}\widehat{\rho}_i(t) &= \widehat{\eta}_i \|x(t)\| + \left\|\left(B_i^T X_i B_i\right)^{-1} B_i^T X_i N_{1i}\right\| \\ &\quad \times \left(\|H_{ai}x(t)\| + \|H_{adi}x(t-\tau(t))\|\right) \\ &\quad + \left\|\left(B_i^T X_i B_i\right)^{-1} B_i^T X_i B_{vi}\right\| \|v(t)\|,\end{aligned} \tag{54}$$

and adaptive law as

$$\dot{\widehat{\eta}}_i = \beta_i \|s(t)\| \|x(t)\|, \tag{55}$$

with μ_i, β_i as small positive constants for each $i \in S$.

Proof. Choose the following Lyapunov function:

$$V(t) = \frac{1}{2}s^T(t)\left(B_i^T X_i B_i\right)^{-1} s(t) + \frac{1}{2}\beta_i^{-1}\widetilde{\eta}_i^2(t), \tag{56}$$

where $\widetilde{\eta}_i(t) = \widehat{\eta}_i(t) - \eta_i$. In line with the proof of Theorem 8, it can easily obtained that

$$\begin{aligned}\dot{V}(t) &\le \|s(x(t),i)\| \left\|\left(B_i^T X_i B_i\right)^{-1} B_i^T X_i N_{1i}\right\| \\ &\quad \times \left(\|H_{ai}x(t)\| + \|H_{adi}x(t-\tau(t))\|\right) \\ &\quad - \widehat{\rho}_i(t)\|s(x(t),i)\|_1 \\ &\quad + \|s(x(t),i)\| \left\|\left(B_i^T X_i B_i\right)^{-1} B_i^T X_i B_{vi}\right\| \|v(t)\| \\ &\quad - \mu_i \|s(x(t),i)\|^2 + \eta_i \|s(x(t),i)\| \|x(t)\| \\ &\quad + \widetilde{\eta}_i(t)\|s(x(t),i)\| \|x(t)\|.\end{aligned} \tag{57}$$

Noting that $\|s(x(t),i)\|_1 \geq \|s(x(t),i)\|$ and substituting (54) into (57), one has

$$\dot{V}(t) \leq -\mu_i \|s(x(t),i)\|^2 < 0, \quad \text{for } \|s(x(t),i)\| \neq 0. \quad (58)$$

This means that the trajectories of system (5)–(7) can be driven to remain in the sliding surface from the initial time. This completes the proof. □

4. Simulation Example

Consider the uncertain stochastic delay system (3) with $N = 2$ and the following parameters.

Mode 1. We have the following:

$$A_1 = \begin{bmatrix} -1 & 0.6 & -2.4 \\ 2 & 0.1 & -0.5 \\ 0.1 & 2.2 & 0.5 \end{bmatrix}, \qquad A_{d1} = \begin{bmatrix} 0.2 & 0.1 & 0.1 \\ -0.1 & 0.1 & 0 \\ 0 & 0.1 & -0.5 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 & -2 \\ 4 & 3.5 \\ 2.4 & 5.5 \end{bmatrix}, \qquad B_{v1} = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.3 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -0.2 & 0.1 \\ 0 & 1 \\ 0.1 & 0.04 \end{bmatrix}^T, \qquad E_1 = \begin{bmatrix} 0.4 & 0.4 & 0 \\ 0.5 & 0.1 & 0.2 \\ 0.1 & 0.2 & 0.3 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.1 \end{bmatrix}, \qquad N_{11} = \begin{bmatrix} 0 \\ 0.1 \\ 0.1 \end{bmatrix},$$

$$N_{21} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \qquad H_{a1} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0 \end{bmatrix},$$

$$H_{ad1} = \begin{bmatrix} 0 \\ 0.1 \\ 0.2 \end{bmatrix}, \qquad H_{e1} = \begin{bmatrix} 0.01 \\ 0.1 \\ 0.1 \end{bmatrix},$$

$$H_{c1} = \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 1 & -0.1 & 0.3 \\ -0.2 & 0.3 & 0 \end{bmatrix}, \qquad H_{cd1} = \begin{bmatrix} -0.2 & 0 & 0.2 \\ 0.1 & 0.1 & 0.1 \\ 0.2 & -0.1 & 0 \end{bmatrix},$$

$$C_{d1} = \begin{bmatrix} 0 & 0.3 \\ 0.2 & 0.2 \\ 0.01 & 0.1 \end{bmatrix}^T,$$

$$f(x(t),t,1) = \left[0.6\sin\sqrt{x_1^2(t) + x_2^2(t)} \quad 0.6x_2(t)\right]^T,$$

$$v(t) = \frac{3}{1+t^2}, \qquad F_{11}(t) = 0.5\sin t,$$

$$F_{21}(t) = \left[0.2\sin t \quad 0.2\cos t \quad 0\right]. \quad (59)$$

Mode 2. We have the following:

$$A_2 = \begin{bmatrix} 1 & 0.8 & 1 \\ 0 & 0.5 & -0.6 \\ 0.3 & 0.4 & -0.5 \end{bmatrix}, \qquad A_{d2} = \begin{bmatrix} -0.2 & 0.2 & 0.3 \\ 0 & 0.5 & 0.1 \\ 0.1 & 0.4 & 0.5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1.2 & 3 \\ 4 & 0 \\ 0.4 & 2 \end{bmatrix}, \qquad B_{v2} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.2 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.1 \\ 0.4 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.2 & -0.1 \\ 0 & 1 \\ -0.1 & 0.02 \end{bmatrix}^T,$$

$$E_2 = \begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0 & 0.8 & 0.3 \\ 0.2 & 0 & 0.3 \end{bmatrix}, \qquad F_2 = \begin{bmatrix} 0.3 \\ 0.4 \\ 0.2 \end{bmatrix},$$

$$N_{21} = \begin{bmatrix} 0 \\ 0.2 \\ -0.1 \end{bmatrix}, \qquad N_{22} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$

$$H_{a2} = \begin{bmatrix} 0.2 \\ 0.4 \\ 0 \end{bmatrix},$$

$$H_{ad2} = \begin{bmatrix} 0.1 \\ 0 \\ 0.1 \end{bmatrix}, \qquad H_{e2} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0 \end{bmatrix},$$

$$H_{c2} = \begin{bmatrix} 0.2 & 0.1 & 0.2 \\ 0.1 & -0.1 & 0.2 \\ -0.1 & 0.2 & 0 \end{bmatrix}, \qquad H_{cd2} = \begin{bmatrix} -0.1 & 0 & 0.1 \\ 0.2 & 0.1 & 0.2 \\ 0.1 & -0.1 & 0 \end{bmatrix},$$

$$C_{d2} = \begin{bmatrix} 0.1 & 0.2 \\ -0.2 & 0.1 \\ 0.01 & -0.1 \end{bmatrix}^T,$$

$$f(x(t),t,2) = \left[\sin x_1(t) \quad \sqrt{2\left|x_2(t)x_3(t)\right|}\right]^T,$$

$$F_{12}(t) = 0.5\cos t, \qquad F_{22}(t) = \left[0.2\cos t \quad 0.2\sin t \quad 0\right]. \quad (60)$$

Moreover, the time-varying delay $\tau(t) = 0.5\sin t + 0.5$ with $d = 1$, $h = 0.5$, and the transition rate matrix π of Markov chain $\{r_t\}$ is defined as $\pi = \left[\begin{smallmatrix} -1 & 1 \\ 1.5 & -1.5 \end{smallmatrix}\right]$. Choose $\gamma = 1$. Now, for the bounds $\eta_1 = 0.6$ and $\eta_2 = 1$, solving LMIs (22)-(23) yields

$$X_1 = \begin{bmatrix} 1.7215 & -1.0069 & 1.1293 \\ -1.0069 & 0.9575 & -0.8057 \\ 1.1293 & -0.8057 & 1.1127 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 1.9635 & -0.6287 & -1.3014 \\ 0.6287 & 0.7440 & -0.0362 \\ -1.3014 & -0.0362 & 4.0151 \end{bmatrix},$$

$$\varepsilon_{11} = 1.0867, \qquad \varepsilon_{12} = 1.2007,$$
$$\varepsilon_{21} = 4.2260, \qquad \varepsilon_{22} = 2.2260, \qquad \varepsilon_{31} = 1.2260,$$
$$\varepsilon_{32} = 1.5600, \qquad \varepsilon_{41} = 3.2260, \qquad \varepsilon_{42} = 2.1200,$$
$$\beta \approx 1.4797 \times 10^{-11},$$
$$Q = \begin{bmatrix} 6.7459 & -1.5885 & 3.5946 \\ -1.5885 & 3.3264 & -0.1067 \\ 3.5946 & -0.1067 & 4.4201 \end{bmatrix}. \tag{61}$$

Furthermore, controller gain K_1 and K_2 are chosen as

$$K_1 = \begin{bmatrix} -1.5 & -4.5 & -4 \\ 3 & 3.5 & -1.2 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 1 & -3 & 2 \\ -3.5 & 0.2 & -0.8 \end{bmatrix}. \tag{62}$$

Then, the desired SMC law can be obtained as follows.

Mode 1. We have the following:

$$\begin{aligned} u(t) = {} & \begin{bmatrix} -1.5 & -4.5 & -4 \\ 3 & 3.5 & -1.2 \end{bmatrix} x(t) - \begin{bmatrix} 0.3642 & 0.1127 \\ 0.1127 & 0.4264 \end{bmatrix} \\ & \times s(x(t), 1) \\ & - \begin{bmatrix} 0.0479 & 0.0496 & 0.0252 \\ -0.0483 & -0.0143 & -0.0699 \end{bmatrix} x(t - \tau(t)) \\ & - [0.6 \|x(t)\| + 0.02 (\| [0.1 \;\; 0.2 \;\; 0] x(t) \| \\ & \qquad + \| [0 \;\; 0.1 \;\; 0.2] x(t - \tau(t)) \|) \\ & \quad +0.0867] \\ & \times \operatorname{sgn}(s(x(t), 1)). \end{aligned} \tag{63}$$

Mode 2. We have the following:

$$\begin{aligned} u(t) = {} & \begin{bmatrix} 1 & -3 & 2 \\ -3.5 & 0.2 & -0.8 \end{bmatrix} x(t) \\ & - \begin{bmatrix} 0.7037 & -0.1691 \\ -0.1691 & 0.6103 \end{bmatrix} s(x(t), 2) \\ & - \begin{bmatrix} 0.0124 & 0.1419 & 0.0416 \\ -0.0120 & 0.0909 & 0.1627 \end{bmatrix} x(t - \tau(t)) \\ & - [\|x(t)\| + 0.0599 (\| [0.2 \;\; 0.4 \;\; 0] x(t) \| \\ & \qquad + \| [0.1 \;\; 0 \;\; 0.1] x(t - \tau(t)) \|) \\ & \quad +0.0763] \operatorname{sgn}(s(x(t), 2)), \end{aligned} \tag{64}$$

where $\mu_1 = \mu_2 = 0.5$.

Using the discretization approach similar to that in [21], the simulation results are given in Figures 1, 2, 3, 4, 5, and 6 which show the effective of the proposed methods. Figures 1–6 demonstrated the simulation results for every mode, respectively.

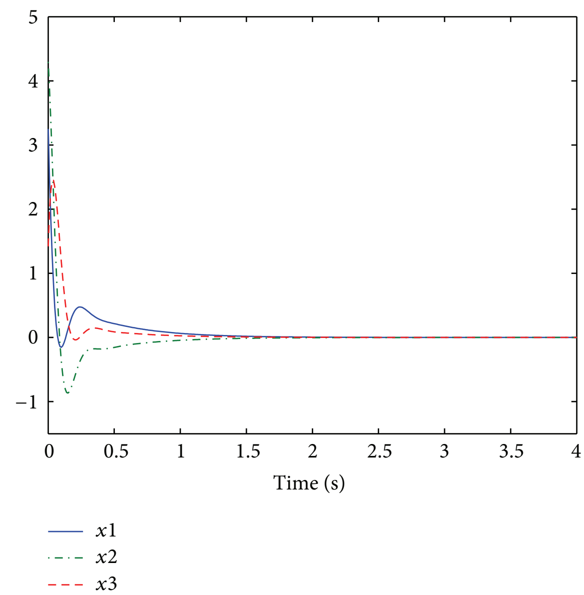

Figure 1: Mode 1: state vector $x(t)$.

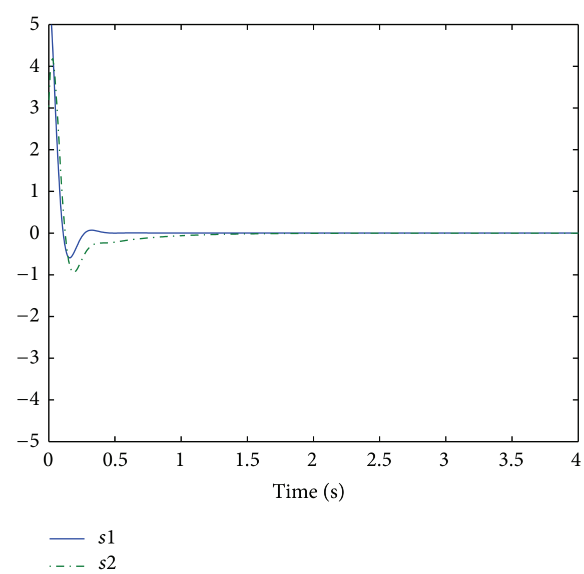

Figure 2: Mode 1: sliding mode $s(t)$.

5. Conclusions

This work has investigated the problem of SMC problem for stochastic systems with Markovian switching and time-varying delays. A sufficient condition for the stochastic stability of sliding motion has been proposed in terms of LMIs. An SMC law has been synthesized such that the state trajectories of the closed-lop systems are globally driven onto the specified switching surface corresponding to every mode and reduces the effect of the disturbance input on the controlled output to a prescribed level irrespective of all the admissible uncertainties. It is observed that the effects of Markovian switching and time delay have been considered

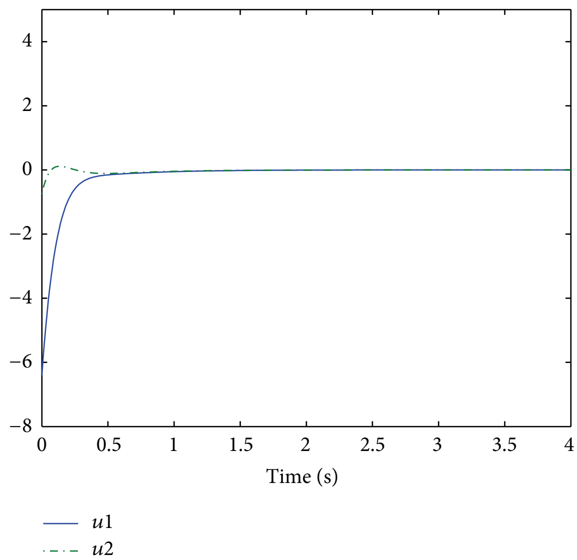

FIGURE 3: Mode 1: control signal $u(t)$.

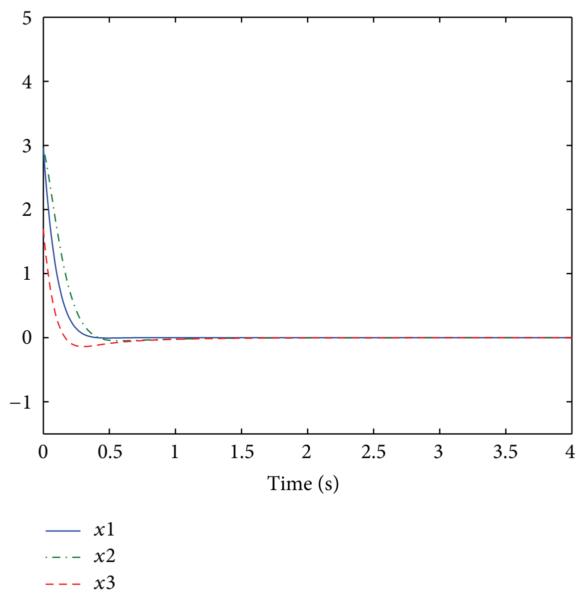

FIGURE 4: Mode 2: state vector $x(t)$.

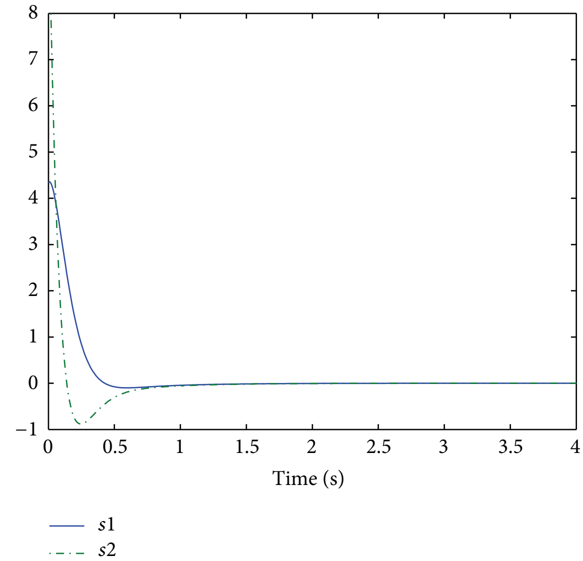

FIGURE 5: Mode 2: sliding mode $s(t)$.

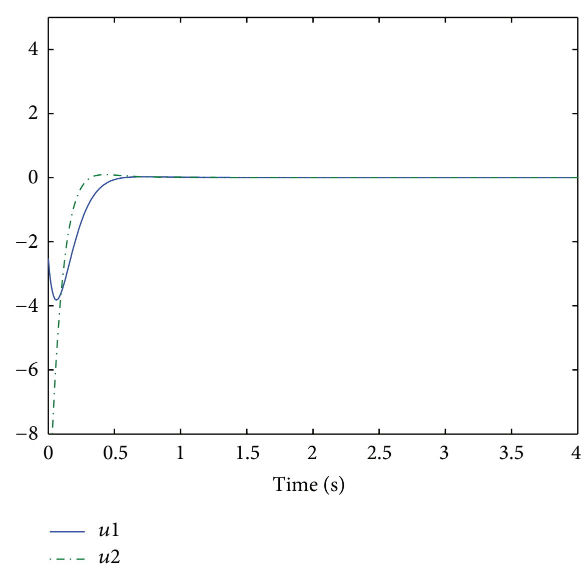

FIGURE 6: Mode 2: control signal $u(t)$.

in the design of both sliding surface and SMC law. Future works will consider the problem of SMC problem for singular systems with Markovian switching and time-varying delays. In that case, due to the existence of singular matrix, the sliding mode control law design is much more complicated.

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