Research Article

Adaptive Synchronization of Nonlinearly Parameterized Complex Dynamical Networks with Unknown Time-Varying Parameters

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A new adaptive learning control approach is proposed for a class of nonlinearly parameterized complex dynamical networks with unknown time-varying parameters. By using the parameter separation and reparameterization technique, the adaptive learning laws of periodically time-varying and constant parameters and an adaptive control strategy are designed to ensure the asymptotic convergence of the synchronization error in the sense of square error norm. Then, a sufficient condition of the synchronization is given by constructing a composite energy function. Finally, an example of the complex network is used to verify the effectiveness of proposed approach.

1. Introduction

Since small world network and scale-free network were discovered in the past decade, complex dynamical networks have become a focus issue and have been studied intensively in various disciplines, such as sociology, biology, mathematics, and engineering [1, 2]. As nonlinear dynamics and topological structure of the complex dynamical networks influence its dynamical behaviors, and synchronization is one of the basic forms of cooperative behavior, many of the basic mechanisms have a direct relationship with the synchronization. Research on synchronization of networks has an important significance on understanding of cooperative behavior mechanism and complex phenomena in nature and society. What is more is that it also has a very broad application prospects on nuclear magnetic resonance, wireless sensor network, and multirobot coordination. On the other hand, it can also provide the necessary theoretical basis simultaneously in multiple mobile robot systems, unmanned aircraft systems and distributed sensor array in the field of intelligent autonomous coordination control of complex systems. Up to now, synchronization can be roughly divided into complete

synchronization and generalized synchronization. Generalized synchronization includes cluster synchronization, partial synchronization, and functional project synchronization.

However, there are many of real-world network systems such as biological networks, mobile communications networks, and social networks, whose structure will change over time, coupling parameter, and other system parameters. For example, the feed-forward loops (FFLs) are typical network motifs in many real world biological networks. The structures, functions, as well as noise characteristics of FFLs are dependent on the network parameters. In [3], a global relative parameter sensitivities (GRPSs) were proposed for FFLs in the genetic networks. The proposed GRPS approach sheds some light on the potential real world applications, such as the synthetic genetic circuits, predicting the effect of interventions in medicine and biotechnology, and so on. The existence of time-varying coupling parameter increases the difficulty of researching the problem of synchronization for complex networks. In the existing results [4–19], the method of synchronization is divided into two categories which are adaptive coupling and adaptive control approach. [20] studied a time-varying complex dynamical network model and its controlled synchronization criteria. It showed that synchronization of such a time-varying dynamical network was completely determined by the inner-coupling matrix and by the eigenvalues and the corresponding eigenvectors of the coupling configuration matrix of the network. [21] investigated pinning adaptive synchronization of a general complex dynamical network and indeed gave the positive answers to two questions which were how many nodes and how much coupling strength should a network with fixed structure and coupling be chosen. However, in [20, 21], they considered the controlled complex dynamical network with linearly diffusive couplings; [22] researched adaptive synchronization of an uncertain complex dynamical network. Although it considered complex dynamical network with uncertain nonlinear diffusive couplings, the uncertainty of the system with unknown time-varying parameters was not considered. [23] discussed a global payoff-based strategy updating model for studying cooperative behavior of a networked population. It characterized the interactions among individuals by time varying parameter. It is obvious that the coupling of actual complex networks is mostly complex, and the coupling parameters of the network may be not sure or change with some varying factors. Among the above results, none of them is concerning with the adaptive synchronization of complex network systems with unknown time-varying nonlinear coupling.

On the other hand, in recent years, adaptive learning control [24], a new adaptive control scheme, has been proposed to deal with the unknown periodic time-varying parameters. It used various methods to estimate the unknown constant or time-varying parameters of the system in order to obtain control input. The convergence of tracking error was proved by using the composite energy function. It is worth mentioning that learning control based on the composite energy function plays an important role in dealing with the estimates of periodically time-varying parameters.

Motivated by the above observations, in this paper, the problem of adaptive synchronization is investigated for nonlinearly parameterized complex dynamical networks with unknown time-varying parameters via adaptive learning control method. The main contribution of the paper is that, for the first time, the problem of adaptive synchronization for nonlinearly parameterized complex dynamical networks with unknown time-varying parameters is solved. By combining the parameter separation [25, 26] and reparameterization technique, the adaptive learning laws of periodically time-varying and constant parameters and the proposed adaptive controllers guarantee the asymptotic convergence of the synchronization error in the L_T^2 norm and that all closed-loop signals are bounded in the L_T^2 norm. Simulation results are provided to show the effectiveness of the proposed approach.

2. Problem Formulation and Preliminaries

In this section, we introduce the network model considered in this paper and give some useful mathematical preliminaries.

Consider a dynamical network consisting of N identical nodes with nonlinear couplings, in which each node is an n-dimensional dynamical system. The state equations of the network are

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} a_{ij}\Gamma g(x_j(t), \varphi_i(t)) \quad i = 1, 2 \ldots, N, \tag{2.1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^{\mathrm{T}} \in R^n$ represents the state vector of the ith node. $f : R^n \times R \to R^n$ is a smooth nonlinear vector-valued function. $\varphi_i(t)$ represents the unknown time-varying function. $g : R^n \times R \times R \to R^n$ is an unknown continuous nonlinear vector-valued function. $a_{ij} = a_{ji}$ is the coupling element. If there exists a connection between node i and node j $(i \neq j)$, then $a_{ij} = a_{ji} > 0$, else $a_{ij} = a_{ji} = 0$. The inner coupling matrix $\Gamma = \mathrm{diag}(\gamma_1, \ldots, \gamma_n)$ is nonnegative definite.

Let

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{pmatrix}, \tag{2.2}$$

where $A^{\mathrm{T}} = A$ is an unknown irreducible real symmetric matrix. In this model, it is only required that the elements of the coupling matrix A satisfy

$$a_{ii} = -\sum_{j=1, j \neq i}^{N} a_{ij} < 0, \quad i, j = 1, 2, \ldots, N. \tag{2.3}$$

The system (2.1) is said to be asymptotic synchronization in L_T^2 norm if

$$\lim_{t \to \infty} \int_{t-T}^{t} \|x_i(\tau) - s(\tau)\|^2 d\tau = 0, \quad i = 1, 2, \ldots, N, \tag{2.4}$$

where $\| \cdot \|$ stands for the Euclidean vector norm. Synchronous evolution $s(t)$ is an arbitrary desired state, which is also an isolated node of the network (2.1) with $\dot{s}(t) = f(s(t))$. We assume that for the system $\dot{s}(t) = f(s(t))$ there exists stable equilibrium point, stable periodic orbit, or even chaotic attractor.

Remark 2.1. It is well known that multiagent system is a special kind of complex networks. The consensus has similar meaning with synchronization, and consensus is a fundamental natural phenomenon in nature. [27] researched the cluster consensus of discrete-time multiagent systems. Based on Markov chains and nonnegative matrix analysis, two novel cluster consensus criteria were obtained for MAS with fixed and switching topology,

respectively. However, in this paper, we consider a new synchronization problem for continuous-time complex dynamic network with nonlinearly time-varying coupling parameter. This problem is quite different from [27], and the challenge problem is due to the time-varying nonlinearly coupling parameter in (2.1).

Now, the following assumptions are introduced.

Assumption 2.2. In the network (2.1), suppose that there exists $l_i > 0$, satisfying

$$[x_i(t) - s(t)]^T [f(x_i(t)) - f(s(t))] \le l_i [x_i(t) - s(t)]^T [x_i(t) - s(t)], \tag{2.5}$$

where $x_i(t)$ and $s(t)$ are time varying vectors.

Lemma 2.3 (separation principle). *For the unknown continuous function $g(\cdot, \cdot)$, the following inequality holds*

$$\left\| g(x_j(t), \varphi_i(t)) - g(s(t), \varphi_i(t)) \right\|^2 \le \left\| x_j(t) - s(t) \right\|^2 h(x_j(t), s(t)) \lambda(\varphi_i(t)), \tag{2.6}$$

where $h(\cdot, \cdot)$ is known nonnegative continuous function and $\lambda(\cdot)$ is unknown nonnegative continuous function.

Assumption 2.4. In the network (2.1), $\varphi_i(t)$ is unknown time-varying function with a known period T, in that way $\lambda(\varphi_i(t))$ is also a periodic function. Suppose $\lambda(\varphi_i(t)) = \phi_i(t) + \theta_i$, where $\phi_i(t)$ is unknown continuous function and θ_i is unknown constant parameter.

Assumption 2.5. In the network (2.1), the inner coupling matrix Γ, coupling matrix A, and $h(\cdot, \cdot)$ satisfy

$$\|\Gamma\| = \gamma, \quad h(x_j(t), s(t)) < H, \quad |a_{ij}| \le a, \quad \forall i, j = 1, 2, \dots, N, \tag{2.7}$$

where γ, H, a are positive constants.

Lemma 2.6 (Young's inequality). *For any vectors $x, y \in R^n$, and any $c > 0$, the following matrix inequality holds:*

$$x^T y \le c x^T x + \frac{1}{4c} y^T y. \tag{2.8}$$

3. Adaptive Controller Design

To achieve the control objective (2.4), we need an adaptive control strategy to nodes in the network (2.1). Then the controlled network is given by

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} a_{ij} \Gamma g(x_j(t), \varphi_i(t)) + u_i(t), \quad i = 1, 2, \dots, N, \tag{3.1}$$

where $u_i(t) = [u_{i1}(t), u_{i2}(t), \ldots, u_{in}(t)]^T \in R^n$, $i = 1, 2, \ldots, N$ are the adaptive controllers to be designed. Let synchronization error be $e_i(t) = x_i(t) - s(t)$, $i = 1, 2, \ldots, N$ and we have the following dynamical error equations:

$$\dot{e}_i(t) = f(x_i(t)) - f(s(t)) + \sum_{j=1}^{N} a_{ij} \Gamma\left(g(x_j(t), \varphi_i(t)) - g(s(t), \varphi_i(t))\right) + u_i(t). \tag{3.2}$$

Then we design controllers by

$$u_i(t) = -Na\gamma H\left(\widehat{\phi}_i(t) + \widehat{\theta}_i(t)\right)(x_i(t) - s(t)), \tag{3.3}$$

where γ, a, H satisfy condition (2.7), N is the node number of the network (2.1), and $\widehat{\phi}_i(t), \widehat{\theta}_i(t)$ are estimations to $\phi_i(t)$, θ_i, respectively. For convenience, we denote

$$\widetilde{\phi}_i(t) = \phi_i(t) - \widehat{\phi}_i(t), \qquad \widetilde{\theta}_i(t) = \theta_i - \widehat{\theta}_i(t). \tag{3.4}$$

The constant parameter update law and the time-varying periodic adaptive learning law are designed as follows:

$$\dot{\widehat{\theta}}_i(t) = r_i e_i^T(t) e_i(t), \tag{3.5}$$

$$\widehat{\phi}_i(t) = \begin{cases} \widehat{\phi}_i(t-T) + q_i e_i^T(t) e_i(t), & t \in [T, +\infty), \\ q_{i0}(t) e_i^T(t) e_i(t), & t \in [0, T), \\ 0, & t \in [-T, 0), \end{cases} \tag{3.6}$$

where $r_i, q_i > 0$ are constants and $q_{i0}(t)$ is a continuous and strictly increasing nonnegative function satisfying $q_{i0}(0) = 0$, $q_{i0}(T) = q_i$, which ensure $\widehat{\phi}_i(t)$ is continuous in $t = iT$, $i = 1, 2, \ldots, N$.

4. Convergence Analysis

In the section, we will give the convergence of the proposed algorithm in the following theorem.

Theorem 4.1. *Under Assumptions 2.2–2.5, the control law* (3.3) *with update law* (3.5) *and the periodic adaptive law* (3.6), *guarantees the L_T^2 asymptotic synchronization of the controlled network* (3.1), *while keeping all closed-loop signals are L_T^2 bounded.*

Proof. To facilitate the convergence analysis, we choose a Lyapunov-Krasovskii function as follows:

$$
V(t) = \begin{cases} \dfrac{1}{2}\displaystyle\sum_{i=1}^{N}e_i^{\mathrm{T}}e_i + \dfrac{Na\gamma H}{2}\sum_{i=1}^{N}\int_0^t q_i^{-1}\widetilde{\phi}_i^2(\tau)d\tau + \dfrac{Na\gamma H}{2}\sum_{i=1}^{N}r_i^{-1}\Big(\widetilde{\theta}_i(t)+L_i\Big)^2, & t\in[0,T), \\[18pt] \dfrac{1}{2}\displaystyle\sum_{i=1}^{N}e_i^{\mathrm{T}}e_i + \dfrac{Na\gamma H}{2}\sum_{i=1}^{N}\int_{t-T}^t q_i^{-1}\widetilde{\phi}_i^2(\tau)d\tau + \dfrac{Na\gamma H}{2}\sum_{i=1}^{N}r_i^{-1}\Big(\widetilde{\theta}_i(t)+L_i\Big)^2, & t\in[T,+\infty), \end{cases}
$$

(4.1)

where L_i is sufficiently large positive constant which will be determined.

Firstly, we will prove the finiteness of $V(t)$ in $[0,T)$. Secondly, we will prove the asymptotical convergence of $e_i(t)$. Finally, we will prove all closed-loop signals are L_T^2 bounded.

First step, let us confirm the finiteness property of $V(t)$ for the first period $[0,T)$. From the system dynamics (3.1) and the proposed control laws (3.3), (3.5), and (3.6), it can be seen that the right-hand side of (3.1) is continuous with respect to all arguments. In light of the existence theorem of differential equation, (3.2) has unique solution in an interval $[0,T_1) \subset [0,T)$, with $0 < T_1 \le T$. Therefore, the boundedness of $V(t)$ over $[0,T_1)$ can be guaranteed and we need only focus on the interval $[T_1,T)$.

For any $t \in [T_1,T)$, the time derivative of $V(t)$ for $t \in [T_1,T)$ is given by

$$
\dot{V}(t) = \sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\dot{e}_i(t) + \frac{Na\gamma H}{2}\sum_{i=1}^{N}q_i^{-1}\widetilde{\phi}_i^2(t) + Na\gamma H\sum_{i=1}^{N}r_i^{-1}\Big(\widetilde{\theta}_i(t)+L_i\Big)\dot{\widetilde{\theta}}_i(t), \quad t\in[T_1,T).
$$

(4.2)

Taking the first term on the right-hand side of (4.2), one obtains

$$
\sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\dot{e}_i(t) = \sum_{i=1}^{N}e_i^{\mathrm{T}}(t)\left[f(x_i(t)) - f(s(t)) + \sum_{j=1}^{N}a_{ij}\Gamma\big(g(x_j(t),\varphi_i(t)) - g(s(t),\varphi_i(t))\big) + u_i(t)\right].
$$

(4.3)

Let $g(x_j(t),\varphi_i(t)) - g(s(t),\varphi_i(t)) = \Lambda$, and

$$
\|\Lambda\|^2 \le \|e_j(t)\|^2 h(x_j(t),s(t))\lambda(\varphi_i(t)).
$$

(4.4)

Then according to Assumption 2.2 and Lemma 2.6, we get

$$\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\dot{e}_i(t) \le \sum_{i=1}^{N}\left[l_i e_i^{\mathrm{T}} e_i + \sum_{j=1}^{N} a_{ij} e_i^{\mathrm{T}} \Gamma \Lambda - Na\gamma H\left(\hat{\phi}_i(t) + \hat{\theta}_i(t)\right)e_i^{\mathrm{T}} e_i\right]$$

$$\le \sum_{i=1}^{N}\left[l_i e_i^{\mathrm{T}} e_i + \sum_{j=1}^{N}\left(a_{ij}^2 c e_i^{\mathrm{T}} e_i + \frac{1}{4c}\Lambda^{\mathrm{T}}\Gamma^{\mathrm{T}}\Gamma\Lambda\right) - Na\gamma H\left(\hat{\phi}_i(t) + \hat{\theta}_i(t)\right)e_i^{\mathrm{T}} e_i\right]$$

$$\le \sum_{i=1}^{N}\left[l_i e_i^{\mathrm{T}} e_i + Na^2 c e_i^{\mathrm{T}} e_i + \frac{NH\gamma^2}{4c}e_i^{\mathrm{T}} e_i(\phi_i(t) + \theta_i) - Na\gamma H\left(\hat{\phi}_i(t) + \hat{\theta}_i(t)\right)e_i^{\mathrm{T}} e_i\right],$$

$$(4.5)$$

where c is a positive constant. If choosing $c = \gamma/4a$, we have

$$\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\dot{e}_i(t) \le \sum_{i=1}^{N}\left[l_i + \frac{Na\gamma}{4} + Na\gamma H\left(\tilde{\phi}_i(t) + \tilde{\theta}_i(t)\right)\right]e_i^{\mathrm{T}} e_i. \qquad (4.6)$$

Taking the third term on the right-hand side of (4.2), we have

$$Na\gamma H \sum_{i=1}^{N} r_i^{-1}\left(\tilde{\theta}_i(t) + L_i\right)\dot{\hat{\theta}}_i(t) = -Na\gamma H \sum_{i=1}^{N}\left(\tilde{\theta}_i(t) + L_i\right)e_i^{\mathrm{T}} e_i. \qquad (4.7)$$

Let us focus on the second term on the right-hand side of (4.2). Since $q_{i0}(t)$ is a continuous function and strictly increasing in $[0, T)$, $q_i^{-1} \le q_{i0}^{-1}(t) < \infty$ is ensured in the interval $[T_1, T)$, and

$$\frac{Na\gamma H}{2}\sum_{i=1}^{N} q_i^{-1}\tilde{\phi}_i^2(t) \le \frac{Na\gamma H}{2}\sum_{i=1}^{N} q_{i0}^{-1}\tilde{\phi}_i^2(t)$$

$$= \frac{Na\gamma H}{2}\sum_{i=1}^{N} q_{i0}^{-1}\left[\phi_i^2(t) + 2\hat{\phi}_i^2(t) - 2\phi_i(t)\hat{\phi}_i(t) - \hat{\phi}_i^2(t)\right]$$

$$(4.8)$$

$$\le \frac{Na\gamma H}{2}\sum_{i=1}^{N} q_{i0}^{-1}\left[\phi_i^2(t) + 2\hat{\phi}_i^2(t) - 2\phi_i(t)\hat{\phi}_i(t)\right]$$

$$= \frac{Na\gamma H}{2}\sum_{i=1}^{N} q_{i0}^{-1}\phi_i^2(t) - Na\gamma H \sum_{i=1}^{N}\tilde{\phi}_i(t)e_i^{\mathrm{T}} e_i.$$

Substituting (4.6)–(4.8) into (4.2) yields

$$\dot{V}(t) \le \sum_{i=1}^{N}\left(l_i + \frac{Na\gamma}{4} - Na\gamma H L_i\right)e_i^{\mathrm{T}} e_i + \frac{Na\gamma H}{2}\sum_{i=1}^{N} q_{i0}^{-1}(t)\phi_i^2(t). \qquad (4.9)$$

It is obviously that there exists sufficiently large positive constants L_i such that $l_i + (Na\gamma/4) - Na\gamma H L_i < 0$. According to (4.9), we have

$$\dot{V}(t) \leq \frac{Na\gamma H}{2} \sum_{i=1}^{N} q_{i0}^{-1}(t)\phi_i^2(t). \tag{4.10}$$

Since $\phi_i(t)$ is continuous and periodic, its boundedness can be obtained. The boundedness of $\phi_i(t)$ leads to the boundedness of $\dot{V}(t)$. For $V(T_1)$ is bounded, the finiteness of $V(t)$ is obvious, for all $t \in [T_1, T)$.

Next step, we prove the asymptotical convergence of $e(t)$.

According to (4.1), we can get

$$\Delta V(t) = V(t) - V(t - T)$$

$$= \frac{1}{2} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) - \frac{1}{2} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t - T)e_i(t - T) + \frac{Na\gamma H}{2} \sum_{i=1}^{N} \int_{t-T}^{t} \left[q_i^{-1}\tilde{\phi}_i^2(\tau) - q_i^{-1}\tilde{\phi}_i^2(\tau - T) \right] d\tau$$

$$+ \frac{Na\gamma H}{2} \sum_{i=1}^{N} r_i^{-1} \left(\tilde{\theta}_i(t) + L_i \right)^2 - \frac{Na\gamma H}{2} \sum_{i=1}^{N} r_i^{-1} \left(\tilde{\theta}_i(t - T) + L_i \right)^2. \tag{4.11}$$

Looking into the first two terms on the right-hand side of (4.11), with Newton-Leibniz formula, we obtain

$$\frac{1}{2} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) - \frac{1}{2} \sum_{i=1}^{N} e_i^{\mathrm{T}}(t - T)e_i(t - T) = \sum_{i=1}^{N} \int_{t-T}^{t} e_i^{\mathrm{T}}(\tau)\dot{e}_i(\tau)d\tau$$

$$\leq \sum_{i=1}^{N} \int_{t-T}^{t} \left[l_i + \frac{Na\gamma}{4} + Na\gamma H \left(\tilde{\phi}_i + \tilde{\theta}_i \right) \right] e_i^{\mathrm{T}}(\tau)e_i(\tau)d\tau. \tag{4.12}$$

Using the algebraic relation

$$(a - b)^{\mathrm{T}} H (a - b) - (a - c)^{\mathrm{T}} H (a - c) = (c - b)^{\mathrm{T}} H [2(a - b) + (b - c)], \tag{4.13}$$

where $a, b, c \in R^p$, $H \in R^{p \times p}$, and taking the third term on the right-hand side of (4.11), one obtains

$$\frac{Na\gamma H}{2} \sum_{i=1}^{N} \int_{t-T}^{t} \left[q_i^{-1}\tilde{\phi}_i^2(\tau) - q_i^{-1}\tilde{\phi}_i^2(\tau - T) \right] d\tau = -Na\gamma H \sum_{i=1}^{N} \int_{t-T}^{t} \left[\tilde{\phi}_i(\tau) + \frac{1}{2} q_i e_i^{\mathrm{T}} e_i \right] e_i^{\mathrm{T}} e_i d\tau. \tag{4.14}$$

The last two terms of right-hand side of (4.11) can be simplified as follows:

$$\frac{Na\gamma H}{2}\sum_{i=1}^{N}r_i^{-1}\left(\tilde{\theta}_i(t)+L_i\right)^2-\frac{Na\gamma H}{2}\sum_{i=1}^{N}r_i^{-1}\left(\tilde{\theta}_i(t-T)+L_i\right)^2$$

$$=Na\gamma H\sum_{i=1}^{N}\int_{t-T}^{t}r_i^{-1}\left(\tilde{\theta}_i(\tau)+L_i\right)\dot{\tilde{\theta}}_i(\tau)d\tau \tag{4.15}$$

$$=-Na\gamma H\sum_{i=1}^{N}\int_{t-T}^{t}\left(\tilde{\theta}_i(\tau)+L_i\right)e_i^{\mathrm{T}}(\tau)e_i(\tau)d\tau.$$

Substituting (4.12)–(4.15) into (4.11), we can attain

$$\Delta V(t)\leq-\sum_{i=1}^{N}\int_{t-T}^{t}\left(Na\gamma HL_i-l_i-\frac{Na\gamma}{4}\right)e_i^{\mathrm{T}}(\tau)e_i(\tau)d\tau. \tag{4.16}$$

Choosing $L_i>(l_i/Na\gamma H)+(1/4H)$, we can obtain

$$\Delta V(t)\ <0. \tag{4.17}$$

Applying (4.16) repeatedly for any $t\in[lT,(l+1)T]$, $l=1,2,\ldots$, and denoting $t_0=t-lT$, we have

$$V(t)=V(t_0)+\sum_{j=0}^{l-1}\Delta V(t-jT). \tag{4.18}$$

Considering $t_0\in[0,T)$ and the positive of $V(t)$, according to (4.18), we obtain

$$\lim_{t\to\infty}V(t)<\max_{t_0\in[0,T)}V(t_0)-\lim_{l\to\infty}\sum_{j=0}^{l-1}\sum_{i=1}^{N}\int_{t-(j+1)T}^{t-jT}\left(Na\gamma HL_i-l_i-\frac{Na\gamma}{4}\right)e_i^{\mathrm{T}}(\tau)e_i(\tau)d\tau. \tag{4.19}$$

Since $V(t_0)$ is bounded in the interval $[0,T)$, according to the convergence theorem of the sum of series and (4.19), the error $e(t)$ converges to zero asymptotically in L_T^2 norm. That is to say, we have

$$\lim_{t\to\infty}\int_{t-T}^{t}e_i^{\mathrm{T}}(\tau)e_i(\tau)d\tau=0. \tag{4.20}$$

Finally, we prove that all the closed-loop signals are bounded in L_T^2 norm sense. For all $t \in [T, \infty)$, the derivative of $V(t)$ is

$$\dot{V}(t) = \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\dot{e}_i(t) + \frac{Na\gamma H}{2}\sum_{i=1}^{N} q_i^{-1}\tilde{\phi}_i^2(t) - \frac{Na\gamma H}{2}\sum_{i=1}^{N} q_i^{-1}\tilde{\phi}_i^2(t-T)$$

$$+ Na\gamma H \sum_{i=1}^{N} r_i^{-1}\left(\tilde{\theta}_i(t) + L_i\right)\dot{\tilde{\theta}}_i(t) \tag{4.21}$$

$$\leq -\frac{Na\gamma H}{2}\sum_{i=1}^{N} q_i\left(e_i^{\mathrm{T}}e_i\right)^2 - \sum_{i=1}^{N}\left(Na\gamma HL_i - l_i - \frac{Na\gamma}{4}\right)e_i^{\mathrm{T}}e_i.$$

By (4.21), one can obtain

$$V(t) \leq V(T) - \frac{Na\gamma H}{2}\sum_{i=1}^{N}\int_T^t q_i\left(e_i^{\mathrm{T}}(\tau)e_i(\tau)\right)^2 d\tau - \sum_{i=1}^{N}\int_T^t\left(Na\gamma HL_i - l_i - \frac{Na\gamma}{4}\right)e_i^{\mathrm{T}}e_i d\tau.$$

$$\tag{4.22}$$

Choosing $L_i > (l_i/Na\gamma H) + (1/4H)$, we have

$$V(t) < V(T). \tag{4.23}$$

From the boundedness of $V(t)$ and (4.1), we can conclude that e_i, $\int_{t-T}^t \tilde{\phi}_i^2(\tau)d\tau$, $\tilde{\theta}_i(t)$ are all bounded. Since $\phi_i(t)$ is continuous periodic function, it implies the L_T^2 boundedness of $\hat{\phi}_i(t)$ and $\hat{\theta}_i(t)$. According to (3.3), the L_T^2 boundedness of the control input $u_i(t)$ is obtained. Since $e_i(t)$ is bounded, the boundedness of $x_i(t)$ is received. So the proof is completed. $\square$

5. Simulation Examples

To demonstrate the theoretical result obtained in Section 3, the Chuas chaotic circuit is used as a dynamical node of the network.

Consider the Chuas chaos circuit

$$\dot{x}_1 = p(-x_1 + x_2 - g(x_1)),$$
$$\dot{x}_2 = x_1 - x_2 + x_3, \tag{5.1}$$
$$\dot{x}_3 = -qx_2,$$

where $g(x_1) = m_0 x_1 + (1/2)(m_1 - m_0)(|x_1 + 1| - |x_1 - 1|)$ with $p = 10$, $q = 14.7$, $m_0 = -0.68$ and $m_1 = -1.27$.

We take the system (5.1) as identical nodes of the network, which is given by

$$\begin{pmatrix}\dot{x}_{i1}\\\dot{x}_{i2}\\\dot{x}_{i3}\end{pmatrix} = \begin{pmatrix}p(-x_{i1} + x_{i2} - g(x_{i1}))\\x_{i1} - x_{i2} + x_{i3}\\-qx_{i2}\end{pmatrix} + \sum_{j=1}^{N} a_{ij}\Gamma\exp\left(-\varphi_i(t)\begin{pmatrix}x_{j1}^2(t)\\x_{j2}^2(t)\\x_{j3}^2(t)\end{pmatrix}\right) + u_i(t), \quad i = 1, 2, \ldots, N,$$

$$\tag{5.2}$$

where

$$A = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 1 & -3 & 2 & 0 & 0 \\ 0 & 0 & 2 & -4 & 1 & 1 \\ 0 & 0 & 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 1 & 4 & -5 \end{pmatrix} \qquad \Gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5.3}$$

Nonlinearly parameterized function satisfies

$$\left\| \exp\left(-\varphi_i(t) \begin{pmatrix} x_{j1}^2(t) \\ x_{j2}^2(t) \\ x_{j3}^2(t) \end{pmatrix} \right) - \exp\left(-\varphi_i(t) \begin{pmatrix} s^2(t) \\ s^2(t) \\ s^2(t) \end{pmatrix} \right) \right\|^2 \leq \|e_j(t)\|^2 2\varphi_i(t)\exp(-1). \tag{5.4}$$

The parameters are selected as follows:

$$N = 6, \quad T = 6, \quad a = 5, \quad \gamma = 1, \quad H = 5,$$

$$\phi_1(t) = 0.2\sin\frac{2\pi t}{3}, \quad \phi_2(t) = 2\cos\pi t, \quad \phi_3(t) = -\sin\frac{2\pi t}{3}, \quad \phi_4(t) = \cos\pi t, \tag{5.5}$$

$$\phi_5(t) = -2\sin\frac{2\pi t}{3}, \quad \phi_6(t) = 2\sin\frac{2\pi t}{3}, \quad \theta = (2,10,8,3,2,5)^{\mathrm{T}}.$$

In the following simulations, we choose

$$q_1 = 0.0001, \quad q_2 = 0.0003, \quad q_3 = 0.0002, \quad q_4 = 0.0005, \quad q_5 = 0.0001, \quad q_6 = 0.0002,$$

$$q_{10}(t) = \frac{t}{6}q_1, \quad q_{20}(t) = \frac{t}{6}q_2, \quad q_{30}(t) = \frac{t}{6}q_3, \quad q_{40}(t) = \frac{t}{6}q_4, \quad q_{50}(t) = \frac{t}{6}q_5, \quad q_{60}(t) = \frac{t}{6}q_6,$$
$$\tag{5.6}$$

$r_1 = r_2 = r_3 = r_4 = r_5 = r_6 = 0.0005$. The initially estimated value of the unknown parameter $\phi(t)$ is $\hat{\phi}(0) = (0,0,0,0,0,0)^{\mathrm{T}}$, and the initial states are chosen as

$$x_1(0) = (2,0,-1)^{\mathrm{T}}, \quad x_2(0) = (3,1,-2)^{\mathrm{T}}, \quad x_3(0) = (-3,-2,1)^{\mathrm{T}}, \quad x_4(0) = (-1,0,-2)^{\mathrm{T}},$$
$$\tag{5.7}$$
$$x_5(0) = (-1,1,0)^{\mathrm{T}}, \quad x_6(0) = (3,-2,0.8)^{\mathrm{T}}, \quad s(0) = (-0.2,0.2,0.5)^{\mathrm{T}}.$$

According to Theorem 4.1, the complex dynamical network (5.2) can be synchronized by applying the adaptive controllers (3.3) and the adaptive learning laws of parameters (3.5) and (3.6). Figure 1 shows the error evolutions under the designed controllers. We clearly show that the states of the network (5.2) asymptotically synchronize in L_T^2 norm with the states of the desired orbit (5.1) showed in Figure 2. Figure 3 depicts the time evolution of the controllers, and Figure 4 shows the evolution of the estimated time-varying parameters.
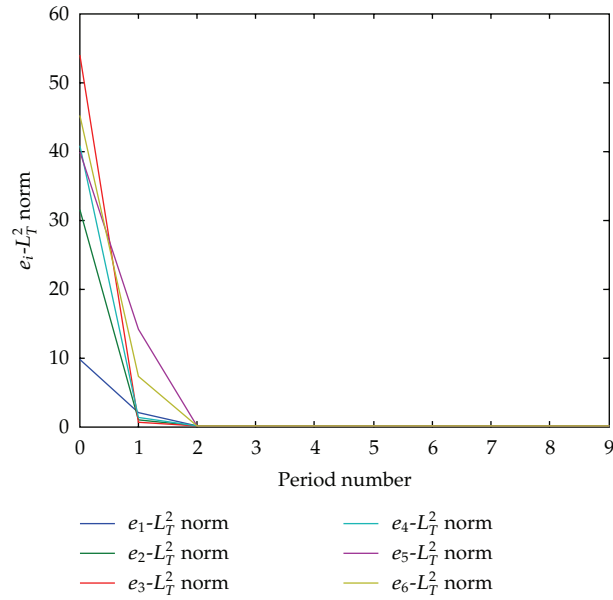
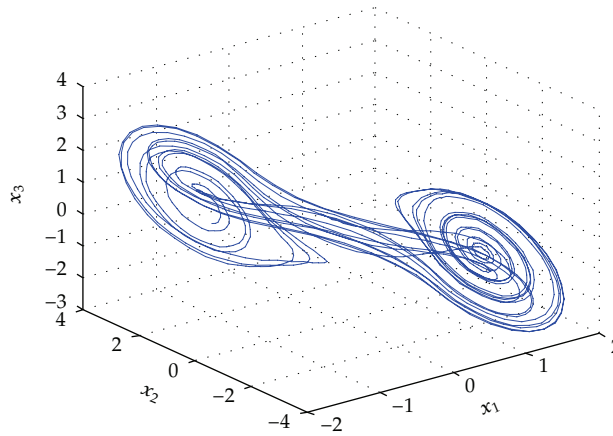**Figure 1:** The synchronization errors in L_T^2 norm.

Figure 2: The desired orbit.

Figure 5 displays the evolution of the estimated constant parameters. Obviously, Figures 3–5 show that all the control signals and estimated parameters are also bounded.

6. Conclusion

In this paper, a new adaptive learning control method applied in nonlinearly parameterized complex dynamical networks with unknown time-varying parameters is presented. By
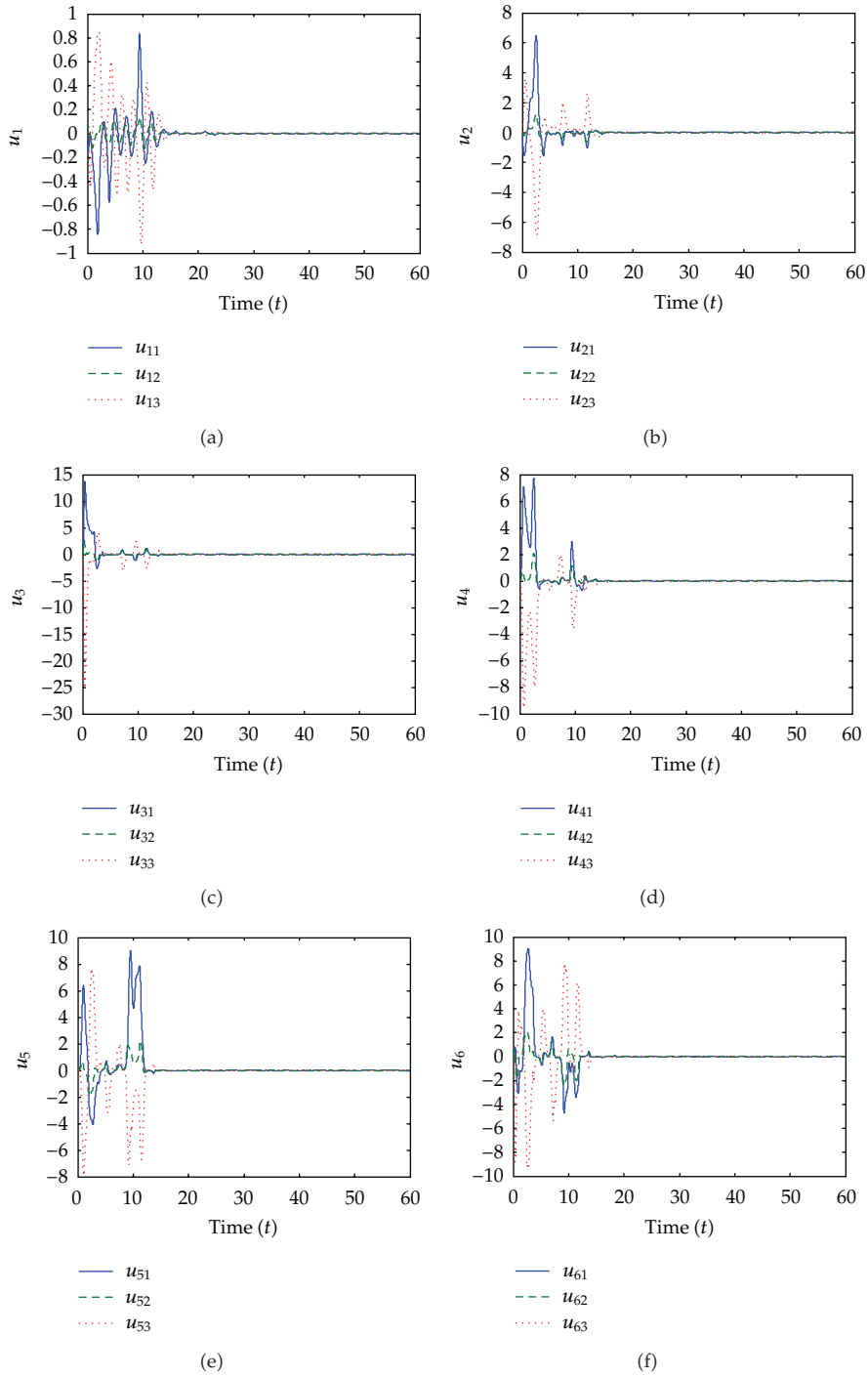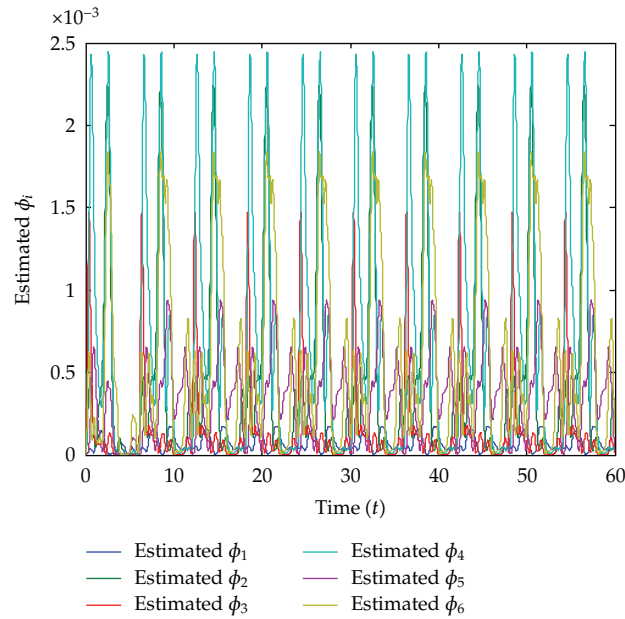
Figure 3: The curve of control inputs u_i.

Figure 4: The evaluations of parameters ϕ_i.
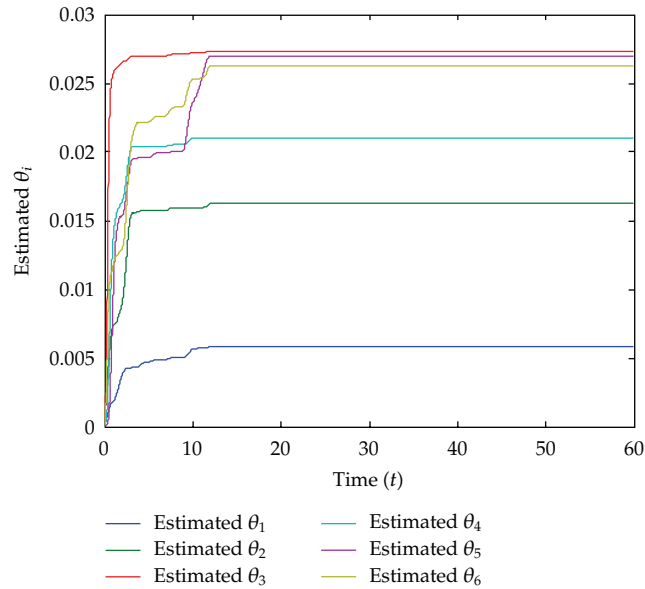
Figure 5: The evaluations of parameters θ_i.

using the parameter separation technique and constructing a composite energy function, the adaptive learning laws of parameters and the adaptive control strategy are designed to ensure the asymptotic convergence of the synchronization error in the sense of square error norm. An example of the complex network is finally used to verify the proposed theoretical result.
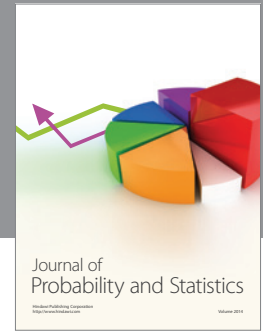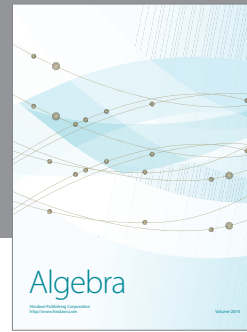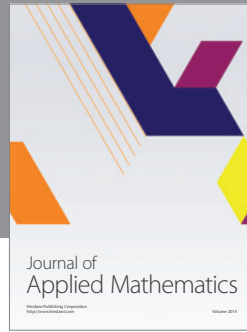
Acknowledgments

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