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Research Article

Oscillation Criteria for Linear Neutral Delay Differential Equations of First Order

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Some new sufficient conditions for oscillation of all solutions of the first-order linear neutral delay differential equations are obtained. Our new results improve many well-known results in the literature. Some examples are inserted to illustrate our results.

1. Introduction

A neutral delay differential equation (NDDE) is a differential equation in which the highest-order derivative of the unknown function is evaluated both at the present state at time t and at the past state at time $t-k$ for some positive constant k.

In the last two decades, there has been an increasing interest in obtaining sufficient conditions for the oscillation and/or nonoscillation of solutions of neutral delay differential equations. Particularly, we mention the papers by Ladas and Sficas [1], Chuanxi and Ladas [2], Ruan [3], Elabbasy and Saker [4], Kulenović et al. [5], and Karpuz and Öcalan [6] who investigated NDDEs with variable coefficients. To a large extent, this is due to its theoretical interest as well as to its importance in applications. It suffices to note that NDDEs appear in the study of networks containing lossless transmission lines (as in high-speed computers where the lossless transmission lines are used to interconnect switching circuits) in population dynamics and also in many applications in epidemics and infection diseases. We refer reader to [1–18] for relevant studies on this subject.

In this paper, we consider the linear first-order NDDE of the type

$$(x(t) - px(t-\tau))' + q_1 x(t) + q_2(t)\, x(t-\sigma) = 0; \quad t \geq t_0, \tag{1}$$

where $p, q_1, \tau, \sigma \in (0, \infty)$ and $q_2(t) \in C[[t_0, \infty), \mathbb{R}]$. When $q_1 \equiv 0$ and $q_2(t) = q, q$ is a constant, Jaroš [9] established some new oscillation conditions for all solutions of (1), and his technique was based on the study of the characteristic equation

$$\lambda - \lambda p e^{-\lambda\tau} + q e^{-\lambda\sigma} = 0. \tag{2}$$

Zhang [19], Ladas and Sficas [1], Grammatikopoulos et al. [10], and Yu et al. [8] considered (1) when $q_1 \equiv 0$, and they obtained some sufficient conditions for oscillation of (1). The purpose of this work is to present some new sufficient conditions under which all solutions of (1) are oscillatory. In order to achieve this object, we are first concerned with NDDE (1) with constant coefficients (when $q_2(t) \equiv q_2, q_2$ is a constant). That is,

$$(x(t) - px(t-\tau))' + q_1 x(t) + q_2 x(t-\sigma) = 0, \quad t \geq 0. \tag{3}$$

Some illustrating examples are given. In some sense, the established results extend and improve some previous investigations such as [1, 8–10, 19].

As usual, a solution of (1) is said to be oscillatory if it has arbitrarily large zeros and nonoscillatory if it is eventually positive or eventually negative. A function $x(t)$ is called eventually positive (or negative) if there exists t_0 such that $x(t) > 0$ (or $x(t) < 0$) for all $t \geq t_0$. Equation (1) is called oscillatory if all its solutions are oscillatory; otherwise, it is called nonoscillatory.

2. Main Results

In this section, we give some new sufficient conditions for the oscillation of all solutions of (1) and (3). This is done by using the following well-known lemmas which are from [11, 12].

Lemma 1. *Consider the NDDE*

$$\left(x(t) + px(t-\tau)\right)' + \sum_{i=1}^{n} q_i x\left(t-\sigma_i\right) = 0, \quad t \geq t_0, \tag{4}$$

where $\tau \geq 0, q_i > 0,$ *and* $\sigma_i \geq 0$ *for all* $i = 1, 2, \ldots, n$.

Let $x(t)$ *be a positive solution of* (4). *Set*

$$z(t) = x(t) - px(t-\tau). \tag{5}$$

If $p \geq -1$, *then* $z(t)$ *is a positive and decreasing solution of* (4); *that is,*

$$z'(t) + pz'(t-\tau) + \sum_{i=1}^{n} q_i z\left(t-\sigma_i\right) = 0, \quad t \geq t_0. \tag{6}$$

Lemma 2. *Let* p *and* τ *be positive constants. Let* $x(t)$ *be an eventually positive solution of the delay differential inequality*

$$x'(t) + px(t-\tau) \leq 0. \tag{7}$$

Then for t *sufficiently large,*

$$x(t-\tau) \leq Bx(t), \tag{8}$$

where

$$B = \frac{4}{(p\tau)^2}. \tag{9}$$

Our main results can now be given as follows.

Theorem 3. *Consider NDDE* (3). *Assume that*

(i) $q_2 \in [0, \infty), \sigma \geq \tau$, $0 < pe^{q_1\tau} < 1$ *and*

(ii) $\tau(q_1 pe^{q_1\tau} + q_2 e^{q_1\sigma}) > (1 - pe^{q_1\tau}\overline{m})^2/\overline{m}$,

where $\overline{m}$ *is the unique real root of the equation*

$$1 - pe^{q_1\tau} m = \ln m, \quad 1 \leq m \leq \frac{1}{pe^{-q_1\tau}}. \tag{10}$$

Then all solutions of (3) *are oscillatory.*

Proof. Assume, for the sake of a contradiction, that (3) has a nonoscillatory solution $x(t)$. Without loss of generality, assume that $x(t) > 0 \ \ \forall t \geq t_0 > 0$. Let

$$x(t) = e^{-q_1 t} y(t). \tag{11}$$

So that $y(t)$ is also a positive solution of (3).

That is,

$$\left(y(t) - p_1 y(t-\tau)\right)' + \sum_{i=1}^{2} a_i y\left(t-\tau_i\right) = 0, \tag{12}$$

where

$$p_1 = pe^{q_1\tau}, \qquad a_1 = q_1 pe^{q_1\tau}, \qquad a_2 = q_2 e^{q_1\sigma}, \\ \tau_1 = \tau, \qquad \tau_2 = \sigma. \tag{13}$$

Set for $t \geq t_0 + 2\tau$

$$z(t) = y(t) - p_1 y(t-\tau), \\ w(t) = \frac{z(t-\tau)}{z(t)}. \tag{14}$$

Thus it follows from Lemma 1 that $z(t)$ is a positive and decreasing solution of

$$z'(t) - p_1 z'(t-\tau) + \sum_{i=1}^{2} a_i z\left(t-\tau_i\right) = 0, \tag{15}$$

and in particular (as $\sigma > \tau$ implies that $t-\tau_i \leq t-\tau$, $i = 1, 2$), it follows that

$$z'(t) - p_1 z'(t-\tau) + \left(a_1 + a_2\right) z(t-\tau) \leq 0. \tag{16}$$

But we have

$$z'(t-\tau) < 0. \tag{17}$$

This implies that

$$z'(t) + \left(a_1 + a_2\right) z(t-\tau) \leq 0. \tag{18}$$

Applying Lemma 2 with (18) we get

$$z(t-\tau) \leq Bz(t); \quad B = \frac{4}{\tau^2\left(a_1 + a_2\right)^2}. \tag{19}$$

Then $w(t)$ is bounded.

Dividing (16) by $z(t) > 0$ and integrating from $t-\tau$ to t, we get

$$\ln w(t) \geq \left(a_1 + a_2\right) \int_{t-\tau}^{t} w(s)\, ds \\ - p_1 \int_{t-\tau}^{t} w(s) \frac{d}{ds} (\ln z(s-\tau))\, ds. \tag{20}$$

Let $m = \lim_{t\to\infty} \inf w(t)$.

Then, it follows from (20) that for $\varepsilon > 0$ and sufficiently small,

$$\ln(m+\varepsilon) \geq \left(a_1 + a_2\right)(m-\varepsilon)\tau + p_1 (m-\varepsilon)\ln(m-\varepsilon). \tag{21}$$

As ε is arbitrary, so we have

$$\left(a_1 + a_2\right)\tau \leq \frac{\left(1 - p_1 m\right)\ln m}{m}. \tag{22}$$

Let

$$F(m) = \frac{(1 - p_1 m)\ln m}{m}, \quad 1 \le m \le B. \tag{23}$$

Then

$$F'(m) = \frac{1 - p_1 m - \ln m}{m^2}, \quad 1 \le m \le B. \tag{24}$$

Let $\overline{m}$ be the unique real root of the equation

$$1 - p_1 m = \ln m, \quad m \in \left[1, \frac{1}{p_1}\right]. \tag{25}$$

Then

$$\max_{m \ge 1} F(m) = F(\overline{m}) = \frac{(1 - p_1 \overline{m})^2}{\overline{m}}. \tag{26}$$

Hence

$$\left(q_1 p e^{q_1 \tau} + q_2 e^{q_1 \sigma}\right)\tau \le \frac{(1 - p e^{q_1 \tau}\overline{m})^2}{\overline{m}}. \tag{27}$$

This contradicts condition (ii) and then completes the proof. □

Example 4. Consider the NDDE

$$\left(x(t) - \frac{1}{9}x\left(t - \frac{\pi}{2}\right)\right)' + \frac{1}{9}x(t) + x\left(t - \frac{5\pi}{2}\right) = 0, \quad t \ge 0. \tag{28}$$

We note that

$$p = \frac{1}{9}, \qquad q_1 = \frac{1}{9}, \qquad q_2 = 1, \qquad \tau = \frac{\pi}{2}, \qquad \sigma = \frac{5\pi}{2}. \tag{29}$$

Then we have

(i) $0 < p e^{q_1 \tau} = (1/9)e^{\pi/18} < 1$ and $\sigma \ge \tau$,

(ii)

$$\tau\left(q_1 p e^{q_1 \tau} + q_2 e^{q_1 \sigma}\right) = \frac{\pi}{2}\left(\frac{1}{81}e^{\pi/18} + \frac{1}{9}e^{5\pi/18}\right) > \frac{(1 - p e^{q_1 \tau}\overline{m})^2}{\overline{m}} = \frac{\left(1 - (1/9)e^{\pi/18}\right)^2}{2}, \tag{30}$$

where $\overline{m} = 2$ is the unique real root of the equation

$$1 - \frac{1}{9}e^{\pi/18} m = \ln m, \quad m \in \left[1, 9e^{-\pi/18}\right]. \tag{31}$$

Then all the hypotheses of Theorem 3 are satisfied, and therefore every solution of (28) oscillates. (Indeed $x(t) = \sin t$ is such a solution.)

Theorem 5. *Consider the NDDE* (1). *Assume that*

(iii) $0 < p e^{q_1 \tau} < 1$, $\sigma = \tau$, *and* $q_2(t) \in C[[t_0, \infty), (0, \infty)]$ *is periodic with period* τ,

(iv) $\lim_{t \to \infty} \inf \int_{t-\tau}^{t} e^{q_1 \tau}(q_2(s) + q_1 p)\,ds > (1 - p e^{q_1 \tau}\overline{m})^2 / \overline{m}$,

where $\overline{m}$ *is defined as in Theorem 3. Then all solutions of* (1) *are oscillatory.*

Proof. Assume, for the sake of contradiction, that (1) has a nonoscillatory solution $x(t)$. Without loss of generality, assume that $x(t) > 0 \quad \forall t \ge t_0 > 0$. Let

$$x(t) = e^{-q_1 t} y(t), \tag{32}$$

which is oscillation invariant transformation. Then $y(t)$ is a positive solution of the equation

$$\left(y(t) - p_1 y(t - \tau)\right)' + q(t)\, y(t - \tau) = 0, \tag{33}$$

where $p_1 = p e^{q_1 \tau}$ and $q(t) = e^{q_1 \tau}(q_2(t) + q_1 p)$ is periodic with period τ.

Let

$$z(t) = y(t) - p_1 y(t - \tau). \tag{34}$$

Then $z(t)$ is decreasing positive solution of the equation

$$\left(z(t) - p_1 z(t - \tau)\right)' + q(t)\, z(t - \tau) = 0. \tag{35}$$

Set

$$w(t) = \frac{z(t - \tau)}{z(t)}. \tag{36}$$

This implies that $w(t) \ge 1$, since $z(t - \tau) \ge z(t)$.

Dividing both sides of (33) by $z(t)$ and then integrating from $t - \tau$ to t, we obtain that

$$\ln w(t) = \int_{t-\tau}^{t} \frac{q(s)\left(y(s - \tau) - p_1 y(s - 2\tau) + p_1 y(s - 2\tau)\right)}{y(s) - p_1 y(s - \tau)}\,ds. \tag{37}$$

Hence

$$\ln w(t) = \int_{t-\tau}^{t} q(s)\, w(s)\, ds + p_1 \int_{t-\tau}^{t} \left(\frac{q(s)\, y(s - 2\tau)}{y(s) - p_1 y(s - \tau)}\right) ds. \tag{38}$$

Since $q(t)$ is periodic with period τ, then we obtain

$$q(t) = q(t - \tau) = -\frac{\left(y(t - \tau) - p_1 y(t - 2\tau)\right)'}{y(t - 2\tau)}. \tag{39}$$

Substituting in (38) we find, for all $t \ge t_0$,

$$\ln w(t) = \int_{t-\tau}^{t} q(s) w(s) ds - p_1 \int_{t-\tau}^{t} w(s) d \ln (y(s-\tau) - p_1 y(s-2\tau)) ds. \tag{40}$$

Now, we want to prove that $w(t)$ is bounded.

Applying the assumption (iv), we can find $t^* \in (t-\tau, t)$ such that

$$\int_{t-\tau}^{t^*} q(s) ds > \frac{F(\overline{m})}{2}, \qquad \int_{t^*}^{t} q(s) ds > \frac{F(\overline{m})}{2}, \tag{41}$$

where $F(\overline{m})$ is similar as in the proof of Theorem 3.

Integrating (33) from t^* to t we obtain

$$\begin{aligned} y(t^*) - p_1 y(t^* - \tau) &\ge \int_{t^*}^{t} q(s) y(s-\tau) ds \\ &\ge \int_{t^*}^{t} q(s) (y(s-\tau) - p_1 y(s-2\tau)) ds, \end{aligned} \tag{42}$$

Using Bonnet's Theorem and in particular (as $z'(t-\tau) < o$), we get

$$\begin{aligned} &y(t^*) - p_1 y(t^* - \tau) \\ &\quad \ge [y(t-\tau) - p_1 y(t-2\tau)] \cdot \int_{t^*}^{t} q(s) ds. \end{aligned} \tag{43}$$

Integrating (33) from $t-\tau$ to t^*, we get

$$\begin{aligned} &y(t-\tau) - p_1 y(t-2\tau) \\ &\quad \ge \int_{t-\tau}^{t^*} q(s) y(s-\tau) ds \\ &\quad \ge \int_{t-\tau}^{t^*} q(s) (y(s-\tau) - p_1 y(s-2\tau)) ds. \end{aligned} \tag{44}$$

Using Bonnet's Theorem and in particular (as $z'(t-\tau) < o$), we get

$$\begin{aligned} &y(t-\tau) - p_1 y(t-2\tau) \\ &\quad \ge [y(t^*-\tau) - p_1 y(t^* - 2\tau)] \cdot \int_{t-\tau}^{t^*} q(s) ds. \end{aligned} \tag{45}$$

Combining (43) and (45), we conclude

$$\begin{aligned} &y(t^*) - p_1 y(t^* - \tau) \\ &\quad \ge (y(t^*-\tau) - p_1 y(t^* - 2\tau)) \left(\frac{F(\overline{m})}{2} \right)^2, \end{aligned} \tag{46}$$

or

$$w(t^*) = \frac{y(t^*-\tau) - p_1 y(t^*-2\tau)}{y(t^*) - p_1 y(t^*-\tau)} \le \frac{4}{(F(\overline{m}))^2}. \tag{47}$$

Then $w(t)$ is bounded.

Now, let

$$m = \liminf_{t \to \infty} w(t). \tag{48}$$

But we have proved that $w(t)$ is bounded; that is, m is finite.

From (40), we obtain

$$\ln m \ge p_1 m \ln m + \liminf_{t \to \infty} m \int_{t-\tau}^{t} q(s) ds. \tag{49}$$

Therefore, we get

$$\liminf_{t \to \infty} m \int_{t-\tau}^{t} q(s) ds \le \frac{1 - p_1 m}{m} \ln m. \tag{50}$$

Hence

$$\liminf_{t \to \infty} \int_{t-\tau}^{t} q(s) ds \le \frac{(1 - p_1 \overline{m})^2}{\overline{m}}. \tag{51}$$

This contradicts our assumption (iv) and then completes the proof. □

Example 6. Consider the NDDE

$$\begin{aligned} &\left(x(t) - \frac{1}{9} x(t-\pi) \right)' + \frac{1}{18} x(t) \\ &\quad + (1 + \cos 2t) x(t-\pi) = 0, \quad t \ge 0, \end{aligned} \tag{52}$$

where

$$\begin{aligned} p = \frac{1}{9}, \qquad q_1 = \frac{1}{18}, \qquad \tau = \sigma = \pi, \\ q_2(t) = 1 + \cos 2t. \end{aligned} \tag{53}$$

Then we have

(1) $0 < p e^{q_1 \tau} = e^{\pi/18}/9 < 1$;

(2) $q_2(t) = 1 + \cos 2t \in C[[0,\infty),(0,\infty)]$ is periodic with period π and satisfies

$$\begin{aligned} &\liminf_{t \to \infty} \int_{t-\tau}^{t} e^{q_1 \tau} (q_2(s) + q_1 p) ds \\ &\quad = \liminf_{t \to \infty} \int_{t-\pi}^{t} e^{\pi/18} \left((1 + \cos 2s) + \frac{1}{162} \right) ds \\ &\quad = e^{\pi/18} \liminf_{t \to \infty} \left(s + \frac{s}{162} + \frac{1}{2} \sin 2s \Big|_{t-\pi}^{t} \right) \\ &\quad = \infty > \frac{(1 - p e^{q_1 \tau} \overline{m})^2}{\overline{m}} = \frac{\left(1 - (2/9) e^{\pi/18}\right)^2}{2}, \end{aligned} \tag{54}$$

where $\overline{m} = 2$ is the unique real root of the equation

$$1 - \frac{1}{9} e^{\pi/18} m = \ln m; \quad 1 \le m \le 9 e^{-\pi/18}. \tag{55}$$

Therefore (52) satisfies all the hypotheses of Theorem 5. Hence every solution of this equation is oscillatory.

Theorem 7. *Suppose that condition (iii) holds. If*

(v) $\lim_{t\to\infty}\inf\int_{t-\tau}^{t} e^{q_1\tau}[q_2(s)+q_1 p]ds > (1-pe^{q_1\tau})/e$,

then every solution of (1) *is oscillatory.*

Proof. Proceeding as in the proof of Theorem 5, we get (49) which implies that

$$\ln m \geq p_1 \ln m + m \lim_{t\to\infty}\inf\int_{t-\tau}^{t} q(s)\,ds. \tag{56}$$

Hence

$$\lim_{t\to\infty}\inf\int_{t-\tau}^{t} q(s)\,ds \leq \frac{1-p_1}{m}\ln m \leq \frac{1-p_1}{e}. \tag{57}$$

But this is a contradiction of assumption (v), and then the proof is complete. □

Example 8. Consider the NDDE

$$\begin{gathered}\left(x(t)-\frac{1}{5e}x\left(t-\frac{\pi}{2}\right)\right)'+\frac{2}{\pi}x(t)\\ +(e+\sin 4t)\,x\left(t-\frac{\pi}{2}\right)=0, \quad t\geq 0.\end{gathered} \tag{58}$$

Here we have

$$\begin{gathered}p=\frac{1}{5e}, \qquad q_1=\frac{2}{\pi}, \qquad \tau=\sigma=\frac{\pi}{2},\\ q_2(t)=e+\sin 4t.\end{gathered} \tag{59}$$

Note that $q_2(t) = e + \sin 4t$ is positive and periodic with period $\pi/2$, and also

(1) $0 < pe^{q_1\tau} = 1/5 < 1$,

(2)

$$\begin{aligned}&\lim_{t\to\infty}\inf\int_{t-\tau}^{t} e^{q_1\tau}\left[q_2(s)+q_1 p\right]ds\\ &\quad=\lim_{t\to\infty}\inf\int_{t-\tau}^{t} e\left[(e+\sin 4t)+\frac{2}{5e\pi}\right]ds=\infty\\ &\quad>\frac{\left(1-pe^{q_1\tau}\right)}{e}=\frac{4}{5e}.\end{aligned} \tag{60}$$

Then (58) satisfies hypotheses of Theorem 7, and so all its solutions are oscillatory.

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