Hindawi Publishing Corporation
Advances in Optical Technologies
Volume 2011, Article ID 989515, 8 pages
doi:10.1155/2011/989515

Research Article

Ultrashort Optical Pulse Propagation in terms of Analytic Signal

Sh. Amiranashvili[1] and A. Demircan[2]

[1] *Division of Laser Dynamics, Weierstrass Institute for Applied Analysis and Stochastics, Mohrenstrasse 39, 10117 Berlin, Germany*
[2] *Invalidenstrasse 114, 10115 Berlin, Germany*

Correspondence should be addressed to Sh. Amiranashvili, shalva@wias-berlin.de

Received 26 May 2011; Revised 9 August 2011; Accepted 9 August 2011

Academic Editor: Ci-Ling Pan

Copyright © 2011 Sh. Amiranashvili and A. Demircan. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We demonstrate that ultrashort optical pulses propagating in a nonlinear dispersive medium are naturally described through incorporation of analytic signal for the electric field. To this end a second-order nonlinear wave equation is first simplified using a unidirectional approximation. Then the analytic signal is introduced, and all nonresonant nonlinear terms are eliminated. The derived propagation equation accounts for arbitrary dispersion, resonant four-wave mixing processes, weak absorption, and arbitrary pulse duration. The model applies to the complex electric field and is independent of the slowly varying envelope approximation. Still the derived propagation equation posses universal structure of the generalized nonlinear Schrödinger equation (NSE). In particular, it can be solved numerically with only small changes of the standard split-step solver or more complicated spectral algorithms for NSE. We present exemplary numerical solutions describing supercontinuum generation with an ultrashort optical pulse.

1. Introduction

Complex envelope adequately describes linear and nonlinear propagation of a wave packet with many field cycles [1]. A slowly varying envelope approximation (SVEA) reduces the full set of Maxwell equations for the pulse field to a much more simple first-order nonlinear Schrödinger equation (NSE) for the complex envelope [2–4]. On the other hand, SVEA lacks precision when the relevant time scales are comparable to a single cycle period. Nonenvelope pulse propagation regimes include self-focusing [5, 6], optical shocks [7, 8], supercontinuum (SC) generation [9], and dynamics of ultrashort pulses [10–15]. In such situations NSE should be replaced by a more general propagation model.

Several simplified unidirectional propagation equations have been derived for special dispersion profiles. Such models do not use the pulse envelope and apply directly to the pulse field (see [16–20] and a review paper [21]). For a general dispersion profile, pulse propagation is commonly described by a generalized NSE [2, 4] in which a polynomial approximation of dispersion in the frequency domain is used. An arbitrary dispersion is then accounted for by a local dispersion operator in the time domain. To resolve convergence problems [22] also rational approximations and nonlocal dispersion operators may be considered [18, 23]. The nonlinear term in the generalized NSE is also modified to capture an arbitrary pulse duration [8, 24], Raman scattering [25, 26], and diffraction effect [24, 27, 28].

Being an envelope model, the generalized NSE was successfully applied to many propagation problems where the pulse envelope evolves as fast as the pulse field and SVEA cannot be used. This is a paradoxical situation especially because the SVEA is implicitly used in a common definition of the envelope. Indeed, an envelope Ψ of the field E with a central angular frequency ω_0 is usually introduced through the ansatz (see, e.g., [8, 27, 29–33])

$$E = \frac{1}{2}\Psi e^{-i\omega_0 t} + \text{c.c.}, \tag{1}$$

where for the sake of simplicity we (for the moment) consider only time dependence. Now, the imaginary part of $\Psi e^{-i\omega_0 t}$ remains unspecified but evidently affects, for example, the $|\Psi|^2\Psi$ term in the either standard or generalized

NSE. Relation (1) is meaningful if SVEA applies. One can then define Ψ as a sliding time average

$$\Psi + \Psi^* e^{2i\omega_0 t} = 2Ee^{i\omega_0 t} \Longrightarrow \Psi = 2\left\langle Ee^{i\omega_0 t}\right\rangle, \tag{2}$$

where $\langle \Psi^* e^{2i\omega_0 t}\rangle \approx 0$ because Ψ is slow on a time scale $2\pi/\omega_0$. Evidently, the sliding time average cannot be used for a few-cycle pulse for which we recall that Ψ is as fast as E itself (see [34] for a critical review of several possible definitions of the envelope).

This contradiction is addressed in the present paper. Specifically we demonstrate that propagation of an ultrashort pulse can naturally be described in terms of analytic signal instead of the envelope. In other words, the real-valued electric field $E(z,t)$ is replaced with a complex-valued one $\mathcal{E}(z,t)$ containing only positive harmonics [1]. The propagation equation for the analytic signal is structurally similar to NSE and can be solved with a small adaptation of the existing NSE solvers. Still the proposed propagation model applies directly to the electric field and avoids questionable use of definition (1). Useful applications of the analytic signal concept to the theory of nonlinear oscillations can be found in [34].

The present paper is organized as follows. Following [31, 35–37] we first derive a set of simplified equations for the field harmonics $E_\omega(z)$. Then we introduce an analytic signal and demonstrate how to remove all nonresonant nonlinear terms. The remaining resonant nonlinear terms have a simple "envelope" structure without use of the SVEA or the envelope as such. Finally the analytic signal approach is illustrated by exemplary numerical solutions.

2. Basic Equations

We consider a periodic sequence of linearly polarized electromagnetic pulses propagating along the z-axis in a one-dimensional dispersive nonlinear medium. The pulse field $E(z,t)$ is governed by the following nonlinear wave equation:

$$\partial_z^2 E - \frac{1}{c^2}\partial_t^2\left(E + \hat{\chi}^{(1)}E + \chi E^3\right) = 0 \tag{3}$$

in which $\hat{\chi}^{(1)}$ is a nonlocal linear susceptibility operator and a constant factor χ represents an instant nonlinear susceptibility of the third order. An inverse symmetry is assumed such that the quadratic nonlinear term is neglected. To quantify $\hat{\chi}^{(1)}$ we write $E(z,t)$ in the frequency domain:

$$E(z,t) = \sum_\omega E_\omega(z)e^{-i\omega t} \quad \text{with } \omega \in \frac{2\pi}{T}\mathbb{Z}, \tag{4}$$

where T is the period of the pulse sequence and $E_\omega(z) = \int_{-T/2}^{+T/2} E(z,t)e^{i\omega t}(dt/T)$. In what follows we assume that $E_{\omega=0} = 0$, that is, the time-averaged electric field vanishes. Now, $\hat{\chi}^{(1)}E$ is given by a convolution

$$\left(\hat{\chi}^{(1)}E\right)_\omega = \chi^{(1)}(\omega)E_\omega. \tag{5}$$

The linear susceptibility $\chi^{(1)}(\omega)$ yields the dielectric constant $\epsilon(\omega) = 1 + \chi^{(1)}(\omega)$ and the propagation parameter

$$k(\omega) = \frac{\omega}{c}\sqrt{\epsilon(\omega)} = \beta(\omega) + i\alpha(\omega), \tag{6}$$

where $\beta(\omega)$ and $\alpha(\omega)$ are odd and even functions, respectively. In the following we consider a small absorption limit such that $|\beta(\omega)| \gg \alpha(\omega) \geq 0$ in a transparency window to which an essential part of the pulse spectrum must belong.

To proceed we write the nonlinear wave equation (3) in the frequency domain:

$$[\partial_z^2 + \beta^2 + 2i\alpha\beta]E_\omega + \frac{\omega^2\chi}{c^2}\sum_{\omega_1+\omega_2+\omega_3=\omega} E_{\omega_1}E_{\omega_2}E_{\omega_3} = 0, \tag{7}$$

where the term $\alpha^2 E_\omega$ is neglected. The summation on the right hand side is performed only over the suitable triads $\{\omega_1, \omega_2, \omega_3\}$. In what follows such summations will be abbreviated as $\sum_{123|\omega} E_{\omega_1}E_{\omega_2}E_{\omega_3}$.

Equation (7) is the starting point of our considerations. It will be simplified in a weakly nonlinear limit. Specifically we introduce a smallness parameter $\epsilon \ll 1$ and assume the following scaling:

$$\chi E^2 = O(\epsilon), \qquad \alpha = O(\epsilon), \tag{8}$$

for all fields and frequencies of interest. In particular, both $\alpha^2 E$ and $\chi^{(5)}E^5$ terms neglected in (7) are estimated as $O(\epsilon^2 E)$, the latter quantity defines which terms should further be neglected when reducing (7). In the next sections (7) is simplified using an unidirectional approximation, introducing a proper complex electric field and eliminating the nonresonant terms.

3. Unidirectional Approximation

As explained above, both the nonlinear and the absorption terms in (7) are taken small. In a first step, neglecting the small terms, we write (7) as

$$[\partial_z^2 + \beta^2(\omega)]E_\omega = O(\epsilon E). \tag{9}$$

The unidirectional approximation deals with two classes of special solutions of (7), namely, the forward and the backward solutions, where

$$[i\partial_z + \beta(\omega)]E_\omega = O(\epsilon E), \qquad [i\partial_z - \beta(\omega)]E_\omega = O(\epsilon E), \tag{10}$$

respectively. We further consider the forward wave solution for which the term $O(\epsilon E)$ describes small contributions of the backward waves permanently generated by the nonlinear term in (7). To calculate this contribution we apply an exact identity:

$$\begin{aligned}[\partial_z^2 + \beta^2]E_\omega &= 2\beta[i\partial_z + \beta]E_\omega - [i\partial_z + \beta]^2 E_\omega \\ &= 2\beta[i\partial_z + \beta]E_\omega + O(\epsilon^2 E),\end{aligned} \tag{11}$$

where the last term should be neglected because such terms were neglected when deriving (7). The latter is then transformed to the unidirectional form

$$i\partial_z E_\omega + \beta(\omega)E_\omega = -i\alpha(\omega)E_\omega - \frac{\omega^2\chi}{2c^2\beta(\omega)}\sum_{123|\omega} E_{\omega_1}E_{\omega_2}E_{\omega_3} \tag{12}$$

where both the left- and the right-hand sides are scaled as $O(\epsilon E)$ cf. (8).

The propagation model (12) was first suggested in [35] and then generalized in [31, 38] for an arbitrary polarization. As we will see below, it can be significantly simplified by elimination of the non-resonant nonlinear terms.

4. Resonances

A natural approach to (12) is to simplify it by a suitable change of variables. In the spirit of the perturbation theory, we use a power expansion and formally define

$$\widetilde{E}_\omega = E_\omega + \frac{\omega^2\chi}{2c^2\beta(\omega)} \sum_{123|\omega} \frac{E_{\omega_1}E_{\omega_2}E_{\omega_3}}{\beta(\omega)-\beta(\omega_1)-\beta(\omega_2)-\beta(\omega_3)}. \tag{13}$$

We apply $i\partial_z+\beta$ to both sides of (13). Equations (8) and (12) yield

$$[i\partial_z+\beta(\omega)]E_{\omega_1}E_{\omega_2}E_{\omega_3} = [\beta(\omega)-\beta(\omega_1)-\beta(\omega_2)-\beta(\omega_3)] \times E_{\omega_1}E_{\omega_2}E_{\omega_3} + O(\epsilon^2 E). \tag{14}$$

We see that

$$[i\partial_z+\beta]\widetilde{E}_\omega = [i\partial_z+\beta]E_\omega + \frac{\omega^2\chi}{2c^2\beta} \sum_{123|\omega} E_{\omega_1}E_{\omega_2}E_{\omega_3} + O(\epsilon^2 E). \tag{15}$$

Using the last equation together with definition (13), (12) can be transformed to the form

$$[i\partial_z+\beta]\widetilde{E}_\omega = -i\alpha\widetilde{E}_\omega + O\left(\epsilon^2\widetilde{E}\right). \tag{16}$$

Comparing the latter equation with (12) we see that the cubic nonlinear term is formally eliminated. Strictly speaking, the contribution of the cubic term is "shifted" to the higher $O(\epsilon^2 E)$ order in the spirit of the canonical perturbation theory [39, 40].

It is a good point to stress that such an elimination is possible only for non-resonant triads $\{\omega_1, \omega_2, \omega_3\}$. The resonant frequencies are defined by the conditions

$$\begin{aligned} \omega_1+\omega_2+\omega_3 &= \omega, \\ \beta(\omega_1)+\beta(\omega_2)+\beta(\omega_3) &= \beta(\omega). \end{aligned} \tag{17}$$

Exemplary solutions of the above resonance conditions in the three-dimensional frequency space are shown by thick solid lines in Figure 1 for fluoride glass. In the vicinity of the resonant lines the transformation (13) is singular and contributions of the nonlinear terms in (12) cannot be eliminated. Far from the resonant curves, the nonlinear terms in (12) lead only to small forced oscillations of $E_\omega(z)$ and can be neglected.

In what follows we assume that for $\omega > 0$ all solutions of (17) contain one negative and two positive frequencies. In other words, only $2 \rightleftarrows 2$ four-wave mixing (FWM) processes

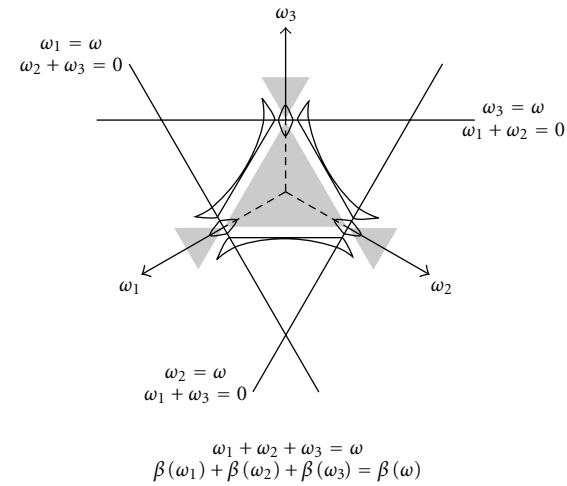


Figure 1: Three-dimensional frequency space $(\omega_1, \omega_2, \omega_3)$. The thick curves show solutions of the resonance conditions (17) for a bulk fluoride glass (ω corresponds to 0.8 μm, $\epsilon(\omega)$ is taken from [22]). The dashed regions correspond to $3 \rightleftarrows 1$ four-wave mixing processes which are neglected in this paper. The nondashed part of the resonance curves corresponds to $2 \rightleftarrows 2$ processes and is taken into account.

are taken into account. The contribution of $3 \rightleftarrows 1$ FWM processes, for example, when all frequencies in the resonance conditions (17) are positive, is neglected. In such a situation (12) cannot be linearized completely, but can be considerably simplified as explained in the next section.

5. Analytic Signal

To simplify (12) we first write it in such a way that contributions of the positive and negative frequencies are explicitly distinguished. To this end we introduce a complex-valued analytic signal $\mathcal{E}(z,t)$ for the electric field [1, 34]. In contrast to the real-valued $E(z,t)$, the analytic signal contains only positive harmonics

$$\mathcal{E}(z,t) = 2\sum_{\omega>0} E_\omega(z)e^{i\omega t}, \qquad E(z,t) = \frac{\mathcal{E}(z,t)+\mathcal{E}^*(z,t)}{2} \tag{18}$$

such that

$$E_{\omega>0} = \frac{1}{2}\mathcal{E}_\omega, \qquad E_{\omega<0} = E^*_{-\omega} = \frac{1}{2}\mathcal{E}^*_{-\omega}. \tag{19}$$

In other words,

$$E_\omega = \frac{\mathcal{E}_\omega + \mathcal{E}^*_{-\omega}}{2}, \tag{20}$$

because $\mathcal{E}_{\omega<0} = 0$ by construction. Inserting (20) into the product $E_{\omega_1}E_{\omega_2}E_{\omega_3}$ in (12) and performing multiplication, we see that all FWM processes are now separated. The reason for such a simplification is that the analytic signal is related to classical creation and annihilation operators as explained in [41] for a more general bidirectional nonlinear wave

equation. After neglecting all but $2 \rightleftarrows 2$ FWM processes, the unidirectional (12) finally takes the form

$$[i\partial_z + \beta(\omega) + i\alpha(\omega)]\mathcal{E}_\omega + \frac{3\omega^2\chi}{8c^2\beta(\omega)}\left(\mathfrak{P}[|\mathcal{E}|^2\mathcal{E}]\right)_\omega = 0 \tag{21}$$

where $(\cdots)_\omega$ denotes a spectral component, $\omega > 0$, and $\mathfrak{P}[|\mathcal{E}|^2\mathcal{E}]$ denotes a positive frequency part of $|\mathcal{E}|^2\mathcal{E}$.

Equation (21) is our main result. It compromises properties of both the spectral propagation models and the envelope models. For instance, an arbitrary $\beta(\omega)$ is captured like in the spectral model (12); on the other hand a familiar invariance with respect to the phase shifts ($\mathcal{E}_\omega \rightarrow \mathcal{E}_\omega e^{i\theta}$ with $\theta = \text{const}$) is retained like in the standard NSE.

For the vanishing dissipation term $\alpha(\omega)$, the analytic signal equation was derived in [41]. We want to emphasize that dissipation enters (21) in a nontrivial way: the nonlinear term is not affected. If one carelessly replaces $\beta(\omega)$ with $k(\omega) = \beta(\omega) + i\alpha(\omega)$ in the cubic term, the resulting model will show an unphysical nonlinear gain which will finally spoil pulse propagation.

The nonlinear term in (21) is just a positive frequency part of the standard NSE-type nonlinearity. Still the analytic signal $\mathcal{E}(z,t)$ directly represents the electric field $E = \text{Re}[\mathcal{E}]$ and is independent of SVEA. The negative frequency part of $|\mathcal{E}|^2\mathcal{E}$ corresponds to backscattered light that is generated in the course of pulse propagation. The feedback effect of the backscattered light is neglected in the unidirectional approximation. Finally we note that the analytic signal equation (21) can be solved numerically with only small changes of the standard NSE solvers. Exemplary numerical solutions are given in the next section.

6. Numerical Solutions

The numerical solutions of (21) are described in this section.We use a dealiased pseudospectral method, which originates from the computational fluid dynamics [42]. This method calculates all linear operators and derivatives in the frequency domain and performs the nonlinear multiplications in the time domain, with the transformations between the domains achieved by the fast Fourier transform. The integration for the linear and nonlinear part is performed in the frequency domain by a precise Runge-Kutta integration scheme of order eight with adaptive stepsize control depending on the accuracy as described in [43].

Considering ultrashort optical pulses with the carrier frequencies of several hundreds THz, we use the time step of at least $\Delta t = 0.6$ fs. Depending on the initial pulse width, we use a resolution of 2^{14} and 2^{15} harmonics for a periodic time window $T = 5$ ps and $T = 10$ ps, respectively. Several test calculations were performed for a better resolution, 2^{17}. The increase of the harmonics number does not affect the results.

In the following we study the nonlinear propagation of femtosecond pulses in the anomalous dispersion regime of a microstructured fiber, where complex and comprehensive behavior can be observed. Depending on the input pulse power and width, the interplay of linear and nonlinear effects such as self-phase modulation (SPM), FWM, and soliton dynamics can lead to the generation of octave-spanning spectra. It is well known that the physical mechanism of the dramatic spectral broadening process is related to the break-up of higher-order solitons [35]. The soliton fission is caused by the formation of fundamental soliton pulses and the generation of a nonsolitonic dispersive wave into the phase-matched wavelength, leading to a spectrum broadened over an octave, even if the injected pulse energy is less than a few nanojoules. Besides soliton propagation, the modulation instability (MI) is another general feature in the anomalous dispersion regime, which affects the propagation of an optical pulse. The MI is a well understood instability phenomenon of the NSE, which results from the interplay between SPM and group velocity dispersion. In [44] the ability of the MI to generate SC and the dominance of the MI for short pulses have been demonstrated. In [45] it has been shown that soliton fission dominates for low input power and short pulses (100 fs) and the modulation instability has a strong impact for high input powers at arbitrary pulse widths.

For our simulations the dispersion profile of the highly nonlinear microstructured fiber is taken from [46]. As an initial condition we consider an input pulse electric field having a central angular frequency ω_0,

$$\sqrt{\chi}E(z,t)|_{z=0} = \frac{1}{2}\Psi(t)e^{-i\omega_0 t} + \text{c.c.}, \tag{22}$$

and a hyperbolic-secant shape for the initial envelope $\Psi(t) = \Psi_0 \cosh^{-1}(t/t_0)$ with the dimensionless amplitude $\Psi_0 = 0.03$ and temporal width $t_0 = 10$–100 fs. The value of ω_0 corresponds to a pump wavelength $\lambda_0 = 810$ nm; this wavelength is in the vicinity of the zero dispersion wavelength in the anomalous dispersion regime.

Figure 2 shows the density plots in the (ω, z)-plane of the spectral evolution for different input pulse widths. The spectra are shown on a logarithmic scale to illustrate the fine structure of the spectrum generated. For a 100 fs pulse spectral broadening in the range $z = 1$–10 mm is mainly dominated by SPM. However, the significant features of the MI can be observed. The underlying MI acts in the initial stage on the pulse and leads to the generation of a Stokes and an anti-Stokes component. Figure 2(a) shows the appearance of two sidebands after $z = 3.7$ mm. This demonstrates that also for short pulses with durations of 100 fs the modulation instability is present and can have an impact on the propagation dynamics.

The simulations in Figures 2(b) and 2(c) illustrate the typical scenario of spectral broadening by soliton-related dynamics, for input pulses with $t_0 = 50$ fs and $t_0 = 10$ fs. Three different stages are clearly observed. The initial stage of propagation is dominated by symmetrical spectral broadening induced by SPM. An extreme spectral broadening is then caused by pulse contraction due to the first step of soliton propagation (Figures 3(b) and 3(d) at $z = 7.9$ mm for 50 fs and $z = 2.1$ mm for 10 fs). In the second stage the spectral broadening becomes asymmetric and energy is shifted to the blue side of the spectrum, due to soliton fission accompanied

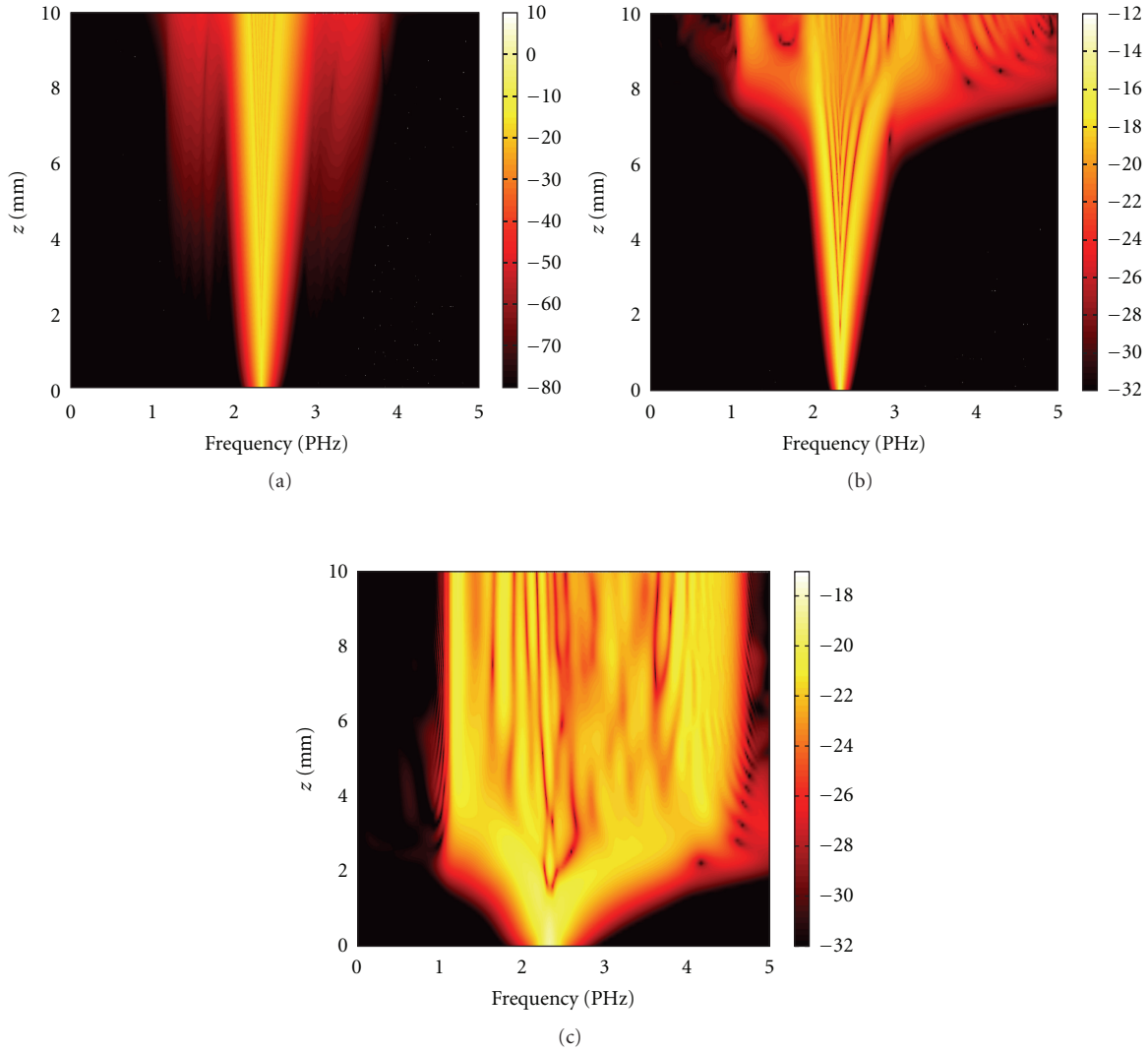


Figure 2: Density plots of the spectral evolution for (a) $t_0 = 100$ fs pulse with typical signature of the modulation instability, (b) $t_0 = 50$ fs, and (c) $t_0 = 10$ fs pulses generating SC by soliton fission. The spectra are shown in logarithmic scale (dB).

by the excitation of dispersive waves. This is associated with the development of distinct temporal peaks that sit upon a broader low-amplitude background (Figures 3(e) and 3(f) at $z = 9.9$ mm). The extension of the spectrum to the blue side is related to the dispersion profile of the optical fiber and to the input pulse power. The spectral broadening is limited by the broadening of the temporal waveform of the pulses. In the third stage the spectral width is already saturated, but FWM generates complicated substructures. The appearance of the fine structure is an essential phenomenon and is extremely sensitive with respect to the initial pulse energy.

Equation (21) clearly reproduces all essential features of the SC evolution seen in a number of experiments [9, 47] and in simulations with the generalized NSE, namely, SPM and FWM, MI, soliton fission, and generation of dispersive waves by solitons. Moreover, it goes beyond the envelope approximation and allows for an arbitrary pulse duration.

7. Conclusions

Let us summarize our results. Propagation of spectrally broad ultrashort optical pulses is considered. We first derived the so-called forward Maxwell (12) following [35] but taking into account a small dissipation effect. Then we simplified the forward Maxwell equation by eliminating all non-resonant nonlinear terms. To this end we incorporated an analytic signal representation. The latter is useful as it is intrinsically related to classical creation and annihilation operators [41, 48, 49]. The resulting non-envelope model (21) applies to the analytic signal for the pulse electric field in the frequency domain. The model (21) combines advantages of both envelope and non-envelope approaches; it accounts for arbitrary dispersion, resonant four-wave mixing processes, weak absorption, and arbitrary pulse duration. It is of interest that the linear absorption does not affect the nonlinear term in (21). This is an important

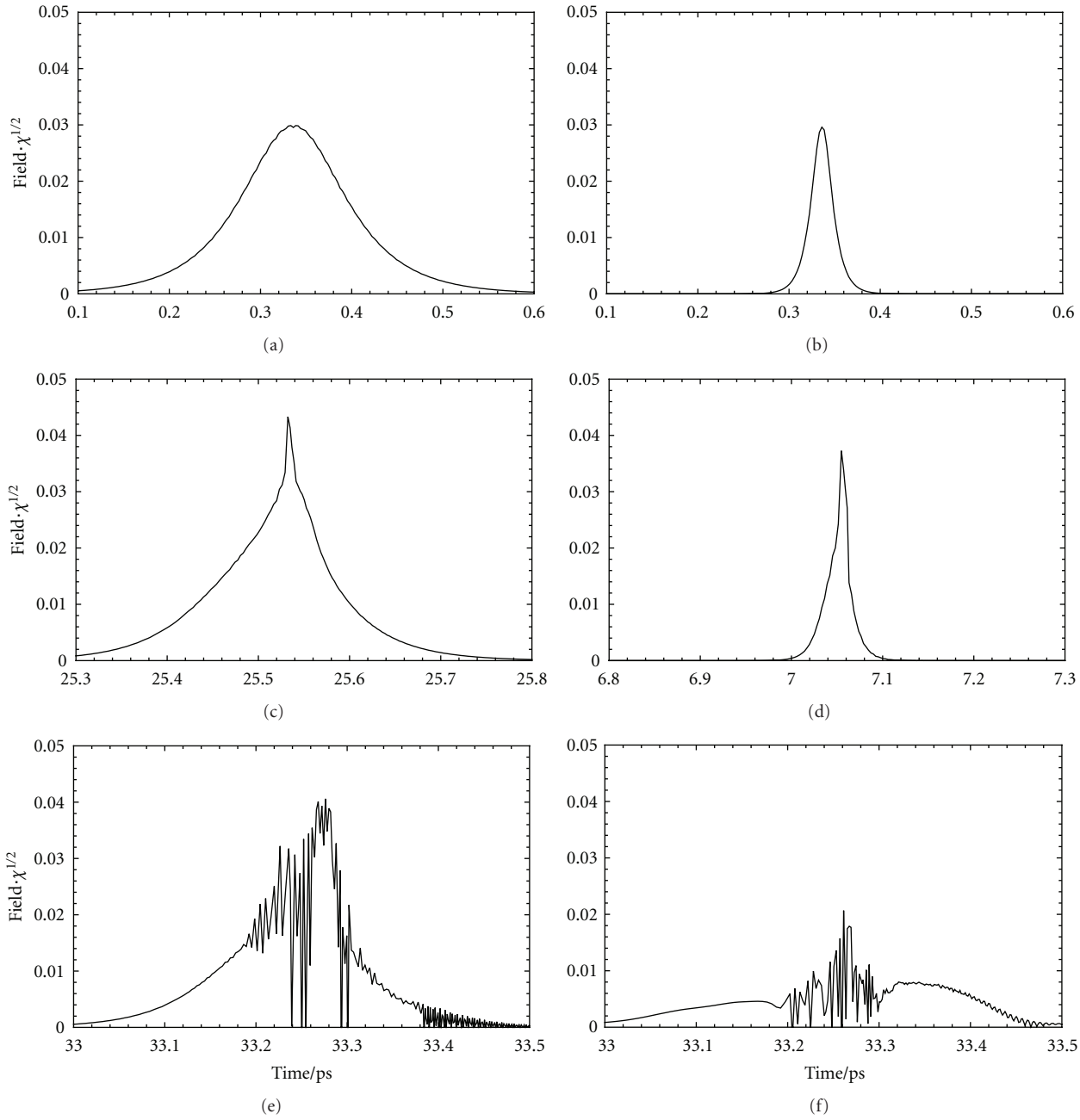


FIGURE 3: Temporal evolution for selected propagation distances for a 50 fs pulse at (a) $z = 0.1$ mm, (c) $z = 7.6$ mm, and (e) $z = 9.9$ mm and for a 10 fs pulse at (b) $z = 0.1$ mm, (d) $z = 2.1$ mm, and (f) $z = 9.9$ mm. The values of $|\mathcal{E}(z,t)|$ are shown.

issue because careless use of the complex refractive index, for example, in (21), leads to an unphysical nonlinear gain. Finally, the existing numerical approaches to the envelope propagation equations can be easily adopted to solve the non-envelope (21).

Acknowledgments

The work of Sh. Amiranashvili was supported by the DFG Research Center MATHEON under project D 14. Helpful discussions with U. Bandelow, U. Leonhardt, M. Lichtner, and A. Mielke are gratefully acknowledged.

References

[1] M. Born and E. Wolf, *Principles of Optics*, Cambridge University Press, Cambridge, UK, 7th edition, 1999.

[2] Y. Kodama and A. Hasegawa, "Nonlinear pulse propagation in a monomode dielectric guide," *IEEE Journal of Quantum Electronics*, vol. 23, no. 5, pp. 510–524, 1987.

[3] G. P. Agrawal, *Nonlinear Fiber Optics*, Academic Press, New York, NY, USA, 4th edition, 2007.

[4] R. W. Boyd, *Nonlinear Optics*, Academic Press, New York, NY, USA, 3rd edition, 2008.

[5] J. K. Ranka and A. L. Gaeta, "Breakdown of the slowly varying envelope approximation in the self-focusing of ultrashort pulses," *Optics Letters*, vol. 23, no. 7, pp. 534–536, 1998.

[6] J. E. Rothenberg, "Space-time focusing: breakdown of the slowly varying envelope approximation in the self-focusing of femtosecond pulses," *Optics Letters*, vol. 17, no. 19, pp. 1340–1342, 1992.

[7] F. DeMartini, C. H. Townes, T. K. Gustafson, and P. L. Kelley, "Self-steepening of light pulses," *Physical Review*, vol. 164, no. 2, pp. 312–323, 1967.

[8] P. Kinsler and G. H. C. New, "Few-cycle pulse propagation," *Physical Review A*, vol. 67, no. 2, article 023813, 2003.

[9] J. M. Dudley, G. Genty, and S. Coen, "Supercontinuum generation in photonic crystal fiber," *Reviews of Modern Physics*, vol. 78, no. 4, pp. 1135–1183, 2006.

[10] K. Akimoto, "Properties and applications of ultra-short electromagnetic mono- and sub-cycle waves," *Journal of the Physical Society of Japan*, vol. 65, no. 7, pp. 2020–2032, 1996.

[11] A. V. Kim, M. Y. Ryabikin, and A. M. Sergeev, "From femtosecond to attosecond pulses," *Uspekhi Fizicheskikh Nauk*, vol. 42, no. 1, pp. 54–61, 1999.

[12] T. Brabec and F. Krausz, "Intense few-cycle laser fields: frontiers of nonlinear optics," *Reviews of Modern Physics*, vol. 72, no. 2, pp. 545–591, 2000.

[13] P. M. Paul, E. S. Toma, P. Breger et al., "Observation of a train of attosecond pulses from high harmonic generation," *Science*, vol. 292, no. 5522, pp. 1689–1692, 2001.

[14] M. Hentschel, R. Kienberger, C. Spielmann et al., "Attosecond metrology," *Nature*, vol. 414, no. 6863, pp. 509–513, 2001.

[15] M. Drescher, M. Hentschel, R. Kienberger et al., "X-ray pulses approaching the attosecond frontier," *Science*, vol. 291, no. 5510, pp. 1923–1927, 2001.

[16] S. A. Kozlov and S. V. Sazonov, "Nonlinear propagation of optical pulses of a few oscillations duration in dielectric media," *Journal of Experimental and Theoretical Physics*, vol. 84, no. 2, pp. 221–228, 1997.

[17] T. Schäfer and C. E. Wayne, "Propagation of ultra-short optical pulses in cubic nonlinear media," *Physica D*, vol. 196, no. 1-2, pp. 90–105, 2004.

[18] V. G. Bespalov, S. A. Kozlov, Y. A. Shpolyanskiy, and I. A. Walmsley, "Simplified field wave equations for the nonlinear propagation of extremely short light pulses," *Physical Review A*, vol. 66, no. 1, article 013811, 2002.

[19] S. Amiranashvili, A. G. Vladimirov, and U. Bandelow, "A model equation for ultrashort optical pulses around the zero dispersion frequency," *European Physical Journal D*, vol. 58, no. 2, pp. 219–226, 2010.

[20] H. Leblond and D. Mihalache, "Few-optical-cycle solitons: modified Korteweg-de Vries sine-Gordon equation versus other non-slowly-varying-envelope-approximation models," *Physical Review A*, vol. 79, no. 6, article 063835, 2009.

[21] A. I. Maimistov, "Some models of propagation of extremely short electromagnetic pulses in a nonlinear medium," *Quantum Electronics*, vol. 30, no. 4, pp. 287–304, 2000.

[22] K. E. Oughstun and H. Xiao, "Failure of the quasimonochromatic approximation for ultrashort pulse propagation in a dispersive, attenuative medium," *Physical Review Letters*, vol. 78, no. 4, pp. 642–645, 1997.

[23] S. Amiranashvili, U. Bandelow, and A. Mielke, "Padé approximant for refractive index and nonlocal envelope equations," *Optics Communications*, vol. 283, no. 3, pp. 480–485, 2010.

[24] T. Brabec and F. Krausz, "Nonlinear optical pulse propagation in the single-cycle regime," *Physical Review Letters*, vol. 78, no. 17, pp. 3282–3285, 1997.

[25] K. J. Blow and D. Wood, "Theoretical description of transient stimulated Raman scattering in optical fibers," *IEEE Journal of Quantum Electronics*, vol. 25, no. 12, pp. 2665–2673, 1989.

[26] N. Karasawa, S. Nakamura, N. Nakagawa et al., "Comparison between theory and experiment of nonlinear propagation for a-few-cycle and ultrabroadband optical pulses in a fused-silica fiber," *IEEE Journal of Quantum Electronics*, vol. 37, no. 3, pp. 398–404, 2001.

[27] M. A. Porras, "Propagation of single-cycle pulsed light beams in dispersive media," *Physical Review A*, vol. 60, no. 6, pp. 5069–5073, 1999.

[28] M. Geissler, G. Tempea, A. Scrinzi, M. Schnürer, F. Krausz, and T. Brabec, "Light propagation in field-ionizing media: extreme nonlinear optics," *Physical Review Letters*, vol. 83, no. 15, pp. 2930–2933, 1999.

[29] P. Kinsler and G. H. C. New, "Few-cycle soliton propagation," *Physical Review A*, vol. 69, no. 1, article 013805, 2004.

[30] X. Fu, L. Qian, S. Wen, and D. Fan, "Propagation of the ultrashort pulsed beam with ultrabroad bandwidth in the dispersive medium," *Physical Review A*, vol. 68, no. 6, article 063818, 2003.

[31] M. Kolesik, J. V. Moloney, and M. Mlejnek, "Unidirectional optical pulse propagation equation," *Physical Review Letters*, vol. 89, no. 28, article 283902, 2002.

[32] J. R. Gulley and W. M. Dennis, "Ultrashort-pulse propagation through free-carrier plasmas," *Physical Review A*, vol. 81, no. 3, article 033818, 2010.

[33] J. Moses and W. W. Wise, "Controllable self-steepening of ultrashort pulses in quadratic nonlinear media," *Physical Review Letters*, vol. 97, no. 7, article 073903, 2006.

[34] D. E. Vakman and L. A. Vainshtein, "Amplitude, phase, frequency—fundamental concepts of oscillation theory," *Uspekhi Fizicheskikh Nauk*, vol. 20, no. 12, pp. 1002–1016, 1977.

[35] A. V. Husakou and J. Herrmann, "Supercontinuum generation of higher-order solitons by fission in photonic crystal fibers," *Physical Review Letters*, vol. 87, no. 20, article 203901, 2001.

[36] A. Ferrando, M. Zacarés, P. F. de Córdoba, D. Binosi, and Á. Montera, "Forward-backward equations for nonlinear propagation in axially invariant optical systems," *Physical Review E*, vol. 71, no. 1, article 016601, 2005.

[37] P. Kinsler, S. B. P. Radnor, and G. H. C. New, "Theory of directional pulse propagation," *Physical Review A*, vol. 72, no. 6, article 063807, 2005.

[38] M. Kolesik and J. V. Moloney, "Nonlinear optical pulse propagation simulation: from Maxwell's to unidirectional equations," *Physical Review E*, vol. 70, no. 3, article 036604, 2004.

[39] A. J. Lichtenberg and M. A. Lieberman, *Regular and Chaotic Dynamics*, vol. 38 of *Applied Mathematical Sciences*, Springer, New York, NY, USA, 2nd edition, 1982.

[40] V. E. Zakharov, V. S. L'vov, and G. Falkovich, *Kolmogorov Spectra of Turbulence 1. Wave Turbulence*, Springer, Berlin, Germany, 1992.

[41] S. Amiranashvili and A. Demircan, "Hamiltonian structure of propagation equations for ultrashort optical pulses," *Physical Review A*, vol. 82, no. 1, article 013812, 2010.

[42] C. Canuto, M. Y. Hussaini, A. Quarteroni, and T. A. Zang, *Spectral Methods: Fundamentals in Single Domains*, Springer, Berlin, Germany, 3rd edition, 2006.

[43] E. Hairer, S. P. Nørsett, and G. Wanner, *Solving Ordinary Differential Equations I: Nonstiff Problems*, Springer, Berlin, Germany, 2nd edition, 2000.

[44] A. Demircan and U. Bandelow, "Supercontinuum generation by the modulation instability," *Optics Communications*, vol. 244, no. 1, pp. 181–185, 2005.

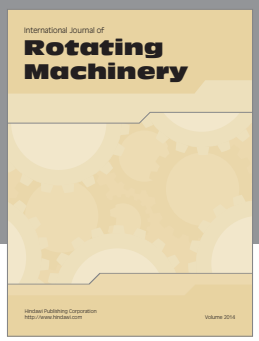
[45] A. Demircan and U. Bandelow, "Analysis of the interplay between soliton fission and modulation instability in supercontinuum generation," *Applied Physics B*, vol. 86, no. 1, pp. 31–39, 2007.

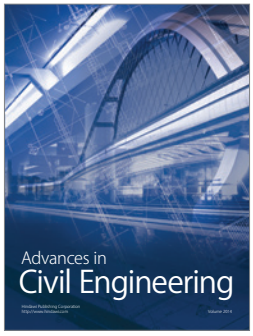
[46] I. Cristiani, R. Tediosi, L. Tartara, and V. Degiorgio, "Dispersive wave generation by solitons in microstructured optical fibers," *Optics Express*, vol. 12, no. 1, pp. 124–135, 2004.

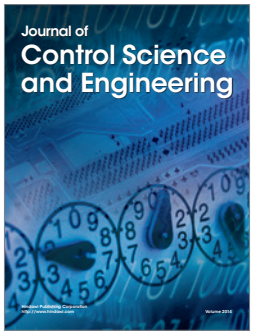
[47] J. K. Ranka, R. S. Windeler, and A. J. Stentz, "Visible continuum generation in air-silica microstructure optical fibers with anomalous dispersion at 800 nm," *Optics Letters*, vol. 25, no. 1, pp. 25–27, 2000.

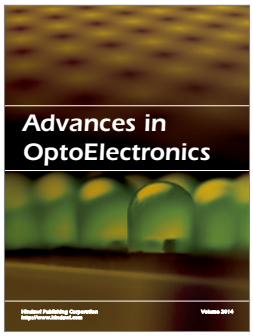
[48] V. E. Zakharov, S. L. Musher, and A. M. Rubenchik, "Hamiltonian approach to the description of non-linear plasma phenomena," *Physics Reports*, vol. 129, no. 5, pp. 285–366, 1985.

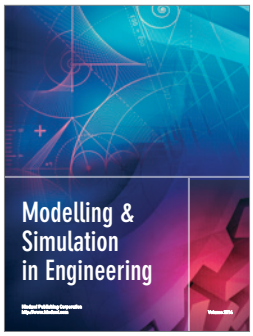
[49] V. E. Zakharov and E. A. Kuznetsov, "Hamiltonian formalism for nonlinear waves," *Uspekhi Fizicheskikh Nauk*, vol. 40, no. 11, pp. 1087–1116, 1997.

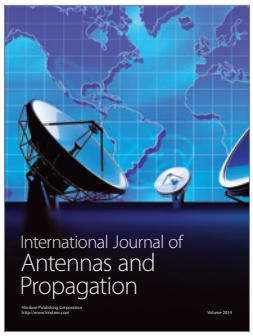


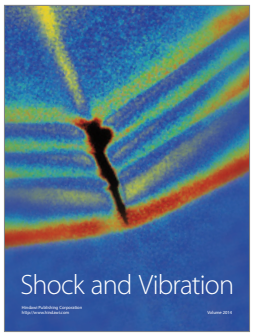


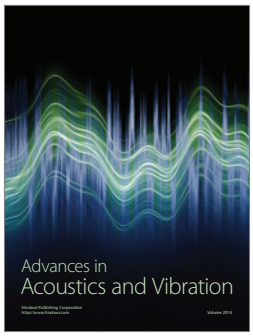












Hindawi

Submit your manuscripts at
http://www.hindawi.com