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Magnetic behaviour of SO(5) superconductors

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Abstract. The distinction between type I and type II superconductivity is re-examined in the context of the SO(5) model recently put forth by Zhang. Whereas in conventional superconductivity only one parameter (the Ginzburg-Landau parameter κ) characterizes the model, in the SO(5) model there are two essential parameters. These can be chosen to be κ and another parameter, β, related to the doping. There is a more complicated relation between κ and the behaviour of a superconductor in a magnetic field. In particular, one can find type I superconductivity even when κ is large, for appropriate values of β.

INTRODUCTION

In this talk, recent work on magnetic properties of the SO(5) model of high-temperature superconductivity (HTSC) will be presented. After reviewing the case of conventional superconductivity and some relevant facts of HTSC, the SO(5) model will be introduced. The behaviour of a system described by this model when placed in a magnetic field will be analysed. The main conclusion is that in strongly underdoped superconductors, the critical value of the Ginzburg-Landau (GL) parameter κ can be much larger than the conventional value. Thus, a large value of κ can be associated with a type I superconductor. The application of these ideas to HTSC will be discussed briefly. This work forms the bulk of Refs. [1, 2].

PRELIMINARIES

Conventional superconductivity

In this section we briefly review some features of conventional superconductivity (SC). This material is well-known, and can be found in greater detail in almost any introductory SC textbook, such as Tinkham [3].

The phase transition in a conventional superconductor can be described at low energies by an effective theory, known as a Ginzburg-Landau (GL) theory, written in terms of the SC order parameter ϕ (a complex field representing the Cooper pair amplitude) and the electromagnetic field. This GL theory can be expressed in terms of a Helmholtz

free energy, which takes the following form:

$$F = \int d\mathbf{x} \left\{ f_n - \frac{a_1^2}{2}|\phi|^2 + \frac{b}{4}|\phi|^4 + \frac{1}{2m^*}\left|\left(-i\hbar\nabla - \frac{e^*\mathbf{A}}{c}\right)\phi\right|^2 + \frac{\mathbf{h}^2}{8\pi}\right\}.$$

Here, f_n is a constant, $\mathbf{h} = \nabla \times \mathbf{A}$ is the microscopic magnetic field and a_1, b are parameters. The minimum of the potential is $|\phi|^2 = a_1^2/b \equiv v^2$.

There are two characteristic length scales in this model: the coherence length $\xi = (\hbar^2/m^* a_1^2)^{1/2}$ and the magnetic field penetration depth $\lambda = (m^* c^2/4\pi e^{*^2} v^2)^{1/2}$. These are, roughly, the Compton wavelengths of the scalar and electromagnetic fields, respectively. By scaling out all dimensionful quantities, one finds that the behaviour of a SC described by the above free energy is determined by one dimensionless parameter, the GL parameter $\kappa = \lambda/\xi$. Some typical values for this parameter appear in Table 1.

TABLE 1. Typical parameter values for various categories of superconductors.

	λ (Å)	ξ (Å)	κ
Simple metal	300	1000	.3
Alloy	2000	50	40
High-T_c	2000	20	100

There are two very different classes of (conventional) SCs, depending on the value of κ. If $\kappa < 1/\sqrt{2}$, the material is said to be type I, while if $\kappa > 1/\sqrt{2}$, it is said to be type II. The behaviour when a magnetic field is applied to a superconductor differs greatly for these two classes.

There are several ways to see this. One way is to consider a configuration where a magnetic field equal to the so-called thermodynamic critical field H_c^0 is applied to the superconductor.[1] The sign of the energy of a surface separating SC and normal regions is the telling quantity. If it is positive, the system would prefer to minimize the amount of surface for a given magnetic flux; this is achieved if the flux penetrates in a macroscopic region. In contrast, if the surface energy is negative, the flux will form a lattice of flux tubes of the minimum allowable flux. One can calculate numerically the surface energy of a boundary between SC and normal regions. Even quantitatively, one can argue that for small κ the surface energy is positive, while for large κ it is negative (see Figure 1).

An alternative way to determine the distinction between type I and type II superconductors is to consider the energetics of vortices of varying winding number. Since the magnetic flux of a vortex is proportional to its winding number, the energy per unit winding number indicates whether it is energetically favourable for a given amount of flux to penetrate in many unit-winding-number vortices or in one large vortex. The former will

[1] If a weak magnetic field is applied to a superconductor, the field is expelled; if a strong field is applied, SC is destroyed and the field penetrates the material. The critical field is the transitional value, i.e., that where the (Gibbs) free energy of the normal phase in the magnetic field is equal to that of the SC phase in the field's absence.

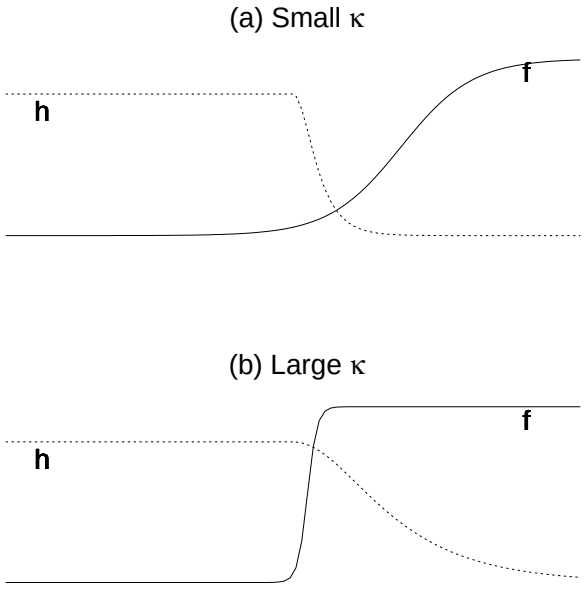

FIGURE 1. Field profiles at SC/normal surface for (a) Type I and (b) Type II superconductors. f is a rescaled ϕ. The slow variation of f when $\kappa \ll 1$ gives rise to a positive contribution to the surface energy; the slow variation of h has the opposite effect in the opposite limit. (Figure from "Magnetic Properties of SO(5) Superconductivity" by M. Juneau, R. MacKenzie and M.-A. Vachon, in Annals of Physics, Volume 298, 421, copyright 2002, Elsevier Science (USA), reproduced by permission of the publisher.)

be the case if the energy per unit winding number is of positive slope, while the latter will be true if it is of negative slope (see [1]).

The magnetization curves of type I and type II superconductors also highlight their different behaviour in a magnetic field (see Figure 2). As the magnetic field is increased in a type I superconductor, the field is completely expelled until H_c^0 is reached, at which point SC is destroyed macroscopically. In contrast, in a type II superconductor, at a lower field H_{c1}^0 the field starts to penetrate the superconductor in vortices; when the upper critical field H_{c2}^0 is reached, SC is finally destroyed. These critical fields vary as a function of κ; one finds $H_{c2}^0 = \sqrt{2}\kappa H_c^0$. The transition point between type I and type II SC occurs when these two critical fields are equal, which occurs at $\kappa = \kappa_c = 1/\sqrt{2}$.

High-temperature superconductivity

In this section, a couple of relevant facts of HTSC are presented. The key observation which leads to the SO(5) model of HTSC is that these materials exhibit two very different phases at low temperature, depending on the degree of doping. At sufficiently high doping, one sees SC, while at lower values of the doping (including the undoped case) the materials are antiferromagnetic (AF), as shown in Figure 3.

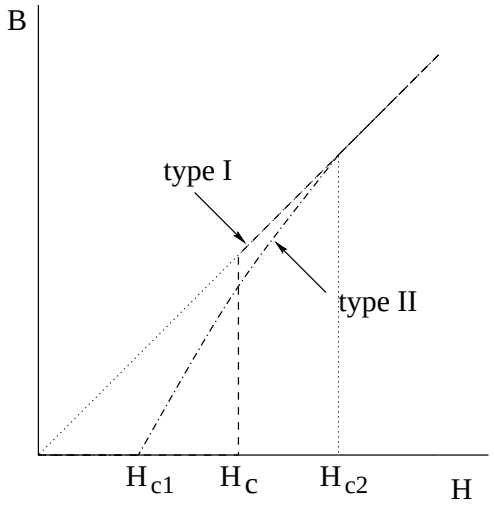

FIGURE 2. *B* vs. *H* for type I and type II superconductors.

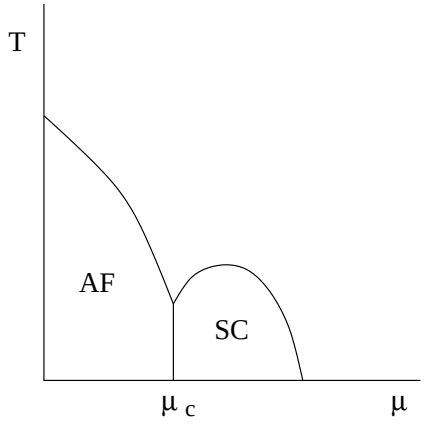

FIGURE 3. Approximate phase diagram for high-temperature superconductors and for SO(5) superconductivity.

An important property of HTSCs is that, as mentioned above (see Table 1), they are "highly type II", i.e., κ is very much larger than its critical value. (Values in the vicinity of 100 are typical; the lowest value we have seen reported is 17.)

However, as we shall see presently, in the SO(5) model, κ alone does not determine the magnetic behaviour (i.e., the type) of a superconductor. Indeed, it is possible that, in spite of having a very large κ, HTSCs (if described by the SO(5) model) might exhibit type I behaviour under certain conditions.

SO(5) SUPERCONDUCTIVITY

Motivation; Ginzburg-Landau model

As mentioned above, the presence of both AF and SC in HTSCs suggests the possibility of a sort of unification of these phenomena, both of which involve spontaneous symmetry breaking. This possibility was put forth by Zhang [4], who wrote down a model in terms of a five-component real order parameter. The five components are the real and imaginary components of the SC order parameter ϕ and the three components of the AF order parameter η. A GL theory which has an approximate SO(5) rotational symmetry can then be written down; the free energy is

$$F = \int d\mathbf{x} \left\{ \frac{\mathbf{h}^2}{8\pi} + \frac{\hbar^2}{2m^*} \left| \left(\nabla + \frac{ie^*\mathbf{A}}{\hbar c} \right) \phi \right|^2 + \frac{\hbar^2}{2m^*} (\nabla\eta)^2 + V(\phi,\eta) \right\},$$

where the potential is

$$V(\phi,\eta) = -\frac{{a_1}^2}{2}\phi^2 - \frac{{a_2}^2}{2}\eta^2 + \frac{b}{4}(\phi^2+\eta^2)^2.$$

There are now three relevant length scales: λ, ξ and ξ' (the characteristic length of the η field). Rescaling now reduces the number of essential parameters to two, which can be taken to be κ and $\beta \equiv ({a_2}^2/{a_1}^2)$. The latter is related to the doping; $\beta = 1$ at the AF-SC boundary, and $\beta < 1$ in the SC phase.

The potential for $\beta < 1$ is shown in Figure 4. There are two important features of the potential. First, it is minimized at a nonzero value of ϕ and $\eta = 0$, so the ground state is indeed SC. Second, if ϕ is somehow forced to be zero, then η will be nonzero.

In fact, there are two situations when ϕ is indeed forced to be zero: firstly, in the core of a vortex [4, 5, 6, 1], and secondly, if the superconductor is placed in a sufficiently strong magnetic field.

Magnetic properties

This "induced antiferromagnetism" can have a dramatic effect on the critical fields, and on the type of superconductor described by the model. The effect on the thermodynamic critical field H_c arises because it is found by comparing the Gibbs free energies of SC and AF (rather than normal) states. The other critical fields $H_{c1,2}$ are affected because vortex energetics are affected by the AF core.

Both H_c and H_{c2} can be calculated analytically [2]:

$$H_c = H_c^0\sqrt{1-\beta^2};$$
$$H_{c2} = H_{c2}^0(1-\beta) = \sqrt{2}\kappa H_c^0(1-\beta).$$

As in the conventional case, equality of these critical fields indicates the boundary of type I/II behaviour. We can thus obtain a curve in the β-κ plane which represents the

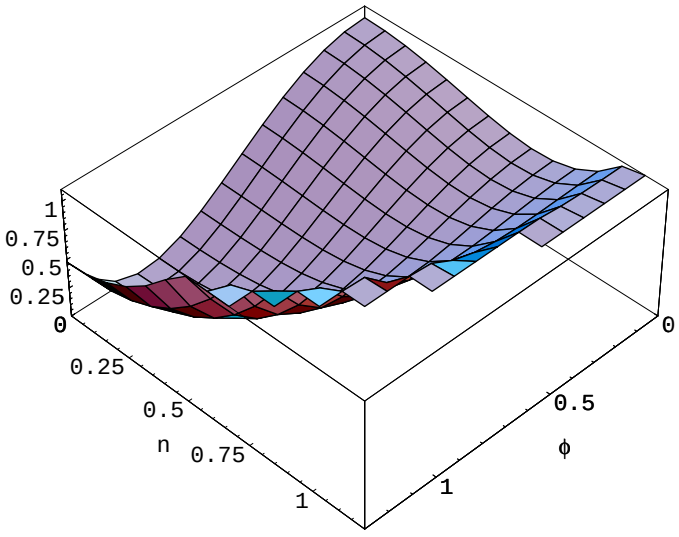

FIGURE 4. SO(5) potential as a function of ϕ and η.

boundary separating type I and type II behaviour (Figure 5). Analytically, this curve is given by

$$\kappa_c(\beta) = \frac{1}{\sqrt{2}}\sqrt{\frac{1+\beta}{1-\beta}}.$$

This expression is confirmed (numerically) by analysis of surface energy and vortex energetics.

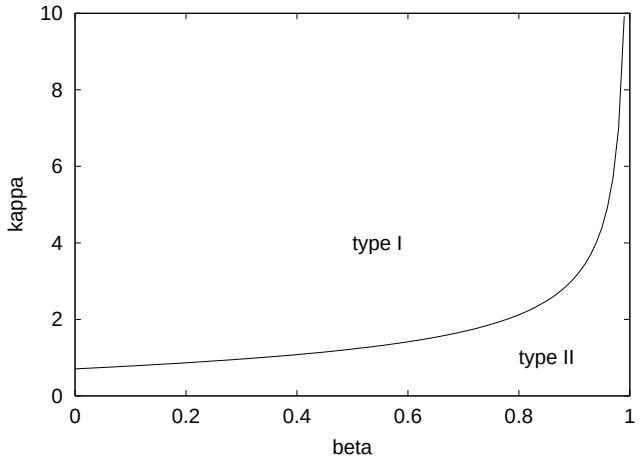

FIGURE 5. Curve delineating type I vs type II superconductivity in the κ-β plane. (Figure from "Magnetic Properties of SO(5) Superconductivity" by M. Juneau, R. MacKenzie and M.-A. Vachon, in Annals of Physics, Volume 298, 421, copyright 2002, Elsevier Science (USA), reproduced by permission of the publisher.)

Application to high-temperature superconductivity

In HTSC, as mentioned above, κ is typically of order 100; thus, HTSCs are considered highly type II. However, as the previous section demonstrates, in the SO(5) model β also plays a role in the nature of the superconductor. In particular, for any $\kappa > 1/\sqrt{2}$, if β is sufficiently large, the material is type I. We can invert the above expression for $\kappa_c(\beta)$ to obtain the following expression for $\beta_c(\kappa)$, valid if $\kappa \gg 1$ $\beta_c(\kappa) \simeq 1 - \kappa^{-2}$. For example, if $\kappa = 100$, the material is a type I superconductor if $\beta < 0.9999$ while it is type II if $\beta > 0.9999$. (Since $\beta < 1$ in the SC state, there is only a minute range of β corresponding to type II.) If $\kappa = 17$ (the smallest value reported for a HTSC), $\beta > 0.996$ for the material to be type II – still a small window, but much greater than that for $\kappa = 100$.

The parameter β is related to measurable quantities: $\beta = 1 - 8m^*\hbar^{-2}\xi(\mu^2 - \mu_c^2)$, where μ, μ_c are the chemical potential and the critical chemical potential (that at the SC-AF boundary) and ξ is the charge susceptibility.

Thus, we have the possibility of a fairly dramatic test of the SO(5) model; however, the experimental situation in the underdoped region appears rather delicate. For example, the appearance of inhomogeneities (stripe formation, phase separation) could mask the appearance of type I behaviour. Nonetheless, since the degree to which a superconductor is type II is reduced as the doping is reduced, one could hope to see signs of a reduction in the rigidity of the vortex lattice.

ACKNOWLEDGMENTS

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