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Application of a probabilistic model of rainfall-induced shallow landslides to complex hollows

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Abstract. Recently, D'Odorico and Fagherazzi (2003) proposed "A probabilistic model of rainfall-triggered shallow landslides in hollows" (Water Resour. Res., 39, 2003). Their model describes the long-term evolution of colluvial deposits through a probabilistic soil mass balance at a point. Further building blocks of the model are: an infinite-slope stability analysis; a steady-state kinematic wave model (KW) of hollow groundwater hydrology; and a statistical model relating intensity, duration, and frequency of extreme precipitation. Here we extend the work of D'Odorico and Fagherazzi (2003) by incorporating a more realistic description of hollow hydrology (hillslope storage Boussinesq model, HSB) such that this model can also be applied to more gentle slopes and hollows with different plan shapes. We show that results obtained using the KW and HSB models are significantly different as in the KW model the diffusion term is ignored. We generalize our results by examining the stability of several hollow types with different plan shapes (different convergence degree). For each hollow type, the minimum value of the landslide-triggering saturated depth corresponding to the triggering precipitation (critical recharge rate) is computed for steep and gentle hollows. Long term analysis of shallow landslides by the presented model illustrates that all hollows show a quite different behavior from the stability view point. In hollows with more convergence, landslide occurrence is limited by the supply of deposits (supply limited regime) or rainfall events (event limited regime) while hollows with low convergence degree are unconditionally stable regardless of the soil thickness or rainfall intensity. Overall, our results show that in addition to the effect of slope angle, plan shape (convergence degree) also controls the subsurface

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flow and this process affects the probability distribution of landslide occurrence in different hollows. Finally, we conclude that incorporating a more realistic description of hollow hydrology (instead of the KW model) in landslide probability models is necessary, especially for hollows with high convergence degree which are more susceptible to landsliding.

1 Introduction

The relationship between the return period of rainfall and shallow landslides has attracted the interest of numerous researchers (e.g. Dietrich and Dunne, 1978; Montgomery et al., 1998; Iverson, 2000; Borga et al., 2002; D'Odorico et al., 2005; Rosso et al., 2006) because rainfall is the most frequent landslide-triggering factor in many regions in the world. In steep soil-mantled landscapes, landslides tend to occur in topographic hollows due to convergence of water and accumulation of colluvial soils that leads to a cycle of periodic filling and excavation by landsliding (Dietrich and Dunne, 1978).

Shallow landsliding is a stochastic process, and understanding what controls the return period is crucial for risk assessment (Sidle et al., 1985; Iida, 1999; D'Odorico and Fagherazzi, 2003). Observations of repeated landslides in certain areas indicate that for some slopes and soil properties there exists a threshold of soil thickness, beyond which failure must occur, provided the slope gradient is greater than the angle of internal friction of the failure surface (Sidle and Ochiai, 2006). Therefore, to estimate the long-term susceptibility to shallow landsliding, a combined model of soil depth development and rainstorm occurrence is needed, since both of these factors control the recurrence interval of shallow landsliding (Iida, 2004).

Recently D'Odorico and Fagherazzi (2003) have presented a probabilistic model of rainfall-triggered shallow landslides in hollows and showed that landslide frequency is linked to the rainfall intensity-duration-frequency characteristics of the region. They developed a stochastic model that computes the temporal evolution of regolith thickness in a hollow and hollow hydrologic response to rainfall based on a steady-state kinematic wave model for subsurface flow. In this research, we will use some elements of this model (stochastic soil mass balance) to simulate the soil production (colluvial deposits) and soil erosion (landslides) in time for hollows with complex shapes. Although our model is similar to that presented by D'Odorico and Fagherazzi (2003) in that it is a probabilistic model of rainfall-induced shallow landslides, there is an important difference. Convergent plan shapes or concave profile curvatures cause the kinematic wave model to perform relatively poorly even in steep slopes (Hilberts et al., 2004). Troch et al. (2003) observed that hillslope plan shape rather than mean bedrock slope angle determines the validity of the kinematic wave approximation to describe the subsurface flow process along complex hillslopes. Therefore, incorporating a more realistic description of hollow hydrology in the stochastic landslide model is needed, as hollows are generally convergent (e.g. Hack, 1965; Reneau and Dietrich, 1987) and hollows with more convergence have more potential for landslide occurrence (e.g. Anderson et al., 1991).

To investigate the role of rain infiltration on landslide triggering, some investigators (e.g. Iverson, 2000) have employed the Richards equation to assess the effects of transient rainfall on the timing, rate and location of landslides. However, the Richards equation is highly complex and requires the solution of large systems of equations even for small problems (Paniconi et al., 2003). Troch et al. (2003) introduced the hillslope-storage Boussinesq (HSB) model to describe subsurface flow and saturation along geometrically complex hillslopes. This model is formulated by expressing the continuity and Darcy equations in terms of soil storage as the dependent variable. The resulting HSB model shows that the dynamic response of complex hillslopes during drainage and recharge events depends very much on the slope angle, plan shape and profile curvature (Troch et al., 2003; Hilberts et al., 2004; Berne et al., 2005; Hilberts et al., 2007). The HSB model can be linearized and further reduced to an advection-diffusion equation for subsurface flow in hillslopes with constant bedrock slopes and exponential width functions (Berne et al., 2005).

To relax the KW assumptions, in this paper we substitute the linearized steady-state HSB model in the work of D'Odorico and Fagherazzi (2003) for complex hollows (hollows with different length, slope angle and convergence degree). In fact, using an exponential width function, hollows with different convergence degrees are presented and for each hollow the critical soil depth, the minimum value of landslide-triggering saturated depth and the minimum rainfall intensity needed to trigger a landslide along the hollow length are computed. Moreover, the temporal evolution of colluvium thickness is studied through a stochastic soil mass balance. By considering the soil production function and hydrologic conditions in the different hollows, the stability of each hollow is analyzed by applying the infinite slope stability method. The aim of the generalized model is to investigate the relation between rainfall characteristics (intensity and duration), water table depth and slope stability of colluvial deposits in complex hollows.

2 Model formulation

2.1 Hollow geometry

Topography influences shallow landslide initiation through both concentration of subsurface flow and the effect of slope gradient on slope stability (Montgomery and Dietrich, 1994). Slope failure often occurs in areas of convergent topography where subsurface soil water flow paths give rise to excess pore-water pressures downslope (Anderson et al., 1991; Wilkinson et al., 2002; Talebi et al., 2008). From the topography view point, in most models of slope stability only the slope angle is considered. Although slope gradient is an important factor in landslide initiation, other geometric characteristics (such as profile curvature and plan shape) also control the hydrological process (Hilberts et al., 2004) and as such affect hillslope stability (Talebi et al., 2007). The plan shape defines topographic convergence, which is an important control on subsurface flow concentration. Several investigations (e.g. Fernandes et al., 1994; Montgomery et al., 1997; Tsuboyama et al., 2000; Troch et al., 2002; Hilberts et al., 2004) have shown that hillslopes with convergent plan shape tend to concentrate subsurface water into small areas of the slope, thereby generating rapid pore water pressure increases during rain storms.

We consider only hollows with moderate to steep slopes and shallow, permeable soils overlying a straight bedrock where subsurface storm flow is the dominant flow mechanism. Shallow soils are most prone to rain-induced landslides. It is assumed that the plan shape of the hollow can be described using an exponential width function:

$$
w(x) = w_0 e^{ax} \to A(x) = \frac{w_0}{a} \left(e^{aL} - e^{ax} \right),\tag{1}
$$

where w is the hollow width (deposits) along the x direction, x is the distance from the outlet of the hollow (parallel to the bedrock), w_o is the hollow width at the outlet, $A(x)$ is the hollow area upstream of x, L is the hollow length and a is a plan shape parameter (see Fig. 1). Allowing this plan shape parameter to assume either a positive, zero, or negative value, one can define several basic geometric relief forms: $a > 0$ for convergent, $a < 0$ for divergent and $a=0$ for parallel shapes. As hollows are generally convergent (e.g. Hack, 1965; Reneau and Dietrich, 1987), we will assume a wide range of positive numbers for convergent hollows.

As the purpose of this study is to investigate the effect of hollow geometry and hydrology on landslide probability, we employ the subsurface flow similarity parameter for complex hollows proposed by Berne et al. (2005). This dimensionless parameter, the hillslope Péclet number, is defined for subsurface flow as the ratio between the characteristic diffusive time and the characteristic advective time, taken from the middle of the hillslope:

$$
Pe = \left(\frac{L}{2pD}\right) \tan \beta - \left(\frac{aL}{2}\right),\tag{2}
$$

where p is a linearization parameter, D is the soil depth and β is the bedrock slope angle. As can be seen, Pe is a function of three independent dimensionless groups: $L/(2pD)$, tan β and $aL/2$; $L/(2pD)$ represents the ratio of the half length and the average depth of the aquifer (related to the hollow hydrology), and tan β and $aL/2$ define the hollow geometry (see Berne et al., 2005).

2.2 Hollow stability

Planar infinite slope analysis has been widely applied to the determination of natural slope stability, particularly where the thickness of the soil mantle is small compared with the slope length and where landslides are due to the failure of a soil mantle that overlies a sloping drainage barrier (Borga et al., 2002). Iida (1999) used the same approach in his stochastic hydro-geomorphological model for shallow landsliding due to rainstorms. He states that the two-layer model of soil (regolith) and bedrock, which assumes a potential landsliding (soil) layer, is suitable for the slope stability analysis in case of shallow landsliding. In this study the slope stability model is based on a Mohr-Coulomb failure law applied to an infinite planar slope. The failure condition can be expressed as (e.g. Montgometry and Dietrich, 1994; D'Odorico and Fagherazzi, 2003):

$$
\gamma_{\rm sat} D \sin \beta = c_t + (\gamma_{\rm sat} D \cos \beta - \gamma_w h \cos \beta) \tan \phi \tag{3}
$$

where γ_{sat} and γ_w are the specific weights of saturated soil and water respectively, β is the bedrock slope angle, ϕ is the soil repose angle, c_t is the soil cohesion and h is the saturated water depth, with both h and D (deposit thickness) being measured perpendicularly to the bedrock.

By solving Eq. (3) for h, the minimum value of landslide-triggering saturated depth (h_{cr}) can be obtained as (D'Odorico and Fagherazzi, 2003):

$$
h_{cr} = \frac{\gamma_{\rm sat}}{\gamma_w} D \left(1 - \frac{\tan \beta}{\tan \phi} \right) + \frac{c_t}{\gamma_w \tan \phi \cos \beta} \tag{4}
$$

When the soil depth (D) is equal to h_{cr} , the critical soil depth or immunity depth (D_{cr}) is given as follows (e.g. Iida, 1999; D'Odorico and Fagherazzi, 2003):

$$
D_{cr} = \frac{c_t}{\gamma_w \tan \phi \cos \beta + \gamma_{\rm sat} \cos \beta (\tan \beta - \tan \phi)}
$$
(5)

Bedrock Side slopes Colluvium Side slopes α β *h_max D W*

Fig. 1. Three-dimensional view of a convergent hollow on top of a straight bedrock profile (modified from D'Odorico and Fagherazzi, 2003).

As long as $D < D_{cr}$, no shallow landslide will occur as the depth h of the saturated layer cannot reach the critical value h_{cr} , even following an intense rainstorm. For this reason the period during which $D < D_{cr}$ may be named "immunity period" (Iida, 1999; D'Odorico and Fagherazzi, 2003). In gentle slopes (in contrast to steep slopes), an increase in colluvium thickness increases stability. Hence, for gentle slopes the likelihood of landslide occurrence is maximum when $D=D_{cr}$ and decreases for larger values of D (e.g. Iida, 1999; D'Odorico and Fagherazzi, 2003). For steep slopes the occurrence of a rainstorm can lead to landsliding as soon as the soil depth starts to exceed the critical depth.

The hydrogeomorphological significance of these equations is as follows:

- When $D < D_{cr}$, no shallow landsliding occurs and the slope is stable (independent of rainfall).
- **–** When $D > D_{cr}$, the water table depth (h) can exceed h_{cr} during a rainstorm, potentially leading to shallow landsliding.

In the case of relatively steep slopes ($\beta > \phi$), h_{cr} decreases linearly (i.e. stability decreases) with an increase of soil depth D (see Eq. (4). The soil depth D_{max} for which shallow landsliding can occur without saturated throughflow (corresponding to h_{cr} =0) is (Iida, 1999):

$$
D_{\text{max}} = \frac{c_t}{\gamma_{\text{sat}} \cos \beta (\tan \beta - \tan \phi)}
$$
(6)

In practice, D_{max} is never reached because the soil depth increases slowly with time (see Sect. 2.5) and periodic rainstorms will produce at least some saturated subsurface flow and consequently destabilization at thicknesses less than

Parameter name	Symbol	Units	Value in this paper
Saturated hydraulic conductivity	$k_{\rm S}$	$m d^{-1}$	65
Effective porosity			0.30
Soil cohesion	c_t	$kN m^{-2}$	11.0
Soil repose angle	Φ	Ω	33
Saturated unit weight of soil	γ_{sat}	$kN m^{-3}$	20.0
Unit weight of water	γ_w	kN m ^{-3}	9.81
Diffusivity coefficient	D_c	m^2 yr ⁻¹	0.0032
Side slope angle	α	\circ	$arctan(\tan \frac{\beta}{0.8})$

Table 1. Hydrological and geotechnical model parameters (Published data from Montgomery et al., 1997).

Table 2. Geometric characteristics of four hollows used in this study (Published data from Montgomery et al., 1997; D'Odorico and Fagherazzi, 2003). L and a are determined from Eq. (1), D_{cr} from Eq. (5) and T_{im} from Eq. (32).

Parameter	Symbol	Units			3	4
Drainage area	А	m ²	3700	860	7500	4500
Bedrock slope angle	β	\circ	43	43	30	30
Outlet width	w_0	m	12	6	12	12
Length	L	m	77	37	110	85
Convergence degree	\overline{a}	m^{-1}	0.030	0.061	0.026	0.029
Immunity depth	D_{cr}	m	1.25	1.25	2.58	2.58
Immunity period	T_{im}	vr	682	682	6389	6389

 D_{max} . Therefore, according to this model, shallow landsliding occurs when the soil depth D ranges between D_{cr} and D_{max} . Note that in the case of relatively gentle slopes $(\beta < \phi)$, h_{cr} increases linearly (i.e. stability increases) with an increase of D, hence no upper limit to the soil depth (D_{max}) exists. This means that for gentle slopes, the likelihood of landslide occurrence is maximum when $D=D_{cr}$.

2.3 Hollow hydrology

Hillslope hydrological response has traditionally been studied by means of hydraulic groundwater theory (Troch et al., 2003). In many regions, groundwater flow is the main source of streamflow between rainfall events. The basic macroscopic equation describing the movement of water in the soil is known as the three-dimensional Richards' equation. It is highly complex and requires the solution of relatively large systems of equations even for small problems (Paniconi et al., 2003).

To incorporate the hydrological process in hillslope stability analysis, many researchers (e.g. Montgomery and Dietrich, 1994; Wu and Sidle, 1995; D'Odorico and Fagherazzi, 2003) have used kinematic wave hydrology (KW). However, when water table gradients are high and bedrock slopes are relatively small, diffusive effects become important. In such hillslopes (e.g. convergent and gentle hillslopes) the KW model shows a relatively poor match to the Richards' model (Hilberts et al., 2004). Therefore, we propose to relax the KW assumption in hillslope stability analysis.

Troch et al. (2003) reformulated the continuity and Darcy equations in terms of storage along the hillslope, which leads to the hillslope storage Boussinesq (HSB) equation for subsurface flow in hillslopes. Extending Brutsaert's (1994) analysis, they linearized this equation as:

$$
\frac{\partial S}{\partial t} = K \frac{\partial^2 S}{\partial x^2} + U \frac{\partial S}{\partial x} + Nw \tag{7}
$$

with $K = \frac{k_s pD \cos \beta}{f}$ and $U = \frac{k_s \sin \beta}{f} - aK$, where S is the subsurface saturated storage, N is the recharge to the ground water table, k_s is the saturated hydraulic conductivity and f is the drainable porosity (note that the value of p is determined iteratively as we assume pD to be equal to the average water table height \int $\mathbf 0$ $S(x)dx/(Af)$ where A is the hollow drainage area). The assumptions are that the recharge rate of subsurface flow is equal to the rainfall intensity and that water flows parallel to bedrock. Comparison between the hillslope-storage Boussinesq and Richards' equation models for various scenarios and hillslope configurations shows that the HSB model is able to capture the general features of the storage and outflow responses of complex hillslopes (Paniconi et al., 2003; Hilberts et al., 2004). Berne et al. (2005)

Table 3. Characteristics of hollows based on the kinematic wave assumption.

Parameter	Symbol	Units				
Time of concentration	1 _c	hr	12.5	6.0	24.3	18.8
Critical rainfall intensity	R_{cr}	$mm\ hr^{-1}$	7.5	16.1	5.6	9.3
Return period of triggering rainfall		yr	98	1581	216	6623
Slope stability regime		$\overline{}$	Supply-limited	Event-limited	Supply-limited	Event-limited to unconditionally stable

Table 4. Characteristics of hollows based on the HSB model.

derived the steady-state solution of Eq. (7), with a zerostorage downstream boundary condition and a zero-flux upstream boundary condition, for a given recharge N as:

$$
S(x) = \frac{Nw_0}{a} \left[\frac{e^{aL}}{U} \left(1 - e^{-\frac{U}{K}x} \right) + \frac{1}{(Ka+U)} \left(e^{-\frac{U}{K}x} - e^{ax} \right) \right] \tag{8}
$$

For parallel hillslopes $(a=0)$, this reduces to:

$$
S(x) = \frac{Nw_0}{U} \left[\left(\frac{K}{U} + L \right) \left(1 - e^{-\frac{U}{K}x} \right) - x \right]
$$
(9)

According to the definition of the storage S, the mean groundwater table height (over the hillslope width) is:

$$
\overline{h}(x) = \frac{S(x)}{fw(x)} = \frac{Ne^{-ax}}{af} \times \left[\frac{e^{aL}}{U} \left(1 - e^{-\frac{U}{K}x} \right) + \frac{1}{(Ka+U)} \left(e^{-\frac{U}{K}x} - e^{ax} \right) \right]
$$
(10)

Again, for parallel hillslopes this reduces to:

$$
\overline{h}(x) = \frac{N}{fU} \left[\left(\frac{K}{U} + L \right) \left(1 - e^{-\frac{U}{K}x} \right) - x \right] \tag{11}
$$

The x-coordinate x_m where the mean groundwater table height is maximum (the critical point for slope stability), can be obtained by solving $\overline{h}'(x_m) = 0$ (see Berne et al., 2005):

$$
x_m = \frac{K}{U} \ln \left[1 + \frac{U}{Ka} \left(1 - e^{-aL} \right) \right]
$$
 (12)

which for parallel hillslopes reduces to $x_m = \frac{K}{U} \ln \left(1 + \frac{UL}{K} \right)$.

Now, by substituting the Eq. (12) into Eqs. (10) or (11), we can obtain the maximum groundwater table depth in each hillslope (which is critical for landslide occurrence):

$$
\overline{h}(x_m) = \frac{N}{fa(aK+U)} \left\{ e^{aL} \left[1 + \frac{U}{aK} \left(1 - e^{-aL} \right) \right]^{-\frac{aK}{U}} - 1 \right\}
$$
(13)

which for parallel hillslopes reduces to:

$$
\overline{h}(x_m) = \frac{N}{fU} \left[L - \frac{K}{U} \ln \left(1 + \frac{UL}{K} \right) \right]
$$
(14)

Equating $\overline{h}(x_m)$ and h_{cr} , the critical rainfall intensity for triggering landslides (R_{cr}) can now be calculated as:

$$
R_{cr} = \frac{h_{cr}fa\left(aK+U\right)}{\left\{e^{aL}\left[1+\frac{U}{aK}\left(1-e^{-aL}\right)\right]^{-\frac{aK}{U}}-1\right\}}
$$
(15)

which for parallel hillslopes $(a=0)$ reduces to:

$$
R_{cr} = \frac{fUh_{cr}}{L - \frac{K}{U}\ln\left(1 + \frac{UL}{K}\right)}\tag{16}
$$

Note that Eqs. (15) and (16) illustrate how R_{cr} (the minimum rainfall intensity needed to trigger a landslide) is a function of the deposit thickness (D_{cr}) .

To compare with the linearized HSB model, we also derive the steady-state solution of Eq. (7) for a given recharge N under the KW assumption $(K=0)$ as:

$$
S(x) = \frac{Nw_0}{Ua} \left(e^{aL} - e^{ax} \right) \Rightarrow \overline{h}(x) =
$$

$$
\frac{S(x)}{fw(x)} = \frac{N}{Uaf} \left(e^{a(L-x)} - 1 \right)
$$
 (17)

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As $x_m=0$ (in the KW case), then:

$$
\overline{h}(x_m) = \frac{N}{Uaf} \left(e^{aL} - 1 \right)
$$
\n(18)

Now, the critical rainfall intensity for triggering landslides (R_{cr}) can be calculated as:

$$
R_{cr} = \frac{Uafh_{cr}}{e^{aL} - 1} \tag{19}
$$

Note that for parallel hillslopes $(a=0)$, these equations reduce to:

$$
S(x) = \frac{Nw_0}{U} (L - x) \Rightarrow \overline{h}(x) = \frac{N}{Uf} (L - x) \quad , \tag{20}
$$

$$
\overline{h}(x_m) = \frac{NL}{Uf} \tag{21}
$$

and

$$
R_{cr} = \frac{Uf h_{cr}}{L} = \frac{h_{cr} k_s \sin \beta}{L}
$$
 (22)

D'Odorico and Fagherazzi (2003) have assumed the critical rainfall intensity to be equal to $R_{cr}=h_{cr}k_sw_0 \sin \beta/A$ for all hollow shapes, based on the KW assumption. As can be seen, this equation is similar to Eq. (21), which has been derived here based on the KW assumption and for parallel hollows. As we will show, the results of the KW and HSB models for hollow hydrology differ significantly (especially for hollows with a high convergence degree) and this affects landslide probability.

The analysis of landslide frequency also requires the estimation of the duration of the triggering rainfall. For this purpose, D'Odorico and Fagherazzi (2003) applied the rational method (e.g. Chow et al., 1988) to the subsurface flow in hollows to determine the most critical storm duration for a given return period. The rational method assumes that the time of concentration (T_c) is the most critical storm duration. Thus the maximum saturated depth generated by storms of a given frequency is due to events of duration T_c . Here, we update the way in which the time of concentration is calculated to make it fully consistent with the linearized steady-state HSB model. Hence, the concentration time can be expressed as:

$$
T_c = \int_{x_m}^{L} \frac{S(x)}{Q(x)} dx
$$
\n(23)

with

$$
Q(x) = \begin{cases} \frac{Nw_0}{a} (e^{aL} - e^{ax}) (a \neq 0) \\ Nw_0 (L - x) (a = 0) \end{cases}
$$
 (24)

In the KW limit this reduces to $T_c = L/U$, which for parallel hillslopes can be written as $T_c = L_f/(k_s \sin \beta)$. D'Odorico and Fagherazzi (2003) expressed the concentration time as $T_c = C \sqrt{A}/(k_s \sin \beta)$, where C is a dimensionless coefficient accounting for other factors affecting the concentration time

and A is the hollow contributing area. This suggests that an equivalent hollow length can be estimated as $L = C \sqrt{A/f}$. According to the exponential width function, Eq. (1), the contributing area is $A=w_0\left(e^{aL}-1\right)/a$. This provides an implicit equation to estimate the degree of convergence of an equivalent exponential hollow from given values of A , w_0 , and L.

2.4 Return period of the triggering rainfall

Rainfall is considered to be the most important factor in triggering slope failure. To accomplish a hazard analysis of the landslide phenomenon, a probability analysis of intense rainfall occurrence for different return periods is needed. The objective of rainfall frequency analysis is to estimate the amount of rainfall falling at a given point for a specified duration and return period. The frequency of extreme rainfall is usually defined by reference to the annual maximum series, which comprises the largest values observed in each year. The Gumbel distribution has been the most common probabilistic model used in modelling hydrological extremes (Brutsaert, 2005). Since landslides are triggered by extreme rainfalls, following D'Odorico and Fagherazzi (2003), we use a Gumbel distribution to express the dependence between annual maximum rainfall intensity for events of duration T_c and return period T_r as follows:

$$
\frac{1}{T_r} = \lambda = 1 - \exp\left[-\exp\left(-\frac{R(T_c) - u}{v}\right)\right]
$$
 (25)

where T_r is the return period, $R(T_c)$ is the annual maximum rainfall intensity of duration T_c , u and v are the parameters of the Gumbel distribution and λ is the probability that the maximum intensity exceeds $R(T_c)$ in a given year. As our model is applied to a parameter set (for four realistic hollows) derived from published data from the Oregon Coastal range (e.g. Montgomery et al., 1997; Torres et al., 1998; Stock and Dietrich, 2003; D'Odorico and Fagherazzi, 2003), based on rainfall data available for the same region (Montgomery et al., 1997) the relation between u , v and T_c is found to be $u/v=2.6$ and $v=4.75T_c^{-0.6}$. From the value of T_c in a hollow, the parameters u and v are computed and the return period of the critical rainfall intensity will be determined. Note that $\lambda = 1/T_r$ is a function of the soil depth due to the dependence between the intensity of the triggering precipitation, R_{cr} , and D_{cr} (see Eqs. 15, 16, 19, and 22).

2.5 Temporal evolution of deposit thickness

The temporal evolution of colluvial deposits in hollows can be characterized by a continuous process of deposit accretion, and a discontinuous random process of denudation caused by rainfall-triggered landslides, which scour to the bedrock large portions of the hollow (D'Odorico and Fagherazzi, 2003). The temporal evolution of colluvium thickness can thus be studied through a stochastic soil mass balance,

accounting for the supply of debris from the adjacent slopes and for random denudation due to landsliding. The description of the probabilistic soil mass balance model we apply in this study largely follows that of D'Odorico and Fagherazzi (2003).

Many studies analyzed the soil production and landscape evolution to investigate the spatial and temporal patterns of soil thickness (e.g. Kirkby, 1985; Dietrich et al., 1986; Heimsath et al., 1997; Heimsath et al., 2001). Based on the conservation of mass equation for a tipped triangular trough and slope-dependent transport, Dietrich et al. (1986) presented an expression for the rate of colluvium accumulation in hollows. They showed that the rate of accumulation is a function of the side-slope gradient and the difference between the side-slope and hollow gradient. For a hollow composed of a tipped triangular trough and two planar side slopes, the accretion of colluvial deposits can be obtained as (Dietrich et al., 1986):

$$
D = \left[2 D_c \cos \beta \left(\tan^2 \alpha - \tan^2 \beta\right) t\right]^{0.5},\tag{26}
$$

where D_c is the soil creep diffusivity, α is the angle between the side slopes and a horizontal plane and t is time. Dietrich et al. (1986) also showed that basin form, consisting of noses, side slopes, and a hollow appears to be well represented by the geometry of a tipped triangular trough and typically the ratio of hollow slope to side slope is about 0.8. If we assume that α and β do not vary substantially with time, then Eq. (24) can be expressed as (D'Odorico and Fagherazzi, 2003):

$$
D = \sqrt{Mt}; \quad M = 2D_c \cos \beta \left(\tan^2 \alpha - \tan^2 \beta\right) \tag{27}
$$

with *M* being independent of time. The differentiation of Eq. (25) with respect to time leads to:

$$
\frac{dD}{dt} = l(D) = \frac{M}{2D} \tag{28}
$$

showing that the rate of colluvium accretion decreases with the depth, D, of the deposit (D'Odorico and Fagherazzi, 2003). Now, the overall temporal evolution of the deposit thickness (D) can be expressed as:

$$
\frac{dD}{dt} = l(D) - J(D, t) \tag{29}
$$

where $l(D)$ is a depth-dependent function of net colluvium accretion expressed by Eq. (28) and $J(D, t)$ is the rate of soil removed by debris flow and shallow landslides. The latter is modelled as a stochastic Poisson process (D'Odorico and Fagherazzi, 2003):

$$
J(D, t) = \zeta(D) \sum_{i} \delta(t - t_i)
$$
 (30)

where

$$
\zeta(D) = \begin{cases} 0; 0 \le D \le D_{cr} \\ D; D > D_{cr} \end{cases}
$$
\n(31)

In this equation δ represents a Dirac- δ function and the sequence t is such that the interarrival time of the triggering precipitation, $\tau = t_{i+1} - t_i$, is an exponentially distributed random variable. As a result, the temporal variability of colluvium thickness is controlled by the rates of colluvium accretion and erosion (i.e., landslides), and both of them depend on the actual state (i.e., deposit thickness) of the system. Note that the time needed to accumulate a colluvium thickness $D=D_{cr}$ (T_{im}) is computed as:

$$
T_{im} = D_{cr}^2 / \left[2D_c \cos \beta \left(\tan^2 \alpha - \tan^2 \beta \right) \right]
$$
 (32)

2.6 Numerical simulation of landslide occurrence

To simulate the dynamics of complex hollows, the following steps are performed:

- **–** The deposit thickness of a simulated hollow is D=0 at $t=0$.
- **–** The linearization parameter (p) is determined iteratively as pD is assumed to be equal to the average water table depth in each hollow (\overline{h}) and \overline{h} is calculated using S (saturated storage, Eqs. (8) and (9)). Note that N is also computed iteratively by substituting D_{cr} (instead of h_{cr}) in Eqs. (15) and (16).
- The time of concentration (T_c) of each hollow is determined by Eq. (23).
- **–** Gumbel rainfall parameters (u and v) are estimated for extreme precipitation of duration T_c .
- $-$ The minimum saturated depth (h_{cr}) able to trigger a landslide is calculated from Eq. (4).
- $-$ The critical rainfall intensity (R_{cr}) corresponding to h_{cr} is computed from Eqs. (15) and (16).
- **–** The probability that R is exceeded in a given year is estimated by Eq. (25).
- **–** A random number to determine if a landslide occurs is drawn; if a triggering storm occurs, the landslide scours the hollow entirely.
- **–** The deposit thickness D increases by transport from uphill based on Eq. (28). Note that in this model, a landslide occurs when $D_{\text{max}} > D \ge D_{cr}$.

The presented model, which is an extension of that of D'Odorico and Fagherazzi (2003), simulates the long term evolution of soil depth. The extension lies in the fact that the probability distribution of scar depth, landslide return period and colluvium thickness is calculated for complex hollows based on a more realistic description of hollow hydrology (the linearized HSB model). As the aim of this paper is to investigate the effect of geometry and hydrology of hollows on

Fig. 2. Long term simulation of deposit thickness for the four hollows in Table 1 (from first to fourth row, respectively). Left column: KW; right column: HSB model.

landslide probability, we compare the different approaches of hollow hydrology (KW and HSB models) in hollows with different geometries.

The model is applied to a parameter set (for four hollows) derived from published data from the Oregon Coastal range (e.g. Montgomery et al., 1997; Torres et al., 1998; Stock and Dietrich, 2003; D'Odorico and Fagherazzi, 2003). To apply the exponential width function to these four hollows (which are convergent, see Fig. 2, Montgomery et al., 1997), Eq. (1) is fitted using the total contributing areas (A) , the outlet width (w_o) and the dimensionless coefficient affecting concentration time (C coefficient in Eq. 6, D'Odorico and Fagherazzi, 2003), as was explained previously after Eq. (23) . The hollow length assuming exponential geometry is estimated by $L = C \sqrt{A/f}$. Now, from the equivalent hollow length (L) , the plan shape parameter (a) is calculated using the Eq. (2). Finally, by computing pD (see Eq. 1) in each hollow (iteratively), the subsurface saturated storage is obtained by the HSB model (Eqs. 8 and 9).

Table 1 lists the values of the hydrological and geotechnical variables used to perform stability analyses in the different hollows and Table 2 shows the geometric characteristics of these four hollows. To generalize the obtained results, we also apply the model for a wide range of hollows with different geometric characteristics and different hydrology conditions (different Péclet numbers).

3 Results and discussion

Based on Eq. (5), the critical soil depth (D_{cr}) for the two steep hollows ($\beta > \phi$) is found to be 1.25 m and for the two gentle hollows ($\beta < \phi$) 2.58 m (see Table 2). Tables 3 and 4 show the results of the landslide probability analysis for the KW and the HSB model, respectively. They illustrate how the hydrological properties and stability of hollows change as a function of hollow geometry. As can be seen, the values of the concentration time (T_c) are slightly longer for the HSB model than for the KW model. This is because in the KW model the diffusion term is ignored $(K=0$ in Eq. 7). As a result, other parameters (R_{cr} and T_r) for all hollows are also larger for the HSB model. This affects the stability regime, especially in gentle and convergent hollows (see Table 4).

Based on the obtained results (Tables 3 and 4), the immunity period $(T_{im}, i.e.$ the time needed to accumulate a colluvium thickness $D=D_{cr}$) of hollows 1 and 3 are significantly longer than the return period of the triggering rainfall (T_r) . This means that landslide occurrence is limited by the supply of debris from the adjacent slopes, rather than by the occurrence of triggering rainfall. As soon as the soil depth reaches D_{cr} , a landslide will occur shortly. D'Odorico and Fagherazzi (2003) denote this regime as "supply limited", indicating that the landslide return period depends first and foremost on soil production. On the other hand, in hollow 2 (where T_{im} is the same as for hollow 1), a higher rainfall intensity is needed to trigger landslides $(T_{im} < T_r)$. In that case landslides occur when an extreme rainfall intensity is able to produce the critical saturated depth (h_{cr}) required for landslide occurrence. This is called the event-limited regime. Hence, it can be concluded that hollow geometry is an important control on subsurface flow response (Troch et al., 2003; Hilbert et al., 2004) and this process affects slope stability (Talebi et al., 2007).

Figure 2 shows long term simulations of deposit thickness evolution in the four hollows (from top to bottom) and illustrates how shallow landsliding occurs when the soil thickness (D) ranges between D_{cr} and D_{max} . In this figure, left and right columns show the time series of deposit thickness for the KW and HSB models, respectively. As can be seen, the landslide probability analysis for the HSB model (using a more realistic description of hollow hydrology) shows significant differences with respect to the results of the KW model (especially in gentle and convergent hollows, Fig. 2, last row). Comparison of the results reported in Tables 3 and 4 with Fig. 2 also illustrate that the KW model looses its ability

Fig. 3. Probability distribution of scar depth (colluvium thickness when a landslide occurs) for hollows 1, 2 and 3, respectively. Left column: KW; right column: HSB model.

in gentle hollows, such as in hollow no. 4, where the stability regime has also been changed. Figure 2 also indicates how, as a function of the hollow geometry from steep slopes (top) to gentle slopes (bottom), the landslide probability is changed as well. For instance in hollow 4 (where $T_r \gg T_{im}$), landslides never occur and the system can be termed "unconditionallystable".

Figure 3 illustrates the probability distribution of colluvium thickness when a landslide occurs as simulated by the KW model (left column) and the HSB model (right column) (D_{slide}) . Note that hollow 4 lies in the unconditionally-stable regime, hence the distribution of D_{slide} can only be presented for the three remaining hollows. These histograms show that not only the different hollows have different distributions of scar depth, but also the results of the KW and HSB models are significantly different. The average (m) and standard deviation (sd) of the histograms in each row show these differences clearly. As can be seen, the probability distribution of D_{slide} is concentrated close to the immunity depth (D_{cr}) for the supply-limited case, whereas it is concentrated at significantly larger depths for the event-limited cases.

Fig. 4. Probability distribution of landslide return period for hollows 1, 2 and 3, respectively. Left column: KW; right column: HSB model.

Figures 4 and 5 indicate how the probability distributions of the interarrival of the landslide-producing rain events (T_{slide}) and the corresponding rainfall intensities (R_{slide}) vary for the different hollows. As in the previous figure, the left and right columns show the results of the KW model and the HSB model, respectively. These results show that in hollow 1 (which has less convergence and a larger area), T_{slide} is close to T_{im} (supply limited regime), while in hollow 2 (which has more convergence and a smaller area), T_{slide} moves in the direction of T_r (event limited regime). Comparison of the left and right columns in Fig. 5 also indicates that the values of R_{slide} (the rainfall intensity generating a landslide) are significantly different for the KW and HSB models. This is because the computation of the concentration time of the HSB model (Eq. 23) includes the effect of diffusion.

Figure 6 shows the probability distribution of colluvium thickness for the different hollows corresponding to the KW (left column) and the HSB model (right column). As can be seen, as soon as the soil depth reaches the immunity depth, landslides begin to occur. Hollow 2 (second row in Fig. 6) shows a significant difference between the KW and HSB

Fig. 5. Probability distribution of the landslide triggering rainfall intensity for hollows 1, 2 and 3, respectively. Left column: KW; right column: HSB model.

models. This is because in hollow 2 (which lies in the eventlimited regime), the value of T_r/T_{im} for the KW model is less than for the HSB model, indicating that landslides occur a longer time after D_{cr} for the HSB model. Hence, including a diffusion term in the steady-state hydrological model has a noticeable influence on the landslide probability.

In order to quantify the effect of variations of hollow geometrical parameters (e.g. D , L , A , a and β) and hollow hydrological parameters (e.g. S , h and p) and to generalize the results obtained, different landslide regimes have been investigated for a wide range of hollow geometrical and hydrological parameters for gentle and steep slopes (see Fig. 7). As can be seen, by changing the length (L) , slope (β) and shape (a) of several hollows, a wide range of Péclet numbers (Eq. 2, dimensionless parameter for hollow geometry and hydrology), corresponding to the immunity period (T_{im}) and return period of triggering rainfall (T_r) has been obtained. In particular, we have investigated the relationship between the Péclet number of a large number of hollows and the ratio of T_r/T_{im} . Based on our model, different regimes can occur, which de-

Fig. 6. Probability distribution of colluvium thickness for hollows 1, 2 and 3, respectively. Left column: KW; right column: HSB model.

pend on the ratio between T_r and T_{im} (see Table 4). Figure 7 summarizes the results of this paper and shows how the presented model allows to identify different landslide regimes as a function of hollow geometry, hydrology and climatology. Therefore, the Péclet number (Pe) as the index of geometry and hydrology, T_{im} as the index of temporal variability of colluvium thickness and T_r as the index of climatology can be used to investigate the probability distribution of shallow landslides in the different hollows. It should be noted, however, that the Péclet number seems to only have a secondary effect on the return time of landsliding, the most important parameter presumably being the concentration time.

4 Conclusions

The aim of this paper was to investigate the effect of hollow geometry and hydrology on the probability distribution of landslides in complex hollows (hollows with different length, slope and convergence degree). For that purpose and to relax

the KW assumptions, we substituted a more realistic description of hollow hydrology (the linearized steady-state HSB model) in the work of D'Odorico and Fagherazzi (2003). The obtained model constitutes a probabilistic model of rainfallinduced shallow landsliding in complex hollows and allows to investigate the relation between the return period of rainfall, deposit thickness and landslide occurrence. The main assumptions of the presented model are:

- **–** infinite planar slope stability analysis;
- **–** steady-state hydrology;
- **–** statistical model relating depth-duration-frequency of extreme precipitation based on Gumbel extreme value distribution;
- **–** growth of colluvial deposits in hollow only due to transport of soil from uphill, not from physical weathering of underlying bedrock;
- **–** and landslides scour hollow to bedrock.

Note that similar assumptions regarding hillslope hydrology and stability have been employed by other researchers (e.g. Montgomery et al., 1998; Iida, 1999; D'Odorico and Fagherazzi, 2003; Rosso et al., 2006).

The following conclusions can be drawn from our rainfallinduced landslide stability analysis in response to deposit thickness evolution in complex hollows:

- (i) Although shallow landslides in hollows are mainly triggered by high rainfall intensities, deposit thickness also plays an important role in stability.
- (ii) With other site variables constant, shallow landslides usually occur when the soil depth (deposit thickness) is between D_{cr} and D_{max} (as has been confirmed by other researchers, e.g. Iida, 1999; D'Odorico and Fagherazzi, 2003). In fact, shallow landslides always occur shortly after D_{cr} has been reached.
- (iii) Given a deposit thickness, for each hollow there exists a critical rainfall intensity leading to the highest water table and subsequent landslide occurrence.
- (iv) In general, when the convergence degree of hollows increases, the time period between land slides (T_{slide}) decreases. This means that hollows with a higher convergence degree are generally more susceptible to landsliding.
- (v) Finally, it can be concluded that incorporating a more realistic description of hollow hydrology (instead of the KW model) in landslide probability models is necessary, especially for hollows with a high convergence degree (which are more susceptible to land sliding).

Fig. 7. The relation between hollow geometry (Pe) and hollow stability and landslide regime in steep (top) and gentle hollows (bottom).

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