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A self-consistent derivation of ion drag and Joule heating for atmospheric dynamics in the thermosphere

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Abstract. The thermosphere is subject to additional electric and magnetic forces, not important in the middle and lower atmosphere, due to its partially ionized atmosphere. The effects of charged particles on the neutral atmospheric dynamics are often parameterized by ion drag in the momentum equations and Joule heating in the energy equation. Presented in this paper are a set of more accurate parameterizations for the ion drag and Joule heating for the neutral atmosphere that are functions of the difference between bulk ion velocity and neutral wind. The parameterized expressions also depend on the magnetic field, the Pedersen and Hall conductivities, and the ratio of the ion cyclotron frequency to the ion-neutral collision frequency. The formal relationship between the electromagnetic energy, atmospheric kinetic energy, and Joule heating is illustrated through the conversion terms between these three types of energy. It is shown that there will always be an accompanying conversion of kinetic energy into Joule heating when electromagnetic energy is generated through the dynamo mechanism of the atmospheric neutral wind. Likewise, electromagnetic energy cannot be fully converted into kinetic energy without producing Joule heating in the thermosphere.

Keywords. Meteorology and atmospheric dynamics (Thermospheric dynamics) – Ionosphere (Ionosphere-atmosphere interactions; Modeling and forecasting)

1 Introduction

Thermospheric dynamics is affected by charged particles because the atmosphere is partially ionized. Atmospheric flows consisting of major neutral species and minor charged particles will differ from purely neutral flows because the charged particles are subject to additional electric and magnetic forces. Collisions between atmospheric neutral species and charged particles will yield a momentum exchange term in the momentum equations (ion drag), and an energy conversion term in the thermal energy equation (Joule heating) actualized by the neutral wind. Though the concentrations of the charged particles are far less than those of the neutral atmosphere in the thermosphere below ~1000 km (e.g., Kelley, 1989, Appendix B) they impose a non-negligible ion drag and a Joule heating because the charged particles experiencing the electromagnetic forces may move in a significantly different direction and magnitude than the neutral wind.

At least three different formulations can represent the ion drag and Joule heating imposed on the neutral atmosphere by charged particles: (i) the momentum sink and energy dissipation of charged particles through microscopic collisions with neutral species when there exist velocity differences between the two; (ii) macroscopic Lorentz force and Ohmic dissipation caused by electric currents and electromagnetic fields; or (iii) parameterized drag and heating that are functions of the difference between the bulk ion velocity and neutral wind. The major difference between (i) and (iii) is that velocities of ion and neutral species in (i) represent actual motions, are treated equally, and both need to be solved self-consistently. On the other hand, the motion of the bulk ion velocity in (iii) has been averaged over a gyrocycle, is determined by the constant intrinsic magnetic field, and is prescribed as model parameters for calculating the force terms for the neutral wind. Depending on specific problems, a particular formulation could be more useful for the purpose of an illustration or calculation. The microscopic description of interactions between neutral species and charged particles through collisions leads to a solid foundation of the transport phenomena associated with various physical quantities, including mass, momentum and energy (e.g., Gombosi, 1994; Schunk and Nagy, 2000). When measurements of the electric and magnetic fields are available, the electromagnetic formalism becomes a useful tool for diagnosing the thermospheric heat transfer (e.g., Lu et al., 1995, 1998; Thayer, 1998). On the other hand, the parameterized ion drag that linearly depends on the difference between the bulk ion velocity and neutral wind provides both an insight into the

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physics and a method of straightforward implementation of the ionospheric momentum source in numerical models (e.g., Dickinson et al., 1981). While these different approaches are equally valid in describing essentially the same phenomena, they are neither precisely the same nor always interchangeable. While formalisms of (i) and (ii) are readily available and their differences described in the existing literature (e.g., St.-Maurice and Schunk, 1981), a set of parameterizations for ion drag and Joule heating expressed as bulk ion-neutral wind difference (iii) is presented here.

The purpose of this paper is to derive a set of parameterized expressions for the ion drag and Joule heating that is more accurate than those shown in the literature when the ion-neutral collisions become important. The parameterizations also possess an energetic consistency in their forms. From an energetics perspective, ion drag and Joule heating are associated with changes of three types of energy in the system: the change of the electromagnetic energy caused by electric currents, the change of kinetic energy caused by the imposed drag force, and the change of the internal energy associated with the added heat. The effect of neutral wind on Joule heating is discussed in parallel with analysis of energy conversion terms. Such a detailed discussion has heretofore been lacking in the literature. In Sect. 2, we review ion drag in the macroscopic electromagnetic form and then derive explicit expressions parameterized by the bulk velocity difference. In Sect. 3, we present both formalisms for Joule heating. The energetic consistencies of the derived formalisms and their implications are shown in Sect. 4. Finally, Sect. 5 summarizes the results.

2 Derivation of the parameterized ion drag terms in momentum equations

In this paper, we focus on the thermospheric atmospheric dynamics below ∼1000 km where the number density of neutral particles is far greater than that of ions (e.g., Kelley, 1989, Appendix B). We mainly consider how the atmospheric motions of those major neutral species are affected by the minor charged particles.

2.1 Review of literature

To explicitly derive the bulk parameterization of the ion drag in the momentum equation for the neutral atmosphere we start from the general transport equation for an individual species, s, (e.g., Schunk and Nagy, 2000, p. 54)

$$n_s m_s \frac{D_s \boldsymbol{u}_s}{Dt} + \nabla \cdot P_s - n_s m_s \boldsymbol{g} = n_s e_s (\boldsymbol{E} + \boldsymbol{u}_s \times \boldsymbol{B}) + \frac{\delta \boldsymbol{M}_s}{\delta t}, \quad s = 1, 2, 3, \ldots, \tag{1}$$

where

t	= time
n_s	= species number density
m_s	= species mass
$\boldsymbol{g}$	= acceleration due to gravity
e_s	= species charge (= 0, $\pm e$ with $-e$ being the electron charge)
$\boldsymbol{u}_s$	= average velocity of species s
P_s	= partial pressure tensor for species s
$\boldsymbol{E}$	= electric field
$\boldsymbol{B}$	= magnetic field
$\frac{D_s}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{u}_s \cdot \nabla$	= convective (or total) derivative following $\boldsymbol{u}_s$
$\frac{\delta \boldsymbol{M}_s}{\delta t}$	= momentum source for species s due to collisions

At first glance, to quantitatively study the thermospheric atmospheric motion subject to charged particles one needs at least three momentum equations for three different types of particles in the thermosphere: neutral particles (e_s=0), single-charged ions (e_s=e), and electrons (e_s=$-e$). However, since neutral species remain the major components in the thermosphere it is more sensible to focus on the dynamical equations for a neutral fluid with the parameterized effects of charged particles. An appropriate dynamical system for major species is essential for thermospheric general circulation models since it provides a modeling frame for simulating the physics and chemistry of the rest of the minor tracers, including charged particles (e.g., Dickinson et al., 1975, 1981; Fuller-Rowell and Rees, 1980; Namgaladze et al., 1990).

The momentum equations for the neutral components will be Eq. (1) without the electric and magnetic forcing terms (e_s=0). However, the collisional integral in the transport Eq. (1) for a multi-species gas is too complicated for a rigorous analytical solution. An alternative approach is to start from the so-called magnetohydrodynamic equations that combine the equations for individual species into a gas mixture (e.g., Baumjohann and Treumann, 1996; Schunk and Nagy, 2000). Because the collisional terms describe the internal transfer of momentum from one species to another, the collisional terms cancel when the individual momentum equations of Eq. (1) are summed. Thus, the general magnetohydrodynamic equation for the momentum conservation is (Schunk and Nagy, 2000, p. 195)

$$\rho \frac{D\boldsymbol{u}}{Dt} + \nabla \cdot P - \rho \boldsymbol{g} = \boldsymbol{J} \times \boldsymbol{B}, \tag{2}$$

where ρ is the total mass density, $\boldsymbol{u}$ is the average velocity, P is the total pressure tensor, $\boldsymbol{J}$ is the total current density, and $D/Dt = \partial/\partial t + \boldsymbol{u} \cdot \nabla$ is the convective derivative following the average velocity $\boldsymbol{u}$. We have also neglected the force term by the electric field, $\rho_c \boldsymbol{E}$, due to the quasi-neutrality approximation for the charge density (ρ_c=0) (Goldston and Rutherford, 1995, Sect. 8.2). The notation in Eqs. (1) and (2) mostly follows Schunk and Nagy (2000), to which readers can refer to for more detailed definitions.

In Eq. (2), ρ, $\boldsymbol{u}$, and P are weighted by the total densities of both neutral species and charged particles whereas the force term on the right-hand side associated with $\boldsymbol{J}$ is weighted only by the density of charged particles. Therefore, for a gas mixture where charged particles are considered minor tracers the left-hand side of Eq. (2) can be approximately simplified by only retaining terms contributed from the major species of the neutral atmosphere

$$\rho_n \frac{D_n \boldsymbol{u}_n}{Dt} + \nabla \cdot P_n - \rho_n \boldsymbol{g} = \boldsymbol{J} \times \boldsymbol{B}, \tag{3}$$

where all quantities on the left-hand side have the same definitions as in Eq. (2) and the subscript n refers to the neutral atmosphere. The right-hand side of Eq. (3) is the ion drag imposed on the neutral atmosphere as a result of the motions of the charged particles.

The technique of scale analysis in simplifying a set of fluid equations in the lower atmosphere proposes that a term of much smaller magnitude in comparison with one or more remaining terms can be dropped (e.g., Haltiner and Williams, 1980, Ch. 3; Holton, 1992, Sect. 2.4). Two basic rules one needs to follow when applying scale analysis to simplify equations are: (i) small terms need to be dropped one by one; and (ii) at least two terms need to be kept at the end of simplification. The reason for having the first rule is that two terms in an equation may form a near exact balance, so that the sum of the two is much smaller than the rest of the terms, although the absolute magnitude of the two individual terms is much greater than the others. Elimination of those two terms simultaneously may lead to an inappropriate equation that represents the erroneous residual of two major physical processes described by the two nearly balanced terms. The second rule prevents the occurrence of the so-called "one-term dominance" that always leads to a contradictory solution of a vanishing dominant term. Our simplification from Eq. (2) to Eq. (3) for a gas mixture satisfies these two basic rules since the dropped terms weighted by the minor tracers are much smaller than the corresponding terms weighted by the major species. Since the right-hand side of Eq. (2) had already been weighted by the minor tracers it cannot be simplified further in Eq. (3) without a reference term that can be compared to in magnitude.

Note that the left-hand side of Eq. (2) would be identical to Eq. (3) if the momentum Eq. (1) were written for the neutral atmosphere (e=0), but the right-hand side would contain the collisional term $\delta \boldsymbol{M}_n/\delta t$. $\delta \boldsymbol{M}_n/\delta t$ describes the exact microscopic transform of momentum (approach (i)) from the charged particles to the neutrals whereas $\boldsymbol{J}\times\boldsymbol{B}$ can be considered a macroscopic drag imposed on the neutral fluid (approach (ii)). Our resulting Eq. (3) based on the scale analysis to a gas mixture is consistent with the recent approach by Vasyliunas and Song (2005) who arrived at the same conclusion based on the scale analysis of a system of two separate fluids.

2.2 Derivation of the parameterized ion drag

To calculate the ion drag term $\boldsymbol{J}\times\boldsymbol{B}$ ($=\boldsymbol{J}_\perp\times\boldsymbol{B}$ because $\boldsymbol{J}_\parallel\times\boldsymbol{B}$=0), we first note that the current density perpendicular to the magnetic field ($\boldsymbol{J}_\perp$) is given by (e.g., Schunk and Nagy, 2000, p. 131)

$$\boldsymbol{J}_\perp = \sigma_P(\boldsymbol{E}_\perp + \boldsymbol{u}_n \times \boldsymbol{B}) + \sigma_H \boldsymbol{b} \times (\boldsymbol{E}_\perp + \boldsymbol{u}_n \times \boldsymbol{B}), \tag{4}$$

where $\boldsymbol{E}_\perp$ is the electric field perpendicular to $\boldsymbol{B}$ with $\boldsymbol{b}$ being the unit vector along $\boldsymbol{B}$. σ_P and σ_H are the Pedersen and Hall conductivities, respectively. σ_P and σ_H represent the measures of the charged particle mobility parallel and perpendicular to $\boldsymbol{E}_\perp$, respectively. We have again adopted the quasi-neutrality approximation for the charge density. The perpendicular electric field $\boldsymbol{E}_\perp$ under a strong intrinsic magnetic field, such as that in the Earth's thermosphere, is related to the cross-$\boldsymbol{B}$ current and can be approximately expressed as (e.g., Schunk and Nagy, 2000, p. 130)

$$\boldsymbol{E}_\perp = -(\boldsymbol{u}_i \times \boldsymbol{B}) + \frac{m_i \nu_i}{e}(\boldsymbol{u}_i - \boldsymbol{u}_n), \tag{5}$$

where $\boldsymbol{u}_i$ is the ion velocity and ν_i is the ion-neutral collision frequency. Adoption of Eqs. (4) and (5) for $\boldsymbol{J}$ and $\boldsymbol{E}$ implies that the ion velocities in Eq. (1) are determined by the strong intrinsic magnetic field. Note that ion and neutral velocities are treated equally as unknown dependent variables in Eq. (1). However, using the bulk flow system of Eqs. (3), (4) and (5), the bulk ion velocity is determined diagnostically from Eq. (5). Substituting Eqs. (4) and (5) into the right-hand side of Eq. (3) we obtain the ion drag term

$$\boldsymbol{F}_{I-D} \equiv \boldsymbol{J} \times \boldsymbol{B} = \mu_1 \boldsymbol{u}'_\perp + \mu_2(\boldsymbol{b} \times \boldsymbol{u}'), \tag{6}$$

where

$$\mu_1 = B^2(\sigma_P + \kappa_i^{-1}\sigma_H), \tag{7a}$$

$$\mu_2 = B^2(\sigma_H - \kappa_i^{-1}\sigma_P), \tag{7b}$$

and

$$\boldsymbol{u}' = (u', v', w') = (u_i - u_n)\boldsymbol{e}_x + (v_i - v_n)\boldsymbol{e}_y + (w_i - w_n)\boldsymbol{e}_z \tag{8}$$

is the difference between the ion velocity and neutral wind. Also, $\boldsymbol{u}'_\perp \equiv \boldsymbol{u}' - (\boldsymbol{b}\cdot\boldsymbol{u}')\boldsymbol{b}$ is the perpendicular component of the velocity difference projected onto a plane perpendicular to the magnetic field: $\boldsymbol{u}'_\perp\cdot\boldsymbol{b}$=0. In Eq. (7), κ_i is the ratio of the ion cyclotron frequency ($\omega_{ci}=eB/m_i$) to the ion-neutral collision frequency (ν_i). In Eq. (8), $\boldsymbol{e}_x$, $\boldsymbol{e}_y$, and $\boldsymbol{e}_z$ are the unit vectors directed eastward (x), northward (y), and upward (z), respectively. The subscripts i and n on the velocity components denote ion and neutral, respectively. Note that the second term in Eq. (6) is also perpendicular to $\boldsymbol{b}$. Therefore, the ion drag as parameterized by second expression in Eq. (6) is perpendicular to $\boldsymbol{b}$, which is consistent with its macroscopic definition of $\boldsymbol{J}\times\boldsymbol{B}$. We also note that the coefficients μ_1 and μ_2 in Eq. (6) are different from those shown in the literature (e.g., Rees 1989, p. 206).

Given $\boldsymbol{E}$, $\boldsymbol{B}$, $\boldsymbol{u}_n$ and other ionospheric parameters, one first needs to solve Eq. (5) for ion velocity $\boldsymbol{u}_i$ in order to derive $\boldsymbol{F}_{I-D}$ from Eq. (6) (see below and Appendix A for numerical solutions for $\boldsymbol{u}_i$). However, since $\boldsymbol{F}_{I-D}$ is perpendicular to $\boldsymbol{b}$, it is possible to rewrite it in terms of the E-cross-B drift velocity $\boldsymbol{u}_E \equiv \boldsymbol{E}\times\boldsymbol{B}/B^2$, which can be calculated directly from $\boldsymbol{E}$ and $\boldsymbol{B}$ fields. In terms of $\boldsymbol{u}_E$ Eq. (5) can be written as

$$\boldsymbol{u}_E = \boldsymbol{u}_{i\perp} - \kappa_i^{-1}\left(\boldsymbol{b}\times\boldsymbol{u}'\right), \tag{9}$$

where $\boldsymbol{u}_{i\perp}$ denotes the corresponding perpendicular component of $\boldsymbol{u}_i$. Substituting Eq. (9) into Eq. (6) with some algebraic manipulations we arrive at a similar form for the ion drag parameterization

$$\boldsymbol{F}_{I-D} = \mu_1^*\boldsymbol{u}''_\perp + \mu_2^*(\boldsymbol{b}\times\boldsymbol{u}''), \tag{10}$$

where $\boldsymbol{u}''=\boldsymbol{u}_E-\boldsymbol{u}_n$ is the velocity difference between the E-cross-B drift velocity and neutral wind, $\boldsymbol{u}''_\perp$ is the corresponding perpendicular component: $\boldsymbol{u}''_\perp=\boldsymbol{u}''-(\boldsymbol{b}\cdot\boldsymbol{u}'')\boldsymbol{b}$. Two coefficients in Eq. (10) are simply the first terms in Eqs. (7a, b):

$$\mu_1^* = \sigma_P B^2, \tag{11a}$$

$$\mu_2^* = \sigma_H B^2. \tag{11b}$$

Equation (10) is the form often used for the atmospheric dynamics in the thermosphere (e.g., Dickinson et al., 1981).

It is worthwhile to compare the difference in their mathematical forms of ion drag between the microscopic form $\delta\boldsymbol{M}_s/\delta t$ in Eq. (1) and the parameterized form of Eqs. (6) and (10). When there exists only one (dominant) kind of ion that produces ion drag on the neutral wind, $\delta\boldsymbol{M}_s/\delta t$ is proportional to $\boldsymbol{u}'$ ($=\boldsymbol{u}_i-\boldsymbol{u}_n$) with a scalar coefficient (e.g., Schunk and Nagy, 2000, p. 82). Therefore, the ion drag vector $\boldsymbol{F}_{I-D}$ in Eq. (1) is always parallel to the velocity difference vector $\boldsymbol{u}'$. On the other hand, $\boldsymbol{F}_{I-D}$ and $\boldsymbol{u}'$ in the parameterized forms of Eqs. (6) and (10) are no longer parallel unless $\boldsymbol{u}'$ or $\boldsymbol{u}''$ is perpendicular to the magnetic field. The apparent paradox is caused by differing definitions of ion velocity. The ion velocity in Eq. (1) represents the absolute movement of ions, including the gyro-motion that defines the ion-cyclotron frequency and the ion-neutral collision frequency. On the other hand, $\boldsymbol{u}_i$ in Eq. (6) or $\boldsymbol{u}_E$ in Eq. (10) is determined by a steady state relationship such as Eq. (5) that represents the bulk ion velocity, which has been averaged over a gyrocycle. It should also be pointed out that the general macroscopic expression for $\boldsymbol{F}_{I-D}$ ($=\boldsymbol{J}\times\boldsymbol{B}$) on the right-hand side of Eq. (2) is universally correct by definition. Depending on whether $\boldsymbol{J}$ and $\boldsymbol{B}$ are derived by time-dependent Maxwell equations or by a steady state Eq. (5), $\boldsymbol{J}\times\boldsymbol{B}$ can be reduced to either the microscopic form of Eq. (1) or the parameterized form of Eq. (6) or Eq. (10).

It is also noted that below ∼200 km the second collision term on the right-hand side of Eq. (5) becomes non-negligible (e.g., Kelley, 1989, Sect. 2.2). Therefore, it is important to recognize the different definitions of $\boldsymbol{u}'$ and $\boldsymbol{u}''$ in Eqs. (6) and (10) because their coefficients are different though the functional forms are very similar. In Fig. 1, we show the Pedersen and Hall conductivities that are calculated from the NRLMSISE-00 model atmosphere (Picone et al., 2002) and IRI ionospheric model (Bilitza et al., 1993) at 65° N magnetic latitude with the A_p index and the 10.7-cm solar radio flux (in units of $10^{-22}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$) of 4.0 and 150.0, respectively. Results at midnight are provided in the upper panel while those in the lower panel are at noon. Also shown in the figure are the coefficients, μ_1 and μ_2, that are normalized by B^2 and used in Eq. (6) for the ion drag calculations. Figure 1 shows increasing differences with decreasing altitude between μ_1 and $\mu_1^*(=\sigma_P B^2)$ below ∼200 km due to the contribution from the second term in Eq. (5). It also shows significant differences between μ_2 and $\mu_2^*(=\sigma_H B^2)$ because the second term in Eq. (7b) is greater than the first, making μ_2 negative. It can be shown that for $\kappa_i^{-1}\ll 1$ that corresponds to a very low collision frequency, the two terms in Eq. (7b) nearly cancel each other so that $\mu_2\approx 0$.

2.3 Parameterized ion drag in a dipole magnetic field

The Earth's magnetic field in the thermosphere can be approximated by a magnetic dipole. To simplify the derivations with clearer physics, we first assume coincident geographic and geomagnetic poles. In this case, the unit vector of the magnetic field, $\boldsymbol{b}$, can be expressed as (e.g., Schunk and Nagy 2000, p. 314)

$$\boldsymbol{b} = -(\cos I)\boldsymbol{e}_y - (\sin I)\boldsymbol{e}_z, \tag{12}$$

where I is the dip angle between the magnetic field $\boldsymbol{B}$ and the local horizontal direction. Substituting Eqs. (8) and (12) into Eq. (6), we finally obtain the parameterized ion drag term for the momentum equation of the neutral atmosphere in the thermosphere

$$\boldsymbol{F}_{I-D} = \left(\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z\right) \begin{bmatrix} \mu_1 & \mu_2 \sin I & -\mu_2 \cos I \\ -\mu_2 \sin I & \mu_1 \sin^2 I & -\mu_1 \sin I \cos I \\ \mu_2 \cos I & -\mu_1 \sin I \cos I & \mu_1 \cos^2 I \end{bmatrix} \begin{pmatrix} u' \\ v' \\ w' \end{pmatrix}. \tag{13}$$

Note that the 3×3 matrix in Eq. (13) is neither symmetric nor anti-symmetric. In most regions of the thermosphere, the magnitudes of the horizontal velocities are far greater than the magnitudes of the vertical ones. Therefore, the four terms as sketched on the upper-left corner of Eq. (13) are often used in the horizontal momentum equations for the atmospheric dynamics in the thermosphere (e.g., Dickinson et al., 1981; Rees 1989, p. 207). Because we have adopted the axial-centered dipole approximation (12) for the magnetic field, the 2×2 sub-set of Eq. (13) for the horizontal ion drag happens to be anti-symmetric. Also note from Eq. (13) that the major contribution to the ion drag in the vertical direction will come from terms associated with the horizontal wind.

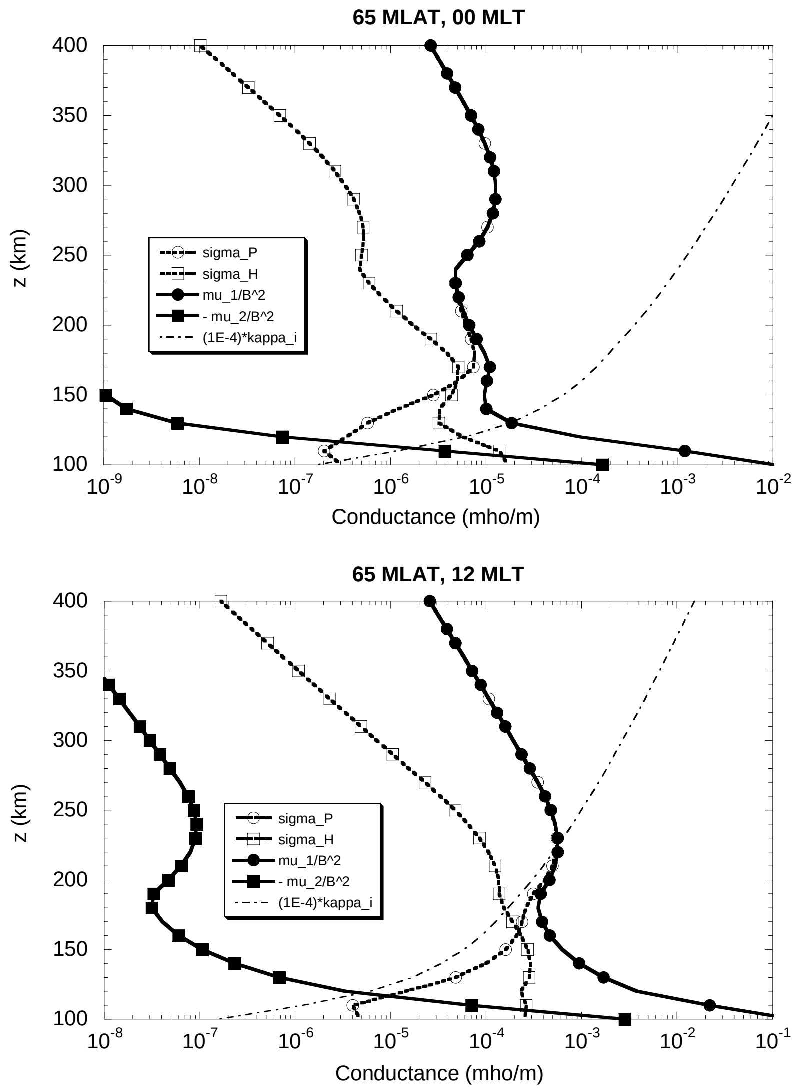

Fig. 1. Pedersen and Hall conductivities (σ_P and σ_H, dashed lines) used in Eq. (10) as the coefficients for calculating ion drag and the improved coefficients (μ_1/B^2 and $-\mu_2/B^2$, solid lines) in current parameterization of Eqs. (6) and (7) for ion drag. The thin dash-dotted lines represent the ratio of the ion cyclotron frequency to the ion-neutral collision frequency scaled by 10^{-4}. The computations are done based on the NRLMSISE-00 model atmosphere at 65° N magnetic latitude and the IRI90 ionospheric model. The A_p index and the 10.7-cm solar radio flux (in units of $10^{-22}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$) used in computations are 4.0 and 150.0, respectively. The upper and lower panels are for the midnight and noon magnetic local times, respectively.

For a displaced magnetic pole with declination angle δ, the unit vector of the magnetic field is given by (e.g., Roble and Dickinson, 1974)

$$\boldsymbol{b} = \pm(\cos I \sin\delta)\boldsymbol{e}_x - (\cos I \cos\delta)\boldsymbol{e}_y - (\sin I)\boldsymbol{e}_z \quad , \tag{14}$$

where "+" and "–" are for the northern and southern geomagnetic hemispheres, respectively. The ion drag corresponding to Eq. (14) is given by

$$\boldsymbol{F}_{I-D} = \left(\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z\right) \left[\begin{array}{cc|c} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu_{yx} & \mu_{yy} & \mu_{yz} \\ \hline \mu_{zx} & \mu_{zy} & \mu_{zz} \end{array}\right] \begin{pmatrix} u' \\ v' \\ w' \end{pmatrix}, \tag{15}$$

where

$$\mu_{xx} = \mu_1(1 - \sin^2\delta\cos^2 I), \tag{16a}$$

$$\mu_{yy} = \mu_1(1 - \cos^2\delta\cos^2 I), \tag{16b}$$

$$\mu_{zz} = \mu_1(1 - \sin^2 I), \tag{16c}$$

$$\mu_{xy} = \pm\mu_1 \sin\delta\cos\delta\cos^2 I + \mu_2 \sin I, \tag{16d}$$

$$\mu_{yx} = \pm\mu_1 \sin\delta\cos\delta\cos^2 I - \mu_2 \sin I, \tag{16e}$$

$$\mu_{xz} = \pm\mu_1 \sin\delta\sin I\cos I - \mu_2\cos\delta\cos I, \tag{16f}$$

$$\mu_{zx} = \pm\mu_1 \sin\delta\sin I\cos I + \mu_2\cos\delta\cos I, \tag{16g}$$

$$\mu_{yz} = -\mu_1 \cos\delta\sin I\cos I \mp \mu_2\sin\delta\cos I, \tag{16h}$$

$$\mu_{zy} = -\mu_1 \cos\delta\sin I\cos I \pm \mu_2\sin\delta\cos I. \tag{16i}$$

Note that $\mu_{yx}\neq-\mu_{xy}$ for a non-vanishing declination angle δ. Therefore, in general, the ion drag tensor is not anti-symmetric even for the simplified 2-D case for the horizontal momentum equations.

3 Derivation of the parameterized Joule heating in the energy equation

Under approach (i), the Joule heating is simply the second moment of ion-neutral velocity difference in each species equation (Schunk and Nagy 2000, p. 54). Similar to the derivation of the parameterized ion drag in momentum equations we start from the energy equation of magnetohydrodynamics (approach (ii)) for the derivation of the parameterized Joule heating (Schunk and Nagy, 2000, p. 196, p. 274; Rees, 1989, p. 124) for a bi-atomic gas (N_2 and O_2)

$$\frac{D}{Dt}\left(\frac{5}{2}p\right)+\frac{5}{2}p\nabla\cdot\boldsymbol{u}+\boldsymbol{\tau}:\nabla\boldsymbol{u}+\nabla\cdot\boldsymbol{q}=\boldsymbol{J}\cdot(\boldsymbol{E}+\boldsymbol{u}\times\boldsymbol{B}),\tag{17}$$

where p is the total atmospheric pressure, $\boldsymbol{q}$ is the total heat flow vector, $\boldsymbol{\tau}$ is the total stress tensor, and $\boldsymbol{\tau}:\nabla\boldsymbol{u}(=\sum_{\alpha,\beta}\tau_{\alpha\beta}[\partial u_\alpha/\partial x_\beta])$ denotes the double dot product of two tensors $\boldsymbol{\tau}$ and $\nabla\boldsymbol{u}$. By use of an appropriate equation of state one can relate the pressure variation with the temperature variation in the energy equation.

The terms on the left-hand side of Eq. (17) are again all weighted by the density of both neutral species and charged particles. On the other hand, the right-hand-side terms are proportional to the density of charged particles only. Following the similar approximation for the momentum Eq. (3) we arrive at the following energy equation for the neutral atmosphere

$$\frac{D_n}{Dt}\left(\frac{5}{2}p_n\right)+\frac{5}{2}p_n\nabla\cdot\boldsymbol{u}_n+\boldsymbol{\tau}_n:\nabla\boldsymbol{u}_n+\nabla\cdot\boldsymbol{q}_n$$
$$=\boldsymbol{J}_\perp\cdot(\boldsymbol{E}_\perp+\boldsymbol{u}_n\times\boldsymbol{B}),\tag{18}$$

where the subscript n denotes the physical state for the neutral atmosphere. The right-hand side of Eq. (18) defines the Joule heating for a neutral atmosphere because it is a thermal energy source that depends on electromagnetic fields and electric currents. Note that we have also made two approximations on the right-hand side of Eq. (17) to arrive at Eq. (18): (i) the total velocity on the second term has been replaced by the neutral wind because charged particles are considered minor tracers in the thermosphere; and (ii) the first term $\boldsymbol{J}\cdot\boldsymbol{E}$ has been approximated by $\boldsymbol{J}_\perp\cdot\boldsymbol{E}_\perp$ because the electric potential drop along the magnetic field is usually negligibly small in the thermosphere (e.g., Kelley, 1989, p. 43). These two approximations have been made by the basic rules of scale analysis described in the paragraph following Eq. (3). We will show below that making these two approximations is also necessary in order to have a dynamic system that is energetically consistent with the derivation of ion drag above. In addition, these two approximations lead to a non-negative Joule heating for the neutral atmosphere as shown below in Eq. (19b). Therefore, the approach of scale analysis used in simplifying equations is systematic and self-consistent (e.g., Haltiner and Williams, 1980, Ch. 3).

Substituting Eqs. (4) and (5) into the right-hand side of Eq. (18) we obtain the parameterized Joule heating in the thermosphere

$$Q_J\equiv\boldsymbol{J}_\perp\cdot(\boldsymbol{E}_\perp+\boldsymbol{u}_n\times\boldsymbol{B})=\mu_1^*\left[(1+\kappa_i^{-2})|\boldsymbol{u}'|^2-(\boldsymbol{b}\cdot\boldsymbol{u}')^2\right],\tag{19a}$$

or in its explicitly non-negative form

$$Q_J=\mu_1^*\left[\left|\boldsymbol{u}'_\perp\right|^2+\kappa_i^{-2}\left|\boldsymbol{u}'\right|^2\right],\tag{19b}$$

where μ_1^* is given by Eq. (11). Equation (19) indicates that the Joule heating for the neutral atmosphere is proportional to the Pederson conductivity, is independent of the Hall conductivity, and is non-negative.

By use of the relationship Eq. (9) between the bulk ion velocity $\boldsymbol{u}_i$ and the E-cross-B drift velocity $\boldsymbol{u}_E$ we are also able to derive Q_J that contains the E-cross-B drift velocity $\boldsymbol{u}_E$:

$$Q_J=\mu_1^*\left[\kappa_i^2\left|\boldsymbol{u}_E-\boldsymbol{u}_{i\perp}\right|^2+\kappa_i^{-2}\left|\boldsymbol{u}'\right|^2\right].\tag{20}$$

Unlike Eq. (10) for the ion drag parameterization, it is impossible to eliminate the explicit dependence of $\boldsymbol{u}_i$ in Q_J after introducing $\boldsymbol{u}_E$. It is worth noting that both $\boldsymbol{F}_{I-D}$ and Q_J vanish when the ion velocity coincides with the neutral wind ($\boldsymbol{u}_i=\boldsymbol{u}_n$). Physically, this means that no momentum and energy exchanges occur between ions and neutral particles as they are moving in the same direction at the same speed. Such a physical argument is also explicitly shown in the parameterized forms of Eqs. (6) and (19) for $\boldsymbol{F}_{I-D}$ and Q_J, respectively. We will have both $\boldsymbol{F}_{I-D}=0$ and $Q_J=0$ if $\boldsymbol{u}'$ vanishes. However, the E-cross-B drift velocity $\boldsymbol{u}_E$ is different from the ion velocity $\boldsymbol{u}_i$ if the second term in Eq. (5) or Eq. (9) is non-negligible. For example, by definition, $\boldsymbol{u}_E$ is always perpendicular to both $\boldsymbol{E}$ and $\boldsymbol{B}$ whereas the collisions between ions and neutral particles will drive the direction of $\boldsymbol{u}_i$ vector away from that of $\boldsymbol{u}_E$. As a result, a vanishing $\boldsymbol{u}'$ will not generally lead to a vanishing $\boldsymbol{u}''$ if the collision term in Eq. (5) is included. Therefore, in general, one would not expect to be able to transform an expression of $\boldsymbol{F}_{I-D}$ or Q_J from a purely $\boldsymbol{u}'$-dependent form into a $\boldsymbol{u}''$-dependent one without additional $\boldsymbol{u}_i$-dependent terms. The transform realized between Eqs. (6) and (10) happens to be a special case since the parameterized $\boldsymbol{F}_{I-D}$ is perpendicular to $\boldsymbol{B}$ and so is the E-cross-B drift velocity $\boldsymbol{u}_E$.

From Eq. (20) we can find two limiting cases that the Joule heating can be formally expressed in forms without $\boldsymbol{u}_i$ terms: (i) when the plasma becomes collisionless so that the second term in Eq. (9) is negligible and the ion velocity coincides with the E-cross-B drift velocity ($\boldsymbol{u}_E=\boldsymbol{u}_{i\perp}=\boldsymbol{u}_i$); (ii) when the plasma becomes collision-dominant so that the ion velocity coincides with the neutral wind ($\boldsymbol{u}_i=\boldsymbol{u}_n$). Under these assumptions, the Joule heating can be formally expressed as

$$Q_J=\mu_1^*\kappa_i^{-2}\left|\boldsymbol{u}'\right|^2=\mu_1^*\kappa_i^{-2}\left|\boldsymbol{u}_E-\boldsymbol{u}_n\right|^2\quad\text{as}\quad\kappa_i\to\infty,\tag{21}$$

$$Q_J=\mu_1^*\kappa_i^2\left|\boldsymbol{u}_E-\boldsymbol{u}_{i\perp}\right|^2=\mu_1^*\kappa_i^2\left|\boldsymbol{u}_E-\boldsymbol{u}_{n\perp}\right|^2\quad\text{as}\quad\kappa_i\to 0.\tag{22}$$

From the vertical profile of κ_i (e.g., Kelley, 1989, p. 39) one may expect Eqs. (21) and (22) to be good approximations in regions of high altitude, say >200 km, and low altitude, say <100 km, respectively. However, we should note that the two assumptions used that led to Eqs. (21) and (22) will lead to vanishing Q_J. Therefore, one needs to know the actual values of bulk ion velocity in order to have an appropriate evaluation of Joule heating. For reference we show in Appendix A the explicit equations for solving $\boldsymbol{u}_i$ components in a dynamical model.

Explicit expressions for Q_J by the velocity components can be derived by substituting Eqs. (8) and (12) into Eq. (19) to yield

$$Q_J=\mu_1^*\left[(1+\kappa_i^{-2})u'^2+\kappa_i^{-2}(v'^2+w'^2)+(v'\sin I-w'\cos I)^2\right]. \quad (23)$$

For a displaced magnetic pole it can be shown that

$$Q_J=\mu_1^*\Big[(1+\kappa_i^{-2}-\sin^2\delta\cos^2 I)u'^2+(1+\kappa_i^{-2}-\cos^2\delta\cos^2 I)v'^2 +(\cos^2 I+\kappa_i^{-2})w'^2\pm 2\sin\delta\cos\delta\cos^2 I\,u'v' \pm 2\sin\delta\sin I\cos I\,u'w'-2\cos\delta\sin I\cos I\,v'w'\Big]. \quad (24)$$

To conclude our derivations of $\boldsymbol{F}_{I-D}$ and Q_J parameterizations we emphasize that both the ion drag and Joule heating are functions of the difference between the bulk ion velocity and neutral wind. The dynamic effects of the charged particles on the neutral wind vanish if the ion velocity coincides with the neutral wind ($\boldsymbol{u}_i=\boldsymbol{u}_n$).

4 Atmospheric energetics of the parameterized ion drag and Joule heating

The total energy of the whole atmosphere under adiabatic motion is conserved. From an energetics point of view, various terms in the momentum and thermal energy equations represent conversions of energy from one form to another (e.g., Dutton, 1986, Ch. 11; Holton, 1992, Sect. 10.4). Often, a simplification of the equations of motion for a specific problem requires that the energy conversion relations still hold. To demonstrate the energy consistency of the system that includes charged particles we first note from Eq. (3) that the induced change in kinetic energy density caused by ion drag is

$$\left[\rho_n\frac{D_n}{Dt}\left(\frac{1}{2}\boldsymbol{u}_n^2\right)\right]_{I-D}=\boldsymbol{u}_n\cdot(\boldsymbol{J}\times\boldsymbol{B})=-\boldsymbol{J}_\perp\cdot(\boldsymbol{u}_n\times\boldsymbol{B}). \quad (25)$$

For a system that includes currents and electromagnetic fields, energy conservation is described by Poynting's theorem (e.g., Jackson, 1975, p. 237; Thayer and Vickrey, 1992),

$$\frac{\partial W}{\partial t}+\nabla\cdot\boldsymbol{S}=-\boldsymbol{J}\cdot\boldsymbol{E}\approx-\boldsymbol{J}_\perp\cdot\boldsymbol{E}_\perp, \quad (26)$$

where W is the electromagnetic energy density, and $\boldsymbol{S}=(\boldsymbol{E}\times\boldsymbol{B})/\mu_0$ is the Poynting vector that represents the electromagnetic energy flux density with μ_0 being the permeability of free space. The term on the right-hand side of

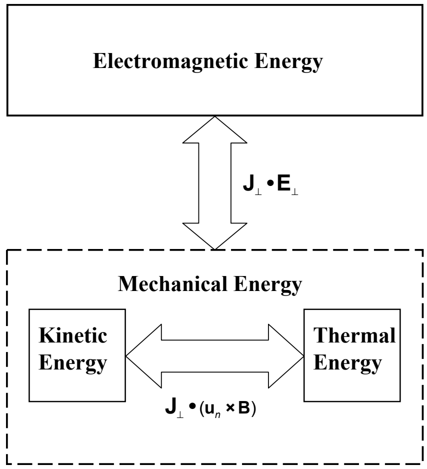

Fig. 2. Relationships of energy conversion among three types of energy in the thermosphere.

Eq. (26) represents the conversion of electromagnetic energy into mechanical energy, which consists of both thermal and kinetic energies for the atmosphere. This can be seen from the sum of the right-hand sides of Eqs. (18) and (25) which equals the negative of the right-hand side of Eq. (26).

By assuming the neutral atmosphere to be a major species in the thermosphere we derived the parameterized ion drag (Eq. 13) (or Eq. 15) and Joule heating (Eq. 23) (or Eq. 24), which are functions of the velocity difference between the bulk ion velocity and neutral wind. It can be explicitly shown that the energetics of the parameterization remains consistent. In other words, the sum of the kinetic energy due to ion drag and Joule heating for the neutral atmosphere as parameterized by the velocities and conductivities is equal to the production ($-\boldsymbol{J}_\perp\cdot\boldsymbol{E}_\perp>0$) or the loss ($-\boldsymbol{J}_\perp\cdot\boldsymbol{E}_\perp<0$) of electromagnetic energy when the same form of approximations Eqs. (4) and (5) for $\boldsymbol{J}_\perp$ and $\boldsymbol{E}_\perp$ are used. According to Eq. (19b), the Joule heating under the current parameterization is positive unless the ion velocity coincides with the neutral wind. Therefore, there will be an accompanying conversion of kinetic energy into Joule heating when electromagnetic energy is generated by the atmospheric neutral wind dynamo. Likewise, the electromagnetic energy cannot be fully converted into kinetic energy without producing Joule heating in the thermosphere.

Joule heating as parameterized in Eq. (18) consists of two terms

$$\boldsymbol{J}_\perp\cdot\boldsymbol{E}_\perp= \sigma_P\,|\boldsymbol{E}_\perp|^2-\sigma_P\boldsymbol{u}_n\cdot(\boldsymbol{E}_\perp\times\boldsymbol{B})+\sigma_H(\boldsymbol{u}_n\times\boldsymbol{b})\cdot(\boldsymbol{E}_\perp\times\boldsymbol{B}), \quad (27a)$$

$$\boldsymbol{J}_\perp\cdot(\boldsymbol{u}_n\times\boldsymbol{B})= \sigma_P\,|\boldsymbol{u}_n\times\boldsymbol{B}|^2-\sigma_P\boldsymbol{u}_n\cdot(\boldsymbol{E}_\perp\times\boldsymbol{B})-\sigma_H(\boldsymbol{u}_n\times\boldsymbol{b})\cdot(\boldsymbol{E}_\perp\times\boldsymbol{B}). \quad (27b)$$

The first term ($\boldsymbol{J}_\perp\cdot\boldsymbol{E}_\perp$), which also appears in Eq. (26) and mathematically represents the conversion between electromagnetic energy and thermal energy, can be considered the conversion between electromagnetic energy and mechanical energy (Fig. 2). The second term ($\boldsymbol{J}_{\perp}\cdot(\boldsymbol{u}_n\times\boldsymbol{B})$), which also appears in Eq. (25) and mathematically represents the conversion between thermal energy and kinetic energy, determines the partition between thermal energy and kinetic energy within the mechanical energy. These two energetic relationships as represented by two terms in Eq. (27) are shown schematically in Fig. 2. When Eqs. (25), (18), and (26) associated with all three types of energy (thermal, kinetic, and electromagnetic) are listed in parallel, the relationships among them become clear and self-consistent through the conversion terms. These clear relationships suggest that the second formulation of the ion drag and Joule heating in terms of macroscopic Lorentz force and Ohmic dissipation, Eqs. (3) and (18), is the best for understanding atmospheric energetics. In addition, the two approximations used to derive the right-hand side of Eq. (18) from Eq. (17) become reasonable under the requirement of the energetics consistency as described in Fig. 2. It provides a rationale for including neutral wind $\boldsymbol{u}_n$ (rather than the average wind $\boldsymbol{u}$ in the primitive energy equation) in modifying the traditional definition of Joule heating that consists only of $\sigma_P\,|\boldsymbol{E}_{\perp}|^2$ contained in $\boldsymbol{J}_{\perp}\cdot\boldsymbol{E}_{\perp}$.

Note that the last terms in Eqs. (27a) and (27b) cancel each other. Therefore, Hall currents that are perpendicular to the electric field do not contribute to Joule heating, which is also explicitly shown in its more compact expression (Eq. 20). However, on the basis of the above analysis, Hall currents contribute to the conversions among three types of energy. Adding Eqs. (27a) and (27b) and rearranging the terms, we can rewrite Joule heating as (Lu et al., 1995)

$$Q_J = Q_{J1} + Q_{J2} \equiv \sigma_P\,|\boldsymbol{E}_{\perp}|^2 + \left\{\sigma_P\,|\boldsymbol{u}_n\times\boldsymbol{B}|^2 - 2\sigma_P\boldsymbol{u}_n\cdot(\boldsymbol{E}_{\perp}\times\boldsymbol{B})\right\}, \tag{28}$$

where the first term (Q_{J1}) is independent of the neutral wind and is called "convection heating" (Lu et al., 1995) or "local Joule heating" (Thayer, 1998). The terms in braces (Q_{J2}) are called "wind heating" (Lu et al., 1995). Unlike expressions (27a) and (27b), which emphasize the conversion among the different types of energy, Eq. (28) emphasizes the effect of neutral wind on Joule heating. Comparing Eq. (28) with Eq. (19b) we can also express the convective heating in terms of the ion velocity $\boldsymbol{u}_i$

$$\sigma_P\,|\boldsymbol{E}_{\perp}|^2 = \mu_1^*\left[|\boldsymbol{u}_{i\perp}|^2 + \kappa_i^{-2}|\boldsymbol{u}_i|^2\right]. \tag{29}$$

Because of the difficulty in directly measuring neutral wind in the thermosphere (e.g., Kelley, 1989, p. 66), the contributions of the neutral wind to Joule heating were often neglected in earlier studies. As a result, neutral wind effects are often neglected in textbook definitions of Joule heating. In some textbooks, authors defined either $\sigma_P\,|\boldsymbol{E}_{\perp}|^2$ (e.g., Kelley 1989, p. 270; Baumjohann and Treumann 1996, p. 88) or $\boldsymbol{J}\cdot\boldsymbol{E}$ (e.g., Kato 1980, p. 186) as Joule heating whereas in others (e.g., Rees 1989, p. 127; Schunk and Nagy 2000, p. 402) authors only provided a descriptive role of Joule heating in the thermal energy balance but gave no explicit expressions for its definition. Recent calculations have shown that the magnitude of wind heating, Q_{J2}, could be comparable to that of convection heating, Q_{J1} (e.g., Lu et al., 1995; Thayer 1998). Therefore, it is necessary to have a more precise and self-consistent definition for Joule heating that includes the effect of neutral wind.

In this paper, both ion drag and Joule heating have been consistently derived from the general magnetohydrodynamic Eqs. (2) and (18), respectively. An alternative approach of deriving Joule heating including the neutral wind effect is to simply replace $\boldsymbol{E}_{\perp}$ in the traditional formula $\sigma_P\,|\boldsymbol{E}_{\perp}|^2$ by $\boldsymbol{E'}_{\perp}=\boldsymbol{E}_{\perp}+\boldsymbol{u}_n\times\boldsymbol{B}$ for the simple reasons (e.g., Jackson, 1975, p. 212): the ions are moving through the neutral gas and it is the electric field that is measured in the reference frame following the neutral wind that counts (e.g., Kivelson and Russell, 1995, p. 494). However, under this argument, the rest equations, such as the momentum equation or the equation for the kinetic energy, also need to be revised into the same moving frame while discussing the system energetics, which would make the study of the thermospheric dynamics more complicated.

The parameterized expressions of Eqs. (13) and (23) or Eqs. (15) and (24) can be directly applied to thermospheric general circulation models. The main advantages of adopting current set of parameterizations are: (i) both ion drag and Joule heating are self-consistently dependent on the same quantity of the difference between the bulk ion velocity and neutral wind; (ii) the parameterizations are energetically conserved so changes in one type of energy in a dynamical model will be exactly compensated by the changes in the rest two types; (iii) the bulk ion velocity can be explicitly derived from the modeled neutral wind (Appendix A) and can be compared with the radar measurements in the thermosphere.

5 Concluding remarks

In this paper, we have derived ion drag term in the momentum equation and the Joule heating term in the thermal energy equation for thermosphere atmospheric dynamics in a consistent fashion, both being parameterized as functions of the difference between the bulk ion velocity and neutral wind. The parameterized expressions also depend on the magnetic field, the Pedersen and Hall conductivities, and the ratio of the ion cyclotron frequency to the ion-neutral collision frequency. Our derivation explicitly shows two different types of approximations in parallel: approach (ii) a macroscopic formulation in terms of electrodynamic force and Ohmic dissipation by Eqs. (3) and (18), respectively, and approach (iii) a parameterized ion drag and Joule heating in terms of bulk velocity difference by Eqs. (6) and (19), respectively. The atmospheric energetics was examined for both sets of expressions. It is shown explicitly that Joule heating linearly depends on Pedersen conductivity and is always positive unless the neutral wind coincides with the bulk ion velocity and as a result of which both ion drag and Joule heating vanish. The Hall currents contribute to the energetics conversion among thermal, kinetic, and electromagnetic energies but have no effect on Joule heating.

We have also shown that there is an accompanying conversion of kinetic energy into Joule heating when electromagnetic energy is generated through the dynamo mechanism of the atmospheric neutral wind. Likewise, electromagnetic energy cannot be fully converted into kinetic energy without producing Joule heating in the thermosphere. The partition between kinetic energy and Joule heating is analyzed by the conversion terms. A similar analysis is also presented for the effect of the neutral wind on Joule heating. When the self-consistent and energetically conserved parameterizations are applied to thermospheric general circulation models the bulk ion velocity derived from the model neutral wind as shown below in Appendix A can be compared with the radar measurements in the thermosphere.

Appendix A
Numerical solutions of the bulk ion velocity

Equation (9) that relates the E-cross-B drift velocity $\boldsymbol{u}_E$ to the bulk ion velocity $\boldsymbol{u}_i$ and neutral wind $\boldsymbol{u}_n$ can be rewritten as

$$\kappa_i \left[\boldsymbol{u}_i - (\boldsymbol{b}\cdot\boldsymbol{u}_i)\,\boldsymbol{b}\right] - \boldsymbol{b}\times\boldsymbol{u}_i = \kappa_i \boldsymbol{u}_E - \boldsymbol{b}\times\boldsymbol{u}_n. \tag{A1}$$

Equation (A1) is a set of linear equations with respect to $\boldsymbol{u}_i$ components with the right-hand side being the force terms. Assuming the general case of a displaced magnetic pole Eq. (14) we can derive the matrix form of Eq. (A1) for the bulk ion velocity components $\boldsymbol{u}_i=[u_i, v_i, w_i]^T$

$$\left[\begin{array}{cc|c} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ \hline a_{zx} & a_{zy} & a_{zz} \end{array}\right] \begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix}, \tag{A2}$$

where the coefficient matrix is similar to ion-drag matrix shown in Eqs. (15) and (16) with μ_1 and μ_2 replaced by κ_i and –1, respectively:

$$a_{xx} = \kappa_i(1-\sin^2\delta\cos^2 I), \tag{A3a}$$

$$a_{yy} = \kappa_i(1-\cos^2\delta\cos^2 I), \tag{A3b}$$

$$a_{zz} = \kappa_i(1-\sin^2 I), \tag{A3c}$$

$$a_{xy} = \pm\kappa_i \sin\delta\cos\delta\cos^2 I - \sin I, \tag{A3d}$$

$$a_{yx} = \pm\kappa_i \sin\delta\cos\delta\cos^2 I + \sin I, \tag{A3e}$$

$$a_{xz} = \pm\kappa_i \sin\delta\sin I\cos I + \cos\delta\cos I, \tag{A3f}$$

$$a_{zx} = \pm\kappa_i \sin\delta\sin I\cos I - \cos\delta\cos I, \tag{A3g}$$

$$a_{yz} = -\kappa_i \cos\delta\sin I\cos I \pm \sin\delta\cos I, \tag{A3h}$$

$$a_{zy} = -\kappa_i \cos\delta\sin I\cos I \mp \sin\delta\cos I. \tag{A3i}$$

The components of the right-hand-side force term are given by

$$f_x = \kappa_i u_E - (\sin I)v_n + (\cos\delta\cos I)w_n, \tag{A4a}$$

$$f_y = \kappa_i v_E \pm (\sin\delta\cos I)w_n + (\sin I)u_n, \tag{A4b}$$

$$f_z = \kappa_i w_E - (\cos\delta\cos I)u_n \mp (\sin\delta\cos I)v_n. \tag{A4c}$$

To derive the horizontal components of bulk ion velocity u_i and v_i, only the four terms as sketched on the upper-left corner of Eq. (A2) are needed. A FORTRAN subroutine that calculates the bulk ion velocity, ion drag and Joule heating is available from X. Zhu (xun.zhu@jhuapl.edu) upon request.

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