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Fixed points of multivalued nonexpansive mappings in Banach spaces

Safeer Hussain Khan[1] and Isa Yildirim[2*]

* Correspondence: isayildirim@atauni.edu.tr
[2]Department of Mathematics, Ataturk University, Erzurum 25240, Turkey
Full list of author information is available at the end of the article

Abstract

In this article, we first give a multivalued version of an iteration scheme of Agarwal et al. We use an idea due to Shahzad and Zegeye which removes a "strong condition" on the mapping involved in the iteration scheme and an observation by Song and Cho about the set of fixed points of that mapping. In this way, we approximate fixed points of a multivalued nonexpansive mapping through an iteration scheme which is independent of but faster than Ishikawa scheme used both by Song and Cho, and Shahzad and Zegeye. Thus our results improve and unify corresponding results in the contemporary literature.

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1. Introduction and preliminaries

Throughout the article, $\mathbb{N}$ denotes the set of positive integers. Let E be a real Banach space. A subset K is called proximinal if for each $x \in E$, there exists an element $k \in K$ such that

$$d(x, k) = \inf\{||x - y|| : y \in K\} = d(x, K)$$

It is known that a weakly compact convex subsets of a Banach space and closed convex subsets of a uniformly convex Banach space are proximinal. We shall denote the family of nonempty bounded proximinal subsets of K by $P(K)$. Consistent with [1], let $CB(K)$ be the class of all nonempty bounded and closed subsets of K. Let H be a Hausdorff metric induced by the metric d of E, that is

$$H(A, B) = \max\{\sup_{x \in A} d(x, B), \sup_{y \in B} d(y, A)\}$$

for every $A, B \in CB(E)$. A multivalued mapping $T : K \to P(K)$ is said to be a *contraction* if there exists a constant $k \in [0, 1)$ such that for any $x, y \in K$,

$$H(Tx, Ty) \leq k||x - y||,$$

and T is said to be *nonexpansive* if

$$H(Tx, Ty) \leq ||x - y||$$

for all $x, y \in K$. A point $x \in K$ is called a fixed point of T if $x \in Tx$.

The study of fixed points for multivalued contractions and nonexpansive mappings using the Hausdorff metric was initiated by Markin [2] (see also [1]). Later, an interesting and rich fixed point theory for such maps was developed which has applications in control theory, convex optimization, differential inclusion, and economics (see, [3] and references cited therein). Moreover, the existence of fixed points for multivalued nonexpansive mappings in uniformly convex Banach spaces was proved by Lim [4].

The theory of multivalued nonexpansive mappings is harder than the corresponding theory of single valued nonexpansive mappings. Different iterative processes have been used to approximate the fixed points of multivalued nonexpansive mappings. Among these iterative processes, Sastry and Babu [5] considered the following.

Let K be a nonempty convex subset of E, $T : K \to P(K)$ a multivalued mapping with $p \in Tp$.

(i) The sequences of Mann iterates is defined by $x_1 \in K$,

$$x_{n+1} = (1 - a_n)x_n + a_n y_n, \tag{1.1}$$

where $y_n \in Tx_n$ is such that $||y_n - p|| = d(p, Tx_n)$, and $\{a_n\}$ is a sequence of numbers in (0, 1) satisfying $\lim_{n\to\infty} a_n = 0$ and $\sum a_n = \infty$.

(ii) The sequence of Ishikawa iterates is defined by $x_1 \in K$,

$$\begin{cases} y_n = (1 - b_n)x_n + b_n z_n, \\ x_{n+1} = (1 - a_n)x_n + a_n u_n, \end{cases} \tag{1.2}$$

where $z_n \in Tx_n$, $u_n \in Ty_n$ are such that $||z_n - p|| = d(p, Tx_n)$ and $||u_n - p|| = d(p, Ty_n)$, and $\{a_n\}$, $\{b_n\}$ are real sequences of numbers with $0 \le a_n, b_n < 1$ satisfying $\lim_{n\to\infty} b_n = 0$ and $\sum a_n b_n = \infty$.

Panyanak [6] generalized the results proved by Sastry and Babu [5].

The following is a useful Lemma due to Nadler [1].

Lemma 1. *Let $A, B \in CB(E)$ and $a \in A$. If $\eta > 0$, then there exists $b \in B$ such that $d(a, b) \le H(A, B) + \eta$.*

Based on the above Lemma, Song and Wang [7] modified the iteration scheme due to Panyanak [6] and improved the results presented therein. Their scheme is given as follows:

Let K be a nonempty convex subset of E, $a_n \in [0, 1]$, $b_n \in [0, 1]$ and $\eta_n \in (0, \infty)$ such that $\lim_{n\to\infty}\eta_n = 0$. Choose $x_1 \in K$ and $z_1 \in Tx_1$. Let

$$y_1 = (1 - b_1)x_1 + b_1 z_1.$$

Choose $u_1 \in Ty_1$ such that $||\, z_1 - u_1 \,|| \le H(Tx_1, Ty_1) + \eta_1$ (see [1,8]). Let

$$x_2 = (1 - a_1)x_1 + a_1 u_1.$$

Choose $z_2 \in Tx_2$ such that $||\, z_2 - u_1 \,|| \le H(Tx_2, Ty_1) + \eta_2$. Take

$$y_2 = (1 - b_2)x_2 + b_2 z_2.$$

Choose $u_2 \in Ty_2$ such that $||\, z_2 - u_2 \,|| \le H(Tx_2, Ty_2) + \eta_2$. Let

$$x_3 = (1 - a_2)x_2 + a_2 u_2.$$

Inductively, we have

$$\begin{cases} y_n = (1 - b_n)x_n + b_n z_n \\ x_{n+1} = (1 - a_n)x_n + a_n u_n \end{cases} \tag{1.3}$$

where $z_n \in Tx_n$, $u_n \in Ty_n$ are such that $||z_n - u_n|| \leq H(Tx_n, Ty_n) + \eta_n$ and $||z_{n+1} - u_n|| \leq H(Tx_{n+1}, Ty_n) + \eta_n$, and $\{a_n\}$,$\{b_n\}$ are real sequences of numbers with $0 \leq a_n$, $b_n < 1$ satisfying $\lim_{n\to\infty} b_n = 0$ and $\sum a_n b_n = \infty$.

It is to be noted that Song and Wang [7] need the condition $Tp = \{p\}$ in order to prove their Theorem 1. Actually, Panyanak [6] proved some results using Ishikawa type iteration process without this condition. Song and Wang [7] showed that without this condition his process was not well-defined. They reconstructed the process using the condition $Tp = \{p\}$ which made it well-defined. Such a condition was also used by Jung [9].

Recently, Shahzad and Zegeye [10] remarked as follows:

"We note that the iteration scheme constructed by Song and Wang [7] involves the estimates which are not easy to be computed and the scheme is more time consuming. We also observe that Song and Wang [7] did not use the above estimates in their proofs and applied Lemma 2.1 (of [10]) without showing $x_n - p$, $y_n - p \in B_R(0)$. The assumption on T namely "$Tp = \{p\}$ for any $p \in F(T)$" is quite strong.... Then we construct an iteration scheme which removes the restriction of T namely $Tp = \{p\}$ for any $p \in F(T)$."

To do this, they defined $P_T(x) = \{y \in Tx : ||x - y|| = d(x, Tx)\}$ for a multivalued mapping $T : K \to P(K)$. They also proved a couple of strong convergence results using Ishikawa type iteration process.

On the other hand, Agarwal et al. [11] introduced the following iteration scheme for single-valued mappings:

$$\begin{cases} x_1 = x \in C, \\ x_{n+1} = (1 - \alpha_n)Tx_n + \alpha_n Ty_n, \\ y_n = (1 - \beta_n)x_n + \beta_n Tx_n, \ n \in \mathbb{N} \end{cases} \tag{1.4}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are in (0, 1). This scheme is independent of both Mann and Ishikawa schemes. They proved that this scheme converges at a rate faster than both Picard iteration scheme $x_{n+1} = Tx_n$ and Mann iteration scheme for contractions. Following their method, it was observed in [12, Example 3.7] that this scheme also converges faster than Ishikawa iteration scheme.

In this paper, we first give a multivalued version of the iteration scheme (1.4) of Agarwal et al. [11] and then use the idea of removal of "$Tp = \{p\}$ for any $p \in F(T)$" due to Shahzad and Zegeye [10] to approximate fixed points of a multivalued nonexpansive mapping T. We also use a result of Song and Cho [13] saying that set of fixed points of T is same as that of P_T , see Lemma 2 below. Moreover, we use the method of direct construction of Cauchy sequence as indicated by Song and Cho [13] (and opposed to [10]) but also used by many other authors including [12,14,15]. Keeping above in mind, we define our iteration scheme as follows:

$$\begin{cases} x_1 \in K, \\ x_{n+1} = (1 - \lambda)v_n + \lambda u_n \\ y_n = (1 - \eta)x_n + \eta v_n, \ n \in \mathbb{N} \end{cases} \tag{1.5}$$

where $v_n \in P_T(x_n)$, $u_n \in P_T(y_n)$ and $0 < \lambda, \eta < 1$. We have used λ, η only for the sake of simplicity but α_n, β_n could be used equally well under suitable conditions. In this way, we approximate fixed points of a multivalued nonexpansive mapping by an iteration scheme which is independent of but faster than Ishikawa scheme. Thus our results improve corresponding results of Shahzad and Zegeye [10], Song and Cho [13] and the results generalized therein.

Now, we give the following definitions.

Definition 1. *A Banach space E is said to satisfy Opial's condition* [16]*if for any sequence* $\{x_n\}$ *in E,* $x_n \rightharpoonup x$ *implies that*

$$\limsup_{n\to\infty} ||x_n - x|| < \limsup_{n\to\infty} ||x_n - y||$$

for all $y \in E$ *with* $y \neq x$.

Examples of Banach spaces satisfying this condition are Hilbert spaces and all l^p spaces $(1 < p < \infty)$. On the other hand, $L^p[0, 2\pi]$ with $1 < p \neq 2$ fail to satisfy Opial's condition.

Definition 2. *A multivalued mapping* $T : K \to P(E)$ *is called demiclosed at* $y \in K$ *if for any sequence* $\{x_n\}$ *in K weakly convergent to an element x and* $y_n \in Tx_n$ *strongly convergent to y, we have* $y \in Tx$.

The following is the multivalued version of condition (I) of Senter and Dotson [17].

Definition 3. *A multivalued nonexpansive mapping* $T : K \to CB(K)$ *where K a subset of E, is said to satisfy condition* (I) *if there exists a nondecreasing function* $f : [0, \infty) \to [0, \infty)$ *with* $f(0) = 0$, $f(r) > 0$ *for all* $r \in (0, \infty)$ *such that* $d(x, Tx) \geq f(d(x, F(T))$ *for all* $x \in K$.

The following very useful theorem is due to Song and Cho [13].

Lemma 2. [13]*Let* $T : K \to P(K)$ *be a multivalued mapping and* $P_T(x) = \{y \in Tx : ||x - y|| = d(x, Tx)\}$. *Then the following are equivalent.*

(1) $x \in F(T)$;

(2) $P_T(x) = \{x\}$;

(3) $x \in F(P_T)$.

Moreover, $F(T) = F(P_T)$.

Next, we state the following helpful lemma.

Lemma 3. [18]*Let E be a uniformly convex Banach space and* $0 < p \leq t_n \leq q < 1$ *for all* $n \in \mathbb{N}$. *Suppose that* $\{x_n\}$ *and* $\{y_n\}$ *are two sequences of E such that* $\limsup_{n\to\infty} ||x_n|| \leq r$, $\limsup_{n\to\infty} ||y_n|| \leq r$ *and* $\lim_{n\to\infty} ||t_n x_n + (1 - t_n)y_n|| = r$ *hold for some* $r \geq 0$. *Then* $\lim_{n\to\infty} ||x_n - y_n|| = 0$.

2. Main results

We start with the following couple of important lemmas.

Lemma 4. *Let E be a normed space and K a nonempty closed convex subset of E. Let* $T : K \to P(K)$ *be a multivalued mapping such that* $F(T) \neq \emptyset$ *and* P_T *is a nonexpansive mapping. Let* $\{x_n\}$ *be the sequence as defined in* (1.5). *Then* $\lim_{n\to\infty} ||x_n - p||$ *exists for all* $p \in F(T)$.

Proof. Let $p \in F(T)$. Then $p \in P_T(p) = \{p\}$ by Lemma 2. It follows from (1.5) that

$$\begin{aligned} ||x_{n+1} - p|| &= ||(1-\lambda)v_n + \lambda u_n - p|| \\ &\leq (1-\lambda)||v_n - p|| + \lambda||u_n - p|| \\ &\leq (1-\lambda)H(P_T(x_n), P_T(p)) + \lambda H(P_T(y_n), P_T(p)) \\ &\leq (1-\lambda)||x_n - p|| + \lambda||y_n - p||. \end{aligned} \tag{2.1}$$

But

$$\begin{aligned} \|y_n - p\| &= \|(1-\eta)x_n + \eta v_n - p\| \\ &\le (1-\eta)\|x_n - p\| + \eta\|v_n - p\| \\ &\le (1-\eta)\|x_n - p\| + \eta H(P_T(x_n), P_T(p)) \\ &\le (1-\eta)\|x_n - p\| + \eta\|x_n - p\| \\ &= \|x_n - p\|. \end{aligned} \tag{2.2}$$

Thus (2.1) becomes

$$\begin{aligned} \|x_{n+1} - p\| &\le (1-\lambda)\|x_n - p\| + \lambda\|x_n - p\| \\ &= \|x_n - p\|, \end{aligned}$$

and $\lim_{n\to\infty} \|x_n - p\|$ exists for each $p \in F(T)$. □

Lemma 5. *Let E be a uniformly convex Banach space and K be a nonempty closed convex subset of E. Let* $T : K \to P(K)$ *be a multivalued mapping such that* $F(T) \neq \varnothing$ *and* P_T *is a nonexpansive mapping. Let* $\{x_n\}$ *be the sequence as defined in* (1.5). *Then* $\lim_{n\to\infty} d(x_n, Tx_n) = 0$.

Proof. From Lemma 4, $\lim_{n\to\infty} \|x_n - p\|$ exists for each $p \in F(T)$. We suppose that $\lim_{n\to\infty} \|x_n - p\| = c$ for some $c \ge 0$.

Since $\limsup_{n\to\infty} \|v_n - p\| \le \limsup_{n\to\infty} H(P_T(x_n), P_T(p)) \le \limsup_{n\to\infty} \|x_n - p\| = c$,

so

$$\limsup_{n\to\infty} \|v_n - p\| \le c. \tag{2.3}$$

Similarly,

$$\limsup_{n\to\infty} \|u_n - p\| \le c.$$

Applying Lemma 3, we get

$$\lim_{n\to\infty} \|v_n - u_n\| = 0.$$

Taking lim sup on both sides of (2.2), we obtain

$$\limsup_{n\to\infty} \|y_n - p\| \le c. \tag{2.4}$$

Also

$$\begin{aligned} \|x_{n+1} - p\| &= \|(1-\lambda)v_n + \lambda u_n - p\| \\ &= \|(v_n - p) + \lambda(u_n - v_n)\| \\ &\le \|v_n - p\| + \|v_n - u_n\| \end{aligned}$$

implies that

$$c \le \liminf \|v_n - p\|. \tag{2.5}$$

Combining (2.3) and (2.5), we have

$$\lim_{n\to\infty} \|v_n - p\| = c.$$

Thus

$$\begin{aligned}||v_n - p|| &\le ||v_n - u_n|| + ||u_n - p|| \\ &\le ||v_n - u_n|| + H(P_T(y_n), P_T(p)) \\ &\le ||v_n - u_n|| + ||y_n - p||\end{aligned}$$

gives

$$c \le \lim\ \inf||y_n - p|| \tag{2.6}$$

and, in turn, by (2.4), we have

$$\lim_{n\to\infty} ||y_n - p|| = c.$$

Applying Lemma 3 once again,

$$\lim_{n\to\infty} ||x_n - v_n|| = 0. \tag{2.7}$$

Since $d(x_n, Tx_n) \le ||x_n - v_n||$, we have

$$\lim_{n\to\infty} d(x_n, Tx_n) = 0.$$

□

Now we approximate fixed points of the mapping T through weak convergence of the sequence $\{x_n\}$ defined in (1.5).

Theorem 1. *Let E be a uniformly convex Banach space satisfying Opial's condition and K a nonempty closed convex subset of E. Let* $T : K \to P(K)$ *be a multivalued mapping such that* $F(T) \neq \emptyset$ *and* P_T *is a nonexpansive mapping. Let* $\{x_n\}$ *be the sequence as defined in* (1.5). *Let* $I - P_T$ *be demiclosed with respect to zero, then* $\{x_n\}$ *converges weakly to a fixed point of T.*

Proof. Let $p \in F(T) = F(P_T)$. From the proof of Lemma 4, $\lim_{n\to\infty} ||x_n - p||$ exists. Now we prove that $\{x_n\}$ has a unique weak subsequential limit in $F(T)$. To prove this, let z_1 and z_2 be weak limits of the subsequences $\{x_{n_i}\}$ and $\{x_{n_j}\}$ of $\{x_n\}$, respectively. By (2.7), there exists $v_n \in Tx_n$ such that $\lim_{n\to\infty} ||x_n - v_n|| = 0$. Since $I - P_T$ is demiclosed with respect to zero, therefore we obtain $z_1 \in F(P_T) = F(T)$. In the same way, we can prove that $z_2 \in F(T)$.

Next, we prove uniqueness. For this, suppose that $z_1 \neq z_2$. Then by Opial's condition, we have

$$\begin{aligned}\lim_{n\to\infty} ||x_n - z_1|| &= \lim_{n_i\to\infty} ||x_{n_i} - z_1|| \\ &< \lim_{n_i\to\infty} ||x_{n_i} - z_2|| \\ &= \lim_{n\to\infty} ||x_n - z_2|| \\ &= \lim_{n_j\to\infty} ||x_{n_j} - z_2|| \\ &< \lim_{n_j\to\infty} ||x_{n_j} - z_1|| \\ &= \lim_{n\to\infty} ||x_n - z_1||,\end{aligned}$$

which is a contradiction. Hence $\{x_n\}$ converges weakly to a point in $F(T)$. □

We now give some strong convergence theorems. Our first strong convergence theorem is valid in general real Banach spaces. We then apply this theorem to obtain a result in uniformly convex Banach spaces. We also use the method of direct construction of Cauchy sequence as indicated by Song and Cho [13] (and opposed to [10]) but used also by many other authors including [12,14,15].

Theorem 2. *Let E be a real Banach space and K a nonempty closed convex subset of E. Let $T : K \to P(K)$ be a multivalued mapping such that $F(T) \neq \emptyset$ and P_T is a nonexpansive mapping. Let $\{x_n\}$ be the sequence as defined in* (1.5), *then $\{x_n\}$ converges strongly to a point of $F(T)$ if and only if* $\liminf_{n\to\infty} d(x_n, F(T)) = 0$.

Proof. The necessity is obvious. Conversely, suppose that $\liminf_{n\to\infty} d(x_n, F(T)) = 0$. As proved in Lemma 4, we have

$$||x_{n+1} - p|| \leq ||x_n - p||,$$

which gives

$$d(x_{n+1}, F(T)) \leq d(x_n, F(T)).$$

This implies that $\lim_{n\to\infty} d(x_n, F(T))$ exists and so by the hypothesis, $\liminf_{n\to\infty} d(x_n, F(T)) = 0$. Therefore we must have $\lim_{n\to\infty} d(x_n, F(T)) = 0$.

Next, we show that $\{x_n\}$ is a Cauchy sequence in K. Let $\varepsilon > 0$ be arbitrarily chosen. Since $\lim_{n\to\infty} d(x_n, F(T)) = 0.$, there exists a constant n_0 such that for all $n \geq n_0$, we have

$$d(x_n, F(T)) < \frac{\varepsilon}{4}.$$

In particular, $\inf\{||x_{n_0} - p|| : p \in F(T)\} < \frac{\varepsilon}{4}$. There must exist a $p^* \in F(T)$ such that

$$||x_{n_0} - p^*|| < \frac{\varepsilon}{2}.$$

Now for $m, n \geq n_0$, we have

$$\begin{aligned} ||x_{n+m} - x_n|| &\leq ||x_{n+m} - p^*|| + ||x_n - p^*|| \\ &\leq 2||x_{n_0} - p^*|| \\ &< 2\left(\frac{\varepsilon}{2}\right) = \varepsilon. \end{aligned}$$

Hence $\{x_n\}$ is a Cauchy sequence in a closed subset K of a Banach space E, and so it must converge in K. Let $\lim_{n\to\infty} x_n = q$. Now

$$\begin{aligned} d(q, P_T q) &\leq ||x_n - q|| + d(x_n, P_T x_n) + H(P_T x_n, P_T q) \\ &\leq ||x_n - q|| + ||x_n - v_n|| + ||x_n - q|| \\ &\to 0 \text{ as } n \to \infty \end{aligned}$$

which gives that $d(q, P_T q) = 0$. But P_T is a nonexpansive mapping so $F(P_T)$ is closed. Therefore, $q \in F(P_T) = F(T)$. □

We now apply the above theorem to obtain the following theorem in uniformly convex Banach spaces where $T : K \to P(K)$ satisfies condition (I).

Theorem 3. *Let E be a uniformly convex Banach space and K a nonempty closed convex subset of E. Let $T : K \to P(K)$ be a multivalued mapping satisfying condition* (I)

such that $F(T) \neq \emptyset$ *and* P_T *is a nonexpansive mapping. Let* $\{x_n\}$ *be the sequence as defined in* (1.5), *then* $\{x_n\}$ *converges strongly to a point of* $F(T)$.

Proof. By Lemma 5, $\lim_{n\to\infty} ||x_n - p||$ exists for all $p \in F(T)$. Let this limit be c for some $c \geq 0$.

If $c = 0$, there is nothing to prove.

Suppose $c > 0$. Now $||x_{n+1} - p|| \leq ||x_n - p||$ implies that

$$\inf_{p\in F(T)} ||x_{n+1} - p|| \leq \inf_{p\in F(T)} ||x_n - p||,$$

which means that $d(x_{n+1}, F(T)) \leq d(x_n, F(T))$ and so $\lim_{n\to\infty} d(x_n, F(T))$ exists. By using condition (I) and Lemma 5, we have

$$\lim_{n\to\infty} f(d(x_n, F(T))) \leq \lim_{n\to\infty} d(x_n, Tx_n) = 0.$$

That is,

$$\lim_{n\to\infty} f(d(x_n, F(T))) = 0.$$

Since f is a nondecreasing function and $f(0) = 0$, it follows that $\lim_{n\to\infty} d(x_n, F(T)) = 0.$. Now applying Theorem 2, we obtain the result. □

Author details
[1]Department of Mathematics, Statistics and Physics,Qatar University, Doha 2713, Qatar [2]Department of Mathematics, Ataturk University, Erzurum 25240, Turkey

Authors' contributions
SHK gave the idea and wrote the initial draft. IY read and agreed upon the draft. SHK then finalized the manuscript. Correspondence was mainly done by IY. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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