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Research Article

Theoretical and Numerical Analysis of 1 : 1 Main Parametric Resonance of Stayed Cable Considering Cable-Beam Coupling

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For the 1:1 main parametric resonances problems of cable-bridge coupling vibration, a main parametric resonances model considering cable-beam coupling is developed and dimensionless parametric resonances differential equations are derived. The main parametric resonances characteristics are discussed by means of multiscale approximation solution methods. Using an actual cable of cable-stayed bridge project for research object, numerical simulation analysis under a variety of conditions is illustrated. The results show that when the coupling system causes 1:1 parametric resonance, nonlinear main parametric resonances in response are unrelated to initial displacement of the cable, but with the increase of deck beam end vertical initial displacement increases, accompanied with a considerable "beat" vibration. When the vertical initial displacement of deck beam end is 10−6 m order of magnitude or even smaller, "beat" vibration phenomenon of cable and beam appears. Displacement amplitude of the cable is small and considerable amplitude vibration may not occur at this time, only making a slight stable "beat" vibration in the vicinity of the equilibrium position, which is different from 2:1 parametric resonance condition of cable-bridge coupling system. Therefore, it is necessary to limit the initial displacement excitation amplitude of beam end and prevent the occurrence of amplitude main parametric excitation resonances.

1. Introduction

The excitation of parametric resonance is dependent on time and is used as a parameter in the vibration equation of the system [1–3]. According to whether the amplitude and frequency of the excitation change with time or not, the parametric resonance of the stay cable is divided into two cases: When the quality of deck or tower is far greater than the cable, without considering the effect of cables on the deck or tower, as an ideal incentive, the incentive is not affected by the response and the system vibration is simplified as spring-mass [4] and cable-mass [5–9] model, in which the input energy of the stay cable is from the support point; when the vibration of cables and bridge deck or tower are coupled with each other, the system will be coupled with resonance in initial condition of cable-deck or cable-tower system. Finally, the large vibration of these two can be aroused, and the system vibration is simplified as the [10–12] model of the cable flexible beam.

In this paper, the displacement time history response and amplitude-frequency characteristics of the cable are mainly discussed in the case of the 1 : 1 main parametric resonance in the cable-beam coupling system, the approximate solutions of motion equations are solved by the method of multiple scales, and the results are verified by numerical simulation. However, the 2 : 1 parametric resonance and 1 : 2 and 1 : 3 superharmonic resonance response problems are not the focus of discussion here.

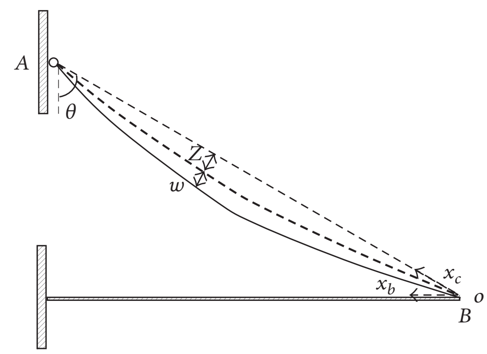

Figure 1: Parametric resonance refined model of cable-beam coupling.

2. Theoretical Model of Parametric Resonance Considering Cable-Beam Coupling

2.1. Differential Equation of Motion with Parametric Resonance Considering Cable-Beam Coupling. Several basic assumptions are made before establishing the equation of motion for the cable-beam composite structure [13]:

(1) The material nonlinearity of the cable and beam is not considered.

(2) The gravity sag curve of cable is considered as a parabola.

(3) The bending stiffness, torsional stiffness, and shear stiffness of the cable are not considered.

(4) The change of cable force along the length direction is not considered.

(5) Cable is always in elastic state during vibration.

(6) The axial deformation of beam is not considered.

On the premise of the above assumptions, the model is established in Figure 1, considering the refined model of the cable-beam coupling.

The motion trajectory of the cable and the deck beam is described by local coordinate system $o\text{-}x_b y_b$ and $o\text{-}x_c y_c$, respectively, as shown in Figure 1. According to the Hamilton principle, the differential equation of motion for the cable-beam coupling structure is [14]

$$\begin{aligned}
& m_c \frac{\partial^2 u_c(x_c,t)}{\partial t^2} + c_c \frac{\partial u_c(x_c,t)}{\partial t} \\
&\quad = \frac{\partial}{\partial s}\left\{T \frac{\partial\left[u_c(x_c,t) + X_c(x_c)\right]}{\partial s}\right\} + m_c g \cos\theta \\
& m_c \frac{\partial^2 v_c(x_c,t)}{\partial t^2} + c_c \frac{\partial v_c(x_c,t)}{\partial t} \\
&\quad = \frac{\partial}{\partial s}\left\{T \frac{\partial\left[v_c(x_c,t) + Y_c(x_c)\right]}{\partial s}\right\} + m_c g \sin\theta \\
& m_b \frac{\partial^2 v_b(x_b,t)}{\partial t^2} + c_b \frac{\partial v_b(x_b,t)}{\partial t} + E_b I_b \frac{\partial^4 v_b(x_b,t)}{\partial x_b^4} \\
&\quad = \frac{\partial}{\partial x_b}\left\{N \frac{\partial\left[v_b(x_b,t) + Y_b(x_b)\right]}{\partial x_b}\right\} - T_d \cos\theta\,\delta(x) \\
&\qquad + m_b g.
\end{aligned} \tag{1}$$

For the convenience of research, the variables with subscripts c and b are defined as the variables of the cable and the deck beam, respectively. Variables u and v indicate the longitudinal and transverse vibration displacement, respectively, and they are the function of the position coordinate x and time t. X and Y represent the longitudinal and transverse sag curves function, respectively; moreover, $Y = 4z_0\,[x/L - (x/L)^2]$. The meaning of other variables in formula (1) is as follows: m, ω, c, ξ, E, L, I, A, z_0, z, and g are the quality of unit length, natural frequency, damping coefficient, damping ratio, elastic modulus, length, moment of inertia, the cross section area, the transverse initial deflection at the midspan, the sag at the midspan, and gravitational acceleration, respectively. Units of each variable are taken from the international unit system. In addition, T is the tangential tension of the stay cable, T_d is the dynamic tension of cable in vibration process, N is axial force of deck beam, s is thr coordinate by the arc length, and $\delta(x)$ is Dirac function:

$$\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & x \neq 0. \end{cases} \tag{2}$$

By using the static equilibrium relation of stay cable, formula (1) can be simplified as follows:

$$\begin{aligned}
& m_c \frac{\partial^2 u_c}{\partial t^2} + c_c \frac{\partial u_c}{\partial t} \\
&\quad - \frac{\partial}{\partial s}\left\{T_0 \frac{\partial u_c}{\partial s} + E_c A_c \varepsilon_c \left(\frac{dX_c}{dx_c} + \frac{\partial v_c}{\partial s}\right)\right\} = 0 \\
& m_c \frac{\partial^2 v_c}{\partial t^2} + c_c \frac{\partial v_c}{\partial t} \\
&\quad - \frac{\partial}{\partial s}\left\{T_0 \frac{\partial v_c}{\partial s} + E_c A_c \varepsilon_c \left(\frac{dY_c}{dx_c} + \frac{\partial v_c}{\partial s}\right)\right\} = 0 \\
& m_b \frac{\partial^2 v_b}{\partial t^2} + c_b \frac{\partial v_b}{\partial t} + E_b I_b \frac{\partial^4 v_b}{\partial x_b^4} \\
&\quad - \frac{\partial}{\partial x_b}\left\{N \frac{\partial v_b}{\partial x_b} + E_b A_b \varepsilon_b \left(\frac{dY_b}{dx_c} + \frac{\partial v_b}{\partial x_b}\right)\right\} \\
&\quad + E_c A_c \varepsilon_c \cos\theta\,\delta(x) = 0,
\end{aligned} \tag{3}$$

where T_0 is the static tangential tension of stay cable in formula (3); moreover, H_0 is static axial tension and ε_c and ε_b are the axial strains of stay cable and beam, respectively. The axial strains can be expressed as follows:

$$\begin{aligned}
\varepsilon_c &= \frac{\partial u_c}{\partial s} + \frac{dY_c}{dx_c}\frac{\partial v_c}{\partial s} + \frac{1}{2}\left(\frac{\partial u_c}{\partial s}\right)^2 + \frac{1}{2}\left(\frac{\partial v_c}{\partial s}\right)^2 \\
\varepsilon_b &= \frac{\partial u_b}{\partial s} + \frac{dY_b}{dx_b}\frac{\partial v_b}{\partial s} + \frac{1}{2}\left(\frac{\partial u_b}{\partial s}\right)^2 + \frac{1}{2}\left(\frac{\partial v_b}{\partial s}\right)^2,
\end{aligned} \tag{4}$$

where u_b represents the longitudinal displacement of beam in formula (4); the two-order small quantity of longitudinal elastic strain for the cable and beam can be neglected, because the longitudinal deformation of the cable and beam is much less than that of transverse deformation. It can be approximate to take $ds \approx dx$ for the stay cable with small sag, and formula (3) can be simplified as

$$\begin{aligned} & m_c \frac{\partial^2 u_c}{\partial t^2} + c_c \frac{\partial u_c}{\partial t} \\ & \quad - \frac{\partial}{\partial x_c} \left\{ E_c A_c \left[\frac{\partial u_c}{\partial x_c} + \frac{dY_c}{dx_c} \frac{\partial v_c}{\partial x_c} + \frac{1}{2} \left(\frac{\partial v_c}{\partial x_c} \right)^2 \right] \frac{\partial v_c}{\partial x_c} \right\} \\ & \quad = 0 \\ & m_c \frac{\partial^2 v_c}{\partial t^2} + c_c \frac{\partial v_c}{\partial t} - \frac{\partial}{\partial x_c} \left\{ H_0 \frac{\partial v_c}{\partial x_c} + E_c A_c \left(\frac{dY_c}{dx_c} + \frac{\partial v_c}{\partial x_c} \right) \right. \\ & \quad \left. \cdot \left[\frac{\partial u_c}{\partial x_c} + \frac{dY_c}{dx_c} \frac{\partial v_c}{\partial x_c} + \frac{1}{2} \left(\frac{\partial v_c}{\partial x_c} \right)^2 \right] \right\} = 0 \\ & m_b \frac{\partial^2 v_b}{\partial t^2} + c_b \frac{\partial v_b}{\partial t} + E_b I_b \frac{\partial^4 v_b}{\partial x_b^4} - N \frac{\partial^2 v_b}{\partial x_b^2} + E_c A_c \left[\frac{\partial u_c}{\partial x_c} \right. \\ & \quad \left. + \frac{dY_c}{dx_c} \frac{\partial v_c}{\partial x_c} + \frac{1}{2} \left(\frac{\partial v_c}{\partial x_c} \right)^2 \right] \cos \theta \delta(x) = 0. \end{aligned} \tag{5}$$

In the case of low modal vibration for the cable, the coupling effect of transverse and longitudinal vibration for the cable is not considered. Meanwhile, the first equation of formula (5) can be simplified as

$$e(t) = \frac{\partial u_c}{\partial x_c} + \frac{dY_c}{dx_c} \frac{\partial v_c}{\partial x_c} + \frac{1}{2} \left(\frac{\partial v_c}{\partial x_c} \right)^2. \tag{6}$$

The second equation of formula (5) can be simplified as

$$\begin{aligned} & m_c \frac{\partial^2 v_c}{\partial t^2} + c_c \frac{\partial v_c}{\partial t} \\ & \quad - \frac{\partial}{\partial x_c} \left\{ H_0 \frac{\partial v_c}{\partial x_c} + E_c A_c \left(\frac{dY_c}{dx_c} + \frac{\partial v_c}{\partial x_c} \right) e(t) \right\} = 0. \end{aligned} \tag{7}$$

From the simplified assumptions, the geometric boundary condition of the stay cable is as follows:

$$\begin{aligned} & u_c(L_c) = 0; \\ & u_c(0) - u_b(0) \cos \theta = 0. \end{aligned} \tag{8}$$

On both sides of formula (6) is integral on $[0, L_c]$ range for x_c at the same time, and it is brought into formula (8), which can be obtained as follows:

$$\begin{aligned} e(t) = {} & \frac{v_b(0,t)}{L_c} \cos \theta \\ & + \frac{1}{L_c} \int_0^{L_c} \left[\frac{dY_c}{dx_c} \frac{\partial v_c}{\partial x_c} + \frac{1}{2} \left(\frac{\partial v_c}{\partial x_c} \right)^2 \right] dx_c. \end{aligned} \tag{9}$$

The axial dynamic force increment of the cable in the process of transverse vibration can be expressed as follows:

$$\begin{aligned} H_d = {} & \frac{E_c A_c}{L_{ce}} \left\{ v_b(0,t) \cos \theta \right. \\ & \left. + \int_0^{L_c} \left[\frac{dY_c}{dx_c} \frac{\partial v_c}{\partial x_c} + \frac{1}{2} \left(\frac{\partial v_c}{\partial x_c} \right)^2 \right] dx_c \right\} \end{aligned} \tag{10}$$

$$H_\omega = \frac{E_c A_c}{L_{ce}} v_b(0,t) \cos \theta \tag{10a}$$

$$H_p = \frac{E_c A_c}{L_{ce}} \int_0^{L_c} \left[\frac{dY_c}{dx_c} \frac{\partial v_c}{\partial x_c} + \frac{1}{2} \left(\frac{\partial v_c}{\partial x_c} \right)^2 \right] dx_c. \tag{10b}$$

The axial force of bridge deck beam in vibration is as follows:

$$N = (H + H_d) \sin \theta \tag{11}$$

Equations (9) and (10) are simultaneous. The third equation of formula (5) is simplified; differential equations of motion for the cable-beam coupled vibration are as follows:

$$\begin{aligned} & m_c \ddot{v}_c + c_c \dot{v}_c - (H_0 + H_d) \frac{\partial^2 v_c}{\partial x_c^2} + H_d \frac{8 z_0}{L_c^{\,2}} = 0 \\ & m_b \ddot{v}_b + c_b \dot{v}_b + E_b I_b \frac{\partial^4 v_b}{\partial x_b^4} - (H_0 + H_d) \sin \theta \frac{\partial^2 v_b}{\partial x_b^2} \\ & \quad + H_d \cos \theta \delta(x) = 0. \end{aligned} \tag{12}$$

2.2. Dimensionless Method of Parametric Resonance Equations Considering Cable-Beam Coupling. The differential equation (12) of motion for cable-beam coupled vibrations is derived by using dimensionless method, and the following dimensionless quantities are defined [15–17]:

$$\begin{aligned} \bar{t} &= \omega_c t; \\ \bar{v}_c &= \frac{v_c}{L_c}; \\ \bar{v}_b &= \frac{v_b}{L_b}; \\ \bar{\omega}_c &= \frac{\omega_{ce}}{\omega_c}; \\ \bar{\omega}_b &= \frac{\omega_b}{\omega_c}, \end{aligned} \tag{13}$$

where ω_c and ω_{ce} are the first natural frequencies of the cable without considering or considering the sag; ω_b is the first natural frequency of bridge deck beam; γ is the sag-span ratio of cable; k^2 is the stiffness coefficient of cable; L_c and L_b are the span of cable and beam, respectively; λ^2 is Irvine parameter; β_b indicates the ratio coefficient of first-order natural frequency for bridge deck beam; $\bar{t}$ denotes the dimensionless time; $\bar{v}_c$ and $\bar{v}_b$ are the transverse relative displacements of the cable and beam, respectively. $\bar{\omega}_c$

and $\overline{\omega}_b$ are the relative frequencies of the cable and beam, respectively.

The expressions of each parameter are

$$\begin{aligned}
\omega_c &= \frac{\pi}{L_c}\sqrt{\frac{H_0}{m_c}};\\
\omega_{ce} &= \omega_c\left(1+\frac{(2/\pi)^4\lambda^2}{2}\right)^{1/2};\\
\omega_b &= \beta_b^2\sqrt{\frac{E_bI_b}{m_bL_b^4}};\\
\xi_c &= \frac{c_c}{2m_c\omega_c};\\
\xi_b &= \frac{c_b}{2m_b\omega_b};\\
\gamma &= \frac{z_0}{L_c};\\
k^2 &= \frac{E_cA_c}{H_0};\\
\lambda^2 &= \frac{64k^2\gamma^2}{1+8\gamma^2};\\
\beta_b^4 &= \omega_b^2\frac{m_bL_b^4}{E_bI_b}.
\end{aligned} \quad (14)$$

Equation (12) is derived by using the dimensionless quantity of the above definition.

$$\begin{aligned}
&\frac{\partial^2\overline{v}_c}{\partial\overline{t}^2}+2\xi_c\overline{\omega}_c\frac{\partial\overline{v}_c}{\partial\overline{t}}-\frac{1}{\pi^2}\left(1+\overline{H}_\omega+\overline{H}_p\right)\frac{\partial^2\overline{v}_c}{\partial\overline{x}_c^{\,2}}\\
&\quad+\left(\overline{H}_\omega+\overline{H}_p\right)\frac{8\gamma}{\pi^2}=0\\
&\frac{\partial^2\overline{v}_b}{\partial\overline{t}^2}+2\xi_b\overline{\omega}_b\frac{\partial\overline{v}_c}{\partial\overline{t}}+\frac{1}{\beta_b^4}\overline{\omega}_b^2\frac{\partial^4\overline{v}_b}{\partial\overline{x}_b^{\,4}}\\
&\quad-\frac{1}{\pi^2}\left(1+\overline{H}_\omega+\overline{H}_p\right)\left(\frac{m_c}{m_b}\right)\left(\frac{L_c}{L_b}\right)^2\sin\theta\frac{\partial^2\overline{v}_b}{\partial\overline{x}_b^{\,2}}\\
&\quad+\frac{1}{\pi^2}\left(\overline{H}_\omega+\overline{H}_p\right)\left(\frac{m_c}{m_b}\right)\left(\frac{L_c^2}{L_b}\right)\cos\theta\delta(x)=0
\end{aligned} \quad (15)$$

$$\overline{H}_\omega=\frac{k^2}{1+8\gamma^2}\overline{v}_b(0,\overline{t})\cos\theta \quad (15a)$$

$$\overline{H}_p=\frac{\lambda^2}{64}\left\{8\int_0^1\frac{1}{\gamma}\overline{v}_c d\overline{x}_c+\frac{1}{2}\int_0^1\left(\frac{1}{\gamma}\frac{\partial\overline{v}_c}{\partial\overline{x}_c}\right)^2 d\overline{x}_c\right\}. \quad (15b)$$

When considering that the first-order mode is only the main vibration, and constraint on the boundary condition of formula (8) is satisfied, the dimensionless displacement function of the cable is as follows:

$$\overline{v}_c(\overline{x}_c,\overline{t})=\overline{v}_b(0,\overline{t})\sin\theta(1-\overline{x}_c)+\sin(\pi\overline{x}_c)q_c(\overline{t}). \quad (16)$$

Moreover, the geometric boundary condition of the fixed end for the bridge deck beam is as follows:

$$\begin{aligned}
v_b(L_b)&=0;\\
\dot{v}_b(L_b)&=0;\\
\ddot{v}_b(0)&=0.
\end{aligned} \quad (17)$$

The physical boundary condition of the connection between cable and bridge deck beam is as follows:

$$E_bI_bv_b'''(0)=-k_cv_b(0), \quad (18)$$

while the dimensionless modal function of in-plane vibration for the deck beam can be expressed as follows:

$$\begin{aligned}
\overline{\psi}_b(\overline{x}_b)&=A_1\sin(\beta_b\overline{x}_b)+A_2\cos(\beta_b\overline{x}_b)\\
&\quad+A_3\sinh(\beta_b\overline{x}_b)+A_4\cosh(\beta_b\overline{x}_b),
\end{aligned} \quad (19)$$

where A_1, A_2, A_3, and A_4 are constant coefficients; they are determined by formulas (17) and (18). Moreover, β_b is determined by the following characteristic equation:

$$\frac{k_c}{E_bI_b}=\beta_b^3\frac{1+\cos\beta_bL_b\cosh\beta_bL_b}{\cos\beta_bL_b\sinh\beta_bL_b-\sin\beta_bL_b\cosh\beta_bL_b}. \quad (20)$$

And the dimensionless displacement function of the deck beam can be expressed as follows:

$$\overline{v}_b(\overline{x}_b,\overline{t})=\overline{\psi}_b(\overline{x}_b)\overline{q}_b(\overline{t}). \quad (21)$$

Formulas (16) and (21) are brought into formula (12) by the Galerkin method, on both sides multiplied by $\sin(\pi\overline{x}_c)$ and $\overline{\psi}_b$ at the same time, and integral on $[0,1]$; it can be obtained by using the mode orthogonality principle:

$$\begin{aligned}
&\ddot{q}_c+2\xi_c\overline{\omega}_c\dot{q}_c+\overline{\omega}_c^2q_c+\alpha_{11}q_cq_b+\alpha_{12}q_cq_b^2+\alpha_2q_c^2\\
&\quad+\alpha_3q_c^3+\alpha_4q_b+\alpha_5q_b^2+\gamma_1\dot{q}_b+\gamma_2\ddot{q}_b=0\\
&\ddot{q}_b+2\xi_b\overline{\omega}_b\dot{q}_b+\overline{\omega}_b^2q_b+\beta_1q_b+\beta_{11}q_cq_b+\beta_{12}q_c^2q_b\\
&\quad+\beta_2q_b^2+\beta_3q_b^3+\beta_4q_c+\beta_5q_c^2=0,
\end{aligned} \quad (22)$$

where $\alpha_{11}, \alpha_{12}, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \gamma_1, \gamma_2, \zeta_1, \zeta_2, \mu_1, \mu_2, \beta_1, \beta_{11}, \beta_{12}, \beta_2, \beta_3, \beta_4$, and β_5 are parameters.

3. Discussion on the Equations with Dimensionless Parametric Resonance

3.1. Approximate Solution of the Method of Multiple Scales. The approximate solution of formula (22) is solved by the method of multiple scales, setting the form of solution as follows:

$$\begin{aligned}
q_c&=\varepsilon q_{c1}(T_0,T_1,T_2)+\varepsilon^2q_{c2}(T_0,T_1,T_2)\\
&\quad+\varepsilon^3q_{c3}(T_0,T_1,T_2)\\
q_b&=\varepsilon q_{b1}(T_0,T_1,T_2)+\varepsilon^2q_{b2}(T_0,T_1,T_2)\\
&\quad+\varepsilon^3q_{b3}(T_0,T_1,T_2).
\end{aligned} \quad (23)$$

Before the multiscale method is adopted to solve the equation, the parameters are as follows:

$$\begin{aligned} \xi_c &= \varepsilon^2 \mu_c; \\ \xi_b &= \varepsilon^2 \mu_b; \\ \alpha_4 &= \varepsilon \eta_1; \\ \gamma_1 &= \varepsilon^2 \eta_2; \\ \gamma_2 &= \varepsilon^2 \eta_3; \\ \beta_1 &= \varepsilon \eta_4; \\ \beta_4 &= \varepsilon \eta_5. \end{aligned} \tag{24}$$

Formula (23) is brought into formula (22), and the same power coefficient of ε is sorted out, which can be obtained as follows:
ε order is

$$\begin{aligned} D_0^2 q_{c1} + \overline{\omega}_c^2 q_{c1} &= 0 \\ D_0^2 q_{b1} + \overline{\omega}_b^2 q_{b1} &= 0. \end{aligned} \tag{25}$$

ε^2 order is

$$\begin{aligned} D_0^2 q_{c2} + \overline{\omega}_c^2 q_{c2} &= -2D_0 D_1 q_{c1} - \alpha_{11} q_{c1} q_{b1} - \alpha_2 q_{c1}^2 \\ &\quad - \eta_1 q_{b1} - \alpha_5 q_{b1}^2 \\ D_0^2 q_{b2} + \overline{\omega}_b^2 q_{b2} &= -2D_0 D_1 q_{b1} - \eta_4 q_{b1} - \beta_{11} q_{c1} q_{b1} \\ &\quad - \beta_2 q_{b1}^2 - \eta_5 q_{c1} - \beta_5 q_{c1}^2. \end{aligned} \tag{26}$$

ε^3 order is

$$\begin{aligned} D_0^2 q_{c3} + \overline{\omega}_c^2 q_{c3} &= -2D_0 D_1 q_{c2} - \left(D_1^2 + 2D_0 D_2\right) q_{c1} \\ &\quad - 2\mu_c \overline{\omega}_c D_0 q_{c1} \\ &\quad - \alpha_{11} \left(q_{c1} q_{b2} + q_{c2} q_{b1}\right) \\ &\quad - \alpha_{12} q_{c1} q_{b1}^2 - 2\alpha_2 q_{c1} q_{c2} - \alpha_3 q_{c1}^3 \\ &\quad - \eta_1 q_{b2} - 2\alpha_5 q_{b1} q_{b2} - \eta_2 D_0 q_{b1} \\ &\quad - \eta_3 D_0^2 q_{b1} \\ D_0^2 q_{b3} + \overline{\omega}_b^2 q_{b3} &= -2D_0 D_1 q_{b2} - \left(D_1^2 + 2D_0 D_2\right) q_{b1} \\ &\quad - 2\mu_b \overline{\omega}_b D_0 q_{b1} \\ &\quad - \beta_{11} \left(q_{c1} q_{b2} + q_{c2} q_{b1}\right) - \eta_4 q_{b2} \\ &\quad - \beta_{12} q_{c1}^2 q_{b1} - 2\beta_2 q_{b1} q_{b2} - \beta_3 q_{b1}^3 \\ &\quad - \eta_5 q_{c2} - 2\beta_5 q_{c1} q_{c2}. \end{aligned} \tag{27}$$

From formula (27), the first solution of the equation can be expressed as

$$\begin{aligned} q_{c1} &= Q_1 \left(T_1, T_2\right) e^{i\overline{\omega}_c T_0} + cc \\ q_{b1} &= B_1 \left(T_1, T_2\right) e^{i\overline{\omega}_b T_0} + cc, \end{aligned} \tag{28}$$

where cc represents the front of the conjugate.

Formula (28) is brought into the second equations of formula (27) and can be obtained as follows:

$$\begin{aligned} D_0^2 q_{c2} + \overline{\omega}_c^2 q_{c2} &= -i2\overline{\omega}_c \frac{\partial Q_1}{\partial T_1} e^{i\overline{\omega}_c T_0} - \alpha_2 Q_1^2 e^{i2\overline{\omega}_c T_0} \\ &\quad - \eta_1 B_1 e^{i\overline{\omega}_b T_0} - \alpha_5 B_1^2 e^{i2\overline{\omega}_b T_0} \\ &\quad - \alpha_{11} Q_1 B_1 e^{i(\overline{\omega}_c + \overline{\omega}_b) T_0} \\ &\quad - \alpha_{11} \overline{Q}_1 B_1 e^{i(\overline{\omega}_b - \overline{\omega}_c) T_0} \\ &\quad - \left(\alpha_2 Q_1 \overline{Q}_1 + \alpha_5 B_1 \overline{B}_1\right) + cc \\ D_0^2 q_{b2} + \overline{\omega}_b^2 q_{b2} &= -i2\overline{\omega}_b \frac{\partial B_1}{\partial T_1} e^{i\overline{\omega}_b T_0} - \eta_4 B_1 e^{i\overline{\omega}_b T_0} \\ &\quad - \beta_2 B_1^2 e^{i2\overline{\omega}_b T_0} - \eta_5 Q_1 e^{i\overline{\omega}_c T_0} \\ &\quad - \beta_5 Q_1^2 e^{i2\overline{\omega}_c T_0} \\ &\quad - \beta_{11} Q_1 B_1 e^{i(\overline{\omega}_c + \overline{\omega}_b) T_0} \\ &\quad - \beta_{11} \overline{Q}_1 B_1 e^{i(\overline{\omega}_b - \overline{\omega}_c) T_0} \\ &\quad - \left(\beta_5 Q_1 \overline{Q}_1 + \beta_2 B_1 \overline{B}_1\right) + cc. \end{aligned} \tag{29}$$

It can be obtained by eliminating the periodic term of formula (29):

$$\begin{aligned} \frac{\partial Q_1}{\partial T_1} &= 0 \\ i2\overline{\omega}_b \frac{\partial B_1}{\partial T_1} + \eta_4 B_1 &= 0. \end{aligned} \tag{30}$$

By using the method that the equation contains a nonperiodic term of multiple scales, when the internal resonance is not considered, the solution of formula (29) is as follows:

$$\begin{aligned} q_{c2} = {} & \frac{\alpha_2 Q_1^2}{3\overline{\omega}_c^2} e^{i2\overline{\omega}_c T_0} + \frac{\eta_1 B_1}{\left(\overline{\omega}_b^2 - \overline{\omega}_c^2\right)} e^{i\overline{\omega}_b T_0} \\ & + \frac{\alpha_5 B_1^2}{\left(4\overline{\omega}_b^2 - \overline{\omega}_c^2\right)} e^{i2\overline{\omega}_b T_0} \\ & + \frac{\alpha_{11} Q_1 B_1}{\left(\overline{\omega}_b^2 + 2\overline{\omega}_c \overline{\omega}_b\right)} e^{i(\overline{\omega}_c + \overline{\omega}_b) T_0} \\ & + \frac{\alpha_{11} \overline{Q}_1 B_1}{\left(\overline{\omega}_b^2 - 2\overline{\omega}_c \overline{\omega}_b\right)} e^{i(\overline{\omega}_b - \overline{\omega}_c) T_0} \end{aligned}$$

Table 1: Parameter of cables and bridge deck beam.

Cable length (m)	Deck beam length (m)	Cable quality (kg·m^{-1})	Bridge deck beam quality (kg·m^{-1})
178	30	120.4	1.6×10^6
Elastic modulus of cable (GPa)	Elastic modulus of bridge deck (GPa)	Cable section area (cm^2)	Angle of cable (°)
190	35	141.24	62.1
Natural frequency of cable ω_c (Hz)	Cable damping ratio ξ_c	Damping ratio of bridge deck ξ_b	Time step $\Delta\bar{t}$
0.768	0.001	0.01	0.01

$$- \frac{\alpha_2 Q_1 \overline{Q}_1 + \alpha_5 B_1 \overline{B}_1}{\overline{\omega}_c^2} + cc$$

$$q_{b2} = \frac{\beta_2 B_1^2}{3\overline{\omega}_b^2} e^{i2\overline{\omega}_b T_0} - \frac{\eta_5 Q_1}{\left(\overline{\omega}_b^2 - \overline{\omega}_c^2\right)} e^{i\overline{\omega}_c T_0} - \frac{\beta_5 Q_1^2}{\left(\overline{\omega}_b^2 - 4\overline{\omega}_c^2\right)} e^{i2\overline{\omega}_c T_0} + \frac{\beta_{11} Q_1 B_1}{\left(\overline{\omega}_c^2 + 2\overline{\omega}_c \overline{\omega}_b\right)} e^{i(\overline{\omega}_c + \overline{\omega}_b)T_0} + \frac{\beta_{11} \overline{Q}_1 B_1}{\left(\overline{\omega}_c^2 - 2\overline{\omega}_c \overline{\omega}_b\right)} e^{i(\overline{\omega}_b - \overline{\omega}_c)T_0} - \frac{\left(\beta_5 Q_1 \overline{Q}_1 + \beta_2 B_1 \overline{B}_1\right)}{\overline{\omega}_b^2} + cc. \tag{31}$$

3.2. Discussion on Resonance Characteristics of Parametric Excitation. From formula (31), $(\eta_1 B_1/(\overline{\omega}_b^2 - \overline{\omega}_c^2))e^{i\overline{\omega}_b T_0}$ items contained in the expression of the displacement modal component q_{c2} of the cable will make the first equation show the nonsingular solution when the frequency ratio between the bridge and the cable is satisfied with $\overline{\omega}_b : \overline{\omega}_c = 1:1$. It can be seen from the second equations that $-(\eta_5 Q_1/(\overline{\omega}_b^2 - \overline{\omega}_c^2))e^{i\overline{\omega}_c T_0}$ items contained in the expression of the displacement modal components q_{b2} of the bridge deck beam will make the second equation show the nonsingular solution, and this moment the cable-beam coupling system will present the 1 : 1 principal parametric resonance phenomenon with displacement modal component q_{c2} and q_{b2} as the main form.

Similarly, when the frequency ratio between the bridge deck beam and the cable is satisfied with the $\overline{\omega}_b : \overline{\omega}_c = 2:1$ and $\overline{\omega}_b : \overline{\omega}_c = 1:2$, $(\alpha_{11} \overline{Q}_1 B_1/(\overline{\omega}_b^2 - 2\overline{\omega}_c \overline{\omega}_b))e^{i(\overline{\omega}_b - \overline{\omega}_c)T_0}$ or $(\alpha_5 B_1^2/(4\overline{\omega}_b^2 - \overline{\omega}_c^2))e^{i2\overline{\omega}_b T_0}$ items of the first equation and $-(\beta_5 Q_1^2/(\overline{\omega}_b^2 - 4\overline{\omega}_c^2))e^{i2\overline{\omega}_c T_0}$ or $(\beta_{11} \overline{Q}_1 B_1/(\overline{\omega}_c^2 - 2\overline{\omega}_c \overline{\omega}_b))e^{i(\overline{\omega}_b - \overline{\omega}_c)T_0}$ items of the second equation of formula (31) led to the equations showing nonsingular solution.

At this moment, the cable-beam coupling system will present the large vibration phenomenon of the 2 : 1 principal parametrical resonance and 1 : 2 superharmonic resonance with displacement modal component q_{c2} and q_{b2} as the main form.

4. Analysis on Numerical Example

4.1. Basic Parameters. The cable in practical engineering of a cable-stayed bridge can be taken as the study object, in order to further verify the characteristics of 1 : 1 main parametrical resonance for cable-beam coupled system. The geometric parameters and material properties of the stay cable are shown in Table 1.

4.2. The 1 : 1 Principal Parametrical Resonance Response Characteristics of Cable-Beam Coupled System

4.2.1. When the Working Conditions $q_c(0) = 5.618 \times 10^{-6}$ and $q_b(0) = 3.33 \times 10^{-4}$. Under this working condition, the actual initial displacements of cable midspan and deck beam end are about 0.001 m and 0.01 m. The dimensionless displacement response time history and amplitude-frequency characteristic curve of coupled system are shown in Figures 2 and 3.

From cable's dimensionless displacement response time history and spectrum curve in Figure 2, stay cable presents large beat vibration near the equilibrium position at its natural frequency ω_c. Due to the small damping of the cable and the slow decay of parametrical resonance displacement response, the sharp vibration has been continued. From the dimensionless displacement response time history and amplitude-frequency characteristic curve of the deck beam end in Figure 3, beam presents the obvious beat vibration near the equilibrium position at initial displacement, the coupling effect is obvious, and the amplitude-frequency characteristic curve of the deck beam end is in single peak value, which says that it does harmonic vibration at its natural frequency.

We could find that, from the dimensionless displacement response amplitude of cable-beam coupled system, due to deck beam's large quality and stiffness, its energy during the vibration is larger than the energy during cable's parametrical resonance process when its excitation amplitude of vertical initial displacement is small; at this moment, the constraints of the cable on the deck beam movement are very small, while the effect of the deck beam on the cable movement is large, so cable presents large beat vibration, and deck beam presents small beat vibration at given initial conditions.

4.2.2. When the Working Conditions $q_c(0) = 5.618 \times 10^{-6}$ and $q_b(0) = 1.66 \times 10^{-3}$. Under this working condition, the actual initial displacements of cable midspan and deck beam end are about 0.001 m and 0.05 m. The dimensionless

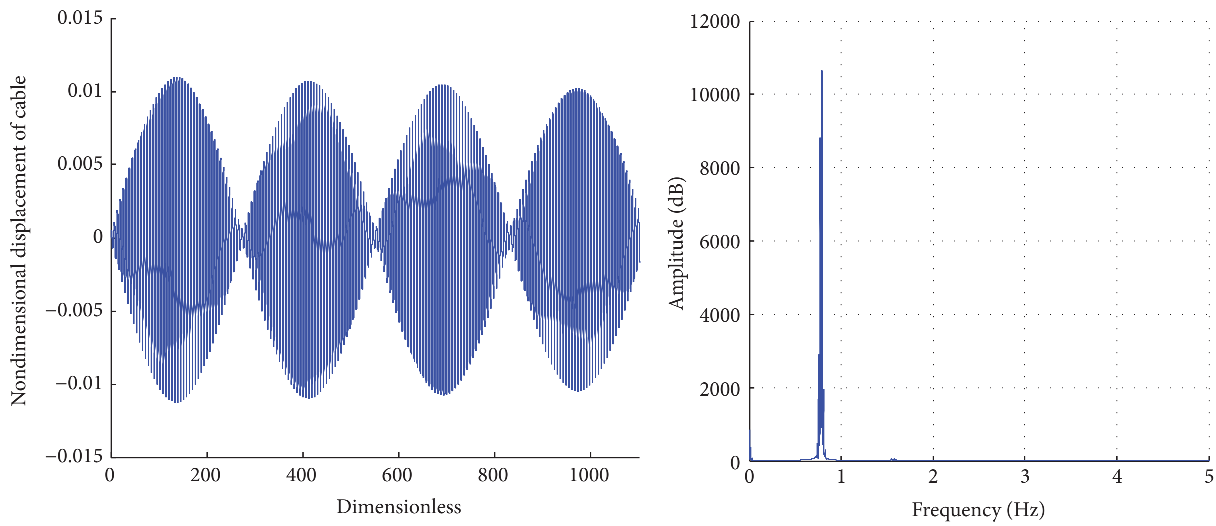

FIGURE 2: Dimensionless displacement response time history and amplitude-frequency characteristic curve of the cable midspan.

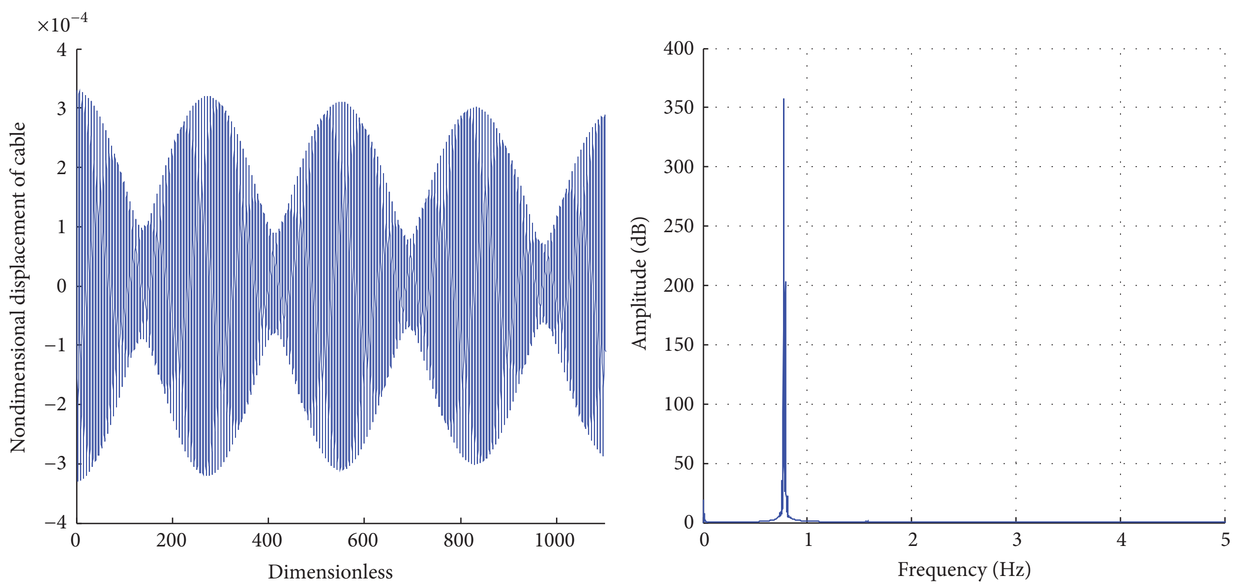

FIGURE 3: Dimensionless displacement response time history and amplitude-frequency characteristic curve of the deck beam end.

displacement response time history and amplitude-frequency characteristic curve of coupling system are shown in Figures 4 and 5.

From Figures 5 and 6, compared with working condition $q_b(0) = 3.3 \times 10^{-4}$, parametrical resonance's coupling effect of cable-beam composite structure is much more obvious under this working condition. Both cable and deck beam present obvious beat vibration phenomenon, but cable's beat vibration has greater displacement response; the maximal displacement amplitude is increased from 1.18% of the cable length to 2.42%, which says that cable midspan's parametrical resonance displacement amplitude is also increased as the excitation amplitude of vertical initial displacement of deck beam end increased; this is fully consistent with the principle of conservation of energy.

4.2.3. When the Working Conditions $q_c(0) = 5.618 \times 10^{-6}$ and $q_b(0) = 3.3 \times 10^{-8}$. Under this working condition, the actual initial displacements of cable midspan and deck beam end are about 0.001 m and 10^{-6} m. The dimensionless displacement response time history and amplitude-frequency characteristic curve of coupling system are shown in Figures 6 and 7.

From Figures 6 and 7, under the condition of beam's small initial displacement, both cable and beam present obvious beat vibration phenomenon; the coupling effect is obvious, but at this point the cable's displacement amplitude is small and there is no significant vibration, which is only a slight stable "beat" vibration in the vicinity of the equilibrium position. Analysis of the reason is as follows: from the multiscale differential equations (29) of stay cable

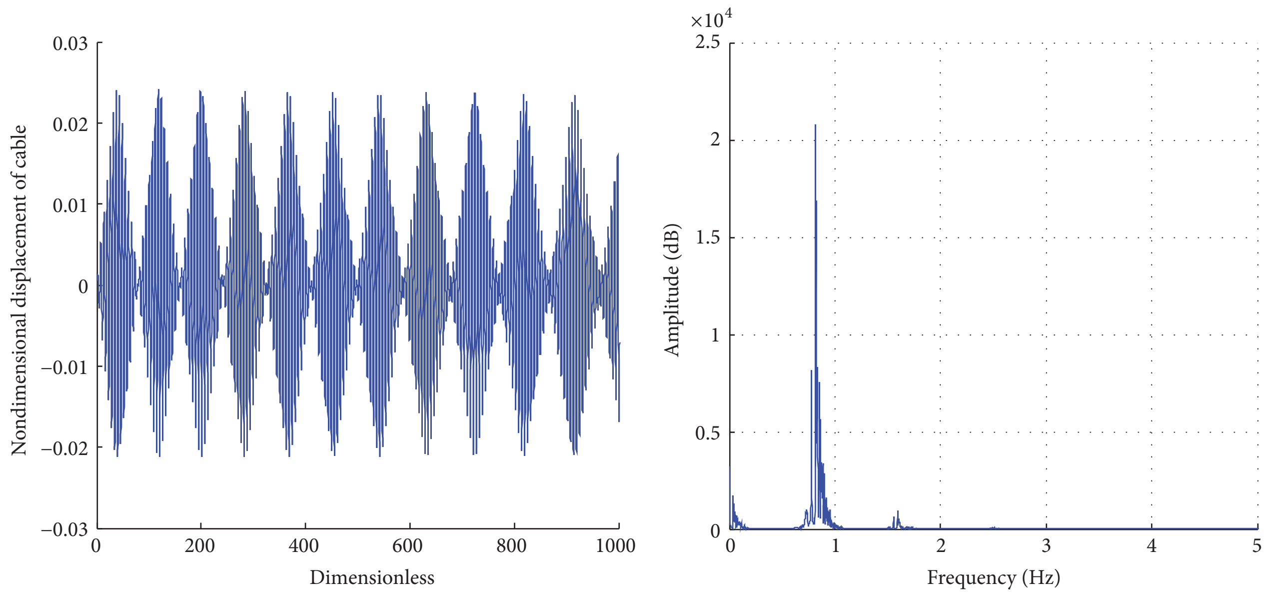

FIGURE 4: Dimensionless displacement response time history and amplitude-frequency characteristic curve of the cable midspan.

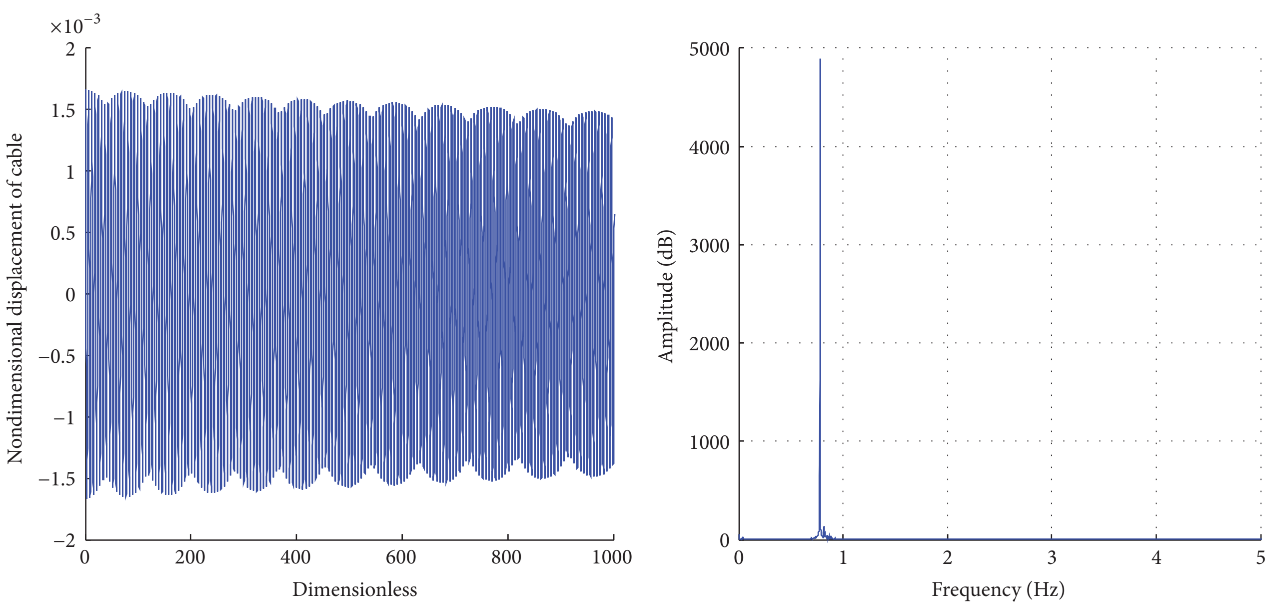

FIGURE 5: Dimensionless displacement response time history and amplitude-frequency characteristic curve of the deck beam end.

and beam's displacement components q_{c2} and q_{b2}, resonance terms of both parametrical resonance and primary resonance $-\alpha_{11}\overline{Q}_1 B_1 e^{i(\overline{\omega}_b-\overline{\omega}_c)T_0}$ and $-\eta_1 B_1 e^{i\overline{\omega}_b T_0}$ are related to the coefficient α_{11} affected by deck beam's excitation amplitude. By derivation, we conclude that the coefficient $32\gamma/\pi^3$ which is the difference between coefficients η_1 and α_{11} is much affected by sag. So the main parametrical resonance will be not significant vibration under beam's small initial condition.

5. Conclusions

(1) Based on the establishment of the model of the cable-beam coupling dimensionless parametrical resonance, the main parametrical resonance characteristics of cable-beam coupled system are discussed emphatically in terms of the frequency ratio relation of the deck beam and cable meets $\overline{\omega}_b : \overline{\omega}_c = 1:1$. Then, the possibility of the occurrence of large main parametrical resonance of cable is proved theoretically.

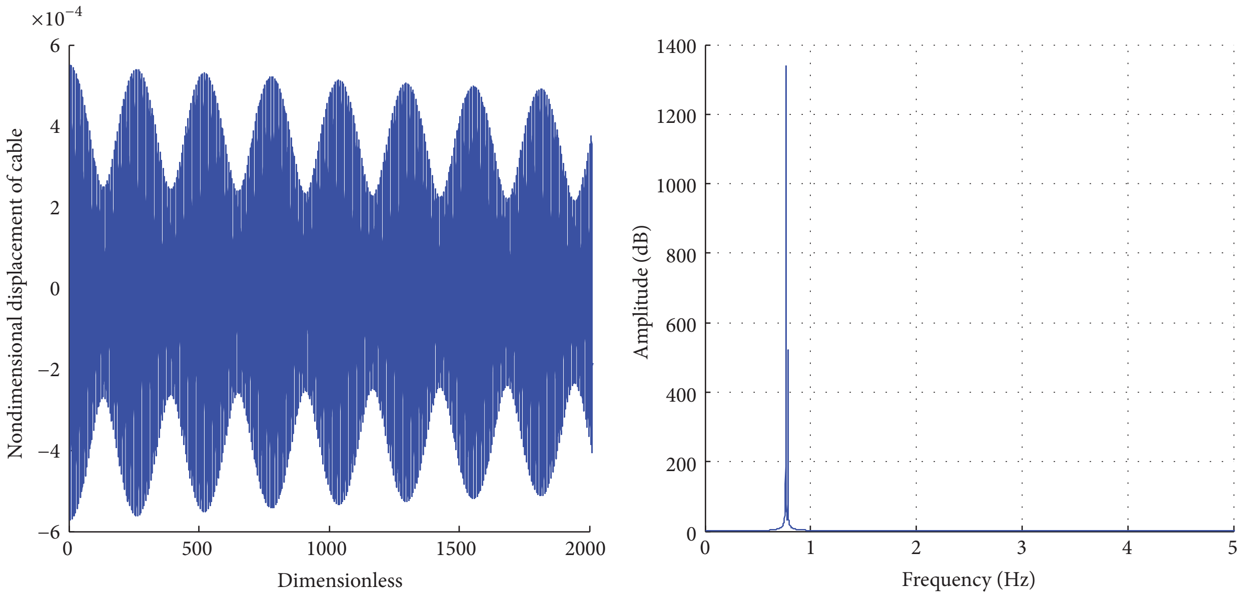

Figure 6: Dimensionless displacement response time history and amplitude-frequency characteristic curve of the cable midspan.

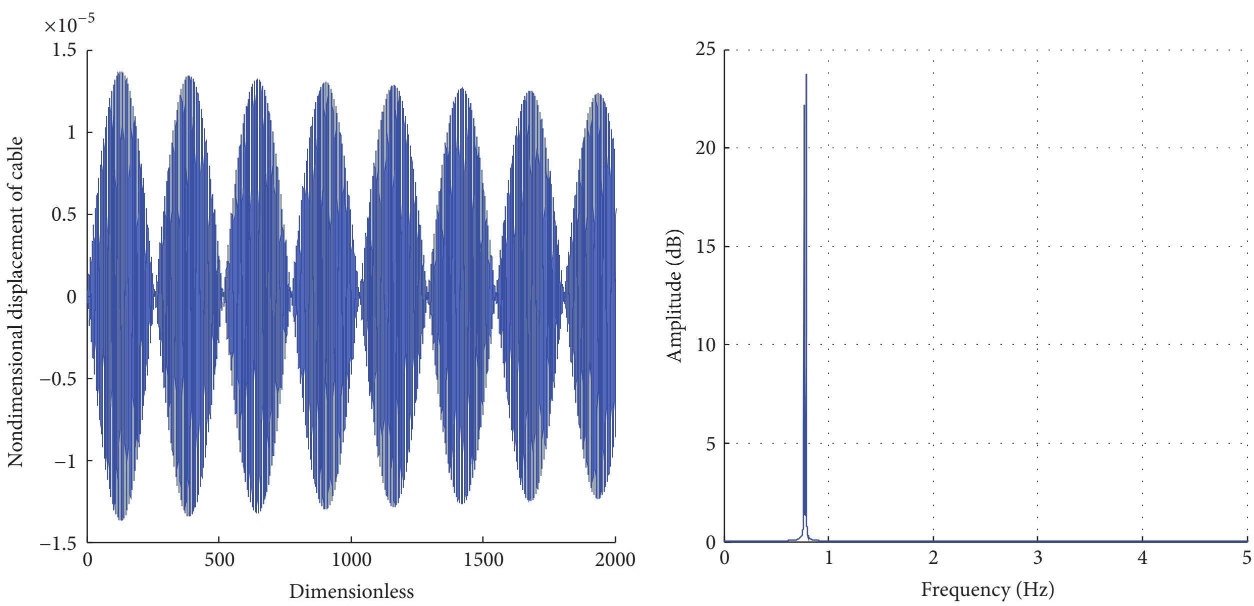

Figure 7: Dimensionless displacement response time history and amplitude-frequency characteristic curve of the deck beam end.

(2) The nonlinear main parametrical resonance response of cable is irrelevant to its initial displacement, which is increased with the bigger initial displacement of the deck beam end; also, the cable midspan displacement is increased obviously and presents obvious beat vibration phenomenon.

(3) When the vertical initial displacement of the deck beam end is 10^{-6} m or even smaller, both the cable and beam present obvious beat vibration phenomenon, but, at this time, the cable displacement amplitude is small and there is no significant vibration, only making a slight stable "beat" vibration in the vicinity of the equilibrium position, which is different from the situation of 2 : 1 parametrical resonance.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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