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Research Article

Reliability Analysis of Load-Sharing K-out-of-N System Considering Component Degradation

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The K-out-of-N configuration is a typical form of redundancy techniques to improve system reliability, where at least K-out-of-N components must work for successful operation of system. When the components are degraded, more components are needed to meet the system requirement, which means that the value of K has to increase. The current reliability analysis methods overestimate the reliability, because using constant K ignores the degradation effect. In a load-sharing system with degrading components, the workload shared on each surviving component will increase after a random component failure, resulting in higher failure rate and increased performance degradation rate. This paper proposes a method combining a tampered failure rate model with a performance degradation model to analyze the reliability of load-sharing K-out-of-N system with degrading components. The proposed method considers the value of K as a variable which is derived by the performance degradation model. Also, the load-sharing effect is evaluated by the tampered failure rate model. Monte-Carlo simulation procedure is used to estimate the discrete probability distribution of K. The case of a solar panel is studied in this paper, and the result shows that the reliability considering component degradation is less than that ignoring component degradation.

1. Introduction

Redundancy technique is widely used to improve system reliability. A typical form of redundancy is a K-out-of-N configuration in which at least K out of N components must work for normal operation of system. When using traditional methods [1–4] to analyze reliability of K-out-of-N system, independence is assumed within the system, which means that a component failure does not affect the failure rate or performance of surviving components. However, in the real world, many systems are load-sharing, such as electric generators sharing an electrical load in a power plant, cables in a suspension bridge, and valves or pumps in a hydraulic system. In the load-sharing system, the workload has to be shared by the remaining components, resulting in an increased load shared on each surviving component [5]. Many empirical studies of mechanical systems [6] and computer systems [7] have proved that the workload strongly affects the component failure rate. Scheuer [8] studied the reliability of K-out-of-N system when component failure induces higher failure rate in survivors. The method is limited in system composed of s-independent and identically distributed components with exponential lifetimes. Liu [9] proposed a generalized accelerated failure-time model (AFTM) for reliability analysis of load-sharing K-out-of-N system with arbitrary distribution load-dependent component lifetime distributions. Amari et al. [10] provided a closed-form analytical solution for the reliability of tampered failure rate load-sharing K-out-of-N system, and Amari and Bergman [11] also used the cumulative exposure model to account for the effect of loading history. The mentioned reliability analysis methods are based on the assumption of binary components, that is to say the component is either failed or working in perfect state. In fact, the performance of component is degrading in lifetime. In order to meet the system performance requirement, the value of K has to monotonously increase in service time. In that case, the reliability of K-out-of-N system considering component degradation may be less than the reliability based on assumption of binary component.

When components have several degraded performance states, the multistate system (MSS) model is introduced to analyze the reliability of K-out-of-N system [12–15]. Amari et al. [16] presented a fast and robust reliability evaluation algorithm for very large multistate K-out-of-N systems. Levitin [17] introduced a new model named multistate vector-K-out-of-N system, which is a generalization of existing multistate K-out-of-N system models. Using current MSS algorithms to calculate reliability, we should know the probability that component performance is located in each state. Many experimental and theoretical researches [18–20] indicate that the performance degradation rate is strangely affected by the load shared on components. In the load-sharing system, the shared load is increasing, and then the performance degradation rate will increase correspondingly. Consequently, it is difficult to get the probability of each performance state.

In the degradation process, the value of K is increasing because of component degradation. The phased-mission system (PMS) model [21–23] is introduced to calculate the reliability of K-out-of-N system with variable K in different phases. Xing et al. [24] proposed an efficient method for reliability evaluation of K-out-of-N systems subject to phased-mission requirements. Unfortunately, when using the PMS model to evaluate reliability, we need to know the specified K in each phase. However, in degradation process, the value of K is determined by the system performance requirement and the degraded performance of each component. As a result, the value of K in each phase is variable. Considering the complexity of degradation law in load-sharing system, it is hard to know the specified K in each phase.

In conclusion, there are some special characteristics of load-sharing K-out-of-N system with degrading components: (1) a component random failure increases the load shared on each remaining component, (2) the failure rate of surviving components will increase after component failures, (3) the rise of load will raise the degradation rate of component, (4) the duration when components have a specified degradation rate is stochastic because a component failure occurs at random time, (5) the value of K is stochastic because the component performance is a random variable at any given time. Some mentioned load-sharing methods deal with characteristics (1) and (2) effectively. What is more, the MSS model is suggested to compute the reliability when component has degraded states, and the PMS model is used to deal with phase-mission requirement. However, characteristics (3), (4), and (5) make the degradation law of component performance very complicated; consequently it is very hard to get some conditions of existed methods.

This paper proposes a method combining a tampered failure rate model with a performance degradation model to analyze the reliability of load-sharing K-out-of-N system with degrading components. The tampered failure rate model is introduced to evaluate the reliability where the rise of load makes the failure rate of surviving components increase. The performance degradation model is derived to calculate degraded component performance in service time. Furthermore, the load-sharing effect on degradation rate and the randomness of duration are included in the performance degradation model. In this way, the discrete probability distribution of K is obtained through performance degradation model coupled with load-sharing effect. Using the discrete probability distribution of K and the reliability computed by the tampered failure rate model when the system has a specified K, the reliability of load-sharing K-out-of-N system with degrading components is calculated accurately.

The remainder of this paper is divided into five sections. In Section 2, the tampered failure rate model is introduced to analyze reliability of load-sharing K-out-of-N system when the value of K is constant. In Section 3, the performance degradation model coupled with load-sharing effect is formulated to evaluate the discrete probability distribution of K. In Section 4, Monte-Carlo simulation procedure is used to estimate the discrete probability distribution of K. The case of a solar panel is studied in Section 5. Conclusions are drawn in Section 6.

2. Reliability of Load-Sharing K-out-of-N System

In a load-sharing system, the workload has to be redistributed among the remaining components after a component failure. Mostly the load is equally shared by each surviving component. Let the total workload be L, and let the total number of components be n. Let z_i be the load on each surviving component when i components have failed. Hence,

$$z_0 = \frac{L}{n},\qquad z_i = \frac{L}{(n-i)}. \tag{1}$$

In order to analyze the reliability of load-sharing system, the load-sharing effect that the rise of shared load on a surviving component raises the failure rate has to be evaluated. The tampered failure rate (TFR) model proposed by Bhattacharyya and Soejoeti [25] can be applied for the load-sharing system. The acceleration of failure when load is raised from lower level to a higher level is reflected in the failure rate function.

Let the component be subject to an ordered sequence of loads, where load z_i $(i = 0, 1, \ldots, n-k)$ is applied during the time interval $[\tau_i, \tau_{i+1}]$ $(\tau_0 = 0)$. According to the TFR model, the failure rate of the component at t is

$$\lambda(t) = \lambda_i(t) = \delta_i \cdot \lambda_0(t) = \delta(z_i) \cdot \lambda_0(t) \quad \text{for } \tau_{i-1} \le t \le \tau_i, \tag{2}$$

where $\lambda_0(t)$ is the baseline failure rate which has nothing to do with load, δ_i is the tampered factor at load z_i, and z_i is the load shared on component at t.

With the assumptions that the load is equally distributed among all surviving identical components and the failure rate of a component varies as described in the TFR model, the reliability of load-sharing K-out-of-N system without component performance degradation could be calculated by analytical method.

2.1. Exponential Case. Firstly, consider a load-sharing K-out-of-N system with components following the exponential lifetime distribution [8]; in other words, the baseline failure rate of the TFR model $\lambda_0(t)$ is constant.

When the system is put into operation, the failure rate of every component is denoted by λ_0. Because there are n working components in the system, the first component failure occurs at failure rate $\alpha_1 = n \cdot \lambda_0$. After the first failure, the remaining $(n-1)$ working components must carry the same workload of system. As a result, the failure rate of each surviving component becomes λ_1, which is commonly higher than λ_0. The second component failure occurs at rate $\alpha_2 = (n-1)\cdot\lambda_1$. When i components have failed, the failure rate of each $(n-i)$ remaining component is denoted by λ_i $(0 \le i \le n-k)$. The ith component failure occurs at failure rate $\alpha_i = (n-i+1)\cdot\lambda_{i-1}$. The system is failed when more than $(n-k)$ components are failed.

The time when the ith component failure occurs in the system is denoted by T_i $(T_0 \equiv 0)$, and the time interval between $(i-1)$th and ith component failure is represented by $X_i = T_i - T_{i-1}$ $(1 \le i \le n-k+1)$. Since all identical components are following the exponential distributions, the X_i follows the exponential distribution with parameter α_i. Hence, the lifetime of system is the $(n-k+1)$th failure time

$$T = T_{n-k+1} = \sum_{i=1}^{n-k+1} X_i. \tag{3}$$

Then, the reliability of K-out-of-N system at t_0 is

$$R(t_0) = P\{T > t_0\}. \tag{4}$$

In order to calculate the distribution of T and the reliability function of load-sharing K-out-of-N system, two typical formulas which can be used are as follows.

Case a. All α_i are equal (say α) [8]:

$$R(t) = \sum_{i=0}^{n-k} \frac{(\alpha t)^i \exp(-\alpha t)}{i!} = gamfc(\alpha t; n-k+1). \tag{5}$$

This case arises when the failure rate of each surviving component is directly proportional to the load it carries, which means that $\delta(z) \propto z$ in the TFR model.

Case b. All α_i are distinct [8]:

$$\begin{aligned} R(t) &= \sum_{i=1}^{n-k+1} A_i \cdot \exp(-\alpha_i t), \\ A_i &\equiv \prod_{\substack{j=1 \\ j\ne i}}^{n-k+1} \frac{\alpha_j}{\alpha_j - \alpha_i} \quad i = 1,2,\ldots,n-k+1. \end{aligned} \tag{6}$$

2.2. General Case. In the case of a load-sharing K-out-of-N system with components following arbitrary lifetime distributions, the baseline failure rate of the TFR model is no longer a constant. A closed-form analytical solution for TFR model with an arbitrary baseline distribution is introduced. The basic idea is to use a time-transformation to convert TFR model with an arbitrary baseline distribution into an equivalent problem with an exponential baseline distribution [10].

Lemma 1. *(1) For any failure distribution $F(t)$, the reliability function is $R(t) = 1 - F(t)$.*

(2) The cumulative failure rate function is

$$\begin{aligned} \Lambda(t) &= \int_0^t \lambda(t)\,dt = \int_0^t \frac{dF(t)}{R(t)} = \int_0^t \frac{-dR(t)}{R(t)} \\ &= -\ln R(t). \end{aligned} \tag{7}$$

(3) The random variable $y = R(t)$ follows a uniform distribution in interval $[0,1]$. The random variable $l = \Lambda(t)$ follows an exponential distribution with mean 1.

(4) For a TFR model with a standard exponential $(\lambda = 1)$ baseline failure time distribution

$$\begin{aligned} \lambda(t) &= \delta_i, \\ \Lambda(t) &= \Lambda(\tau_{i-1}) + \delta_i(t-\tau_{i-1}) \quad \tau_{i-1} \le t < \tau_i. \end{aligned} \tag{8}$$

(5) For a TFR model with a baseline failure rate of $\lambda_0(t)$ and a baseline cumulative failure rate of $\Lambda_0(t)$,

(a) *under the regular scale t,*

$$\begin{aligned} \lambda(t) &= \delta_i \cdot \lambda_0(t), \\ \Lambda(t) &= \Lambda(\tau_{i-1}) + \delta_i \cdot [\Lambda_0(t) - \Lambda_0(\tau_{i-1})] \\ &\qquad \tau_{i-1} \le t < \tau_i; \end{aligned} \tag{9}$$

(b) *under the transformed scale $l = \Lambda_0(t)$.*

Let

$$\begin{aligned} v_i &= \Lambda_0(\tau_i), \\ \lambda_l(l) &= \delta_i, \\ \Lambda_l(l) &= \Lambda_l(v_{i-1}) + \delta_i \cdot [l - v_{i-1}] \quad v_{i-1} \le l < v_i, \end{aligned} \tag{10}$$

where $\Lambda_l(l)$ is the cumulative failure rate in the transformed scale.

According to the above lemmas, it becomes obvious that if the load-sharing effect on the failure rate of an individual component follows a TFR model with $\lambda(t) = \delta_i \cdot \lambda_0(t)$, the reliability of a load-sharing system at t is equivalent to the reliability of corresponding exponential load-sharing model at time $l = \Lambda_0(t)$, where the failure rate of a component is $\lambda_i = \delta_i$ when i components have failed for $i = 0,\ldots,(n-k)$.

3. Degradation Effect in Load-Sharing System

In a load-sharing K-out-of-N system with degrading components, a random component failure raises the load shared on remaining components, leading to the rise of the failure rate and performance degradation rate. Besides, the duration when component has a specified degradation rate is variable because a component failure occurs at random time. Therefore, the component performance is a random variable at any given time, thus the value of K is stochastic in service time.

3.1. Independent Degradation Effect. At the first step, consider the component degradation rate to be independent of load-sharing effect, which means that the load-sharing effect just makes the failure rate higher but the degradation rate has nothing to do with shared load. When the system is just put into operation, the value of K is determined by $K = \lceil C/c_0 \rceil$, where C is the system performance requirement, c_0 is the initial performance of each component, and K is the minimum integer that is greater than or equal to the quotient. When the system has operated for a period of time t, the degraded performance of component is denoted by $c(t) = c_0 D(t)$, where $D(t)$ is degradation law of component performance. $D(t)$ is a monotone decreasing function, and $D(0) = 1$, $D(+\infty) = 0$. For example, the common degradation laws are exponential law denoted by $D(t) = 1 - at^m$ and power law denoted by $D(t) = \exp(-at)$. At time t, the value of K is determined by $K(t) = \lceil C/c(t) \rceil$. Since $D(t)$ is a monotone decreasing function, it is quite clear that $c(t) < c_0$. Thus, $K(t) \geq K$, while the condition for equality is $\lceil C/c(t) \rceil = \lceil C/c_0 \rceil$. Hence, the reliability of K-out-of-N system with degrading component is calculated in a general expression as

$$R'(t) = \sum_{i=0}^{n-K(t)} C_n^i \left[1 - R_0(t)\right]^i \left[R_0(t)\right]^{n-i}. \tag{11}$$

Correspondingly, using the TFR model to analyze the load-sharing K-out-of-N system with degrading components, the value of K should be replaced by $K(t)$.

Obviously, $R'(t) \leq R(t)$ because of $K(t) \geq K$, meaning that the reliability of K-out-of-N system considering component degradation is less than the reliability ignoring component degradation.

3.2. Degradation Coupled with Load-Sharing Effect. When the component performance degradation is related to load-sharing effect, a random failure of component will raise the degradation rate of surviving component. Since a component failure occurs at random time, the duration when components have a specified degradation rate is variable. Therefore, the variable K denoted by $\widetilde{K}(t)$ is stochastic at any given time.

In a load-sharing system, the degradation law of component performance $D(t)$ is not only a function of time, but also a function of the shared load. The generalization of degradation law is expressed as a function of time and load, denoted by $\widetilde{D}(t,z)$. The degradation rate of component performance with load-sharing effect is $\widetilde{d}(t,z) = \partial \widetilde{D}(t,z)/\partial t$. After ith component failure, the performance degradation rate of surviving components is denoted by $\widetilde{d}_i(t,z_i) = \widetilde{d}[t, L/(n-i)]$.

In order to analyze the reliability of load-sharing K-out-of-N system with degrading components at t_0, $\widetilde{K}(t_0)$ should be evaluated previously, which is determined by the degraded performance of component at t_0. With the understanding of degradation rate $\widetilde{d}_i(t,z_i)$ after ith component failure, the degraded performance at t_0 is estimated by using the duration when the component has a specified degradation rate, denoted by X_i. The duration X_i is the interval between $(i-1)$th component failure and ith component failure. In other words, X_i is the minimum order statistic of the components failure time of the surviving $(n-i+1)$ components under the condition that the $(n-i+1)$ components do not fail at T_{i-1}.

Lemma 2.

(1) Conditional Distribution Function

(a) *The reliability of component at $(T_i + \Delta t)$ is*

$$\begin{aligned} R(T_i + \Delta t) &= \exp\left[-\int_0^{T_i+\Delta t} \lambda(t)\,dt\right] \\ &= \exp\left[-\int_0^{T_1} \lambda_0(t)\,dt - \int_{T_1}^{T_2} \lambda_1(t)\,dt - \cdots \right. \\ &\quad \left. - \int_{T_{i-1}}^{T_i} \lambda_{i-1}(t)\,dt - \int_{T_i}^{T_i+\Delta t} \lambda_i(t)\,dt\right] \\ &= \exp\left[-\sum_{j=0}^{i-1} \int_{T_j}^{T_{j+1}} \lambda_j(t)\,dt - \int_{T_i}^{T_i+\Delta t} \lambda_i(t)\,dt\right]. \end{aligned} \tag{12}$$

(b) *The reliability at T_i is*

$$R(T_i) = \exp\left[-\sum_{j=0}^{i-1} \int_{T_j}^{T_{j+1}} \lambda_j(t)\,dt\right]. \tag{13}$$

(c) *The cumulative distribution function of component failure time under the condition that the component did not fail at T_i is*

$$\begin{aligned} F(T_i + \Delta t \mid T_i) &= 1 - R(T_i + \Delta t \mid T_i) \\ &= 1 - \frac{R(T_i + \Delta t)}{R(T_i)} \\ &= 1 - \exp\left[-\int_{T_i}^{T_i+\Delta t} \lambda_i(t)\,dt\right] \\ &= 1 - \exp\left[-\int_0^{\Delta t} \lambda_i(t + T_i)\,dt\right]. \end{aligned} \tag{14}$$

(2) Minimum Order Statistic

(a) *Let the cumulative distribution function of a population be $F(t)$, and probability distribution function is $f(t)$. For samples with size n, probability distribution function of jth order statistic $Y_{(j)}$ is*

$$f_{Y(j)}(t) = nC_{n-1}^{j-1} \left[F(t)\right]^{j-1} \left[1 - F(t)\right]^{n-j} f(t). \tag{15}$$

(b) *The probability distribution function of the minimum order statistic is*

$$f_{Y(1)}(t) = nf(t)\left[1 - F(t)\right]^{n-1}. \tag{16}$$

(c) *The cumulative distribution function of the minimum order statistic is*

$$F_{Y(1)}(t) = \int_0^t f_{Y(1)}(t)\,dt = \int_0^t nf(t)[1-F(t)]^{n-1}\,dt = 1-[1-F(t)]^n. \quad (17)$$

Hence, the cumulative distribution function of X_i is expressed as

$$\begin{aligned} F_{X_i}(\Delta t) &= 1-\left[1-F\left(T_{i-1}+\Delta t \mid T_{i-1}\right)\right]^{n-i+1} \\ &= 1-\left[R\left(T_{i-1}+\Delta t \mid T_{i-1}\right)\right]^{n-i+1} \\ &= 1-\exp\left[-(n-i+1)\int_0^{\Delta t}\lambda_{i-1}\left(t+T_{i-1}\right)dt\right]. \end{aligned} \quad (18)$$

In a special situation that the baseline distribution of TFR is exponential distribution, the distribution of X_i is an exponential distribution with parameter $(n-i+1)\lambda_{i-1}$, which agrees with the conclusion in previous paper Section 2.1.

As i components have failed in the system, the degradation rate of each surviving component is $\tilde{d}_i(t,z_i)$. Moreover, the time interval between ith component failure and $(i+1)$th failure is X_{i+1}. Therefore, the component performance degradation in time interval $[T_i, T_{i+1}]$ is

$$\Delta c_i = c_0\int_{T_i}^{T_{i+1}}\tilde{d}_i(t,z_i)\,dt = c_0\int_0^{X_{i+1}}\tilde{d}_i(T_i+t,z_i)\,dt. \quad (19)$$

The degraded component performance at t_0 is calculated as

$$\begin{aligned} \tilde{c}(t_0) &= c_0 + \sum_{j=0}^{m-1}\Delta c_j + c_0\int_{T_m}^{t_0}\tilde{d}_m(t,z_m)\,dt \\ &= c_0 + c_0\sum_{j=0}^{m-1}\int_{T_j}^{T_{j+1}}\tilde{d}_j\left(t,z_j\right)dt \\ &\quad + c_0\int_{T_m}^{t_0}\tilde{d}_m(t,z_m)\,dt. \end{aligned} \quad (20)$$

According to $X_{j+1} = T_{j+1} - T_j$, $T_j = \sum_{i=0}^{j} X_{i+1}$ $(T_0 \equiv 0)$ and $z_j = L/(n-j)$ for $j = 0,1,2,\ldots,m$, m is determined by $T_m \le t_0 < T_{m+1}$,

$$\int_{T_j}^{T_{j+1}}\tilde{d}_j\left(t,z_j\right)dt = \int_0^{X_{j+1}}\tilde{d}_j\left(t+T_j,\frac{L}{n-j}\right)dt. \quad (21)$$

Hence,

$$\begin{aligned} \tilde{c}(t_0) &= c_0 + c_0\sum_{j=0}^{m-1}\int_0^{X_{j+1}}\tilde{d}_j\left(t+T_j,\frac{L}{n-j}\right)dt \\ &\quad + c_0\int_0^{t_0-T_m}\tilde{d}_m\left(t+T_m,\frac{L}{n-m}\right)dt, \end{aligned} \quad (22)$$

where the distribution of X_j is $F_{X_j}(\Delta t)$ and the degradation rate at each phase is $\tilde{d}_j(t,z_j)$.

According to (22), the distribution of the degraded component performance $\tilde{c}(t_0)$ is calculated on basis of the distribution of X_j denoted by $F_{X_j}(\Delta t)$ and degradation rate $\tilde{d}_j(t,z_j)$. Then, the discrete probability distribution of $\widetilde{K}(t_0)$ could be obtained through

$$\widetilde{K}(t_0) = \left\lceil\frac{C}{\tilde{c}(t_0)}\right\rceil = \left\lceil\frac{C}{c_0 + c_0\sum_{j=0}^{m-1}\int_0^{X_{j+1}}\tilde{d}_j\left(t+T_j, L/n-j\right)dt + c_0\int_0^{t_0-T_m}\tilde{d}_m\left(t+T_m, L/n-m\right)dt}\right\rceil. \quad (23)$$

Using the probability distribution of $\widetilde{K}(t_0)$ estimated by the performance degradation model and the reliability calculated by the TFR model when the system has a specified K, the reliability of load-sharing K-out-of-N system with degrading components is computed in formula as

$$\widetilde{R}(t_0) = \sum p\left(\widetilde{K}_j\right)R_{\widetilde{K}_j}(t_0), \quad (24)$$

where $p(\widetilde{K}_j)$ is the probability when the $\widetilde{K}(t_0)$ is equal to $\widetilde{K}_j$ and $R_{\widetilde{K}_j}(t_0)$ is the reliability of K-out-of-N system when the $\widetilde{K}(t_0)$ is equal to $\widetilde{K}_j$. Using the proposed formula to evaluate the reliability of load-sharing K-out-of-N system with degrading components, the load-sharing effect on component failure rate and the degradation effect coupled with load-sharing effect are all included in the model. Therefore, the reliability is calculated more accurately.

4. Monte-Carlo Simulation

In the load-sharing system, the failure rate and degradation rate are variable during the lifetime, and the duration in each phase is stochastic because a component failure occurs at random time. The analytic expression of degraded performance could be obtained only if the failure rate or degradation rate is subject to some specified formats, such as components following the exponential lifetime distribution or degradation rate being constant. This paper employs Monte-Carlo simulation to estimate the discrete probability distribution of K. The simulation procedure is shown in Figure 1.

In order to evaluate the discrete probability distribution of K, the system configuration must be set firstly: as the total component number denoted by n, the total workload denoted by L, the system performance requirement denoted by C, the initial performance of each component denoted by

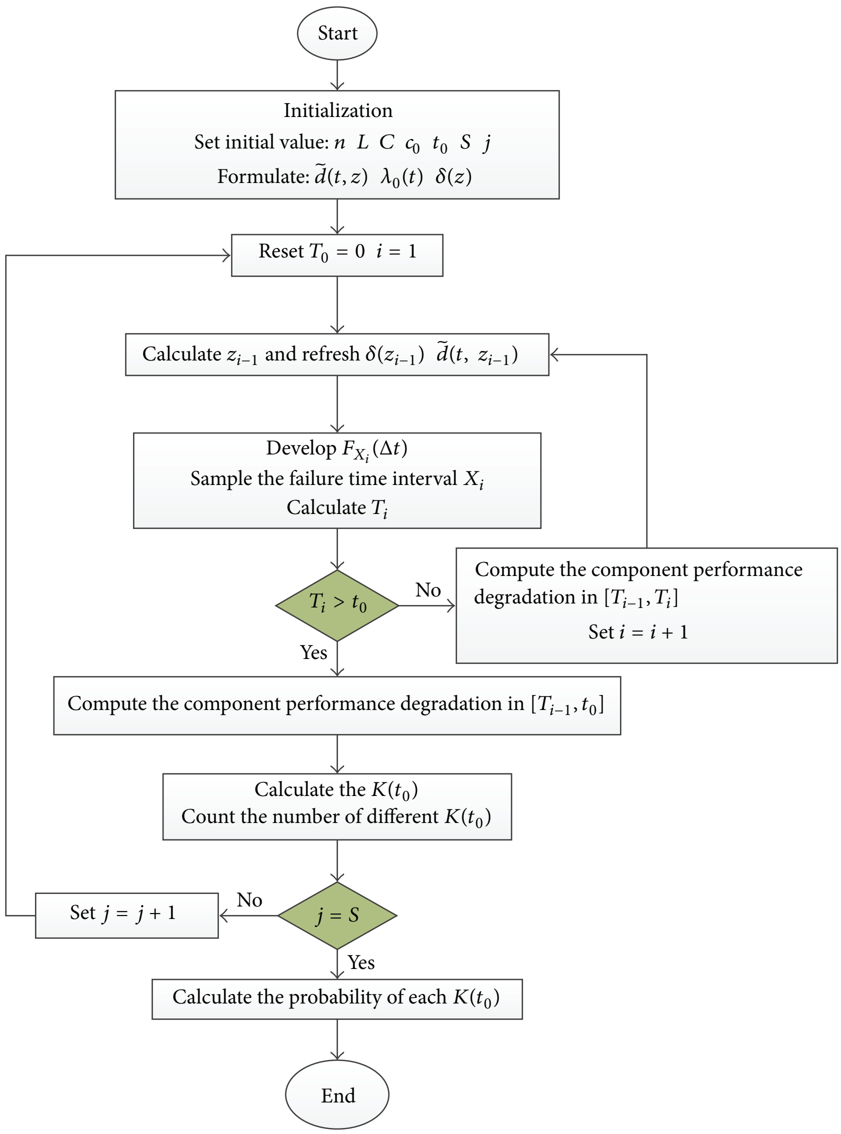

Figure 1: The Monte-Carlo simulation procedure.

c_0, and the system required operation time denoted by t_0. Furthermore, the baseline failure rate function denoted by $\lambda_0(t)$, the degradation rate function denoted by $\tilde{d}(t,z)$, and the tampered factor function denoted by $\delta(z)$ all need to be formulated in advance. Set the required number of simulation cycle S and initialize cyclic variable $j = 1$, and then carry out the Monte-Carlo simulation.

At the beginning of a simulation cycle, reset the start moment $T_0 = 0$ and sampling variable $i = 1$. At the ith sampling in a cycle of the simulation, there are $(n-i+1)$ remaining components. The workload shared on each component is denoted by z_{i-1}. Then the tampered factor can be calculated by $\delta(z)$, and the failure rate under current load is obtained through the baseline failure rate function multiplied by the tampered factor. Besides, the degradation rate under current load is computed by $\tilde{d}(t,z)$. According to the current failure rate and last component failure time T_{i-1}, the distribution of X_i between $(i-1)$th and ith component failure denoted by $F_{X_i}(\Delta t)$ is obtained. Using the distribution $F_{X_i}(\Delta t)$, the sampling formula of X_i denoted by $F_{X_i}^{-1}(\eta)$ is derived, where η follows uniform distribution in the interval $[0,1]$. Generating the X_i from $F_{X_i}^{-1}(\eta)$, the ith component failure time is $T_i = T_{i-1} + X_i$. If T_i is less than t_0, compute the component performance degradation in time interval $[T_{i-1}, T_i]$, and then start the next sampling. When T_i reaches system required operation time t_0, compute the component performance degradation in time interval $[T_{i-1}, t_0]$, and then the degraded component performance at t_0 is obtained. The $\widetilde{K}(t_0)$ is determined by the system performance requirement C and the degraded component performance $\tilde{c}(t_0)$, and then carry out the next cycle of the simulation. When the number of simulation cycles meets the requirement denoted by S, count the number when $\widetilde{K}(t_0)$ is equal to different values. Then the probability of each $\widetilde{K}(t_0)$ is estimated by dividing the number of different $\widetilde{K}(t_0)$ by the number of simulation cycles S.

5. Case Study

Satellites or space station which needs to work for long periods in aerospace must be equipped with suitable and reliable solar panels to provide power for the normal operation of equipment. Once a solar panel cannot supply required power, the spacecraft will lose its function, and it may become "space junk." As the critical module, there are strict requirements about the performance and reliability of solar panel. A solar panel module consists of many solar cells. Factors of the space environment like space radiation, temperature cycling, and charge-discharge cycles will destroy some solar cells. Besides, the performance of solar cells will degrade in lifetime. The failure solar cell cannot be repaired in outer space timely, so the workload of solar panel is shared by the remaining solar cells, and surviving solar cells should supply the required power of system.

A kind of solar panel of geostationary orbit satellite has 20 solar cells. The satellite needs 10 kW output power (the system performance requirement) from solar panel to ensure normal operation of other subsystems. The initial output power of each solar cell is 1 kW (initial performance of component). The solar panel is subject to 10 A steady current (the total workload), and it is shared by 20 solar cells evenly. The baseline failure time distribution is Weibull one with shape parameter $\beta = 2$ and location parameter $\eta = 200000$ h. The tampered factor is $\delta(z) = z^{1.5}$ under the operating current z. The degradation law of solar cell output power is $\widetilde{D}(t,z) = \exp(at/z^4)$, where $a = -1/6000000$. According to the requirement, the solar panel should supply sufficient power for 15 years (consider one year as 365 days, and then 15 years are equal to 131400 hours). The output power of a solar cell is degrading with service time. The random failure of a solar cell will increase the current shared on remaining solar cells, resulting in higher failure rate of solar cells, and the degradation rate of solar cells output power will increase correspondingly. The value of K is determined by the output power of surviving solar cells and the output power requirement of solar panel. It is obvious that the solar panels are a typical load-sharing K-out-of-N system with component performance degradation. With the proposed method in this paper, the reliability of solar panels could be estimated accurately.

Firstly, we should calculate the discrete probability of K by performance degradation model.

According to the Monte-Carlo simulation method, the time interval X_i between ith and $(i-1)$th solar cell failure is subject to

$$F_{X_i}(\Delta t) = 1 - \exp\left[-(n-i+1)\int_0^{\Delta t} \lambda_{i-1}(t+T_{i-1})\,dt\right]. \quad (25)$$

Using the given value $n = 20$, $L = 10$, $\lambda_0(t)$, and $\delta(z_{i-1})$, the distribution of X_i is expressed as

$$F_{X_i}(\Delta t) = 1 - \exp\left\{-10^{1.5}\cdot(21-i)^{-0.5} \cdot\left[\left(\frac{T_{i-1}+\Delta t}{200000}\right)^2 - \left(\frac{T_{i-1}}{200000}\right)^2\right]\right\}. \quad (26)$$

Hence, the sampling formula of X_i is derived:

$$\Delta t = 200000 \cdot\left[\left(\frac{T_{i-1}}{200000}\right)^2 - \frac{(21-i)^{0.5}}{10^{1.5}}\ln(1-\eta)\right]^{1/2} - T_{i-1}, \quad (27)$$

where η is subject to uniform distribution in $[0,1]$.

The degradation rate of a solar cell output power is

$$\widetilde{d}(t,z) = \frac{\partial\widetilde{D}(t,z)}{\partial t} = \frac{a}{z^4}\exp\left(\frac{at}{z^4}\right), \quad (28)$$

where $a = -1/6000000$. The required output power of the solar panel is $C = 10$, and the initial output power of a solar cell is $c_0 = 1$.

The output power degradation in time interval $[T_i, T_{i+1}]$ is

$$\Delta c_i = c_0\int_{T_{i-1}}^{T_i}\widetilde{d}_{i-1}(t, z_{i-1})\,dt = c_0\int_{T_{i-1}}^{T_i}\widetilde{d}\left(t, \frac{L}{n+1-i}\right)dt. \quad (29)$$

With the given value

$$\Delta c_i = -\int_{T_{i-1}}^{T_i}\frac{1}{6000000\,(10/21-i)^4}\exp\left[-\frac{t}{6000000\,(10/21-i)^4}\right]dt. \quad (30)$$

After a cycle of the simulation, the output power of a solar cell $\widetilde{c}(131400)$ at $t = 131400$ h can be calculated. Hence, the value of K is expressed as

$$\widetilde{K}(131400) = \left\lceil\frac{10}{\widetilde{c}(131400)}\right\rceil. \quad (31)$$

Set the number of cycle as $S = 10000$; the count and possibility of $\widetilde{K}(131400)$ are shown in Table 1.

Table 1: The count and possibility of $K(131400)$.

$K(131400)$	12	13	14	15
Count	29	1710	6749	1512
Possibility	0.29%	17.1%	67.49%	15.12%

If the output power of a solar cell does not degrade, the value of K is

$$K = \left\lceil\frac{10}{c_0}\right\rceil = \left\lceil\frac{10}{1}\right\rceil = 10. \quad (32)$$

Table 2: The coefficients of α_i.

α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8	α_9	α_{10}
7.0711	7.2548	7.4536	7.6696	7.9057	8.1650	8.4515	8.7706	9.1287	9.5346
α_{11}	α_{12}	α_{13}	α_{14}	α_{15}	α_{16}	α_{17}	α_{18}	α_{19}	α_{20}
10	10.5409	11.1803	11.9523	12.9099	14.1421	15.8114	18.2574	22.3607	31.6228

Table 3: The coefficients of A_i when $K(131400) = 12$.

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
$9.7723e7$	$-5.7858e8$	$1.4768e9$	$-2.1193e9$	$1.8667e9$	$-1.0311e9$	$3.4787e8$	$-6.5332e7$	$5.2086e6$

From the above results, because the output power of a solar cell is degrading, the value of K at $t = 131400$ h is greater than the situation ignoring solar cell degradation.

Secondly, after getting the discrete probability distribution of $\widetilde{K}(131400)$, the reliability when $\widetilde{K}(131400)$ is a specified value should be calculated by the TFR model.

TFR model of solar panel is shown as follows:

Model: $n = 20$, $L = 10$, $t = 131400$ h, $\beta = 2$, and $\eta = 200000$.

Baseline failure rate:

$$\lambda_0(t) = \frac{\beta}{\eta}\cdot\left(\frac{t}{\eta}\right)^{\beta-1} = \frac{1}{100000}\cdot\frac{t}{200000}. \tag{33}$$

The cumulative failure rate:

$$\Lambda_0(t) = \left(\frac{t}{\eta}\right)^{\beta} = \left(\frac{t}{200000}\right)^{2}. \tag{34}$$

The TFR model of failure rate caused by operating current

$$\lambda(t) = \delta(z)\cdot\lambda_0(t), \tag{35}$$

where the tampered factor is $\delta(z) = z^{1.5}$.

Solution is as follows:

The tampered factor:

$$\delta_i = \delta(z_i) = z_i^{1.5} = \left(\frac{L}{n-i}\right)^{1.5} = \left(\frac{10}{20-i}\right)^{1.5}. \tag{36}$$

The corresponding TFR model with exponential baseline failure distribution is as follows:

Transformed scale:

$$l = \Lambda_0(t) = \left(\frac{131400}{200000}\right)^{2}. \tag{37}$$

Because $\lambda_i = \delta_i = (10/(20-i))^{1.5}$, $\alpha_i = (n-i+1)\lambda_{i-1} = 10^{1.5}(21-i)^{-0.5}$, the coefficients of α_i are shown in Table 2.

When $\widetilde{K}(131400) = 12$,

$$A_i \equiv \prod_{\substack{j=1\\ j\neq i}}^{20-12+1} \frac{\alpha_j}{\alpha_j-\alpha_i} = \prod_{\substack{j=1\\ j\neq i}}^{9} \frac{\alpha_j}{\alpha_j-\alpha_i} \qquad i = 1,2,\ldots,9. \tag{38}$$

The coefficients of A_i when $\widetilde{K}(131400) = 12$ are shown in Table 3:

$$\begin{aligned}\widetilde{R}_{\widetilde{K}=12}(131400) &= \sum_{i=1}^{20-12+1} A_i\cdot\exp\left[-\alpha_i\Lambda_0(131400)\right]\\ &= \sum_{i=1}^{9} A_i\cdot\exp\left[-\left(\frac{1314}{2000}\right)^2\alpha_i\right]\\ &= 0.9912.\end{aligned} \tag{39}$$

In the same way, the reliability when the system has other K is calculated as follows:

$$\begin{aligned}\widetilde{R}_{\widetilde{K}=13}(131400) &= 0.9778,\\ \widetilde{R}_{\widetilde{K}=14}(131400) &= 0.9477,\\ \widetilde{R}_{\widetilde{K}=15}(131400) &= 0.8866.\end{aligned} \tag{40}$$

Therefore, the reliability of solar panel that it could supply sufficient power for 15 years is computed as

$$\widetilde{R}(131400) = \sum p\left(\widetilde{K}_j\right) R_{\widetilde{K}_j}(131400) = 0.9437. \tag{41}$$

If the output power of solar cells does not degrade, meaning that $K = 10$, the coefficients of A_i are shown in Table 4.

Hence, the reliability is

$$\begin{aligned}R(131400) &= \sum_{i=1}^{20-10+1} A_i\cdot\exp\left[-\alpha_i\Lambda_0(131400)\right]\\ &= \sum_{i=1}^{11} A_i\cdot\exp\left[-\left(\frac{1314}{2000}\right)^2\alpha_i\right] = 0.9988.\end{aligned} \tag{42}$$

Comparing $R(131400)$ and $\widetilde{R}(131400)$, we can see that random failures of some solar cells make the operating current of the remaining solar cells increase, also leading to an increase of failure rate and degradation rate. Due to the degradation of solar cells output power, the reliability of solar panel $\widetilde{R}(131400)$ is less than $R(131400)$ which ignores the degradation of solar cell output power.

6. Conclusion

In a load-sharing K-out-of-N system with degrading components, a component random failure raises the load shared on

Table 4: The coefficients of A_i when $K(131400) = 10$.

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}
$1.2913e9$	$-8.8141e9$	$2.6571e10$	$-4.6495e10$	$5.2171e10$	$-3.9115e10$	$1.9777e10$	$-6.6315e9$	$1.4042e9$	$-1.6848e8$	$8.6283e6$

each remaining component, resulting in a higher failure rate and increased degradation rate of surviving components. This paper proposes a method combining a TFR model with a performance degradation model to analyze the reliability of load-sharing K-out-of-N system with degrading components. The TFR model deals with load-sharing effect on failure rate, and the reliability when the system has a specified K is calculated by the TFR model. The performance degradation model is derived to evaluate degradation effect coupled with load-sharing effect, and then the degraded component performance is estimated considering the load-sharing effect on degradation rate. The case of a solar panel is a typical load-sharing K-out-of-N system with degrading components. The results calculated by the proposed method show that the reliability considering component degradation is less than that ignoring component degradation. With utilization of the proposed method, the degradation effect is quantitatively evaluated, and then the reliability of load-sharing K-out-of-N system can be calculated more accurately.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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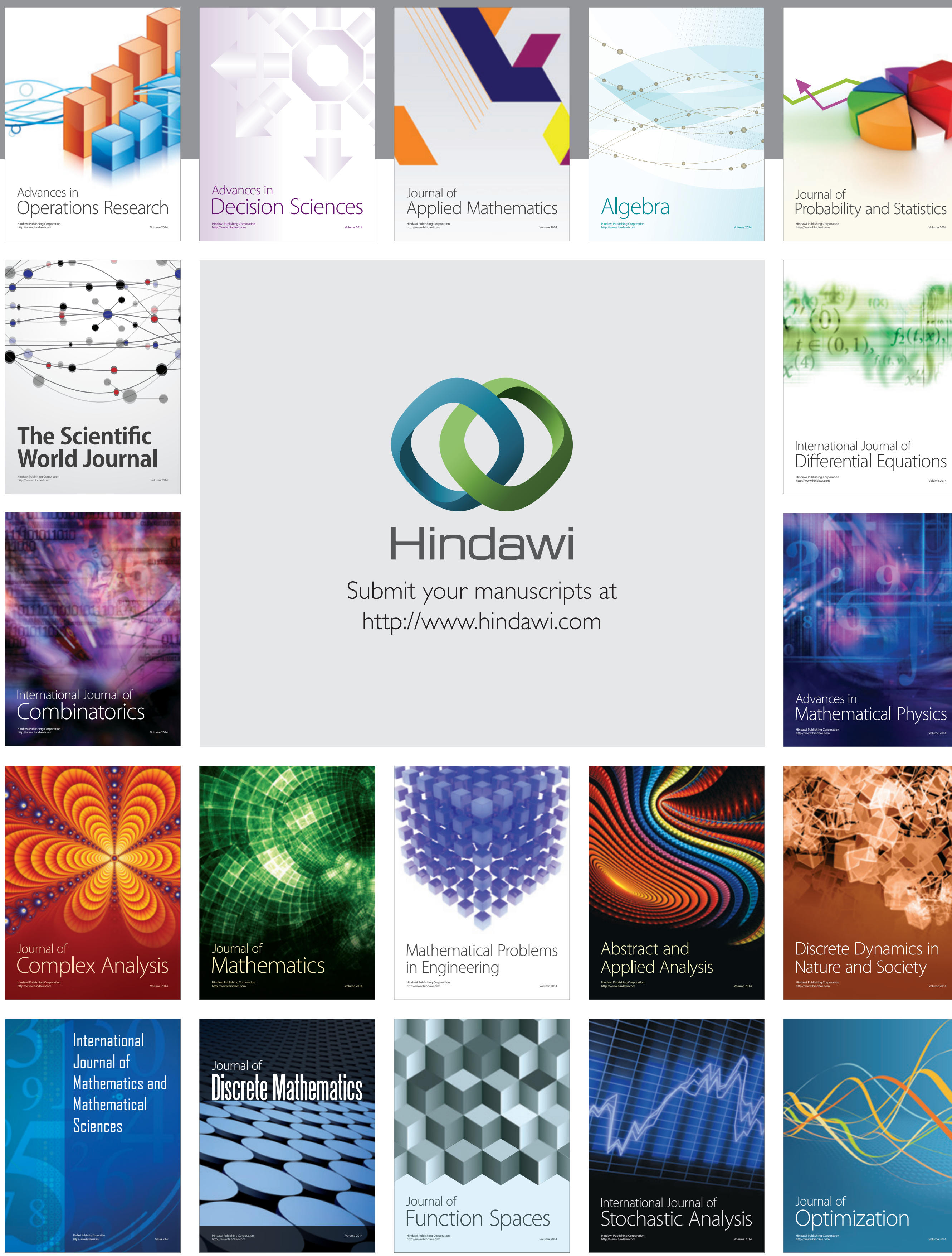
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