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Research Article

Robust H_∞ Filtering for Networked Control Systems with Random Sensor Delay

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The robust H_∞ filtering problem for a class of network-based systems with random sensor delay is investigated. The sensor delay is supposed to be a stochastic variable satisfying Bernoulli binary distribution. Using the Lyapunov function and Wirtinger's inequality approach, the sufficient conditions are derived to ensure that the filtering error systems are exponentially stable with a prescribed H_∞ disturbance attenuation level and the filter design method is proposed in terms of linear matrix inequalities. The effectiveness of the proposed method is illustrated by a numerical example.

1. Introduction

Networked control systems (NCSs) which are new control systems where sensor-controller and controller-actuator signal link is through a real time network [1]. Because of the advantages, such as convenient fault diagnosis, low cost, and simplicity, the NCSs have been widely applied in many application areas such as industrial automation, remote process, and manufacturing plants. However, the insertion of the communication network may cause time delay, so the signal transferred in NCSs loses the stationary, integrity, and determinacy, which makes the analysis of NCSs become complicate. Therefore, increasing attention has been paid to the study of networked control systems (see, e.g., [2–6] and references therein).

On the other hand, the filtering problem for NCSs has attracted constant research [7–11] since it is important in control engineering and signal processing. In [7], the H_∞ filtering for NCSs with multiple packet dropouts is considered. The problem of designing H_∞ filter design for a class of discrete nonlinear NCSs with stochastic time-varying delays and missing measurements is addressed in [9], where sector nonlinearities and parameter uncertainties are also studied. In [10], by using a stochastic sampled-data approach, the problem of distributed H_∞ filtering in sensor networks is considered. And distributed average filtering for sensor networks with sensor saturation is designed by averagely fusing the information of each local node in [11]. However, there are few literatures to analyze the problem of H_∞ filtering for continuous-time NCSs with random sensor delay, which motivates the present study.

In this paper, a delay-dependent H_∞ performance analysis result is derived for the filtering error system and a new random sensor delay model with stochastic parameter matrix is proposed. Combining the reciprocally convex combination technique in [12] and employing Wirtinger's inequality approach, new criteria are derived for H_∞ performance analysis, which reduces the conservatism. Based on the derived criteria for H_∞ performance analysis, the novel H_∞ filter criteria are obtained in terms of LMIs. Finally, a numerical example is presented to show the effectiveness of the proposed approach.

2. Problem Description

Consider the following networked control systems:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bw(t), \\ y(t) &= Cx(t), \\ z(t) &= Lx(t), \end{aligned} \tag{1}$$

where $x(t) \in R^n$ and $y(t) \in R^m$ are the state and measurable output vector, respectively. $z(t) \in R^q$ is the signal to be estimated and $w(t) \in R^p$ is the external disturbance signal belonging to $L_2[0,\infty]$. A, B, C, and L are known matrices with appropriate dimensions.

Consider the following filter for the estimation of $z(t)$:

$$\begin{aligned} \dot{x}_f(t) &= A_f x_f(t) + B_f \tilde{y}(t), \\ z_f(t) &= L_f x_f(t), \end{aligned} \tag{2}$$

where $x_f(t) \in R^n$ and $\tilde{y}(t) \in R^m$ are the filter's state and input vector, respectively. $z_f(t) \in R^q$ is the estimated output. A_f, B_f, and L_f are the filter matrices to be designed.

In the actual networked control systems, the measured output $\tilde{y}(t) \in R^m$ may or may not experience sensor delay, which can be described by two random events:

$$\begin{aligned} &\text{Event 1: } y(t) \text{ does not experience sensor delay,} \\ &\text{Event 2: } y(t) \text{ experiences sensor delay.} \end{aligned} \tag{3}$$

Assume that the occurrences probability of the above given event can be described as the following formula:

$$\begin{aligned} P\{\text{Event 1}\} &= \upsilon_0, \\ P\{\text{Event 2}\} &= 1 - \upsilon_0. \end{aligned} \tag{4}$$

Define a stochastic variable $\upsilon(t)$:

$$\upsilon(t) = \begin{cases} 1, & \text{if Event 1 occurs,} \\ 0, & \text{if Event 2 occurs.} \end{cases} \tag{5}$$

By using Bernoulli distributed sequence, the variable $\upsilon(t)$ can be assumed to follow an exponential distribution of switching, which satisfies

$$\begin{aligned} P\{\upsilon(t) = 1\} &= E\{\upsilon(t)\} = \upsilon_0, \\ P\{\upsilon(t) = 0\} &= E\{1 - \upsilon(t)\} = 1 - \upsilon_0, \end{aligned} \tag{6}$$

where υ_0 is a known constant on $[0,1]$. Considering the random sensor delay, we suppose that the corresponding measurement is defined as

$$\tilde{y}(t) = \upsilon_0 y(t) + (1 - \upsilon_0) y(t - \tau(t)), \tag{7}$$

where $\tau(t)$ stands for time-varying delay which satisfies $\tau_m \leqslant \tau(t) \leqslant \tau_M$.

Define $\xi(t) = [x^T(t) \quad x_f^T(t)]^T$ and $e(t) = z(t) - z_f(t)$; then the filtering error system can be described as follows:

$$\begin{aligned} \dot{\xi}(t) &= \tilde{A}\xi(t) + \upsilon_0 \tilde{B}_d \xi(t) + (1 - \upsilon_0) \tilde{A}_d E \xi(t - \tau(t)) \\ &\quad + \tilde{B} w(t), \\ e(t) &= \tilde{L}\xi(t), \end{aligned} \tag{8}$$

where

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}, \qquad \tilde{B}_d = \begin{bmatrix} 0 & 0 \\ B_f C & 0 \end{bmatrix}, \\ \tilde{B} &= \begin{bmatrix} B \\ 0 \end{bmatrix}, \qquad E = \begin{bmatrix} I & 0 \end{bmatrix}, \\ \tilde{A}_d &= \begin{bmatrix} 0 \\ B_f C \end{bmatrix}, \qquad \tilde{L} = \begin{bmatrix} L & L_f \end{bmatrix}. \end{aligned} \tag{9}$$

Our aim in this paper is to design a robust H_∞ filter in the form of (8) such that

(1) system (1) is robustly exponentially stable, subject to $w(t) = 0$,

(2) under zero initial condition and for the disturbance attenuation level γ, the controlled output $e(t)$ satisfies $\|e(t)\|_2 \leqslant r\|w(t)\|_2$ for $w(t) \in L_2[0,\infty]$, $w(t) \neq 0$.

Throughout this paper, we use the following lemmas.

Lemma 1 (see [13]). *For any positive matrix R, and for differentiable signal x in $[\alpha,\beta] \rightarrow R^n$, the following inequality holds:*

$$\int_\alpha^\beta \dot{x}^T(u) R \dot{x}(u)\, du \geqslant \frac{1}{\beta - \alpha} \begin{bmatrix} x(\beta) \\ x(\alpha) \\ \chi \end{bmatrix}^T W(R) \begin{bmatrix} x(\beta) \\ x(\alpha) \\ \chi \end{bmatrix}, \tag{10}$$

where

$$\chi = \frac{1}{\beta - \alpha} \int_\alpha^\beta w(u)\, du, \tag{11}$$

$$W(R) = \begin{bmatrix} R & -R & 0 \\ * & R & 0 \\ * & * & 0 \end{bmatrix} + \frac{\pi^2}{4} \begin{bmatrix} R & R & -2R \\ * & R & -2R \\ * & * & 4R \end{bmatrix}. \tag{12}$$

Lemma 2 (see [14]). *For any positive matrix $M > 0$, scalar $r > 0$, and a vector function $w : [0,r] \rightarrow R^n$ such that the integration $\int_0^r w(s)^T M w(s) ds$ is well defined, then*

$$r\left(\int_0^r w(s)^T M w(s)\, ds\right) \geqslant \left(\int_0^r w(s)\, ds\right)^T M \left(\int_0^r w(s)\, ds\right). \tag{13}$$

Lemma 3 (see [12]). *Let $F_1, F_2, F_3, \ldots, F_N : R^m \mapsto R$ have positive values for arbitrary value of independent variable in an open subset W of R^m. The reciprocally convex combination of F_i $(i = 1, 2, \ldots, N)$ in W satisfies*

$$\begin{aligned} \min \quad & \sum_{i=1}^{l} \frac{1}{\eta_i} F_i(t) = \sum_{i=1}^{l} F_i(t) + \max \sum_{i=1}^{l} \sum_{j=1, j\neq i}^{l} W_{i,j}(t) \\ \textit{subject to} \quad & \left\{ \eta_i > 0, \sum_{i=1}^{N} \eta_i = 1, W_{i,j}(t) : R^m \longmapsto R, \right. \\ & \left. W_{j,i}(t) = W_{i,j}(t), \begin{bmatrix} F_i(t) & W_{i,j}(t) \\ * & F_j(t) \end{bmatrix} \geqslant 0 \right\}. \end{aligned} \tag{14}$$

3. Main Results

In this section, a H_∞ performance condition for the filtering error system (8) and the robust H_∞ filter design for the system (1) are presented, respectively.

3.1. Performance Analysis of H_∞ Filter

Theorem 4. *Defining $\tau_1 = \tau_m, \tau_2 = (\tau_m+\tau_M)/2, \tau_3 = \tau_M$, and $\delta = \tau_M - \tau_m$, for given positive scalars $0 \leqslant \tau_m < \tau_M$, the filtering error system (8) is robustly exponentially stable with a H_∞ norm bound γ if there exist positive matrices $P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix} > 0$, $R = \begin{bmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{bmatrix} > 0$, $Q_i > 0$ $(i = 1,2,3)$, $S_j > 0$ $(j = 1,2)$, and proper dimensions matric Z_{12} such that*

$$\Omega = \begin{bmatrix} \Omega_1 & \tau_2\Psi^T S_1 & \delta\Psi^T S_2 & \Theta^T \\ * & -S_1 & 0 & 0 \\ * & * & -S_2 & 0 \\ * & * & * & -I \end{bmatrix} < 0, \qquad (15)$$

$$\begin{bmatrix} S_2 & Z_{12} \\ * & S_2 \end{bmatrix} > 0,$$

with

$$\Omega_1 = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & (1-v_0)P_2B_fC & 0 & 0 & \Omega_{18} \\ * & \Omega_{22} & 0 & 0 & (1-v_0)P_3B_fC & 0 & 0 & P_2^TB \\ * & * & \Omega_{33} & \Omega_{34} & 0 & 0 & 0 & 0 \\ * & * & * & -\dfrac{\pi^2}{\tau_2^2}S_1 & 0 & 0 & 0 & R_{12}^TB \\ * & * & * & * & \Omega_{55} & \Omega_{56} & \Omega_{57} & 0 \\ * & * & * & * & * & \Omega_{66} & Z_{12} & 0 \\ * & * & * & * & * & * & \Omega_{77} & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}, \qquad (16)$$

where

$$\begin{aligned}
\Omega_{11} &= A^TP_1 + P_1A + A^TR_{11} + R_{11}A + R_{12} + R_{12}^T \\
&\quad + Q_1 - S_1 - \frac{\pi^2}{4}S_1 + v_0P_2B_fC + v_0C^TB_f^TP_2, \\
\Omega_{12} &= A^TP_2 + P_2A_f + v_0C^TB_f^TP_3, \\
\Omega_{13} &= -R_{12} + S_1 - \frac{\pi^2}{4}S_1, \\
\Omega_{14} &= A^TR_{12} + R_{22} + \frac{\pi^2}{2\tau_2}S_1, \\
\Omega_{18} &= P_1B + R_{11}^TB, \\
\Omega_{22} &= P_3A_f + A_f^TP_3, \\
\Omega_{33} &= Q_3 - Q_2 - S_1 - \frac{\pi^2}{4}S_1, \\
\Omega_{34} &= -R_{22} + \frac{\pi^2}{2\tau_2}S_1, \\
\Omega_{55} &= -S_2 - S_2^T + Z_{12} + Z_{12}^T, \\
\Omega_{56} &= S_2 - Z_{12}, \\
\Omega_{57} &= S_2 - Z_{12}^T, \\
\Omega_{66} &= -Q_1 + Q_2 - S_2, \\
\Omega_{77} &= -Q_3 - S_2,
\end{aligned}$$

$$\begin{aligned}
\Theta &= \begin{bmatrix} L & -L_f & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Psi &= \begin{bmatrix} A & 0 & 0 & 0 & 0 & 0 & 0 & B \end{bmatrix}.
\end{aligned} \qquad (17)$$

Proof. Consider the Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}
V(x_t) &= \xi^T(t)P\xi(t) + \int_{t-\tau_1}^{t} x^T(s)Q_1x(s)\,ds \\
&\quad + \begin{bmatrix} x(t) \\ \int_{t-\tau_2}^{t} x(s)\,ds \end{bmatrix}^T R \begin{bmatrix} x(t) \\ \int_{t-\tau_2}^{t} x(s)\,ds \end{bmatrix} \\
&\quad + \int_{t-\tau_2}^{t-\tau_1} x^T(s)Q_2x(s)\,ds \\
&\quad + \int_{t-\tau_3}^{t-\tau_2} x^T(s)Q_3x(s)\,ds \\
&\quad + \tau_2\int_{t-\tau_2}^{t}\int_{s}^{t} \dot{x}^T(v)S_1\dot{x}(v)\,dv\,ds \\
&\quad + \delta\int_{t-\tau_3}^{t-\tau_1}\int_{s}^{t} \dot{x}^T(v)S_2\dot{x}(v)\,dv\,ds.
\end{aligned} \qquad (18)$$

Calculating the time derivative of $V(x_t)$ along the trajectory of (8) yields

$$\begin{aligned}
\dot{V}(x_t) &= 2\xi^T(t)P\dot{\xi}(t) + x^T(t)Q_1x(t) \\
&\quad - x^T(t-\tau_1)Q_1x(t-\tau_1)
\end{aligned}$$

$$+2\begin{bmatrix}\dot{x}(t)\\ x(t)-x(t-\tau_2)\end{bmatrix}^T R\begin{bmatrix}x(t)\\ \int_{t-\tau_2}^{t} x(t)\end{bmatrix}$$
$$+x^T(t-\tau_1)Q_2x(t-\tau_1)$$
$$-x^T(t-\tau_2)Q_2x(t-\tau_2)$$
$$+x^T(t-\tau_2)Q_3x(t-\tau_2)$$
$$-x^T(t-\tau_3)Q_3x(t-\tau_3)$$
$$+\tau_2^2\dot{x}^T(t)S_1\dot{x}(t)+\delta^2\dot{x}^T(t)S_2\dot{x}(t)$$
$$-\tau_2\int_{t-\tau_2}^{t}\dot{x}^T(t)S_1\dot{x}(t)\,ds$$
$$-\delta\int_{t-\tau_3}^{t-\tau_1}\dot{x}^T(t)S_2\dot{x}(t)\,ds. \quad (19)$$

By utilizing Lemma 1, the integral term $-\tau_2\int_{t-\tau_2}^{t}\dot{x}^T(s)S_1\dot{x}(s)ds$ can be estimated as

$$-\tau_2\int_{t-\tau_2}^{t}\dot{x}^T(s)S_1\dot{x}(s)\,ds \leqslant \begin{bmatrix}x(t)\\ x(t-\tau_2)\\ \frac{1}{\tau_2}\int_{t-\tau_2}^{t}x(s)\,ds\end{bmatrix}^T W(S_1)\begin{bmatrix}x(t)\\ x(t-\tau_2)\\ \frac{1}{\tau_2}\int_{t-\tau_2}^{t}x(s)\,ds\end{bmatrix}, \quad (20)$$

where

$$W(S_1)=-\begin{bmatrix}S_1 & -S_1 & 0\\ * & S_1 & 0\\ * & * & 0\end{bmatrix}-\frac{\pi^2}{4}\begin{bmatrix}S_1 & S_1 & -2S_1\\ * & S_1 & -2S_1\\ * & * & 4S_1\end{bmatrix}. \quad (21)$$

On the other hand, defining $\alpha=(\tau(t)-\tau_1)/\delta$ and $\beta=(\tau_3-\tau(t))/\delta$, by the reciprocally convex combination in Lemma 3, the following inequality holds:

$$-\begin{bmatrix}\sqrt{\frac{\beta}{\alpha}}(x(t-\tau_1)-x(t-\tau(t)))\\ -\sqrt{\frac{\alpha}{\beta}}(x(t-\tau(t))-x(t-\tau_3))\end{bmatrix}^T\begin{bmatrix}S_2 & Z_{12}\\ * & S_2\end{bmatrix}\times\begin{bmatrix}\sqrt{\frac{\beta}{\alpha}}(x(t-\tau_1)-x(t-\tau(t)))\\ -\sqrt{\frac{\alpha}{\beta}}(x(t-\tau(t))-x(t-\tau_3))\end{bmatrix}<0. \quad (22)$$

Note that due to $\tau_1\leqslant\tau(t)\leqslant\tau_3$, according to Lemma 2 and inequalities (22), we have

$$\begin{aligned}-\delta\int_{t-\tau_3}^{t-\tau_1}\dot{x}^T(s)S_2\dot{x}(s)\,ds &= -\delta\int_{t-\tau(t)}^{t-\tau_1}\dot{x}^T(s)S_2\dot{x}(s)\,ds - \delta\int_{t-\tau_3}^{t-\tau(t)}\dot{x}^T(s)S_2\dot{x}(s)\,ds\\
&\leqslant -\frac{\delta}{\tau(t)-\tau_1}\begin{bmatrix}x(t-\tau_1)\\ x(t-\tau(t))\end{bmatrix}^T\begin{bmatrix}S_2 & -S_2\\ * & S_2\end{bmatrix}\times\begin{bmatrix}x(t-\tau_1)\\ x(t-\tau(t))\end{bmatrix}-\frac{\delta}{\tau_3-\tau(t)}\begin{bmatrix}x(t-\tau(t))\\ x(t-\tau_3)\end{bmatrix}^T\\
&\quad\times\begin{bmatrix}Z_2 & -Z_2\\ * & Z_2\end{bmatrix}\begin{bmatrix}x(t-\tau(t))\\ x(t-\tau_3)\end{bmatrix}\\
&\leqslant -\begin{bmatrix}x(t-\tau_1)-x(t-\tau(t))\\ x(t-\tau(t))-x(t-\tau_3)\end{bmatrix}^T\begin{bmatrix}S_2 & Z_{12}\\ * & S_2\end{bmatrix}\times\begin{bmatrix}x(t-\tau_1)-x(t-\tau(t))\\ x(t-\tau(t))-x(t-\tau_3)\end{bmatrix}\\
&=\eta^T(t)\begin{bmatrix}-2S_2+Z_{12}+Z_{12}^T & S_2-Z_{12} & S_2-Z_{12}^T\\ * & -S_2 & Z_{12}\\ * & * & -S_2\end{bmatrix}\eta(t),\end{aligned} \quad (23)$$

where

$$\eta^T(t)=\begin{bmatrix}x^T(t-\tau(t)) & x^T(t-\tau_1) & x^T(t-\tau_3)\end{bmatrix}. \quad (24)$$

Substituting (20)–(23) into (19) and then applying the Schur complement, it can be concluded that

$$\dot{V}(x_t)+e^T(t)e(t)-\gamma^2w^T(t)w(t) \leqslant \zeta^T(t)\left(\Omega_1+\delta^2\Psi^TS_1\Psi+\tau_2^2\Psi^TS_1\Psi+\Theta^T\Theta\right)\zeta(t), \quad (25)$$

where

$$\zeta^T(t)=\Big[\, x^T(t)\ \ x_f^T(t)\ \ x^T(t-\tau_2)\ \ \int_{t-\tau_2}^{t}x(s)\,ds\ \ x^T(t-\tau)\ \ x^T(t-\tau_1)\ \ x^T(t-\tau_3)\ \ w^T(t)\,\Big]. \quad (26)$$

If (25) holds, we have

$$\dot{V}(x_t)+e^T(t)e(t)-\gamma^2w^T(t)w(t)<0. \quad (27)$$

Carrying out integral manipulations on (27) from 0 to ∞ and noting that $V(x_t)|_{t=0}=0$ under zero initial conditions, we obtain

$$\int_0^\infty e^T(s)e(s)\,ds-\int_0^\infty\gamma^2w(s)w(s)\,ds < V(x_t)\big|_{t\to 0}-V(x_t)\big|_{t\to\infty}<0. \quad (28)$$

That is, $\|e(t)\|_2 \leqslant \gamma\|w(t)\|_2$, so the filtering error system has an H_∞ disturbance attenuation level γ under zero initial conditions.

Second, we also can prove that the filtering error system with $w(t) = 0$ is robustly exponentially stable under the condition of Theorem 4. This completes the proof. □

Remark 5. Similar to [15], we divide the delay interval into two subintervals uniformly. However, the new Lyapunov-Krasovskii functional in our paper which not only divides the delay interval into two subintervals but also makes use of the information of $\int_{t-\tau_2}^{t} x(s)ds$ is proposed. The results will be less conservative.

3.2. Design of H_∞ Filter

Theorem 6. *Defining $\tau_1 = \tau_m, \tau_2 = ((\tau_m + \tau_M)/2), \tau_3 = \tau_M$, and $\delta = \tau_M - \tau_m$, for some given constants $0 \leqslant \tau_m < \tau_M, v_0$, and γ, the filtering error system (8) is robustly exponentially stable with a H_∞ norm bound γ if there exist positive matrices $P_1 > 0, X > 0, R = \begin{bmatrix} R_{11} & R_{12} \\ R_{12}^T & R_{22} \end{bmatrix} > 0, Q_i > 0$ $(i = 1, 2, 3)$ and $S_j > 0$ $(j = 1, 2)$ and matrices $\overline{A}_f, \overline{B}_f, \overline{L}_f$, and Z_{12} of appropriate dimensions such that the following LMIs are satisfied:*

$$\widetilde{\Omega} = \begin{bmatrix} \widetilde{\Omega}_1 & \tau_2\Psi^T S_1 & \delta\Psi^T S_2 & \widetilde{\Theta}^T \\ * & -S_1 & 0 & 0 \\ * & * & -S_2 & 0 \\ * & * & * & -I \end{bmatrix} < 0,$$
$$\begin{bmatrix} S_2 & Z_{12} \\ * & S_2 \end{bmatrix} > 0,$$
$$X - P_1 < 0, \tag{29}$$

with

$$\widetilde{\Omega}_1 = \begin{bmatrix} \widetilde{\Omega}_{11} & \widetilde{\Omega}_{12} & \Omega_{13} & \Omega_{14} & (1-v_0)\overline{B}_f C & 0 & 0 & \Omega_{18} \\ * & \widetilde{\Omega}_{22} & 0 & 0 & (1-v_0)\overline{B}_f C & 0 & 0 & XB \\ * & * & \Omega_{33} & \Omega_{34} & 0 & 0 & 0 & 0 \\ * & * & * & -\dfrac{\pi^2}{\tau_2^2}S_1 & 0 & 0 & 0 & R_{12}^T B \\ * & * & * & * & \Omega_{55} & \Omega_{56} & \Omega_{57} & 0 \\ * & * & * & * & * & \Omega_{66} & Z_{12} & 0 \\ * & * & * & * & * & * & \Omega_{77} & 0 \\ * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix}, \tag{30}$$

where

$$\widetilde{\Omega}_{11} = A^T P_1 + P_1 A + A^T R_{11} + R_{11} A + R_{12} + R_{12}^T + Q_1 - S_1 - \frac{\pi^2}{4}S_1 + v_0\overline{B}_f C + v_0 C^T \overline{B}_f^T,$$
$$\widetilde{\Omega}_{12} = A^T X + \overline{A}_f + v_0 C^T \overline{B}_f^T,$$
$$\widetilde{\Omega}_{22} = \overline{A}_f + \overline{A}_f^T,$$
$$\widetilde{\Theta} = \begin{bmatrix} L & -\overline{L}_f & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{31}$$

Then, the H_∞ filtering problem is solvable. Moreover, the parameter matrices of the filter are given by

$$A_f = \overline{A}_f X^{-1}, \quad B_f = \overline{B}_f, \quad L_f = \overline{L}_f X^{-1}. \tag{32}$$

Proof. Defining

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}, \qquad X = P_2 P_3^{-1} P_2^T,$$
$$J = \operatorname{diag}\left\{I, P_2 P_3^{-1}, I, I, \ldots, I\right\}. \tag{33}$$

Applying the Schur complement, it can be concluded that $P > 0$ is equivalent to

$$P_1 - P_2 P_3^{-1} P_2^T = P_1 - X > 0. \tag{34}$$

Set $\overline{A}_f = P_2 A_f P_3^{-1} P_2^T$, $\overline{B}_f = P_2 B_f$, and $\overline{L}_f = L_f P_3^{-1} P_2^T$. Pre- and postmultiplying (15) by J^T and J give

$$J^T \Omega_1 J = \widetilde{\Omega}_1. \tag{35}$$

Thus we can conclude that the filtering error system is robustly exponentially stable with a H_∞ norm bound γ. The transfer function of the filter is defined as

$$T = L_f\left(sI - A_f\right)^{-1} B_f. \tag{36}$$

According to (32), we can get

$$\begin{aligned} T &= L_f\left(sI - A_f\right)^{-1} B_f \\ &= \overline{L}_f P_2^{-T} P_3\left(sI - P_2^{-1}\overline{A}_f P_2^{-T} P\right)^{-1} P_2^{-1}\overline{B}_f \\ &= \overline{L}_f\left(sX - \overline{A}_f\right)^{-1}\overline{B}_f \\ &= \overline{L}_f\left(sI - X^{-1}\overline{A}_f\right)^{-1} X^{-1}\overline{B}_f \\ &= \overline{L}_f X^{-1}\left(sI - \overline{A}_f X^{-1}\right)^{-1}\overline{B}_f. \end{aligned} \tag{37}$$

Therefore, the parameter matrices of the filter can be chosen as in (32). This completes the proof. □

Remark 7. According to LMIs (29), we can find that the variable numbers are fewer than Theorem 2 in [16]; therefore, the filter design method provides a more simple form.

4. Simulation Example

Example 1. Consider the system described by (1) with the following parameters in [16]:

$$A = \begin{bmatrix} 0.5 & 3 \\ -2 & -5 \end{bmatrix}, \qquad B = \begin{bmatrix} -0.5 \\ 0.9 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad L = \begin{bmatrix} 1 & 1 \end{bmatrix}. \tag{38}$$

Assume that $\tau(t)$ satisfies $0.01 \leqslant \tau(t) < 0.2$, $w(t) = 0.2\sin e^{-0.2t}$, and the measured output experiences sensor

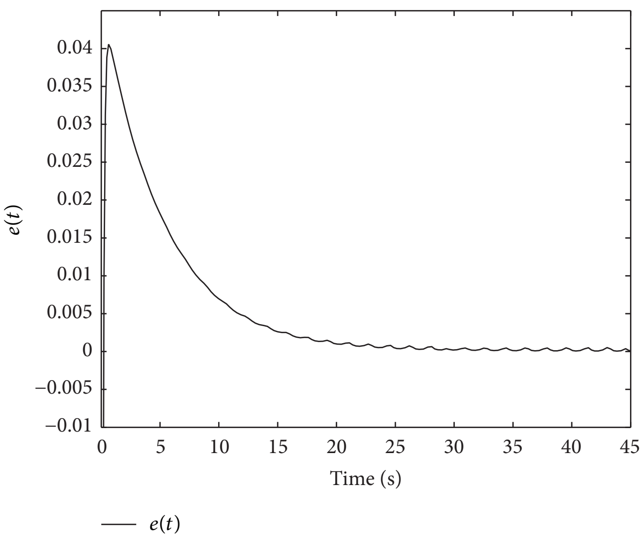

FIGURE 1: The error response $e(t) = z(t) - z_f(t)$.

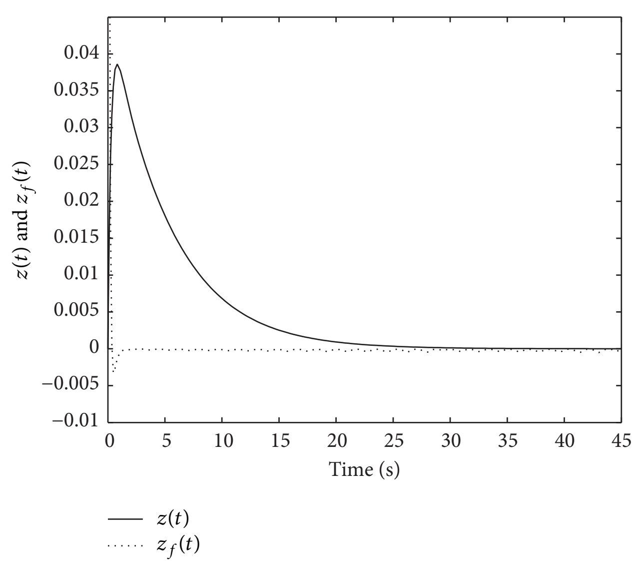

FIGURE 2: The output response of $z(t)$ and $z_f(t)$.

delay, that is, the sensor delay occurrences probability, $\upsilon_0 = 0.5$. The initial conditions $x(t)$ and $x_f(t)$ are $[0.2 \ \ -0.2]^T$ and $[0.03 \ \ -0.05]^T$, respectively. According to Theorem 6 with the help from Matlab LMI toolbox, it can be solved that the desired H_∞ filter parameters are as follows with the performance level $\gamma = 0.2143$:

$$A_f = \begin{bmatrix} -4.5071 & -3.8049 \\ -0.7971 & -9.9490 \end{bmatrix},$$

$$B_f = \begin{bmatrix} -0.0002 \\ -0.0078 \end{bmatrix}, \quad (39)$$

$$C_f = \begin{bmatrix} -1.7957 & -8.2983 \end{bmatrix}.$$

The simulation results are shown in Figures 1 and 2. Figure 1 shows the error response $e(t) = z(t) - z_f(t)$. The output $z(t)$ and $z_f(t)$ are depicted in Figure 2. All the simulations have confirmed that the designed H_∞ filter can stabilize the system (1) with random sensor delay.

5. Conclusion

In this paper, we have studied the network-based robust H_∞ filtering problem for continuous-time systems with random sensor delay. A novel Lyapunov-Krasovskii functional has been constructed to design a filter by means of LMIs, which guarantees a prescribed H_∞ disturbance rejection attenuation level for the filter error system. A numerical example has been provided to show the effectiveness of the proposed filter design method and the input or state delays in the systems should be further considered in the future work.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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