

Research Article

Scattering Analysis of Electromagnetic Materials Using Fast Dipole Method Based on Volume Integral Equation

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The fast dipole method (FDM) is extended to analyze the scattering of dielectric and magnetic materials by solving the volume integral equation (VIE). The FDM is based on the equivalent dipole method (EDM) and can achieve the separation of the field dipole and source dipole, which reduces the complexity of interactions between two far groups (such as group i and group j) from $O(N_i N_j)$ to $O(N_i + N_j)$, where N_i and N_j are the numbers of dipoles in group i and group j , respectively. Targets including left-handed materials (LHMs), which are a kind of dielectric and magnetic materials, are calculated to demonstrate the merits of the FDM. Furthermore, in this study we find that the convergence may become much slower when the targets include LHMs compared with conventional electromagnetic materials. Numerical results about convergence characteristics are presented to show this property.

1. Introduction

The electromagnetic materials have gained wide attention and interest for a long time. The method of moments (MoM) based on the volume integral equation (VIE) [1] is one of the most popular numerical methods to analyze the scattering of the electromagnetic materials. However, the conventional MoM needs $O(N^2)$ CPU time and memory requirement for an iterative solver. The complexity is very expensive for a personal computer with limited resources. Fortunately, many methods have been proposed in the past decades to deal with this problem, such as multilevel fast multipole algorithm (MLFMA) [2–4], the adaptive integral method (AIM) [5, 6], and the conjugate gradient-fast Fourier transform (CG-FFT) [7]. These methods improve the efficiency (computer storage and solution time) of the conventional MoM.

Recently, the equivalent dipole method (EDM) [8–10] is proposed to simplify the impedance matrix element filling procedure for the MoM, in which the Schaubert-Wilton-Glisson (SWG) [11] elements are equivalent to an infinitely small dipole. However, it cannot save memory and reduce iterative time. More recently, the fast dipole method (FDM) [12, 13] is developed to mitigate the problem of the EDM,

which can efficiently calculate the interactions between far groups. It reduces the complexity of interactions between two far groups, such as group i and group j , from $O(N_i N_j)$ to $O(N_i + N_j)$, where N_i and N_j are the numbers of the dipoles in groups i and j , respectively.

In this work, we extended the FDM to analyze the scattering of dielectric and magnetic materials by solving the volume integral equation (VIE). These materials not only contain the conventional materials but also contain the left-handed materials (LHMs) and LHM-RHM (right-handed material) composite materials. We find that it usually converges slowly when solving the targets involving LHMs.

2. Volumetric Integral Equations for Electromagnetic Materials

Consider an arbitrarily shaped 3D scattering electromagnetic object illuminated by a plane wave incident fields $(\mathbf{E}^i, \mathbf{H}^i)$, the permittivity and permeability of the object are characterized by $\epsilon(\mathbf{r})$ and $\mu(\mathbf{r})$. The background space surrounding the dielectric targets with region V is free space with $\epsilon_0 = \mu_0 = 1$. According to the equivalence principle and boundary

conditions, the total electromagnetic field (\mathbf{E}, \mathbf{H}) can be described as follows:

$$\begin{aligned}\mathbf{E} &= \mathbf{E}^i + \mathbf{E}^s(\mathbf{J}_v^e) + \mathbf{E}^s(\mathbf{J}_v^m), \\ \mathbf{H} &= \mathbf{H}^i + \mathbf{H}^s(\mathbf{J}_v^m) + \mathbf{H}^s(\mathbf{J}_v^e),\end{aligned}\quad (1)$$

where \mathbf{E}^s and \mathbf{H}^s denote the scattering electromagnetic fields. $\mathbf{J}_v^e(\mathbf{r})$ and $\mathbf{J}_v^m(\mathbf{r})$ denote equivalent volumetric electric and magnetic currents, respectively. $\mathbf{E}^s(\mathbf{J}_v^e)$, $\mathbf{E}^s(\mathbf{J}_v^m)$, $\mathbf{H}^s(\mathbf{J}_v^m)$, and $\mathbf{H}^s(\mathbf{J}_v^e)$ are the scattered electric field and magnetic field produced by the equivalent volume currents \mathbf{J}_v^e and \mathbf{J}_v^m and can be found in [10]. $\mathbf{J}_v^e(\mathbf{r})$ and $\mathbf{J}_v^m(\mathbf{r})$ can be expressed as

$$\begin{aligned}\mathbf{J}_v^e(\mathbf{r}) &= j\omega\kappa^e(\mathbf{r}) \cdot \mathbf{D}(\mathbf{r}), \\ \mathbf{J}_v^m(\mathbf{r}) &= j\omega\kappa^m(\mathbf{r}) \cdot \mathbf{B}(\mathbf{r}),\end{aligned}\quad (2)$$

where $\kappa^e(\mathbf{r})$ and $\kappa^m(\mathbf{r})$ are the contrast ratios, and

$$\begin{aligned}\kappa^e(\mathbf{r}) &= 1 - \varepsilon_r^{-1}(\mathbf{r}), \\ \kappa^m(\mathbf{r}) &= 1 - \mu_r^{-1}(\mathbf{r}).\end{aligned}\quad (3)$$

The region V is discretized into small tetrahedral elements. Then, the unknown volume currents \mathbf{J}_v^e and \mathbf{J}_v^m can be expanded by a set of SWG basis functions [11] as

$$\begin{aligned}\mathbf{J}_v^e(\mathbf{r}) &= \sum_{n=1}^{N_v} I_n^e \kappa_n^e \mathbf{f}_n(\mathbf{r}), \\ \mathbf{J}_v^m(\mathbf{r}) &= \eta \sum_{n=1}^{N_v} I_n^m \kappa_n^m \mathbf{f}_n(\mathbf{r}),\end{aligned}\quad (4)$$

where I_n^e and I_n^m are the unknown expansion coefficients for the electric and magnetic currents, respectively. κ_n^e and κ_n^m are the contrast ratios in the n th SWG basis function. \mathbf{f}_n denotes the n th volume basis function defined in two adjoining tetrahedrons in the volume meshes associated with common face n . N_v is the number of faces of the tetrahedral meshes. $\eta = \sqrt{\mu_0/\varepsilon_0}$ is the impedance of the free space. Using the Galerkin's method and testing the resultant integral equations with a set of testing functions, as a result, the integral equations are converted into a matrix equation, and can be written in the matrix form as

$$\begin{bmatrix} \mathbf{Z}^{EE} & \mathbf{Z}^{EM} \\ \mathbf{Z}^{ME} & \mathbf{Z}^{MM} \end{bmatrix} \begin{bmatrix} \mathbf{I}^E \\ \mathbf{I}^M \end{bmatrix} = \begin{bmatrix} \mathbf{V}^E \\ \mathbf{V}^M \end{bmatrix}.\quad (5)$$

The impedance elements of the submatrices \mathbf{Z}^{EE} , \mathbf{Z}^{EM} , \mathbf{Z}^{ME} , and \mathbf{Z}^{MM} are represented as

$$\begin{aligned}Z_{mn}^{ee} &= \langle \mathbf{f}_m, \mathbf{E} - \mathbf{E}^s(\mathbf{J}_{vn}^e) \rangle, \\ Z_{mn}^{em} &= \langle \mathbf{f}_m, -\mathbf{E}^s(\mathbf{J}_{vn}^m) \rangle, \\ Z_{mn}^{me} &= \langle \eta \mathbf{f}_m, -\mathbf{H}^s(\mathbf{J}_{vn}^e) \rangle, \\ Z_{mn}^{mm} &= \langle \eta \mathbf{f}_m, \mathbf{H} - \mathbf{H}^s(\mathbf{J}_{vn}^m) \rangle,\end{aligned}\quad (6)$$

where \mathbf{J}_{vn}^e and \mathbf{J}_{vn}^m denote equivalent volume electric and magnetic currents corresponding to the n th SWG element. The elements in voltage vectors \mathbf{V}^E and \mathbf{V}^M are

$$\begin{aligned}V_m^e &= \langle \mathbf{f}_m, \mathbf{E}^i \rangle, \\ V_m^m &= \langle \eta \mathbf{f}_m, \mathbf{H}^i \rangle.\end{aligned}\quad (7)$$

3. Fast Dipole Method for Electromagnetic Materials

The FDM is an efficient way to solve the VIE, which is based on the EDM. In the EDM, each SWG element can be approximated as an infinitely small dipole with an equivalent moment. Referring to [10], the n th volume electric dipole moment \mathbf{m}_n^e and magnetic dipole moment \mathbf{m}_n^m can be represented as

$$\begin{aligned}\mathbf{m}_n^e &= a_n \kappa_n^{e-} (\mathbf{r}_n^{c-} - \mathbf{r}_{ns}^c) + a_n \kappa_n^{e+} (\mathbf{r}_{ns}^c - \mathbf{r}_n^{c+}), \\ \mathbf{m}_n^m &= a_n \kappa_n^{m-} (\mathbf{r}_n^{c-} - \mathbf{r}_{ns}^c) + a_n \kappa_n^{m+} (\mathbf{r}_{ns}^c - \mathbf{r}_n^{c+}),\end{aligned}\quad (8)$$

where $\mathbf{r}_n^{c\pm}$ is the position vector of the centroid of T_n^{\pm} and \mathbf{r}_{ns}^c is the position vector of the centroid of the common face of the T_n^{\pm} . a_n is the area of the n th common face associated with T_n^{\pm} . $\kappa_n^{e\pm}$ and $\kappa_n^{m\pm}$ stand for the contrast ratio of T_n^{\pm} . The impedance matrix elements in (6) can be calculated by the EDM:

$$\begin{aligned}Z_{mn}^{uu} &= jk\eta \mathbf{m}_m' \cdot \overline{\mathbf{G}}(\mathbf{R}) \cdot \mathbf{m}_n^u, \quad u = e, m, \\ Z_{mn}^{me} &= jk\eta \mathbf{m}_m' \cdot \mathbf{G}(\mathbf{R}) \times \mathbf{m}_n^e, \\ Z_{mn}^{em} &= -jk\eta \mathbf{m}_m' \cdot \mathbf{G}(\mathbf{R}) \times \mathbf{m}_n^m,\end{aligned}\quad (9)$$

where

$$\overline{\mathbf{G}}(\mathbf{R}) = \frac{e^{-jkR}}{4\pi R} \left[\overline{\mathbf{I}} \left(1 + \frac{1}{jkR} + \frac{1}{(jkR)^2} \right) - \widehat{\mathbf{R}}\widehat{\mathbf{R}} \left(1 + \frac{3}{jkR} + \frac{3}{(jkR)^2} \right) \right],\quad (10)$$

$$\mathbf{G}(\mathbf{R}) = \frac{e^{-jkR}}{4\pi R} \left(1 + \frac{1}{jkR} \right) \widehat{\mathbf{R}},\quad (11)$$

$$\mathbf{m}_m' = a_m (\mathbf{r}_m^{c-} - \mathbf{r}_m^{c+}).\quad (12)$$

In (10)–(12), $\mathbf{R} = \mathbf{r}_m - \mathbf{r}_n$, $R = |\mathbf{R}|$, and $\widehat{\mathbf{R}} = \mathbf{R}/R$. In addition, $\mathbf{r}_n = (\mathbf{r}_n^{c-} + \mathbf{r}_n^{c+})/2$ and $\mathbf{r}_m = (\mathbf{r}_m^{c-} + \mathbf{r}_m^{c+})/2$. $\mathbf{r}_n^{c\pm}$ ($\mathbf{r}_m^{c\pm}$) is the position vector of the centroid of T_n^{\pm} (T_m^{\pm}). According to [8–10], the EDM can be applied when the distance between the source and the field dipole is greater than $0.15 \lambda_g$ (λ_g is the wavelength in dielectric). It should be noted that λ_g is the maximum wavelength in dielectric when the medium is inhomogeneous.

In [12, 13], the FDM is developed to efficiently calculate the interactions between far groups. In order to describe how the FDM works for dielectric and magnetic materials, we

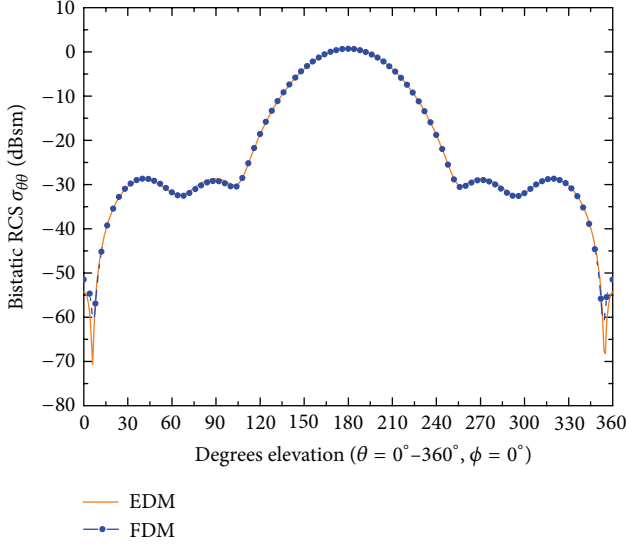


FIGURE 1: Bistatic RCSs ($\theta\theta$ polarization) of a thin plate with $1.0 \text{ m} \times 1.0 \text{ m} \times 0.05 \text{ m}$ and $\epsilon_r = 2, \mu_r = 2$ illuminated by a plane wave with the incident direction of $(\theta, \phi) = (0^\circ, 0^\circ)$ at 0.3 GHz.

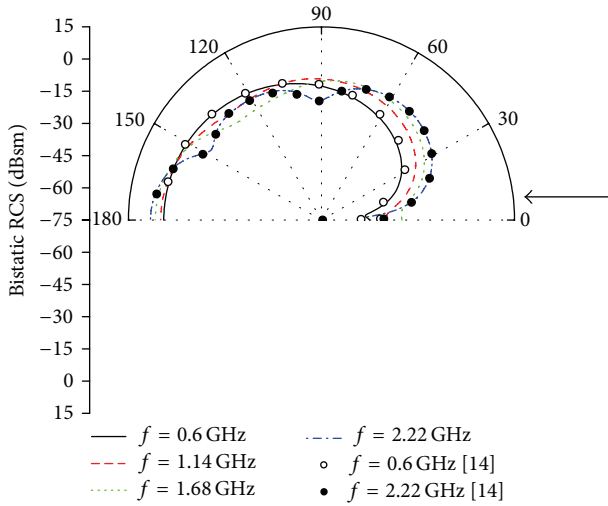


FIGURE 2: Bistatic RCS (E-plane) of an LHM sphere with radius 10 cm and $\epsilon_r = \mu_r = -1 - j0.001$ operating at frequencies of 0.6 GHz, 1.14 GHz, 1.68 GHz, and 2.22 GHz. The arrow indicates the incidence direction of a plane wave.

consider two dipoles m and n , which belong to groups j and i , respectively and suppose the two groups are a far group pair. The distance between the two dipoles can be written as $\mathbf{R} = \mathbf{r}_m - \mathbf{r}_n = \mathbf{r}_{ji} + \mathbf{r}_{mj} + \mathbf{r}_{ni}$, where $\mathbf{r}_{ji} = \mathbf{r}_{o_j} - \mathbf{r}_{o_i}$, $\mathbf{r}_{mj} = \mathbf{r}_m - \mathbf{r}_{o_j}$, and $\mathbf{r}_{ni} = \mathbf{r}_n - \mathbf{r}_{o_i}$, \mathbf{r}_{o_j} and \mathbf{r}_{o_i} is the center vectors of the

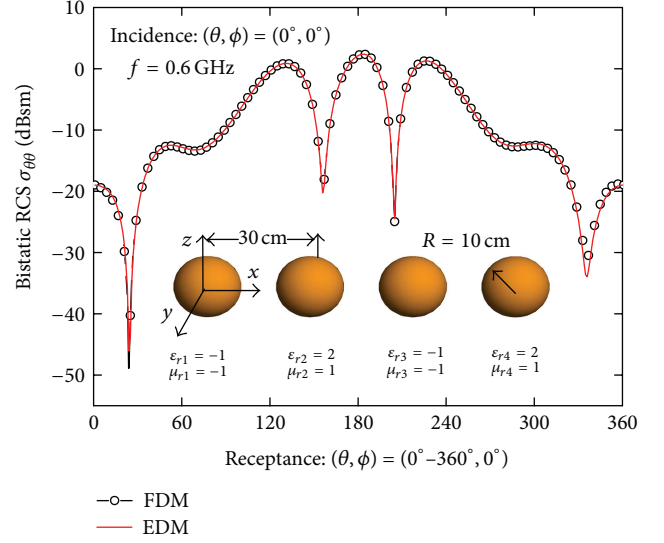


FIGURE 3: Bistatic RCS of a sphere array with $\epsilon_{r1} = -1, \mu_{r1} = -1, \epsilon_{r2} = 2, \mu_{r2} = 1, \epsilon_{r3} = -1, \mu_{r3} = -1, \epsilon_{r4} = 2,$ and $\mu_{r4} = 1$ and performed at 0.6 GHz excited by a plane wave with the incident direction of $(\theta, \phi) = (0^\circ, 0^\circ)$.

groups. According to [13], R^α in the formulae (10) and (11) can be approximated using Taylor series expansion:

$$\begin{aligned} R^\alpha &= |\mathbf{r}_{ji} + \mathbf{r}_{mj} + \mathbf{r}_{ni}|^\alpha \\ &= r_{ji}^\alpha \left[1 + \left(\frac{2\hat{\mathbf{r}}_{ji} \cdot \mathbf{r}_{mj}}{r_{ji}} + \frac{r_{mj}^2}{r_{ji}^2} \right) + \left(\frac{2\hat{\mathbf{r}}_{ij} \cdot \mathbf{r}_{ni}}{r_{ij}} + \frac{r_{ni}^2}{r_{ij}^2} \right) \right. \\ &\quad \left. - \frac{2\mathbf{r}_{mj} \cdot \mathbf{r}_{ni}}{r_{ji}^2} \right]^{\alpha/2} \\ &\approx R_m^{(\alpha)} + R_n^{(\alpha)}, \end{aligned} \quad (13)$$

where

$$\begin{aligned} R_m^{(\alpha)} &= r_{ji}^\alpha \left[\frac{1}{2} + \alpha \left(\frac{\hat{\mathbf{r}}_{ji} \cdot \mathbf{r}_{mj}}{r_{ji}} + \frac{r_{mj}^2}{r_{ji}^2} + \frac{(\alpha-2)(\hat{\mathbf{r}}_{ji} \cdot \mathbf{r}_{mj})^2}{2r_{ji}^2} \right) \right] \\ R_n^{(\alpha)} &= r_{ij}^\alpha \left[\frac{1}{2} + \alpha \left(\frac{\hat{\mathbf{r}}_{ij} \cdot \mathbf{r}_{ni}}{r_{ij}} + \frac{r_{ni}^2}{r_{ij}^2} + \frac{(\alpha-2)(\hat{\mathbf{r}}_{ij} \cdot \mathbf{r}_{ni})^2}{2r_{ij}^2} \right) \right]. \end{aligned} \quad (14)$$

And $\widehat{\mathbf{R}\mathbf{R}}$ can be approximated as

$$\widehat{\mathbf{R}\mathbf{R}} \approx \overline{\mathbf{T}}_m + \overline{\mathbf{T}}_n, \quad (15)$$

where

$$\begin{aligned}\bar{\mathbf{T}}_m &= \frac{1}{r_{ji}^2} \left[\frac{1}{2} \mathbf{r}_{ji} \mathbf{r}_{ji} + \mathbf{s}_m \mathbf{r}_{ji} + \mathbf{r}_{ji} \mathbf{s}_m \right] \\ \bar{\mathbf{T}}_n &= \frac{1}{r_{ij}^2} \left[\frac{1}{2} \mathbf{r}_{ij} \mathbf{r}_{ij} + \mathbf{s}_n \mathbf{r}_{ij} + \mathbf{r}_{ij} \mathbf{s}_n \right] \\ \mathbf{s}_m &= \mathbf{r}_{mj} - \mathbf{r}_{mj} \cdot \hat{\mathbf{r}}_{ji} \hat{\mathbf{r}}_{ji} \\ \mathbf{s}_n &= \mathbf{r}_{ni} - \mathbf{r}_{ni} \cdot \hat{\mathbf{r}}_{ij} \hat{\mathbf{r}}_{ij}.\end{aligned}\quad (16)$$

Then substituting (13) and (15) into (9), the formulae of FDM for electromagnetic materials can be obtained as follows:

$$\begin{aligned}Z_{mn}^{uu} &\approx \frac{jk\eta}{4\pi} \mathbf{M}'_m \cdot \left[\bar{\mathbf{I}} (A_m + A_n) \right. \\ &\quad \left. - (\bar{\mathbf{T}}_m + \bar{\mathbf{T}}_n) (B_m + B_n) \right] \cdot \mathbf{M}_n^u \\ &= \frac{jk\eta}{4\pi} \left[(A_m \mathbf{M}'_m - B_m \mathbf{M}'_m \cdot \bar{\mathbf{T}}_m) \cdot \mathbf{M}_n^u \right. \\ &\quad \left. + \mathbf{M}'_m \cdot (A_n \mathbf{M}_n^u - B_n \bar{\mathbf{T}}_n \cdot \mathbf{M}_n^u) \right. \\ &\quad \left. - (\mathbf{M}'_m \cdot \bar{\mathbf{T}}_m) \cdot (B_n \mathbf{M}_n^u) \right. \\ &\quad \left. - (B_m \mathbf{M}'_m) \cdot (\bar{\mathbf{T}}_n \cdot \mathbf{M}_n^u) \right], \\ &\quad u = e, m, \\ Z_{mn}^{me} &\approx \frac{jk\eta}{4\pi} \mathbf{M}'_m \cdot (C_m + C_n) (\mathbf{r}_{mp} - \mathbf{r}_{np}) \times \mathbf{M}_n^e \\ &= \frac{jk\eta}{4\pi} \left[(C_m \mathbf{M}'_m \times \mathbf{r}_{mp}) \cdot \mathbf{M}_n^e \right. \\ &\quad \left. + (\mathbf{M}'_m \times \mathbf{r}_{mp}) \cdot (C_n \mathbf{M}_n^e), \right. \\ &\quad \left. + (C_m \mathbf{M}'_m) \cdot (\mathbf{M}_n^e \times \mathbf{r}_{np}) \right. \\ &\quad \left. + \mathbf{M}'_m \cdot (C_n \mathbf{M}_n^e \times \mathbf{r}_{np}) \right], \\ Z_{mn}^{em} &\approx -\frac{jk\eta}{4\pi} \mathbf{M}'_m \cdot (C_m + C_n) (\mathbf{r}_{mp} - \mathbf{r}_{np}) \times \mathbf{M}_n^m \\ &= -\frac{jk\eta}{4\pi} \left[(C_m \mathbf{M}'_m \times \mathbf{r}_{mp}) \cdot \mathbf{M}_n^m \right. \\ &\quad \left. + (\mathbf{M}'_m \times \mathbf{r}_{mp}) \cdot (C_n \mathbf{M}_n^m), \right. \\ &\quad \left. + (C_m \mathbf{M}'_m) \cdot (\mathbf{M}_n^m \times \mathbf{r}_{np}) \right. \\ &\quad \left. + \mathbf{M}'_m \cdot (C_n \mathbf{M}_n^m \times \mathbf{r}_{np}) \right],\end{aligned}\quad (17)$$

where $\mathbf{r}_{mp} = \mathbf{r}_m - \mathbf{r}_p$, $\mathbf{r}_{np} = \mathbf{r}_n - \mathbf{r}_p$, $\mathbf{r}_p = (\mathbf{r}_{o_j} + \mathbf{r}_{o_i})/2$,

$$\begin{aligned}A_u &= R_u^{(-1)} + (jk)^{-1} R_u^{(-2)} + (jk)^{-2} R_u^{(-3)}, \\ B_u &= R_u^{(-1)} + 3(jk)^{-1} R_u^{(-2)} + 3(jk)^{-2} R_u^{(-3)}, \\ C_u &= R_u^{(-2)} + (jk)^{-1} R_u^{(-3)}, \\ \mathbf{M}'_m &= \mathbf{m}'_m e^{-jkR_m^{(1)}}, \\ \mathbf{M}_n^u &= \mathbf{m}_n^u e^{-jkR_n^{(1)}}, \quad u = e, m.\end{aligned}\quad (18)$$

In the following part, we will present how to use the FDM to calculate the interactions among the equivalent electric or magnetic dipoles in group i and group j . The effect of all the electric and magnetic dipoles in group i on the m th electric dipoles in group j can be expressed as

$$\begin{aligned}&\sum_{n \in i} (Z_{mn}^{ee} I_n^e + Z_{mn}^{em} I_n^m) \\ &\approx \frac{jk\eta}{4\pi} \left[(A_m \mathbf{M}'_m - B_m \mathbf{M}'_m \cdot \bar{\mathbf{T}}_m) \cdot \sum_{n \in i} I_n^e \mathbf{M}_n^e \right. \\ &\quad \left. + \mathbf{M}'_m \cdot \sum_{n \in i} I_n^e (A_n \mathbf{M}_n^e - B_n \bar{\mathbf{T}}_n \cdot \mathbf{M}_n^e) \right. \\ &\quad \left. - (\mathbf{M}'_m \cdot \bar{\mathbf{T}}_m) \cdot \sum_{n \in i} I_n^e B_n \mathbf{M}_n^e \right. \\ &\quad \left. - (B_m \mathbf{M}'_m) \cdot \sum_{n \in i} I_n^e \bar{\mathbf{T}}_n \cdot \mathbf{M}_n^e \right. \\ &\quad \left. - (C_m \mathbf{M}'_m \times \mathbf{r}_{mp}) \cdot \sum_{n \in i} I_n^m \mathbf{M}_n^m \right. \\ &\quad \left. - (\mathbf{M}'_m \times \mathbf{r}_{mp}) \cdot \sum_{n \in i} I_n^m C_n \mathbf{M}_n^m \right. \\ &\quad \left. - (C_m \mathbf{M}'_m) \cdot \sum_{n \in i} I_n^m \mathbf{M}_n^m \times \mathbf{r}_{np} \right. \\ &\quad \left. - \mathbf{M}'_m \cdot \sum_{n \in i} I_n^m C_n \mathbf{M}_n^m \times \mathbf{r}_{np} \right].\end{aligned}\quad (19)$$

In a similar way, the effect of all the electric and magnetic dipoles in group i on the m th magnetic dipoles in group j can be expressed as

$$\begin{aligned}&\sum_{n \in i} (Z_{mn}^{mm} I_n^m + Z_{mn}^{me} I_n^e) \\ &\approx \frac{jk\eta}{4\pi} \left[(A_m \mathbf{M}'_m - B_m \mathbf{M}'_m \cdot \bar{\mathbf{T}}_m) \cdot \sum_{n \in i} I_n^m \mathbf{M}_n^m \right. \\ &\quad \left. + \mathbf{M}'_m \cdot \sum_{n \in i} I_n^m (A_n \mathbf{M}_n^m - B_n \bar{\mathbf{T}}_n \cdot \mathbf{M}_n^m) \right. \\ &\quad \left. - (\mathbf{M}'_m \cdot \bar{\mathbf{T}}_m) \cdot \sum_{n \in i} I_n^m B_n \mathbf{M}_n^m \right. \\ &\quad \left. - (B_m \mathbf{M}'_m) \cdot \sum_{n \in i} I_n^m \bar{\mathbf{T}}_n \cdot \mathbf{M}_n^m \right. \\ &\quad \left. - (C_m \mathbf{M}'_m \times \mathbf{r}_{mp}) \cdot \sum_{n \in i} I_n^e \mathbf{M}_n^e \right. \\ &\quad \left. - (\mathbf{M}'_m \times \mathbf{r}_{mp}) \cdot \sum_{n \in i} I_n^e C_n \mathbf{M}_n^e \right. \\ &\quad \left. - (C_m \mathbf{M}'_m) \cdot \sum_{n \in i} I_n^e \mathbf{M}_n^e \times \mathbf{r}_{np} \right. \\ &\quad \left. - \mathbf{M}'_m \cdot \sum_{n \in i} I_n^e C_n \mathbf{M}_n^e \times \mathbf{r}_{np} \right].\end{aligned}$$

$$\begin{aligned}
& + (C_m \mathbf{M}'_m \times \mathbf{r}_{mp}) \cdot \sum_{n \in i} I_n^e \mathbf{M}_n^e \\
& + (\mathbf{M}'_m \times \mathbf{r}_{mp}) \cdot \sum_{n \in i} I_n^e C_n \mathbf{M}_n^e \\
& + (C_m \mathbf{M}'_m) \cdot \sum_{n \in i} I_n^e \mathbf{M}_n^e \times \mathbf{r}_{np} \\
& + \mathbf{M}'_m \cdot \sum_{n \in i} I_n^e C_n \mathbf{M}_n^e \times \mathbf{r}_{np} \Big].
\end{aligned} \tag{20}$$

It can be found that each term in (19) and (20) achieves the separation of m and n . Therefore, the summation results $\sum_{n \in i}(\cdot)$ in (19) and (20) are independent of m and can be reused by each dipole in group j . These summation results $\sum_{n \in i}(\cdot)$ only need to be computed once. If we suppose that groups i and j contain N_i and N_j electric or magnetic dipoles, respectively, then the complexity of interactions between the two far groups is reduced from $O(N_i N_j)$ to $O(N_i + N_j)$.

4. Numerical Results and Discussion

All the simulations are performed on a personal computer with the Intel Pentium Dual-Core CPU E2200 with 2.0 GHz (only one core was used) and 2.0 GB RAM. The GMRES iterative solver is employed to obtain an identical residual error, 0.001.

First, we consider the scattering problem of a thin plate with $1.0 \text{ m} \times 1.0 \text{ m} \times 0.05 \text{ m}$ and $\epsilon_r = 2$, $\mu_r = 2$. The plate is divided into 2665 volumetric cells with an average edge length of 0.05 m, and the total number of unknowns is 12420. Totally, 256 nonempty groups with the size of 0.2 are obtained. The bistatic RCS in $\theta\theta$ polarization is compared with the EDM shown in Figure 1. Both the CPU time and memory requirements are shown in Table 1. We can see from Table 1, it costs 185 seconds and 312 MB of memory using the FDM, whereas 674 seconds and 1239 MB of memory are needed using the EDM. That is to say the FDM yields reductions of 72.6% of the total computation time and 74.8% of the memory requirements using EDM.

Furthermore, we consider the scattering problem of a 10 cm sphere with $\epsilon_r = \mu_r = -1 - j0.001$ as an example. To validate our FDM method, the calculated frequencies span the range from 0.6 GHz to 2.22 GHz. Figure 2 gives the bistatic RCSs for E-plane calculated by FDM. The results are in agreement with those from [14] (the RCSs at frequencies of 0.6 GHz and 2.22 GHz in [14] have been shown in Figure 2 as references). We compared the CPU time of the FDM with that of EDM when the object is at frequency of 1.14 GHz in Table 1. At this frequency, the sphere is divided into 1624 volumetric cells with an average edge length of 0.03 m, and the total number of unknowns is 6822. All the unknowns are divided into 1324 nonempty groups and the size of each group is 0.2. We can see from Table 1 that it costs 376 seconds and 173 MB of memory using the FDM, whereas 1753 seconds and 382 MB of memory are needed using the EDM.

TABLE 1: Comparison of CPU time, memory cost, and iterations of RCS solutions for EDM and FDM.

Method	Time (s)	Memory (MB)	Iterations
Problem 1: RHM cube			
EDM	674	1239	13
FDM	185	312	
Problem 2: LHM sphere at 1.14 GHz			
EDM	1753	382	305
FDM	376	173	
Problem 3: LHM-RHM sphere array			
EDM	4204	863	215
FDM	435	272	

Finally, we consider an LHM-RHM composite array. The dimension and parameters of the array are shown in Figure 3. The geometry is discretized into 2413 tetrahedrons using 0.04 m average edge length, thus resulting in total 10340 unknowns. All the unknowns are divided into 1342 nonempty groups and the size of each group is 0.2. $\theta\theta$ polarized RCS is plotted, as shown in Figure 3. It can be seen from Table 1 that it costs 435 seconds and 272 MB of memory using the FDM, whereas 4204 seconds and 863 MB of memory using the EDM are needed. That is to say the FDM can save much more time and memory requirements than the EDM. From Table 1, we also can find that the method converges much more slowly when it is used to analyze the scattering involving the LHM than the RHM. This is because LHM structures usually present numerical resonances that inhibit quick convergence of iterations [15].

5. Conclusion

In this work, the formulations of FDM for analyzing the electromagnetic scattering from electromagnetic materials are given, which saves CPU time and memory requirements compared with the conventional EDM. Several dielectric and magnetic targets include RHM objects and LHM objects, and LHM-RHM composite objects are calculated to demonstrate the merit of this method. Furthermore, the numerical results show that the convergences may become much slower if the LHM is included in the targets. In further work, we will try to construct an efficient preconditioner to alleviate the disadvantage brought by the LHM.

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