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Research Article

Exponential Stability for Discrete-Time Stochastic BAM Neural Networks with Discrete and Distributed Delays

R. Raja,[1] R. Sakthivel,[2] and S. Marshal Anthoni[3]

[1] *Department of Mathematics, Periyar University, Salem 636 011, India*
[2] *Department of Mathematics, Sungkyunkwan University, Suwon 440-746, Republic of Korea*
[3] *Department of Mathematics, Anna University of Technology, Coimbatore 641 047, India*

Correspondence should be addressed to R. Raja, antony.raja67@yahoo.com

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This paper deals with the stability analysis problem for a class of discrete-time stochastic BAM neural networks with discrete and distributed time-varying delays. By constructing a suitable Lyapunov-Krasovskii functional and employing M-matrix theory, we find some sufficient conditions ensuring the global exponential stability of the equilibrium point for stochastic BAM neural networks with time-varying delays. The conditions obtained here are expressed in terms of LMIs whose feasibility can be easily checked by MATLAB LMI Control toolbox. A numerical example is presented to show the effectiveness of the derived LMI-based stability conditions.

1. Introduction

Recently, the study of Bidirectional associative memory neural networks has attracted the attention of many researchers due to its applications in many fields such as pattern recognition, automatic control, associative memory, signal processing, and optimization; see, for example, [1–9]. The (BAM) neural networks model, proposed by Kosko [10, 11], is a two layer nonlinear feedback network model and it was described that the neurons in one layer are fully interconnected to the neurons in the other layer, while there are no interconnections among neurons in the same layer.

Furthermore, due to the finite switching speed of neuron amplifiers and the finite speed of signal propagation time delays are unavoidable in the implementation of neural networks [12–14]. According to the way it occurs, time delay can be classified as two types: discrete and distributed delays. Discrete time-delay is relatively easier to be identified in practice and hence the stability analysis for BAM with discrete delays has been an attractive subject of research in the past few years; see [15, 16]. On the other hand, due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, a neural network usually has a spatial nature. Therefore, it is necessary to introduce continuously distributed delays over a certain duration of time; see [17, 18].

Moreover, in implementations of neural networks, stochastic disturbances are inevitable owing to thermal noise in electronic devices. Practically, the stochastic phenomenon usually appears in the electrical circuit design of neural networks. The stochastic effects can have the ability to destabilize a neural system. Therefore, it is significant and of importance to consider stochastic effects to the stability property of the neural networks with delays. It is noted that most of the BAM neural networks have been assumed to act in a continuous-time manner. However, when it comes to the implementation of discrete-time BAM networks, there are only few works appeared in the literature; see [6, 19–24] and the references cited therein. Therefore, there is a crucial need to study the dynamics of discrete-time BAM neural networks and it becomes more significant from practical point of view. In [19], Gao and Cui discussed the global robust exponential stability of discrete-time interval BAM neural networks with time-varying delays, and in [24], the authors investigated the global exponential stability for discrete-time BAM neural network with time variable delay. In the above said references the stability problem for BAM neural networks is considered only with discrete delay, and distributed delay has not been taken into account and remains challenging. So, our main aim in this work is to make the first attempt to shorten such a gap.

Motivated by the above points, in this paper, we will study the exponential stability problem for a new class of discrete-time stochastic BAM neural networks with both discrete and distributed delays. The existence of the equilibrium point is proved under mild conditions on the activation functions. By constructing an appropriate Lyapunov-Krasovskii functional, a linear matrix inequality (LMI) approach is developed to establish sufficient conditions for the discrete-time BAM neural networks to be globally exponentially stable in the mean square. Here, we note that the LMIs can be easily solved by using Matlab LMI toolbox, and no tuning of parameters is involved. Finally, a numerical example is presented to show the usefulness of the derived LMI-based stability conditions.

Notations. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. I denotes the identity matrix with appropriate dimensions and $\mathrm{diag}(\cdot)$ denotes the diagonal matrix. For real symmetric matrices X and Y, the notation $X \geq Y$ (resp., $X > Y$) means that the matrix $X-Y$ is positive semidefinite (resp., positive definite). $\mathbb{N} = \{1, 2, \ldots, n\}$ and $\|\cdot\|$ stands for the Euclidean norm in $\mathbb{R}^n$. $\lambda_{\max}(X)$ (resp., $\lambda_{\min}(X)$) stands for the maximum (resp., minimum) eigenvalue of the matrix X. The symbol $*$ within a matrix represents the symmetric term of the matrix.

2. Problem Description and Preliminaries

Consider the following discrete-time stochastic BAM neural networks with both discrete and distributed delays of the following form:

$$\begin{aligned}
x_i(k+1) &= \left[a_i x_i(k) + \sum_{j=1}^{n} c_{ji} f_j(y_j(k)) + \sum_{j=1}^{n} w_{ji} g_j(y_j(k-\tau_{ji}(k))) + \sum_{j=1}^{n} m_{ji} \sum_{\mathcal{M}=1}^{+\infty} \mu_{\mathcal{M}} h_j(y_j(k-\mathcal{M})) + I_i\right] \\
&\quad + \delta_{ji}(x_i(k), y_j(k-\tau_{ji}(k)), k) w_1(k), \quad i \in N, \\
y_j(k+1) &= \left[b_j y_j(k) + \sum_{i=1}^{n} d_{ij} \widehat{f}_i(x_i(k)) + \sum_{i=1}^{n} v_{ij} \widehat{g}_i(x_i(k-\sigma_{ij}(k))) + \sum_{i=1}^{n} n_{ij} \sum_{\mathcal{N}=1}^{+\infty} \rho_{\mathcal{N}} \widehat{h}_i(x_i(k-\mathcal{N})) + J_j\right] \\
&\quad + \chi_{ij}(y_j(k), x_i(k-\sigma_{ij}(k)), k) w_2(k), \quad j \in N,
\end{aligned} \tag{2.1}$$

or, in an equivalent form,

$$
\begin{aligned}
x(k+1) &= \left[Ax(k) + Cf(y(k)) + Wg(y(k-\tau(k))) + M\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(y(k-\mathcal{M})) + I\right] \\
&\quad + \delta(x(k), y(k-\tau(k)), k)w_1(k), \\
y(k+1) &= \left[By(k) + D\hat{f}(x(k)) + V\hat{g}(x(k-\sigma(k))) + N\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\hat{h}(x(k-\mathcal{N})) + J\right] \\
&\quad + \chi(y(k), x(k-\sigma(k)), k)w_2(k),
\end{aligned}
\tag{2.2}
$$

for $k = 1, 2, \ldots$, where $x(k)$ and $y(k)$ are the neural state vector; $A = \operatorname{diag}\{a_1, a_2, \ldots, a_n\}$ and $B = \operatorname{diag}\{b_1, b_2, \ldots, b_n\}$ are the state feedback coefficient matrices; $C = [c_{ij}]_{n\times n}, D = [d_{ij}]_{n\times n}, W = [w_{ij}]_{n\times n}, V = [v_{ij}]_{n\times n}, M = [m_{ij}]_{n\times n}$, and $N = [n_{ij}]_{n\times n}$ are, respectively, the connection weight matrices, the discretely delayed connection weight matrices, and distributed delayed connection weight matrices; $\tau(k)$ and $\sigma(k)$ denote the discrete time-varying delays satisfying

$$
\tau_m \le \tau(k) \le \tau_M, \qquad \sigma_m \le \sigma(k) \le \sigma_M, \tag{2.3}
$$

where τ_m, τ_M, σ_m, and σ_M are known positive integer; M, N denotes the distributed time-varying delays. Then

$$
\begin{aligned}
f(y(k)) &= [f_1(y_1(k)), f_2(y_2(k)), \ldots, f_n(y_n(k))]^T, \\
\hat{f}(x(k)) &= \left[\hat{f}_1(x_1(k)), \hat{f}_2(x_2(k)), \ldots, \hat{f}_n(x_n(k))\right]^T, \\
g(y(k)) &= [g_1(y_1(k)), g_2(y_2(k)), \ldots, g_n(y_n(k))]^T, \\
\hat{g}(x(k)) &= [\hat{g}_1(x_1(k)), \hat{g}_2(x_2(k)), \ldots, \hat{g}_n(x_n(k))]^T, \\
h(y(k)) &= [h_1(y_1(k)), h_2(y_2(k)), \ldots, h_n(y_n(k))]^T, \\
\hat{h}(x(k)) &= \left[\hat{h}_1(x_1(k)), \hat{h}_2(x_2(k)), \ldots, \hat{h}_n(x_n(k))\right]^T,
\end{aligned}
\tag{2.4}
$$

denote the neuron activation functions. The constant vectors $J = [J_1, J_2, \ldots, J_n]^T$ and $I = [I_1, I_2, \ldots, I_n]^T$ are the external inputs from outside the system; $\mu_{\mathcal{M}}, (\mathcal{M} = 1, 2, \ldots)$ and $\rho_{\mathcal{N}}, (\mathcal{N} = 1, 2, \ldots)$ are scalar constants, where $w_1(k)$ and $w_2(k)$ are scalar Wiener process (Brownian motion) on the probability space $(\Omega, \mathfrak{F}, \mathfrak{P})$ with

$$
\begin{aligned}
&\mathbb{E}[w_1(k)] = 0, \qquad \mathbb{E}\left[w_1^2(k)\right] = 1, \qquad \mathbb{E}[w_1(i)w_1(j)] = 0 \quad (i \ne j), \\
&\mathbb{E}[w_2(k)] = 0, \qquad \mathbb{E}\left[w_2^2(k)\right] = 1, \qquad \mathbb{E}[w_2(i)w_2(j)] = 0 \quad (i \ne j),
\end{aligned}
\tag{2.5}
$$

with $\mathbb{E}(\cdot)$ being the mathematical expectation operator; $\delta : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ and $\chi : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ are the nonlinear vector function representing the disturbance intensities.

In this paper, we make following assumptions for the neuron activation functions.

Assumption 1. For $j, i \in \{1, 2, \ldots, n\}$, the neuron activation functions $f_j(\cdot)$, $\widehat{f}_i(\cdot)$, $g_j(\cdot)$, $\widehat{g}_i(\cdot)$, $h_j(\cdot)$, and $\widehat{h}_i(\cdot)$ in (2.2) are continuous as well as bounded on $\mathbb{R}$.

Assumption 2. For $j, i \in \{1, 2, \ldots, n\}$, the neuron activation functions in (2.2) satisfy

$$\begin{aligned}
l_j^- &\le \frac{f_j(s_1) - f_j(s_2)}{s_1 - s_2} \le l_j^+, \quad \forall s_1, s_2 \in \mathbb{R}, \qquad u_i^- \le \frac{\widehat{f}_i(t_1) - \widehat{f}_i(t_2)}{t_1 - t_2} \le u_i^+, \quad \forall t_1, t_2 \in \mathbb{R}, \\
m_j^- &\le \frac{g_j(s_1) - g_j(s_2)}{s_1 - s_2} \le m_j^+, \quad \forall s_1, s_2 \in \mathbb{R}, \qquad v_i^- \le \frac{\widehat{g}_i(t_1) - \widehat{g}_i(t_2)}{t_1 - t_2} \le v_i^+, \quad \forall t_1, t_2 \in \mathbb{R}, \\
n_j^- &\le \frac{h_j(s_1) - h_j(s_2)}{s_1 - s_2} \le n_j^+, \quad \forall s_1, s_2 \in \mathbb{R}, \qquad w_i^- \le \frac{\widehat{h}_i(t_1) - \widehat{h}_i(t_2)}{t_1 - t_2} \le w_i^+, \quad \forall t_1,\ t_2 \in \mathbb{R},
\end{aligned} \tag{2.6}$$

where l_j^-, l_j^+, m_j^-, m_j^+, n_j^-, n_j^+, u_i^-, u_i^+, v_i^-, v_i^+, w_i^-, and w_i^+ are some constants.

Remark 2.1. Assumption 2 was first introduced by Liu et al. [25]. The constants l_j^-, l_j^+, m_j^-, m_j^+, n_j^-, n_j^+, u_i^-, u_i^+, v_i^-, v_i^+, w_i^-, and w_i^+ in Assumption 2 are allowed to be positive, negative, or zero. So, the activation functions used in this paper may be nonmonotonic and more general than the usual sigmoid functions and Lipschitz functions. Such conditions are very rude in quantifying the lower and upper bounds of the activation functions; hence we use generalized activation functions, because it is very helpful for using LMI-based technique to reduce the possible conservatism.

In order to simplify our proof, we shift the equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ and $y^* = (y_1^*, y_2^*, \ldots, y_n^*)^T$ of system (2.2) to the origin. Let $u(k) = x(k) - x^*$ and $v(k) = y(k) - y^*$; then system (2.2) can be transformed to

$$\begin{aligned}
u(k+1) &= \left[Au(k) + Cf(v(k)) + Wg(v(k - \tau(k))) + M \sum_{\mathcal{M}=1}^{+\infty} \mu_{\mathcal{M}} h(v(k - \mathcal{M})) \right] \\
&\quad + \delta(u(k), v(k - \tau(k)), k) w_1(k), \\
v(k+1) &= \left[Bv(k) + D\widehat{f}(u(k)) + V\widehat{g}(u(k - \sigma(k))) + N \sum_{\mathcal{N}=1}^{+\infty} \rho_{\mathcal{N}} \widehat{h}(u(k - \mathcal{N})) \right] \\
&\quad + \chi(v(k), u(k - \sigma(k)), k) w_2(k),
\end{aligned} \tag{2.7}$$

where

$$\begin{aligned}
u(k) &= (u_1(k),\ u_2(k),\dots,u_n(k))^T, \qquad v(k) = (v_1(k), v_2(k),\dots,v_n(k))^T,\\
f(v(k)) &= \left[f_1(v_1(k)), f_2(v_2(k)),\dots,f_n(v_n(k))\right]^T = f(y(k)+y^*) - f(y^*),\\
g(v(k)) &= \left[g_1(v_1(k)), g_2(v_2(k)),\dots,g_n(v_n(k))\right]^T = g(y(k)+y^*) - g(y^*),\\
h(v(k)) &= [h_1(v_1(k)), h_2(v_2(k)),\dots,h_n(v_n(k))]^T = h(y(k)+y^*) - h(y^*),\\
\widehat{f}(u(k)) &= \left[\widehat{f}_1(u_1(k)), \widehat{f}_2(u_2(k)),\dots,\widehat{f}_n(u_n(k))\right]^T = \widehat{f}(x(k)+x^*) - \widehat{f}(x^*),\\
\widehat{g}(u(k)) &= \left[\widehat{g}_1(u_1(k)), \widehat{g}_2(u_2(k)),\dots,\widehat{g}_n(u_n(k))\right]^T = \widehat{g}(x(k)+x^*) - \widehat{g}(x^*),\\
\widehat{h}(u(k)) &= \left[\widehat{h}_1(u_1(k)), \widehat{h}_2(u_2(k)),\dots,\widehat{h}_n(u_n(k))\right]^T = \widehat{h}(x(k)+x^*) - \widehat{h}(x^*).
\end{aligned} \tag{2.8}$$

Assumption 3. Obviously, the activation functions f_j, $\widehat{f}_i$, g_j, $\widehat{g}_i$, h_j, and $\widehat{h}_i$ $(i, j \in \mathbb{N})$ satisfy the following condition:

$$\begin{aligned}
&l_j^- \le \frac{f_j(s)}{s} \le l_j^+, \quad \forall s \in \mathbb{R}, \qquad u_i^- \le \frac{\widehat{f}_i(t)}{t} \le u_i^+, \quad \forall t \in \mathbb{R},\\
&m_j^- \le \frac{g_j(s)}{s} \le m_j^+, \quad \forall s \in \mathbb{R}, \qquad v_i^- \le \frac{\widehat{g}_i(t)}{t} \le v_i^+, \quad \forall t \in \mathbb{R},\\
&n_j^- \le \frac{h_j(s)}{s} \le n_j^+, \quad \forall s \in \mathbb{R}, \qquad w_i^- \le \frac{\widehat{h}_i(t)}{t} \le w_i^+, \quad \forall t \in \mathbb{R}.
\end{aligned} \tag{2.9}$$

Assumption 4. The constants $\mu_{\mathcal{M}}, \rho_{\mathcal{N}} \ge 0$ satisfy the following convergent conditions:

$$\begin{aligned}
&\sum_{\mathcal{M}=1}^{+\infty} \mu_{\mathcal{M}} < +\infty, \qquad \sum_{\mathcal{M}=1}^{+\infty} \mathcal{M}\mu_{\mathcal{M}} < +\infty,\\
&\sum_{\mathcal{N}=1}^{+\infty} \rho_{\mathcal{N}} < +\infty, \qquad \sum_{\mathcal{N}=1}^{+\infty} \mathcal{N}\rho_{\mathcal{N}} < +\infty.
\end{aligned} \tag{2.10}$$

Remark 2.2. Assumption 4 ensures that the terms $M\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}} h(v(k-\mathcal{M}))$ and $N\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))$ are convergent, which is significant for the subsequent analysis.

Assumption 5. There exist constant matrices G and K such that

$$\begin{aligned}
&\delta^T(x, y, k)\delta(x, y, k) \le |Gx|^2, \quad \forall x, y \in \mathbb{R}^n,\\
&\chi^T(x, y, k)\chi(x, y, k) \le |Ky|^2, \quad \forall x, y \in \mathbb{R}^n.
\end{aligned} \tag{2.11}$$

The following definition and lemmas will be essential in employing the exponential stability conditions.

Definition 2.3. The delayed discrete-time stochastic BAM neural network (2.7) is said to be globally exponentially stable, if there exist two positive scalars $\nu > 0$ and $0 < \mathcal{G} < 1$ such that

$$\|u(k)\| + \|v(k)\| \leq \nu \mathcal{G}^k \left(\sup_{-\sigma_M \leq s \leq 0} \|u(s)\| + \sup_{-\tau_M \leq s \leq 0} \|v(s)\| \right). \tag{2.12}$$

Lemma 2.4. *Let X and Y be any n-dimensional real vectors and let P be an $n \times n$ positive semidefinite matrix. Then, the following matrix inequality holds:*

$$2X^T PY \leq X^T PX + Y^T PY. \tag{2.13}$$

Lemma 2.5. *Let $M \in \mathbb{R}^{n\times n}$ be a positive semidefinite matrix, $x_i \in \mathbb{R}^n$, and $a_i \geq 0, (i = 1,\ 2, \ldots)$. If the series concerned are convergent, the following inequality holds:*

$$\left(\sum_{i=1}^{+\infty} a_i x_i \right)^T M \left(\sum_{i=1}^{+\infty} a_i x_i \right) \leq \left(\sum_{i=1}^{+\infty} a_i \right) \sum_{i=1}^{+\infty} a_i x_i{}^T M x_i. \tag{2.14}$$

In the rest of the paper, we will focus on the stability analysis of SBAMNN (2.7). By choosing an appropriate Lyapunov-Krasovskii functional, we aim to develop an LMI approach for deriving sufficient conditions under which the SBAMNN (2.7) is globally exponentially stable.

3. Main Results

Now, we are in a position to state our main results in the following theorem.

Theorem 3.1. *Under Assumptions 1–5, the discrete-time stochastic BAM neural network* (2.7) *is globally exponentially stable in the mean square, if there exist constants $\lambda_0 > 0$ and $\epsilon_0 > 0$ if there exist diagonal matrices $\Lambda_1 = \text{diag}\{\lambda_1^{(1)}, \lambda_2^{(1)}, \ldots, \lambda_n^{(1)}\} > 0$, $\Lambda_2 = \text{diag}\{\lambda_1^{(2)}, \lambda_2^{(2)}, \ldots, \lambda_n^{(2)}\} > 0$, $\Gamma_1 = \text{diag}\{\gamma_1^{(1)}, \gamma_2^{(1)}, \ldots, \gamma_n^{(1)}\} > 0$, $\Gamma_2 = \text{diag}\{\gamma_1^{(2)}, \gamma_2^{(2)}, \ldots, \gamma_n^{(2)}\} > 0$, $\Omega_1 = \text{diag}\{\omega_1^{(1)}, \omega_2^{(1)}, \ldots, \omega_n^{(1)}\} > 0$, and $\Omega_2 = \text{diag}\{\omega_1^{(2)}, \omega_2^{(2)}, \ldots, \omega_n^{(2)}\} > 0$ and positive definite matrices $P_1 > 0, P_2 > 0, Q_1 > 0, Q_2 > 0, R_1 > 0,$ and $R_2 > 0$, such that the following LMIs hold:*

$$
\begin{gathered}
P_1 < \lambda_0 I, \\
\Xi_1 = \begin{bmatrix}
\Pi_{11} & 0 & \Lambda_2 U_2 & \Gamma_2 V_2 & 0 & \Omega_2 W_2 & 0 \\
* & -Q_2 & 0 & 0 & 0 & 0 & 0 \\
* & * & \Pi_{33} & 0 & 0 & 0 & 0 \\
* & * & * & -\Gamma_2 & 0 & 0 & 0 \\
* & * & * & * & \Pi_{55} & 0 & 0 \\
* & * & * & * & * & -\Omega_2 & 0 \\
* & * & * & * & * & * & \Pi_{77}
\end{bmatrix} < 0, \\
P_2 < \epsilon_0 I, \\
\Xi_2 = \begin{bmatrix}
\Theta_{11} & 0 & \Lambda_1 L_2 & \Gamma_1 M_2 & 0 & \Omega_1 N_2 & 0 \\
* & -Q_1 & 0 & 0 & 0 & 0 & 0 \\
* & * & \Theta_{33} & 0 & 0 & 0 & 0 \\
* & * & * & -\Gamma_1 & 0 & 0 & 0 \\
* & * & * & * & \Theta_{55} & 0 & 0 \\
* & * & * & * & * & -\Omega_1 & 0 \\
* & * & * & * & * & * & \Theta_{77}
\end{bmatrix} < 0,
\end{gathered}
\tag{3.1}
$$

where

$$
\begin{aligned}
&\Pi_{11} = A^T P_1 A - 2P_1 - \Lambda_2 U_1 - \Gamma_2 V_1 - \Omega_2 W_1 + (1 + \sigma_M - \sigma_m) Q_2 + \overline{\rho} R_2 + \lambda_0 G^T G, \\
&\Pi_{33} = BDP_2 D^T B^T - \Lambda_2 + 2P_2, \qquad \Pi_{55} = BVP_2 V^T B^T + DVP_2 V^T D^T + P_2, \\
&\Pi_{77} = BNP_2 N^T B^T + DNP_2 N^T D^T + VNP_2 N^T V^T - \rho^{-1} R_2, \\
&\Theta_{11} = B^T P_2 B - 2P_2 - \Lambda_1 L_1 - \Gamma_1 M_1 - \Omega_1 N_1 + (1 + \tau_M - \tau_m) Q_1 + \overline{\mu} R_1 + \epsilon_0 K^T K, \\
&\Theta_{33} = ACP_1 C^T A^T - \Lambda_1 + 2P_1, \qquad \Theta_{55} = AWP_1 W^T A^T + CWP_1 W^T C^T + P_1, \\
&\Theta_{77} = AMP_1 M^T A^T + CMP_1 M^T C^T + WMP_1 M^T W^T - \mu^{-1} R_1, \\
&L_1 = \operatorname{diag}\{l_1^+ l_1^-, l_2^+ l_2^-, \dots, l_n^+ l_n^-\}, \qquad L_2 = \operatorname{diag}\left\{\frac{l_1^+ + l_1^-}{2}, \frac{l_2^+ + l_2^-}{2}, \dots, \frac{l_n^+ + l_n^-}{2}\right\}, \\
&M_1 = \operatorname{diag}\{m_1^+ m_1^-, m_2^+ m_2^-, \dots, m_n^+ m_n^-\}, \qquad M_2 = \operatorname{diag}\left\{\frac{m_1^+ + m_1^-}{2}, \frac{m_2^+ + m_2^-}{2}, \dots, \frac{m_n^+ + m_n^-}{2}\right\}, \\
&N_1 = \operatorname{diag}\{n_1^+ n_1^-, n_2^+ n_2^-, \dots, n_n^+ n_n^-\}, \qquad N_2 = \operatorname{diag}\left\{\frac{n_1^+ + n_1^-}{2}, \frac{n_2^+ + n_2^-}{2}, \dots, \frac{n_n^+ + n_n^-}{2}\right\}, \\
&U_1 = \operatorname{diag}\{u_1^+ u_1^-, u_2^+ u_2^-, \dots, u_n^+ u_n^-\}, \qquad U_2 = \operatorname{diag}\left\{\frac{u_1^+ + u_1^-}{2}, \frac{u_2^+ + u_2^-}{2}, \dots, \frac{u_n^+ + u_n^-}{2}\right\},
\end{aligned}
$$

$$V_1 = \operatorname{diag}\{v_1^+ v_1^-, v_2^+ v_2^-, \dots, v_n^+ v_n^-\}, \qquad V_2 = \operatorname{diag}\left\{\frac{v_1^+ + v_1^-}{2}, \frac{v_2^+ + v_2^-}{2}, \dots, \frac{v_n^+ + v_n^-}{2}\right\},$$
$$W_1 = \operatorname{diag}\{w_1^+ w_1^-, w_2^+ w_2^-, \dots, w_n^+ w_n^-\}, \qquad W_2 = \operatorname{diag}\left\{\frac{w_1^+ + w_1^-}{2}, \frac{w_2^+ + w_2^-}{2}, \dots, \frac{w_n^+ + w_n^-}{2}\right\},$$
$$\overline{\mu} = \sum_{k=1}^{+\infty} \mu_k, \qquad \overline{\rho} = \sum_{k=1}^{+\infty} \rho_k. \tag{3.2}$$

Proof. Let us choose the Lyapunov-Krasovskii functional as

$$\begin{aligned}
V_1(k) &= u^T(k) P_1 u(k) + v^T(k) P_2 v(k),\\
V_2(k) &= \sum_{i=k-\tau(k)}^{k-1} v^T(i) Q_1 v(i) + \sum_{i=k-\sigma(k)}^{k-1} u^T(i) Q_2 u(i),\\
V_3(k) &= \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} v^T(i) Q_1 v(i) + \sum_{j=k-\sigma_M+1}^{k-\sigma_m} \sum_{i=j}^{k-1} u^T(i) Q_2 u(i),\\
V_4(k) &= \sum_{i=1}^{+\infty} \mu_i \sum_{j=k-i}^{k-1} v^T(i) R_1 v(i) + \sum_{i=1}^{+\infty} \rho_i \sum_{j=k-i}^{k-1} u^T(i) R_2 u(i).
\end{aligned} \tag{3.3}$$

In order to analyze the global exponential stability of the stochastic BAM neural network, we calculate differences $\Delta V(k)$ of the Lyapunov function $V(k)$, along with the trajectories of the BAM neural network (2.7); then we have

$$\Delta V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \Delta V_4(k), \tag{3.4}$$

where

$$\begin{aligned}
\Delta V_1(k) &= u^T(k+1) P_1 u(k+1) - u^T(k) P_1 u(k) + v^T(k+1) P_2 v(k+1) - v^T(k) P_2 v(k)\\
&= \left[Au(k) + Cf(v(k)) + Wg(v(k-\tau(k))) + M \sum_{\mathcal{M}=1}^{+\infty} \mu_{\mathcal{M}} h(v(k-\mathcal{M})) \right]^T\\
&\quad \times P_1 \left[Au(k) + Cf(v(k)) + Wg(v(k-\tau(k))) + M \sum_{\mathcal{M}=1}^{+\infty} \mu_{\mathcal{M}} h(v(k-\mathcal{M})) \right]\\
&\quad - u^T(k) P_1 u(k)\\
&\quad + \left[Bv(k) + D\widehat{f}(u(k)) + V\widehat{g}(u(k-\sigma(k))) + N \sum_{\mathcal{N}=1}^{+\infty} \rho_{\mathcal{N}} \widehat{h}(u(k-\mathcal{N})) \right]^T
\end{aligned}$$

$$
\begin{aligned}
& \times P_2\left[Bv(k)+D\widehat{f}(u(k))+V\widehat{g}(u(k-\sigma(k)))+N\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right] \\
& -v^T(k)P_2v(k)+\delta^T(u,v,k)P_1\delta(u,v,k)+\chi^T(u,v,k)P_2\chi(u,v,k) \\
\leq\ & u^T(k)(AP_1A-P_1)u(k)+2u^T(k)AP_1Cf(v(k))+2u^T(k)AP_1Wg(v(k-\tau(k))) \\
& +2u^T(k)AP_1M\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right)+f^T(v(k))CP_1Cf(v(k))+2f^T(v(k)) \\
& \times CP_1Wg(v(k-\tau(k)))+2f^T(v(k))CP_1M\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right) \\
& +g^T(v(k-\tau(k)))WP_1Wg(v(k-\tau(k)))+2g^T(v(k-\tau(k)))WP_1M \\
& \times\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right)+\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right)^T MP_1M \\
& \times\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right)+v^T(k)(BP_2B-P_2)v(k)+2v^T(k)BP_2D\widehat{f}(u(k)) \\
& +2v^T(k)BP_2V\widehat{f}(u(k-\sigma(k)))+2v^T(k)BP_2N\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right) \\
& +\widehat{f}^T(u(k))DP_2D\widehat{f}(u(k))+2\widehat{f}^T(u(k))DP_2V\widehat{g}(u(k-\sigma(k)))+2\widehat{f}^T(u(k))DP_2N \\
& \times\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right)+\widehat{g}^T(u(k-\sigma(k)))VP_2V\widehat{g}(u(k-\sigma(k))) \\
& +2\widehat{g}^T(u(k-\sigma(k)))VP_2N\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right)+\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right)N \\
& \times P_2N\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right)+u^T(k)\lambda_0G^TGu(k)+v^T(k)\epsilon_0K^TKv(k).
\end{aligned}
\tag{3.5}
$$

By using Lemma 2.4, we have

$$
\begin{aligned}
& 2u^T(k)AP_1Cf(v(k)) \\
& \quad\leq u^T(k)P_1u(k)+f^T(v(k))ACP_1C^TA^Tf(v(k)), \\
& 2u^T(k)AP_1Wg(v(k-\tau(k))) \\
& \quad\leq u^T(k)P_1u(k)+g^T(v(k-\tau(k)))AWP_1W^TA^Tg(v(k-\tau(k))), \\
& 2u^T(k)AP_1M\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right)\leq u^T(k)P_1u(k) \\
& \quad+\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right)^T AMP_1M^TA^T\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right),
\end{aligned}
$$

$$2f^T(v(k))CP_1Wg(v(k-\tau(k))) \le f^T(v(k))P_1f(v(k)) + g^T(v(k-\tau(k)))CWP_1W^TC^Tg(v(k-\tau(k))),$$

$$2f^T(v(k))CP_1M\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right) \le f^T(v(k))P_1f(v(k)) + \left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right)^T \times CMP_1M^TC^T\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right),$$

$$2g^T(v(k-\tau(k)))WP_1M\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right) \le g^T(v(k-\tau(k)))P_1g(v(k-\tau(k))) + \left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right)^T \times WMP_1M^TW^T\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right),$$

$$2v^T(k)BP_2D\widehat{f}(u(k)) \le v^T(k)P_2v(k) + \widehat{f}^T(u(k))BDP_2D^TB^T\widehat{f}(u(k)),$$

$$2v^T(k)BP_2V\widehat{g}(u(k-\sigma(k))) \le v^T(k)P_2v(k) + \widehat{g}^T(u(k-\sigma(k)))BVP_2V^TB^T\widehat{g}(u(k-\sigma(k))),$$

$$2v^T(k)BP_2N\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right) \le v^T(k)P_2v(k) + \left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right)^T BNP_2N^TB^T\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right),$$

$$2\widehat{f}^T(u(k))DP_2V\widehat{g}(u(k-\sigma(k))) \le \widehat{f}^T(u(k))P_2\widehat{f}(u(k)) + \widehat{g}^T(u(k-\sigma(k)))DVP_2V^TD^T\widehat{g}(u(k-\sigma(k))),$$

$$2\widehat{f}^T(u(k))DP_2N\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right) \le \widehat{f}^T(u(k))P_2\widehat{f}(v(k)) + \left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right)^T \times DNP_2N^TD^T\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right),$$

$$2\widehat{g}^T(u(k-\sigma(k)))VP_2N\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right) \le \widehat{g}^T(u(k-\sigma(k)))P_2\widehat{g}(u(k-\sigma(k))) + \left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right)^T VNP_2N^TV^T\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right),$$

$$
\begin{aligned}
\Delta V_2(k) &= V_2(k+1) - V_2(k) \\
&= \sum_{i=k+1-\tau(k+1)}^{k} v^T(i) Q_1 v(i) - \sum_{i=k-\tau(k)}^{k-1} v^T(i) Q_1 v(i) \\
&\quad + \sum_{i=k+1-\sigma(k+1)}^{k} u^T(i) Q_2 u(i) - \sum_{i=k-\sigma(k)}^{k-1} u^T(i) Q_2 u(i) \\
&= v^T(k) Q_1 v(k) - v^T(k-\tau(k)) Q_1 v(k-\tau(k)) + \sum_{i=k+1-\tau(k+1)}^{k-1} v^T(i) Q_1 v(i) \\
&\quad - \sum_{i=k-\tau(k)+1}^{k-1} v^T(i) Q_1 v(i) + u^T(k) Q_2 u(k) - u^T(k-\sigma(k)) Q_2 u(k-\sigma(k)) \\
&\quad + \sum_{i=k+1-\sigma(k+1)}^{k-1} u^T(i) Q_2 u(i) - \sum_{i=k-\sigma(k)+1}^{k-1} u^T(i) Q_2 u(i) \\
&= v^T(k) Q_1 v(k) - v^T(k-\tau(k)) Q_1 v(k-\tau(k)) + \sum_{i=k-\tau(k+1)+1}^{k-\tau_m} v^T(i) Q_1 v(i) \\
&\quad + \sum_{i=k-\tau_m+1}^{k-1} v^T(i) Q_1 v(i) - \sum_{i=k-\tau(k)+1}^{k-1} v^T(i) Q_1 v(i) \\
&\quad + u^T(k) Q_2 u(k) - u^T(k-\sigma(k)) Q_2 u(k-\sigma(k)) + \sum_{i=k-\sigma(k+1)+1}^{k-\sigma_m} u^T(i) Q_2 u(i) \\
&\quad + \sum_{i=k-\sigma_m+1}^{k-1} u^T(i) Q_2 u(i) - \sum_{i=k-\sigma(k)+1}^{k-1} u^T(i) Q_2 u(i) \\
&= v^T(k) Q_1 v(k) - v^T(k-\tau(k)) Q_1 v(k-\tau(k)) + \sum_{i=k-\tau_M+1}^{k-\tau_m} v^T(i) Q_1 v(i) \\
&\quad + u^T(k) Q_2 u(k) - u^T(k-\sigma(k)) Q_2 u(k-\sigma(k)) + \sum_{i=k-\sigma_M+1}^{k-\sigma_m} u^T(i) Q_2 u(i),
\end{aligned}
\tag{3.6}
$$

$$
\begin{aligned}
\Delta V_3(k) &= V_3(k+1) - V_3(k) \\
&= \sum_{j=k-\tau_M+2}^{k-\tau_m+1} \sum_{i=j}^{k} v^T(i) Q_1 v(i) - \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k} v^T(i) Q_1 v(i) \\
&\quad + \sum_{j=k-\sigma_M+2}^{k-\sigma_m+1} \sum_{i=j}^{k} u^T(i) Q_2 u(i) - \sum_{j=k-\sigma_M+1}^{k-\sigma_m} \sum_{i=j}^{k} u^T(i) Q_2 u(i) \\
&= \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k} v^T(i) Q_1 v(i) - \sum_{j=k-\tau_M+1}^{k-\tau_m} \sum_{i=j}^{k-1} v^T(i) Q_1 v(i)
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j=k-\sigma_M+1}^{k-\sigma_m}\sum_{i=j}^{k}u^T(i)Q_2u(i)-\sum_{j=k-\sigma_M+1}^{k-\sigma_m}\sum_{i=j}^{k-1}u^T(i)Q_2u(i)\\
&=\sum_{j=k-\tau_M+1}^{k-\tau_m}\left[v^T(k)Q_1v(k)-v^T(j)Q_1v(j)\right]+\sum_{j=k-\sigma_M+1}^{k-\sigma_m}\left[u^T(k)Q_2u(k)-u^T(j)Q_2u(j)\right]\\
&\le(\tau_M-\tau_m)v^T(k)Q_1v(k)-\sum_{j=k-\tau_M+1}^{k-\tau_m}v^T(i)Q_1v(i)+(\sigma_M-\sigma_m)u^T(k)Q_2u(k)\\
&-\sum_{j=k-\sigma_M+1}^{k-\sigma_m}u^T(i)Q_2u(i),
\end{aligned}
\tag{3.7}
$$

$$
\begin{aligned}
\Delta V_4(k)&=V_4(k+1)-V_4(k)\\
&=\sum_{i=1}^{+\infty}\mu_i\sum_{j=k-i+1}^{k}v^T(i)R_1v(i)-\sum_{i=1}^{+\infty}\mu_i\sum_{j=k-\mathrm{i}}^{k-1}v^T(i)R_1v(i)\\
&+\sum_{i=1}^{+\infty}\rho_i\sum_{j=k-i+1}^{k}u^T(i)R_2u(i)-\sum_{i=1}^{+\infty}\rho_i\sum_{j=k-i}^{k-1}u^T(i)R_2u(i)\\
&=\sum_{i=1}^{+\infty}\mu_i\left[v^T(k)R_1v(k)-v^T(k-i)R_1v(k-i)\right]+\sum_{i=1}^{+\infty}\rho_i\left[u^T(k)R_2u(k)-u^T(k-i)R_2u(k-i)\right]\\
&\le\overline{\mu}v^T(k)R_1v(k)-\frac{1}{\overline{\mu}}\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right)^TR_1\left(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M}))\right)\\
&+\overline{\rho}u^T(k)R_2u(k)-\frac{1}{\overline{\rho}}\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right)^TR_2\left(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N}))\right).
\end{aligned}
\tag{3.8}
$$

It is clear from (2.9) that

$$\left(f_j(v_j(k))-l_j^+v_j(k)\right)\left(f_j(v_j(k))-l_j^-v_j(k)\right)\le 0, \tag{3.9}$$

$$\left(g_j(v_j(k))-m_j^+v_j(k)\right)\left(g_j(v_j(k))-m_j^-v_j(k)\right)\le 0, \tag{3.10}$$

$$\left(h_j(v_j(k))-n_j^+v_j(k)\right)\left(h_j(v_j(k))-n_j^-v_j(k)\right)\le 0, \tag{3.11}$$

$$\left(\widehat{f}_i(u_i(k))-u_i^+u_i(k)\right)\left(\widehat{f}_i(u_i(k))-u_i^-u_i(k)\right)\le 0, \tag{3.12}$$

$$(\widehat{g}_i(u_i(k))-v_i^+u_i(k))(\widehat{g}_i(u_i(k))-v_i^-u_i(k))\le 0, \tag{3.13}$$

$$\left(\widehat{h}_i(u_i(k))-w_i^+u_i(k)\right)\left(\widehat{h}_i(u_i(k))-w_i^-u_i(k)\right)\le 0, \tag{3.14}$$

which is equivalent to

$$\begin{bmatrix} v(k) \\ f(v(k)) \end{bmatrix}^T \begin{bmatrix} l_j^+ l_j^- e_j e_j^T & -\dfrac{l_j^+ + l_j^-}{2} e_j e_j^T \\ -\dfrac{l_j^+ + l_j^-}{2} e_j e_j^T & e_j e_j^T \end{bmatrix} \begin{bmatrix} v(k) \\ f(v(k)) \end{bmatrix} \leq 0, \quad k = 1,\ 2, \ldots, n, \tag{3.15}$$

where e_k denotes the unit column vector having "1" element on its kth row and zeros elsewhere.

Consequently,

$$\begin{gathered} \sum_{j=1}^{n} \lambda_j^{(1)} \begin{bmatrix} v(k) \\ f(v(k)) \end{bmatrix}^T \begin{bmatrix} l_j^+ l_j^- e_j e_j^T & -\dfrac{l_j^+ + l_j^-}{2} e_j e_j^T \\ -\dfrac{l_j^+ + l_j^-}{2} e_j e_j^T & e_j e_j^T \end{bmatrix} \begin{bmatrix} v(k) \\ f(v(k)) \end{bmatrix} \leq 0, \\ \Longrightarrow \begin{bmatrix} v(k) \\ f(v(k)) \end{bmatrix}^T \begin{bmatrix} \Lambda_1 L_1 & -\Lambda_1 L_2 \\ -\Lambda_1 L_2 & \Lambda_1 \end{bmatrix} \begin{bmatrix} v(k) \\ f(v(k)) \end{bmatrix} \leq 0. \end{gathered} \tag{3.16}$$

Similarly, from (3.10)–(3.14), we have

$$\begin{bmatrix} v(k) \\ g(v(k)) \end{bmatrix}^T \begin{bmatrix} \Gamma_1 M_1 & -\Gamma_1 M_2 \\ -\Gamma_1 M_2 & \Gamma_1 \end{bmatrix} \begin{bmatrix} v(k) \\ g(v(k)) \end{bmatrix} \leq 0, \tag{3.17}$$

$$\begin{bmatrix} v(k) \\ h(v(k)) \end{bmatrix}^T \begin{bmatrix} \Omega_1 N_1 & -\Omega_1 N_2 \\ -\Omega_1 N_2 & \Omega_1 \end{bmatrix} \begin{bmatrix} v(k) \\ h(v(k)) \end{bmatrix} \leq 0, \tag{3.18}$$

$$\begin{bmatrix} u(k) \\ \hat{f}(u(k)) \end{bmatrix}^T \begin{bmatrix} \Lambda_2 U_1 & -\Lambda_2 U_2 \\ -\Lambda_2 U_2 & \Lambda_2 \end{bmatrix} \begin{bmatrix} u(k) \\ \hat{f}(u(k)) \end{bmatrix} \leq 0, \tag{3.19}$$

$$\begin{bmatrix} u(k) \\ \hat{g}(u(k)) \end{bmatrix}^T \begin{bmatrix} \Gamma_2 V_1 & -\Gamma_2 V_2 \\ -\Gamma_2 V_2 & \Gamma_2 \end{bmatrix} \begin{bmatrix} u(k) \\ \hat{g}(u(k)) \end{bmatrix} \leq 0, \tag{3.20}$$

$$\begin{bmatrix} u(k) \\ \hat{h}(u(k)) \end{bmatrix}^T \begin{bmatrix} \Omega_2 W_1 & -\Omega_2 W_2 \\ -\Omega_2 W_2 & \Omega_2 \end{bmatrix} \begin{bmatrix} u(k) \\ \hat{h}(u(k)) \end{bmatrix} \leq 0. \tag{3.21}$$

Then from (3.5)–(3.8) and (3.16)–(3.21), we obtain

$$\begin{aligned} \Delta V(k) \leq{}& u^T(k)\left[AP_1A - 2P_1 - \Lambda_2 U_1 - \Gamma_2 V_1 - \Omega_2 W_1 + (1 + \sigma_M - \sigma_m)Q_2 + \overline{\rho} R_2 + \lambda_0 G^T G\right] u(k) \\ &+ u^T(k)\Lambda_2 U_2 \hat{f}(u(k)) + u^T(k)\Gamma_2 V_2 \hat{g}(u(k)) + u^T(k) \end{aligned}$$

$$
\begin{aligned}
&\times \Omega_2 W_2 \widehat{h}(u(k)) - u^T(k-\sigma(k)) Q_2 u(k-\sigma(k)) + \widehat{f}^T(u(k)) \\
&\times \left[BDP_2 D^T B^T - \Lambda_2 + 2P_2 \right] \widehat{f}(u(k)) - \widehat{g}^T(u(k)) \Gamma_2 \widehat{g}(u(k)) + \widehat{g}^T(u(k-\sigma(k))) \\
&\times \left[BVP_2 V^T B^T + DVP_2 V^T \mathrm{D}^T + P_2 \right] \widehat{g}(u(k-\sigma(k))) - \widehat{h}^T(u(k)) \Omega_2 \widehat{h}(u(k)) \\
&+ \left(\sum_{\mathcal{N}=1}^{+\infty} \rho_{\mathcal{N}} \widehat{h}(u(k-\mathcal{N})) \right)^T \left[BNP_2 N^T B^T + DNP_2 N^T D^T + VNP_2 N^T V^T - \rho^{-1} R_2 \right] \\
&\times \left(\sum_{\mathcal{N}=1}^{+\infty} \rho_{\mathcal{N}} \widehat{h}(u(k-\mathcal{N})) \right) + v^T(k) \\
&\times \left[BP_2 B - 2P_2 - \Lambda_1 L_1 - \Gamma_1 M_1 - \Omega_1 N_1 + (1 + \tau_M - \tau_m) Q_1 + \overline{\mu} R_1 + \epsilon_0 K^T K \right] \\
&+ v^T(k) \Lambda_1 L_2 f(v(k)) + v^T(k) \Gamma_1 M_2 g(v(k)) + v^T(k) \Omega_1 N_2 h(v(k)) \\
&- v^T(k-\tau(k)) Q_1 v(k-\tau(k)) + f^T(v(k)) \left[ACP_1 C^T A^T - \Lambda_1 + 2P_1 \right] f(v(k)) \\
&- g^T(v(k)) \Gamma_1 g(v(k)) + g^T(v(k-\tau(k))) \left[AWP_1 W^T A^T + CWP_1 W^T C^T + P_1 \right] \\
&\times g(v(k-\tau(k))) - h^T(v(k)) \Omega_1 h(v(k)) + \left(\sum_{\mathcal{M}=1}^{+\infty} \mu_{\mathcal{M}} h(v(k-\mathcal{M})) \right)^T \\
&\times \left[AMP_1 M^T A^T + CMP_1 M^T C^T + WMP_1 M^T W^T - \mu^{-1} R_1 \right] \\
&\times \left(\sum_{\mathcal{M}=1}^{+\infty} \mu_{\mathcal{M}} h(v(k-\mathcal{M})) \right) - \begin{bmatrix} v(k) \\ f(v(k)) \end{bmatrix}^T \begin{bmatrix} \Lambda_1 L_1 & -\Lambda_1 L_2 \\ -\Lambda_1 L_2 & \Lambda_1 \end{bmatrix} \begin{bmatrix} v(k) \\ f(v(k)) \end{bmatrix} \\
&- \begin{bmatrix} v(k) \\ g(v(k)) \end{bmatrix}^T \begin{bmatrix} \Gamma_1 M_1 & -\Gamma_1 M_2 \\ -\Gamma_1 M_2 & \Gamma_1 \end{bmatrix} \begin{bmatrix} v(k) \\ g(v(k)) \end{bmatrix} \\
&- \begin{bmatrix} v(k) \\ h(v(k)) \end{bmatrix}^T \begin{bmatrix} \Omega_1 N_1 & -\Omega_1 N_2 \\ -\Omega_1 N_2 & \Omega_1 \end{bmatrix} \begin{bmatrix} v(k) \\ h(v(k)) \end{bmatrix} \\
&- \begin{bmatrix} u(k) \\ \widehat{f}(u(k)) \end{bmatrix}^T \begin{bmatrix} \Lambda_2 U_1 & -\Lambda_2 U_2 \\ -\Lambda_2 U_2 & \Lambda_2 \end{bmatrix} \begin{bmatrix} u(k) \\ \widehat{f}(u(k)) \end{bmatrix} \\
&- \begin{bmatrix} u(k) \\ \widehat{g}(u(k)) \end{bmatrix}^T \begin{bmatrix} \Gamma_2 V_1 & -\Gamma_2 V_2 \\ -\Gamma_2 V_2 & \Gamma_2 \end{bmatrix} \begin{bmatrix} u(k) \\ \widehat{g}(u(k)) \end{bmatrix} \\
&- \begin{bmatrix} u(k) \\ \widehat{h}(u(k)) \end{bmatrix}^T \begin{bmatrix} \Omega_2 W_1 & -\Omega_2 W_2 \\ -\Omega_2 W_2 & \Omega_2 \end{bmatrix} \begin{bmatrix} u(k) \\ \widehat{h}(u(k)) \end{bmatrix} \\
= {}& \xi^T(k) \Xi_1 \xi(k) + \eta^T(k) \Xi_2 \eta(k),
\end{aligned}
\tag{3.22}
$$

where $\xi^T(k) = [u^T(k)u^T(k-\sigma(k))\widehat{f}^T(u(k))\widehat{g}^T(u(k))\widehat{g}^T(u(k-\sigma(k)))\widehat{h}^T(u(k))(\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\widehat{h}(u(k-\mathcal{N})))^T]\eta^T(k) = [v^T(k)v^T(k-\tau(k))f^T(v(k))g^T(v(k))g^T(v(k-\tau(k)))h^T(v(k))(\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(v(k-\mathcal{M})))^T]$.

Therefore, if the LMIs (3.1) hold, it can be concluded that $\Delta V(k) \leq 0$. It follows that $V(k) \leq V(0)$. By (3.22), the SBAMNN is globally asymptotically stable in the mean square.

Now, we are in a position to establish the exponential stability of the SBAMNN (2.7).

Then, there exists a scalar $\beta > 0$ such that

$$\Delta V(k) \leq -\beta\left(\|u(k)\|^2 + \|v(k)\|^2\right). \tag{3.23}$$

From (3.3), it can be verified that

$$\begin{aligned}
V(k) \leq{}& \lambda_{\max}(P_1)\|u(k)\|^2 + \lambda_{\max}(P_2)\|v(k)\|^2 + \lambda_{\max}(Q_1)\sum_{i=k-\tau_M}^{k-1}\|v(i)\|^2 + \lambda_{\max}(Q_2)\sum_{i=k-\sigma_M}^{k-1}\|u(i)\|^2 \\
&+ (\tau_M - \tau_m)\lambda_{\max}(Q_1)\sum_{i=k-\tau_M}^{k-1}\|v(i)\|^2 + (\sigma_M - \sigma_m)\lambda_{\max}(Q_2)\sum_{i=k-\sigma_M}^{k-1}\|v(i)\|^2 \\
={}& \lambda_{\max}(P_1)\|u(k)\|^2 + \lambda_{\max}(P_2)\|v(k)\|^2 + \beta_1\sum_{i=k-\sigma_M}^{k-1}\|u(i)\|^2 + \beta_2\sum_{i=k-\tau_M}^{k-1}\|v(i)\|^2,
\end{aligned} \tag{3.24}$$

where $\beta_1 = (1 + \sigma_M - \sigma_m)\lambda_{\max}(Q_1)$ and $\beta_2 = (1 + \tau_M - \tau_m)\lambda_{\max}(Q_2)$.

Choose a scalar $\theta > 1$, satisfying

$$-\beta\theta + (\theta - 1)(\lambda_{\max}(P_1) + \lambda_{\max}(P_2)) + (\theta - 1)\left(\beta_1\tau_M\theta^{\tau_M} + \beta_2\sigma_M\theta^{\sigma_M}\right) = 0. \tag{3.25}$$

Then by (3.23) and (3.24), we have

$$\begin{aligned}
\theta^{k+1}V(k+1) - \theta^k V(k) &= \theta^{k+1}V(k+1) - \theta^{k+1}V(k) + \theta^{k+1}V(k) - \theta^k V(k) \\
&= \theta^{k+1}\Delta V(k) + \theta^k(\theta - 1)V(k) \\
&\leq \beta_3\theta^k\left(\|u(k)\|^2 + \|v(k)\|^2\right) + \beta_4\theta^k\left(\sum_{i=k-\sigma_M}^{k-1}\|u(i)\|^2 + \sum_{i=k-\tau_M}^{k-1}\|v(i)\|^2\right),
\end{aligned} \tag{3.26}$$

where $\beta_3 = -\beta\theta + (\theta - 1)(\lambda_{\max}(P_1) + \lambda_{\max}(P_2))$ and $\beta_4 = (\theta - 1)(\beta_1 + \beta_2)$.

Therefore, for any integer $N \geq \tau_M + 1$ and $N \geq \sigma_M + 1$, summing up both sides of (3.26) from 0 to $N-1$ with respect to k, we have

$$\theta^N V(N) - V(0) \leq \beta_3 \sum_{k=0}^{N-1} \theta^k \left(\|u(k)\|^2 + \|v(k)\|^2 \right) + \beta_4 \sum_{k=0}^{N-1} \theta^k \left(\sum_{i=k-\sigma_M}^{k-1} \|u(i)\|^2 + \sum_{i=k-\tau_M}^{k-1} \|v(i)\|^2 \right). \tag{3.27}$$

Here, we note that for $\tau_M \geq 1$, $\sigma_M \geq 1$, we have

$$\begin{aligned} \sum_{k=0}^{N-1} \theta^k \left(\sum_{i=k-\sigma_M}^{k-1} \|u(i)\|^2 + \sum_{i=k-\tau_M}^{k-1} \|v(i)\|^2 \right) &= \sigma_M(\sigma_M + 1)\theta^{\sigma_M} \sup_{-\sigma_M \leq i \leq 0} \|u(i)\|^2 + \tau_M(\tau_M + 1)\theta^{\tau_M} \\ &\quad \times \sup_{-\tau_M \leq i \leq 0} \|v(i)\|^2 + \sigma_M \theta^{\sigma_M} \sum_{k=0}^{N-1} \theta^k \|u(k)\|^2 \\ &\quad + \tau_M \theta^{\tau_M} \sum_{k=0}^{N-1} \theta^k \|v(k)\|^2. \end{aligned} \tag{3.28}$$

Substituting (3.28) in (3.27) gives

$$\begin{aligned} \theta^N V(N) &\leq \beta_3 + \beta_4 (\sigma_M \theta^{\sigma_M} + \tau_M \theta^{\tau_M}) \sum_{k=0}^{T-1} \theta^k \left(\|u(k)\|^2 + \|v(k)\|^2 \right) \\ &\quad + \beta_4 [\sigma_M(\sigma_M + 1)\theta^{\sigma_M} + \tau_M(\tau_M + 1)\theta^{\tau_M}] \left(\sup_{-\sigma_M \leq i \leq 0} \|u(i)\|^2 + \sup_{-\tau_M \leq i \leq 0} \|v(i)\|^2 \right) + V(0). \end{aligned} \tag{3.29}$$

We can observe that

$$V(N) \geq \{\lambda_{\min}(P_1), \lambda_{\min}(P_2)\} \left(\|u(N)\|^2 + \|v(N)\|^2 \right). \tag{3.30}$$

It follows easily from (3.24) that

$$V(0) \leq \beta_1 \sigma_M \sup_{-\sigma_M \leq i \leq 0} \|u(i)\|^2 + \beta_2 \tau_M \sup_{-\tau_M \leq i \leq 0} \|v(i)\|^2. \tag{3.31}$$

Then, it follows from (3.25), (3.29), and (3.31) that

$$\|u(N)\| + \|v(N)\| \leq \nu \mathcal{G}^T \left(\sup_{-\sigma_M \leq i \leq 0} \|u(i)\| + \sup_{-\tau_M \leq i \leq 0} \|v(i)\| \right), \tag{3.32}$$

where $\mathcal{G} = \omega^{-1/2}$ and

$$\nu = \sqrt{\frac{\beta_1\sigma_M + \beta_2\tau_M + \beta_4[\sigma_M(\sigma_M+1)\theta^{\sigma_M} + \tau_M(\tau_M+1)\theta^{\tau_M}]}{\lambda_{\min}(P_1), \lambda_{\min}(P_2)}}. \tag{3.33}$$

This indicates that the discrete-time stochastic BAM neural network (2.7) is said to be globally exponentially stable. This completes the proof of this theorem. □

For a deterministic BAM neural network, we have the following system of equations:

$$\begin{aligned} x(k+1) &= Ax(k) + Cf(y(k)) + Wg(y(k-\tau(k))) + M\sum_{\mathcal{M}=1}^{+\infty}\mu_{\mathcal{M}}h(y(k-\mathcal{M})) + I, \\ y(k+1) &= By(k) + D\hat{f}(x(k)) + V\hat{g}(x(k-\sigma(k))) + N\sum_{\mathcal{N}=1}^{+\infty}\rho_{\mathcal{N}}\hat{h}(x(k-\mathcal{N})) + J. \end{aligned} \tag{3.34}$$

Then, by Theorem 3.1, it is very easy to obtain the following theorem.

Theorem 3.2. *Under Assumptions 1–4, the discrete-time BAM neural network* (3.34) *is globally exponentially stable, if there exist diagonal matrices* $\Lambda_1 = \operatorname{diag}\{\lambda_1^{(1)}, \lambda_2^{(1)}, \ldots, \lambda_n^{(1)}\} > 0$, $\Lambda_2 = \operatorname{diag}\{\lambda_1^{(2)}, \lambda_2^{(2)}, \ldots, \lambda_n^{(2)}\} > 0$, $\Gamma_1 = \operatorname{diag}\{\gamma_1^{(1)}, \gamma_2^{(1)}, \ldots, \gamma_n^{(1)}\} > 0$, $\Gamma_2 = \operatorname{diag}\{\gamma_1^{(2)}, \gamma_2^{(2)}, \ldots, \gamma_n^{(2)}\} > 0$, $\Omega_1 = \operatorname{diag}\{\omega_1^{(1)}, \omega_2^{(1)}, \ldots, \omega_n^{(1)}\} > 0$ *and* $\Omega_2 = \operatorname{diag}\{\omega_1^{(2)}, \omega_2^{(2)}, \ldots, \omega_n^{(2)}\} > 0$ *and positive definite matrices* $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, $Q_2 > 0 R_1 > 0$, *and* $R_2 > 0$, *such that the following LMIs hold:*

$$\Xi_3 = \begin{bmatrix} \Psi_{11} & 0 & \Lambda_2 U_2 & \Gamma_2 V_2 & 0 & \Omega_2 W_2 & 0 \\ * & -Q_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 & 0 \\ * & * & * & -\Gamma_2 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & 0 \\ * & * & * & * & * & -\Omega_2 & 0 \\ * & * & * & * & * & * & \Pi_{77} \end{bmatrix} < 0,$$
$$\Xi_4 = \begin{bmatrix} \Phi_{11} & 0 & \Lambda_1 L_2 & \Gamma_1 M_2 & 0 & \Omega_1 N_2 & 0 \\ * & -Q_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Theta_{33} & 0 & 0 & 0 & 0 \\ * & * & * & -\Gamma_1 & 0 & 0 & 0 \\ * & * & * & * & \Theta_{55} & 0 & 0 \\ * & * & * & * & * & -\Omega_1 & 0 \\ * & * & * & * & * & * & \Theta_{77} \end{bmatrix} < 0, \tag{3.35}$$

where

$$\begin{aligned}\Psi_{11} &= A^T P_1 A - 2P_1 - \Lambda_2 U_1 - \Gamma_2 V_1 - \Omega_2 W_1 + (1+\sigma_M - \sigma_m) Q_2 + \overline{\rho} R_2,\\ \Phi_{11} &= B^T P_2 B - 2P_2 - \Lambda_1 L_1 - \Gamma_1 M_1 - \Omega_1 N_1 + (1+\tau_M - \tau_m) Q_1 + \overline{\mu} R_1,\end{aligned} \tag{3.36}$$

and Π_{33}, Π_{55}, Π_{77}, Θ_{33}, Θ_{55}, *and* Θ_{77} *are defined in Theorem 3.1.*

Proof. Similar to the proof of Theorem 3.1, we can derive the stability result. The proof is straightforward and hence omitted. □

If we neglect the distributed delay term in (2.2), it can be reduced to

$$\begin{aligned}x(k+1) &= \left[Ax(k) + Cf\left(y(k)\right) + Wg\left(y(k-\tau(k))\right) + I\right] + \delta\left(x(k), y(k-\tau(k)), k\right) w_1(k),\\ y(k+1) &= \left[By(k) + D\widehat{f}\left(x(k)\right) + V\widehat{g}\left(x(k-\sigma(k))\right) + J\right] + \chi\left(y(k), x(k-\sigma(k)), k\right) w_2(k).\end{aligned} \tag{3.37}$$

For system (3.37), we have the following stability result.

Corollary 3.3. *Under Assumptions 1–5, the discrete-time BAM neural network* (3.37) *is globally exponentially stable, if there exist diagonal matrices* $\Lambda_1 = \operatorname{diag}\{\lambda_1^{(1)}, \lambda_2^{(1)}, \ldots, \lambda_n^{(1)}\} > 0$, $\Lambda_2 = \operatorname{diag}\{\lambda_1^{(2)}, \lambda_2^{(2)}, \ldots, \lambda_n^{(2)}\} > 0$, $\Gamma_1 = \operatorname{diag}\{\gamma_1^{(1)}, \gamma_2^{(1)}, \ldots, \gamma_n^{(1)}\} > 0$, *and* $\Gamma_2 = \operatorname{diag}\{\gamma_1^{(2)}, \gamma_2^{(2)}, \ldots, \gamma_n^{(2)}\} > 0$, *and positive definite matrices* $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, *and* $Q_2 > 0$, *such that the following LMI holds:*

$$\Xi_5 = \begin{bmatrix} \Upsilon_{11} & 0 & \Lambda_2 U_2 & \Gamma_2 V_2 & 0 \\ * & -Q_2 & 0 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 \\ * & * & * & -\Gamma_2 & 0 \\ * & * & * & * & \Pi_{55} \end{bmatrix} < 0,$$
$$\Xi_6 = \begin{bmatrix} \Sigma_{11} & 0 & \Lambda_1 L_2 & \Gamma_1 M_2 & 0 \\ * & -Q_1 & 0 & 0 & 0 \\ * & * & \Theta_{33} & 0 & 0 \\ * & * & * & -\Gamma_1 & 0 \\ * & * & * & * & \Theta_{55} \end{bmatrix} < 0, \tag{3.38}$$

where

$$\begin{aligned}\Upsilon_{11} &= A^T P_1 A - 2P_1 - \Lambda_2 U_1 - \Gamma_2 V_1 + (1+\sigma_M - \sigma_m) Q_2,\\ \Sigma_{11} &= B^T P_2 B - 2P_2 - \Lambda_1 L_1 - \Gamma_1 M_1 + (1+\tau_M - \tau_m) Q_1,\end{aligned} \tag{3.39}$$

and $\Pi_{33}, \Pi_{55}, \Theta_{33},$ *and* Θ_{55} *are defined in Theorem 3.1.*

Table 1: Allowable upper bound for σ_M and τ_M for given σ_m and τ_m.

In [19]	$\sigma_m = \tau_m = 2$	$\sigma_M = \tau_M = 4$
In this paper	$\sigma_m = \tau_m = 2$	$\sigma_M = \tau_M =$ for any large finite value

4. Numerical Example

To illustrate the effectiveness of our stability criterion, we give the following numerical example.

Example 4.1. Consider the SBAM neural networks (2.2) with the following parameters:

$$A = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}, \quad B = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0.4 & 0.2 & -0.1 \\ 0 & 0.2 & 0.3 \\ -0.1 & 0 & 0.2 \end{bmatrix},$$

$$D = \begin{bmatrix} -0.2 & 0.1 & 0 \\ 0.2 & 0.3 & 0.2 \\ 0 & -0.2 & 0.2 \end{bmatrix}, \quad W = \begin{bmatrix} -0.2 & 0.2 & 0.6 \\ 0.3 & 0.1 & 0 \\ 0 & -0.2 & -0.5 \end{bmatrix}, \quad V = \begin{bmatrix} 0.4 & 0.4 & -0.2 \\ 0 & 0.1 & 0.2 \\ -0.3 & 0 & 0.3 \end{bmatrix},$$

$$M = \begin{bmatrix} -0.2 & 0.6 & 0.1 \\ 0.1 & 0.3 & 0 \\ 0 & -0.7 & -0.5 \end{bmatrix}, \quad N = \begin{bmatrix} 0.4 & 0.5 & -0.3 \\ 0 & 0.1 & 0.3 \\ -0.4 & 0 & 0.4 \end{bmatrix}, \quad G = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 0.3 \end{bmatrix},$$

$$K = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}, \quad \tau(k) = 3 + \sin\left(\frac{k\pi}{2}\right), \quad \sigma(k) = 2 - \cos\left(\frac{k\pi}{2}\right), \tag{4.1}$$

$$I = -3\sin\left(\frac{k\pi}{2}\right), \quad J = 2\cos\left(\frac{k\pi}{2}\right), \quad \mu_k = \rho_k = e^{-4k},$$

$$f(y(k)) = g(y(k)) = h(y(k)) = \begin{bmatrix} \tanh(-4y_1(k)) \\ \tanh(-4y_2(k)) \\ \tanh(-y_3(k)) \end{bmatrix},$$

$$\widehat{f}(y(k)) = \widehat{g}(y(k)) = \widehat{h}(y(k)) = \begin{bmatrix} \tanh(-x_1(k)) \\ \tanh(-4x_2(k)) \\ \tanh(-x_3(k)) \end{bmatrix}.$$

It can be verified that $\sigma_m = \tau_m = 3$, $\sigma_M = \tau_M = 4$, $l_1^+ = m_1^+ = n_1^+ = 2$, $l_1^- = m_1^- = n_1^- = -2$, $l_2^+ = m_2^+ = n_2^+ = 2$, $l_2^- = m_2^- = n_2^- = -2$, $l_3^+ = m_3^+ = n_3^+ = 1$, $l_3^- = m_3^- = n_3^- = -1$, $u_1^+ = v_1^+ = w_1^+ = 1$,

$u_1^- = v_1^- = w_1^- = -1$, $u_2^+ = v_2^+ = w_2^+ = 2$, $u_2^- = v_2^- = w_2^- = -2$, $u_3^+ = v_3^+ = w_3^+ = 1$ and $u_3^- = v_3^- = w_3^- = -1$ with

$$L_1 = M_1 = N_1 = \begin{bmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \qquad L_2 = M_2 = N_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$U_1 = V_1 = W_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \qquad U_2 = V_2 = W_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{4.2}$$

By using Matlab LMI toolbox, we solve the LMIs (3.1) in Theorem 3.1 and obtain the feasible solutions as follows:

$$P_1 = \begin{bmatrix} -3.7477 & 0.5120 & -0.2586 \\ 0.5120 & -7.5557 & -0.6515 \\ -0.2586 & -0.6515 & -9.0264 \end{bmatrix}, \qquad P_2 = \begin{bmatrix} -7.6744 & 0.2512 & 0.0154 \\ 0.2512 & -6.2020 & -0.0046 \\ 0.0154 & -0.0046 & -8.4207 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} 2.8411 & 0.0680 & -0.2237 \\ 0.0680 & 3.6480 & -0.2702 \\ -0.2237 & -0.2702 & 2.1788 \end{bmatrix}, \qquad Q_2 = \begin{bmatrix} 2.5375 & 0.5389 & -0.6248 \\ 0.5389 & 3.6343 & -0.3781 \\ -0.6248 & -0.3781 & 2.1497 \end{bmatrix},$$
$$R_1 = \begin{bmatrix} 1.8738 & 0.6069 & 1.2100 \\ 0.6069 & 2.2212 & 1.3303 \\ 1.2100 & 1.3303 & 2.1414 \end{bmatrix}, \qquad R_2 = \begin{bmatrix} -0.5616 & -0.7036 & 2.2034 \\ -0.7036 & 0.8218 & -0.4770 \\ 2.2034 & -0.4770 & -1.2174 \end{bmatrix},$$
$$\Lambda_1 = \begin{bmatrix} -6.5450 & 0 & 0 \\ 0 & -10.4004 & 0 \\ 0 & 0 & -16.6900 \end{bmatrix}, \qquad \Lambda_2 = \begin{bmatrix} -12.0795 & 0 & 0 \\ 0 & -9.8357 & 0 \\ 0 & 0 & -15.5096 \end{bmatrix},$$
$$\Gamma_1 = \begin{bmatrix} 1.7050 & 0 & 0 \\ 0 & 3.6765 & 0 \\ 0 & 0 & 2.1403 \end{bmatrix}, \qquad \Gamma_2 = \begin{bmatrix} 2.3176 & 0 & 0 \\ 0 & 3.2988 & 0 \\ 0 & 0 & 2.0030 \end{bmatrix},$$
$$\Omega_1 = \begin{bmatrix} 1.7050 & 0 & 0 \\ 0 & 3.6765 & 0 \\ 0 & 0 & 2.1403 \end{bmatrix}, \qquad \Omega_2 = \begin{bmatrix} 2.3176 & 0 & 0 \\ 0 & 3.2988 & 0 \\ 0 & 0 & 2.0030 \end{bmatrix},$$
$$\lambda_0 = 1.7593, \qquad \epsilon_0 = 3.6570. \tag{4.3}$$

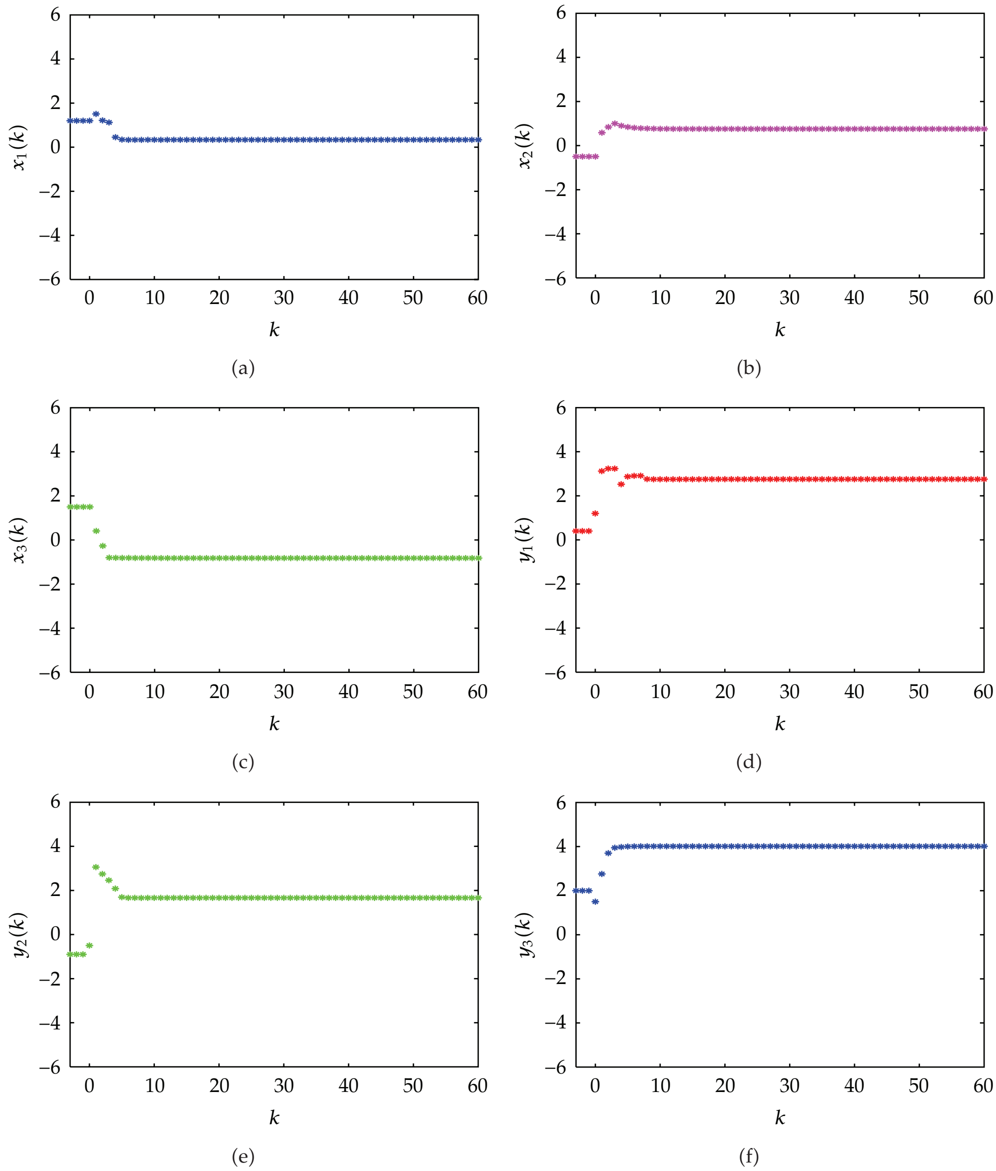

Figure 1: State trajectories of $x_1(k)$, $x_2(k)$, $x_3(k)$, $y_1(k)$, $y_2(k)$, $y_3(k)$ for Example 4.1.

Then, it follows from Theorem 3.1 that the SBAMNN (2.7) with given parameters is globally exponentially stable in the mean square. Our main purpose in this example is to estimate the maximum allowable upper bound delay σ_M and $\tau_M$for given lower bound σ_m and τ_m (Table 1). For instance, if we set $\sigma_m = \tau_m = 2$, the allowable time delay upper bound obtained by Gao and Cui [19] is 4. However, in our paper, we obtained that for any time delay satisfying $0 < \tau(t) \leq \tau_M =$ for any large finite value, $0 < \sigma(t) \leq \sigma_M =$ for any largefinite value. This is much larger than that in [19], which shows the less conservativeness of our developed method (Figure 1).

5. Conclusion

In this paper, we have considered the stability analysis problem for a class of discrete-time stochastic BAM neural networks with both discrete and distributed delays. Employing a Lyapunov-Krasovskii functional and a Linear matrix inequality approach has been developed to establish sufficient conditions for the SBAMNNs to be globally exponentially stable. It has been shown that the delayed SBAMNNs are globally exponentially stable if some LMIs are solvable and the feasibility of such LMIs can be easily checked by using the numerically efficient LMI toolbox in Matlab. A numerical example has been given to demonstrate the effectiveness of the obtained stability conditions.

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