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Spaces of continuous and bounded functions over the field of geometric complex numbers

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Abstract

Following Grossman and Katz (Non-Newtonian Calculus, 1972), we construct the sets $B(A)$ and $C(A)$ of geometric complex-valued bounded and continuous functions, where A denotes the compact subset of the complex plane $\mathbb{C}$. We show that the sets $B(A)$ and $C(A)$ of complex-valued bounded and continuous functions form a vector space with respect to the addition and scalar multiplication in the sense of multiplicative calculus. Finally, we prove that $B(A)$ and $C(A)$ are complete metric spaces.

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1 Introduction

Grossman and Katz [1], introduced the non-Newtonian calculus consisting of the branches of geometric, anageometric and biogeometric calculus, *etc.* Bashirov *et al.* [2] gave results with applications to the well-known properties of derivative and integral in the multiplicative calculus. Uzer [3] extended the multiplicative calculus to the complex-valued functions, was interested in the statements of some fundamental theorems and concepts of multiplicative complex calculus, and demonstrated some analogies between the multiplicative complex calculus and the classical calculus by theoretical and numerical examples. Recently, Çakmak and Başar [4] introduced the field $\mathbb{R}(N)$ of non-Newtonian real numbers and gave the triangle and Minkowski's inequalities in the sense of non-Newtonian calculus. They defined the complete metric spaces $\omega(N)$, $\ell_\infty(N)$, $c(N)$, $c_0(N)$ and $\ell_p(N)$ of all bounded, convergent, null and p-absolutely summable sequences in the sense of non-Newtonian calculus over the field $\mathbb{R}(N)$. Quite recently, Tekin and Başar [5] have introduced the spaces ω^*, ℓ_∞^*, c^*, c_0^* and ℓ_p^* over the non-Newtonian complex field $\mathbb{C}^*$ and obtained the corresponding results for these spaces, where $p \ddot{\geq} \ddot{0}$.

Following Bashirov *et al.* [2, 6, 7] and Uzer [3], Türkmen and Başar [8] obtained corresponding results for multiplicative complex numbers and the concept of multiplicative metric.

Following [8], the main purpose of this paper is the investigation of the space of functions defined by the multiplicative calculus. Following Türkmen and Başar [8], first we define *the set* $\mathbb{C}(G)$ *of multiplicative complex numbers* by

$$\begin{aligned}\mathbb{C}(G) &:= \left\{w = u \oplus i_g \odot v : u, v \in \mathbb{R}(G) \text{ and } i_g = \sqrt{1 \ominus e}^{G}\right\}\\ &= \left\{e^z = e^x \cdot \left(e^y\right)^{\ln e^i} : e^x, e^y \in \mathbb{R}(G) \text{ and } e^i = e^{\sqrt{-1}}\right\}\\ &= \left\{e^{x+iy} : x \in \mathbb{R}; -\pi < y \leq \pi \text{ and } i = \sqrt{-1}\right\}\\ &= \left\{e^z : z \in \mathbb{C}_{\mathrm{str}}\right\},\end{aligned}$$

where $\mathbb{R}(G)$ denotes the set of multiplicative real numbers and

$$\mathbb{C}_{\mathrm{str}} := \{z = x + iy : x \in \mathbb{R}; -\pi < y \leq \pi\}.$$

It is easy to see that $\mathbb{C}(G) = \mathbb{C} \setminus \{0\}$. It is clear from the definition of complex exp function that $\alpha(z) = e^z \neq 0$ for all $z \in \mathbb{C}_{\mathrm{str}}$. Since α-generator is a bijective function, it maps all complex numbers without zero to the set of values.

We suppose throughout that the A is a compact subset of the complex plane $\mathbb{C}$ and $(\mathbb{C}(G), \oplus, \odot)$ denotes the geometric complex field introduced by Türkmen and Başar [8].

We will consider the sets $B(A)$ and $C(A)$ in the following forms:

$$\begin{aligned}B(A) &:= \left\{f : A \to \mathbb{C}(G) \mid \exists K \in \mathbb{R}^+ \ni \forall x \in A, \left|f(x)\right|_G \leq K\right\},\\ C(A) &:= \left\{f : A \to \mathbb{C}(G) \mid f \text{ is continuous on } A\right\}.\end{aligned}$$

For $f, g \in B(A)$ and $\lambda \in \mathbb{C}(G)$, we define the operations addition ($\boxplus$) and scalar multiplication ($\boxdot$) by

$$\begin{array}{cccll}\boxplus & : & B(A) \times B(A) & \longrightarrow & \mathbb{C}(G)\\ & & (f, g) & \longrightarrow & (f \boxplus g)(x) = f(x) \oplus g(x),\\ \boxdot & : & \mathbb{C}(G) \times B(A) & \longrightarrow & \mathbb{C}(G)\\ & & (\lambda, f) & \longrightarrow & (\lambda \boxdot f)(x) = \lambda \odot f(x).\end{array}$$

2 Multiplicative complex field and related properties

Theorem 2.1 *The set $B(A)$ is a vector space with respect to the algebraic operations addition ($\boxplus$) and scalar multiplication ($\boxdot$).*

Proof Let $x \in A$, $f, g \in B(A)$ and $\lambda \in \mathbb{C}(G)$. Then, since $f, g \in B(A)$, there exist positive numbers K_1 and K_2 such that $|f(x)|_G \leq K_1$ and $|g(x)|_G \leq K_2$ for all $x \in A$. Therefore, one can see by the triangle inequality that

$$\begin{aligned}\left|(f \boxplus g)(x)\right|_G &= \left|f(x) \oplus g(x)\right|_G \leq \left|f(x)\right|_G \oplus \left|g(x)\right|_G\\ &\leq K_1 K_2 = K; \quad K \in \mathbb{R}^+.\end{aligned}$$

This means that $f \boxplus g \in B(A)$.

Since the equality $|\alpha_1 \odot \alpha_2|_G = |\alpha_1|_G \odot |\alpha_2|_G$ holds for $\alpha_1, \alpha_2 \in \mathbb{C}(G)$, by using this fact, we observe that

$$\begin{aligned}\left|(\lambda \boxdot f)(x)\right|_G &= \left|\lambda \odot f(x)\right|_G = |\lambda|_G \odot \left|f(x)\right|_G\\ &= \left[\left|f(x)\right|_G\right]^{\ln |\lambda|_G} < \infty.\end{aligned}$$

That is, $\lambda \boxdot f \in B(A)$.

(V1) Addition is commutative, that is,

$$\begin{aligned}(f \boxplus g)(x) &= f(x) \oplus g(x) \\ &= e^{\ln f(x)+\ln g(x)} \\ &= e^{\ln g(x)+\ln f(x)} \\ &= g(x) \oplus f(x) \\ &= (g \boxplus f)(x).\end{aligned}$$

(V2) Addition is associative, *i.e.*,

$$\begin{aligned}\left[(f \boxplus g) \boxplus h\right](x) &= \left[f(x) \oplus g(x)\right] \oplus h(x) \\ &= e^{[\ln f(x)+\ln g(x)]+\ln h(x)} \\ &= e^{\ln f(x)+[\ln g(x)+\ln h(x)]} \\ &= f(x) \oplus \left[g(x) \oplus h(x)\right] \\ &= \left[f \boxplus (g \boxplus h)\right](x).\end{aligned}$$

(V3) An identity element exists for addition. Indeed, since

$$\begin{aligned}&[f \boxplus 0](x) = f(x) \\ &\quad\Longrightarrow\quad f(x) \oplus \dot{0} = f(x) \\ &\quad\Longrightarrow\quad e^{\ln f(x)} e^{\ln \dot{0}} = f(x) \\ &\quad\Longrightarrow\quad f(x) \cdot \dot{0} = f(x) \\ &\quad\Longrightarrow\quad \dot{0} = e^0 = 1 \in B(A),\end{aligned}$$

the identity element is the function 0 such that $0(x) = 1$ for all $x \in A$.

(V4) The inverse element of any $f \in B(A)$ exists such that

$$(f \boxplus g)(x) = f(x) \oplus g(x) = e^{\ln f(x)} e^{\ln g(x)} = f(x) \cdot g(x) = 1,$$

which yields that

$$g(x) = \frac{1}{f(x)} \quad \text{for all } f \in B(A),$$

i.e., the inverse element of $f \in B(A)$ with respect to $\boxplus$ is $g = 1/f$.

(V5) Scalar multiplication distributes to the addition over the field. Indeed, since

$$\begin{aligned}\left[(\lambda \oplus \mu) \boxdot f\right](x) &= (\lambda \oplus \mu) \odot f(x) = e^{(\ln \lambda+\ln \mu)\ln f(x)} = e^{\ln \lambda \ln f(x)+\ln \mu \ln f(x)} \\ &= \left[\lambda \odot f(x)\right] \oplus \left[\mu \odot f(x)\right] \\ &= (\lambda \boxdot f)(x) \boxplus (\mu \boxdot f)(x),\end{aligned}$$

scalar multiplication distributes to the addition over the field.

(V6) Scalar multiplication distributes to vector addition, *i.e.*,

$$\begin{aligned}
\big[\lambda \boxdot (f \boxplus g)\big](x) &= \lambda \odot \big[f(x) \oplus g(x)\big] \\
&= e^{\ln\lambda\cdot[\ln f(x)+\ln g(x)]} \\
&= e^{\ln\lambda\cdot\ln f(x)+\ln\lambda\cdot\ln g(x)} \\
&= \big[\lambda \odot f(x)\big] \oplus \big[\lambda \odot g(x)\big] \\
&= (\lambda \boxdot f)(x) \boxplus (\lambda \boxdot g)(x).
\end{aligned}$$

(V7) Compatibility of scalar multiplication with field multiplication holds:

$$\begin{aligned}
\big[(\lambda \odot \mu) \boxdot f\big](x) &= (\lambda \odot \mu) \odot f(x) \\
&= e^{\ln\lambda\cdot\ln\mu\cdot\ln f(x)} \\
&= e^{\ln\lambda\cdot(\ln\mu\cdot\ln f(x))} \\
&= \lambda \odot \big[\mu \odot f(x)\big] \\
&= \big[\lambda \boxdot (\mu \boxdot f)\big](x).
\end{aligned}$$

(V8) e is the identity element of scalar multiplication. It is easy to see that

$$(e \boxdot f)(x) = e \odot f(x) = e^{\ln e \ln f(x)} = f(x),$$

which says that the identity element of scalar multiplication is e.

From (V1)-(V8) vector space axioms are satisfied. Hence $B(A)$ is a vector space over $\mathbb{C}(G)$ with the algebraic operations addition ($\boxplus$) and scalar multiplication ($\boxdot$). □

Theorem 2.2 *The set $C(A)$ is a subspace of the space $B(A)$ with addition* ($\boxplus$) *and scalar multiplication* ($\boxdot$).

Proof First, we should show that $C(A) \neq \emptyset$ and $C(A) \subset B(A)$.

Since $0(x) = 1$ for all $x \in A$, $0 \in C(A)$, that is, the set $C(A)$ is not empty.

Suppose that $C(A) \not\subset B(A)$. Then there is $f \in C(A)$ such that $|f(x_n)|_G \geq n$ for $x_n \in A$ for all $n \in \mathbb{N}$. Since A is compact, (x_n) is a bounded geometric sequence. So, (x_n) has at least one convergent subsequence (x_{n_k}), say $x_{n_k} \overset{G}{\to} x_0$, as $k \to \infty$. Since A is closed $x_0 \in A$, hence f is continuous at the point x_0. Therefore, for $\epsilon > 1$, there exists at least $\delta > 1$ such that $|f(x) \ominus f(x_0)|_G < \epsilon$ for all $|x \ominus x_0|_G < \delta$. Now, we choose $\epsilon = e$. Thus, we have $||f(x)| - |f(x_0)||_G \leq |f(x) \ominus f(x_0)|_G < \epsilon$ which leads to $|f(x)|_G < e + |f(x_0)|_G$. This contradicts the fact $|f(x_n)|_G \geq n$. Hence, $f \in B(A)$ and the inclusion $C(A) \subset B(A)$ holds.

Let $x \in A, f, g \in C(A)$ and $\lambda, \mu \in \mathbb{C}(G)$. Then we have

$$\begin{aligned}
\big[(\lambda \boxdot f) \boxplus (\mu \boxdot g)\big](x) &= \big[\lambda \odot f(x)\big] \oplus \big[\mu \odot g(x)\big] = e^{\ln\lambda\cdot\ln f(x)+\ln\mu\cdot\ln g(x)} \\
&= \big[e^{\ln f(x)}\big]^{\ln\lambda}\big[e^{\ln g(x)}\big]^{\ln\mu} \\
&= \big[f(x)\big]^{\ln\lambda}\big[g(x)\big]^{\ln\mu} \in C(A).
\end{aligned}$$

Therefore, the algebraic operations $\boxplus$ and $\boxdot$ are closed on $C(A)$.

Axioms (V1)-(V8) on $C(A)$ can be fulfilled in the same way as in the proof of Theorem 2.1. □

3 Geometric metric spaces

For each $f,g \in B(A)$, we define d_G by

$$\begin{array}{llll} d_G: & B(A) \times B(A) & \longrightarrow & \mathbb{R}^+(G) \\ & (f,g) & \longrightarrow & d_G(f,g) = \sup_{x\in A} \left|f(x) \ominus g(x)\right|_G. \end{array} \tag{3.1}$$

Theorem 3.1 *$(B(A), d_G)$ is a complete metric space.*

Proof Let $x \in A$ and $f, g, h \in B(A)$. Now, we check the metric axioms. Let $e^{f_1(y)} = f(x)$, $e^{g_1(y)} = g(x)$, $e^{h_1(y)} = h(x) \in B(A)$ for $x, y \in A$, where f_1, g_1 and h_1 are the complex-valued bounded functions.

(GM1) Non-negative property holds: $\dot{0} = 1$, $\forall f, g \in B(A) \Longrightarrow d_G(f,g) \geq \dot{0}$.

(GM2) $d_G(f,g) = \dot{0} \iff f(x) \ominus g(x) = \dot{0}$.

$\Rightarrow$: From the definition of supremum, we can write

$$\begin{aligned} & d_G(f,g) = 1 \\ & \quad \Longrightarrow \quad \sup_{x\in A} \left|f(x) \ominus g(x)\right|_G = 1 \\ & \quad \Longrightarrow \quad \left|f(x) \ominus g(x)\right|_G \leq 1, \end{aligned} \tag{3.2}$$

and from the definition of geometric absolute value (see [8]),

$$\left|f(x) \ominus g(x)\right|_G \geq 1 = e^0. \tag{3.3}$$

Using (3.2) and (3.3), we have

$$\begin{aligned} & \left|f(x) \ominus g(x)\right|_G = 1 \\ & \quad \Longrightarrow \quad \left|\frac{f(x)}{g(x)}\right|_G = 1 \\ & \quad \Longrightarrow \quad \left|\frac{e^{f_1(y)}}{e^{g_1(y)}}\right|_G = 1 \\ & \quad \Longrightarrow \quad \left|e^{f_1(y)-g_1(y)}\right|_G = 1 \\ & \quad \Longrightarrow \quad e^{|f_1(y)-g_1(y)|} = 1 = e^0 \\ & \quad \Longrightarrow \quad \left|f_1(y) - g_1(y)\right| = 0 \\ & \quad \Longrightarrow \quad f_1(y) - g_1(y) = 0 \\ & \quad \Longrightarrow \quad f_1(y) = g_1(y) \\ & \quad \Longrightarrow \quad e^{f_1(y)} = e^{g_1(y)} \\ & \quad \Longrightarrow \quad f(x) = g(x). \end{aligned}$$

$\Leftarrow$: Conversely, we get

$$
\begin{aligned}
& f(x) = g(x) \\
& \quad \Longrightarrow \quad \left|\frac{f(x)}{g(x)}\right| = 1 \\
& \quad \Longrightarrow \quad \left|f(x) \ominus g(x)\right|_G = 1 \\
& \quad \Longrightarrow \quad \sup_{x\in A}\left|f(x) \ominus g(x)\right|_G = 1 \\
& \quad \Longrightarrow \quad d_G(f,g) = 1.
\end{aligned}
$$

(GM3) Symmetry property holds. From the definition of the relation d_G, we have

$$
\begin{aligned}
d_G(f,g) &= \sup_{x\in A}\left|f(x) \ominus g(x)\right|_G \\
&= \sup_{x\in A}\left|\frac{f(x)}{g(x)}\right|_G = \sup_{y\in A}\left|\frac{e^{f_1(y)}}{e^{g_1(y)}}\right|_G \\
&= \sup_{y\in A}\left|e^{f_1(y)-g_1(y)}\right|_G = \sup_{y\in A} e^{|f_1(y)-g_1(y)|} \\
&= \sup_{y\in A} e^{|g_1(y)-f_1(y)|} = \sup_{y\in A}\left|e^{g_1(y)-f_1(y)}\right|_G \\
&= \sup_{y\in A}\left|\frac{e^{g_1(y)}}{e^{f_1(y)}}\right|_G = \sup_{x\in A}\left|\frac{g(x)}{f(x)}\right|_G \\
&= \sup_{x\in A}\left|g(x) \ominus f(x)\right|_G \\
&= d_G(g,f).
\end{aligned}
$$

(GM4) The triangle inequality holds. Firstly we will get

$$
\begin{aligned}
\left|f(x) \ominus g(x)\right|_G &= \left|\frac{f(x)}{g(x)}\right|_G = \left|\frac{e^{f_1(y)}}{e^{g_1(y)}}\right|_G = \left|e^{f_1(y)-g_1(y)}\right|_G \\
&= e^{|f_1(y)-g_1(y)|} = e^{|f_1(y)-h_1(y)+h_1(y)-g_1(y)|} \\
&\le e^{|f_1(y)-h_1(y)|+|h_1(y)-g_1(y)|} = e^{|f_1(y)-h_1(y)|} \cdot e^{|h_1(y)-g_1(y)|} \\
&= \left|e^{f_1(y)-h_1(y)}\right|_G \cdot \left|e^{h_1(y)-g_1(y)}\right|_G = \left|\frac{e^{f_1(y)}}{e^{h_1(y)}}\right|_G \cdot \left|\frac{e^{h_1(y)}}{e^{g_1(y)}}\right|_G \\
&= \left|\frac{f(x)}{h(x)}\right|_G \cdot \left|\frac{h(x)}{g(x)}\right|_G = \left|f(x) \ominus h(x)\right|_G \oplus \left|h(x) \ominus g(x)\right|_G.
\end{aligned}
$$

Therefore, one can easily see that

$$
\begin{aligned}
\sup_{x\in A}\left|f(x) \ominus g(x)\right|_G &\le \sup_{x\in A}\left[\left|f(x) \ominus h(x)\right|_G \oplus \left|h(x) \ominus g(x)\right|_G\right] \\
&\le \sup_{x\in A}\left|f(x) \ominus h(x)\right|_G \oplus \sup_{x\in A}\left|h(x) \ominus g(x)\right|_G,
\end{aligned}
$$

which leads us to the desired inequality

$$d_G(f,g) \leq d_G(f,h) \oplus d_G(h,g).$$

Properties (GM1)-(GM4) imply that $(B(A), d_G)$ is a metric space.

Suppose that (f_n) is a Cauchy sequence in the metric space $(B(A), d_G)$. Then, for every $\varepsilon > 1$, there is an $n_0 = n_0(\varepsilon)$ such that

$$d_G(f_n, f_m) = \sup_{x\in A}\left|f_n(x) \ominus f_m(x)\right|_G < \varepsilon \tag{3.4}$$

for all $m, n > n_0$. Hence, $\{f_n(x)\}$ is a Cauchy sequence of geometric complex numbers for each fixed $x \in A$. Since $\mathbb{C}(G)$ is complete by Theorem 4.5 of Türkmen and Başar [8], the sequence $\{f_n(x)\}$ is convergent, say $f_n(x) \xrightarrow{G} f(x)$ for $x \in A$, as $n \to \infty$. By letting $m \to \infty$ with $n > n_0$, we derive from (3.4) that $\sup_{x\in A} |f_n(x) - f(x)|_G \leq \epsilon$. Therefore we have $|f_n(x) - f(x)|_G \leq \epsilon$ for all $n > n_0$ and for all $x \in A$. That is to say, for every $\epsilon > 1$, there exists at least $n_0 = n_0(\epsilon) \in \mathbb{N}$ such that $|f_n(x) - f(x)|_G \leq \epsilon$ for all $n > n_0$ and for all $x \in A$. This means that the sequence (f_n) converges uniformly to f as $n \to \infty$.

Additionally, since there exists a $K > 0$ such that

$$\left|f(x)\right|_G = \left|f(x) - f_n(x) + f_n(x)\right|_G \leq \left|f(x) - f_n(x)\right|_G \oplus \left|f_n(x)\right|_G \leq \epsilon \cdot K$$

for all $x \in A$ and for all $n \in \mathbb{N}, f \in B(A)$. That is to say, an arbitrary Cauchy sequence in the metric space $(B(A), d_G)$ is convergent. This completes the proof. □

It is obvious that d_G is an induced metric from the norm $\|\cdot\|_G$, that is,

$$\|f\|_G = d_G(f,0) = \sup_{x\in A}\left|f(x) \ominus 0(x)\right|_G; \quad f \in B(A). \tag{3.5}$$

So, we have the following as a direct consequence of Theorem 3.1.

Corollary 3.2 *$(B(A), \|\cdot\|_G)$ is a Banach space, where $\|\cdot\|_G$ is defined by* (3.5) .

Theorem 3.3 *$(C(A), d'_G)$ is a complete metric space, where d'_G is defined on the space $C(A)$ by*

$$\begin{array}{llll} d'_G: & C(A) \times C(A) & \longrightarrow & \mathbb{R}^+(G) \\ & (f,g) & \longrightarrow & d'_G(f,g) = \max_{x\in A}\left|f(x) \ominus g(x)\right|_G. \end{array}$$

Theorem 3.3 leads to the following result.

Corollary 3.4 *$(C(A), \|\cdot\|_G)$ is a Banach space, where $\|\cdot\|_G$ is defined on the space $C(A)$ by*

$$\|f\|_G = d'_G(f,0) = \max_{x\in A}\left|f(x) \ominus 0(x)\right|_G; \quad f \in C(A).$$

Competing interests
The author declares that he has no competing interests.

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