UNIT ROOT TESTING IN THE PRESENCE OF INNOVATION VARIANCE BREAKS: A SIMPLE SOLUTION WITH INCREASED POWER

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The Dickey-Fuller unit root test is known to suffer severe oversizing in the presence of innovation variance breaks. In this paper, forward and reverse Dickey-Fuller regressions are proposed as a means of correcting this size distortion. The results of Monte Carlo experimentation show such an approach to result in both satisfactory size properties and increased power relative to previously suggested solutions.

1. Introduction

The Dickey-Fuller (DF) test [4] is routinely employed in applied econometric analysis to examine the order of integration of economic time series. Recently, Kim et al. [9] have considered the properties of the DF test when applied to a unit root process, which experiences a break in innovation variance. This analysis is a welcome development, as in contrast to the huge literature on the behaviour of unit root tests in the presence of structural breaks in the level or trend of a time series (see, *inter alia*, Bai et al. [1]; Bai and Perron [2]; Banerjee et al. [3]; Perron [12, 13]), the impact of structural changes in variance has rarely been addressed, especially for integrated processes. (Wichern, Miller, and Hsu [14], Hsu [7], and Inclan [8] are cited by Kim et al. [9] as examples of the few instances where variance breaks have been considered in general circumstances. The only case cited where the impact of variance breaks has been considered in the context of integrated processes is Hamori and Tokihisa [6].) The evidence presented by Kim et al. [9] shows that when the break in variance takes the form of a large *decrease* early in the sample period, the DF can suffer severe size distortion. With a DF testing equation as

in (1.1), the DF τ_μ test is calculated as the *t*-statistic for null hypothesis $H_0 : \rho = 1$,

$$y_t = \mu + \rho y_{t-1} + \xi_t, \qquad V(\xi_t) = \sigma^2. \tag{1.1}$$

Given (1.1), the τ_μ test is subject to severe size distortion when there is a break in σ^2 early in the sample period. This spurious rejection of the unit hypothesis is in sharp contrast to the literature associated with Perron [12] where structural breaks are shown to cause $I(0)$ processes to appear $I(1)$. In response to this, Kim et al. [9] develop an alternative Perron-style [12, 13] unit root test statistic based upon feasible modified generalised least squares, denoted by t_F. Having improved size properties in the presence of variance breaks, this test suffers however from low power in comparison to the τ_μ test as the authors note.

In this paper, the properties of the DF_{max} test of Leybourne [10] are examined in the presence of innovation variance breaks. Considering (1.1) above, the DF_{max} test results from joint application of the τ_μ test to both $\{y_t\}$ and $\{z_t\}$, where $z_t = y_{T-t+1}$ for $t = 1,\dots,T$, with the larger value obtained are denoted by DF_{max}. With early decreases in innovation variance for the forward regression becoming late increases in innovation variance for the reverse regression, the DF_{max} test has an intuitive appeal, as neither late nor increasing breaks result in size distortion.

Using Monte Carlo simulation, the properties of the τ_μ, DF_{max}, and t_F tests are examined in the presence of innovation variance breaks. Crucially, it is found that in the majority of the cases considered, the DF_{max} has a clear power advantage over the t_F test, while exhibiting similar size.

2. Monte Carlo simulation

2.1. Experimental design

To allow a direct comparison with the results of Kim et al. [9] for the t_F test, the following data generation process (DGP) was employed:

$$
\begin{aligned}
& y_t = \rho y_{t-1} + \varepsilon_t, \quad t = 1,\dots,T, \\
& \varepsilon_t = \sigma_t \eta_t, \\
& \eta_t \sim \text{i.i.d. } N(0,1), \\
& \sigma_t = \begin{cases} \sigma_1 & \text{for } t \leqslant \tau^* T, \\ \sigma_2 & \text{for } t > \tau^* T, \end{cases}
\end{aligned} \tag{2.1}
$$

where τ^* represents the break fraction determining the point at which there is an abrupt change in the error variance. (The error series $\{\eta_t\}$ was generated using pseudo i.i.d. $N(0,1)$ random numbers from the RNDNS procedure in the Gauss programming language version 3.2.13, with the initial value (y_0) set equal to zero. All experiments were performed over 40,000 replications with the first 100 observations created discarded to remove the influence of initial conditions.)

Kim et al. [9] show size distortion of the τ_μ test to depend upon the size of the decrease in variance and the time at which it occurs. Denoting the break size (σ_2/σ_1) as δ, the values $\delta \in \{0.25, 0.4, 0.6, 0.8, 1.0\}$ are considered, with $\delta = 1$ denoting no break in variance. Following Kim et al. [9], the values $\tau^* \in \{0.2, 0.4, 0.6, 0.8\}$ are chosen for the break fractions. Similarly, two sample sizes are considered with $T \in \{100, 200\}$. For each of the experimental designs, the τ_μ and $\mathrm{DF_{max}}$ tests are estimated, with the results obtained compared to those of Kim et al. [9] for the t_F test. To consider the sizes of the unit root tests in the presence of variance breaks, the value $\rho = 1$ was imposed in the DGP. To assess the power of the tests, near integrated processes are considered. The values chosen for ρ in these cases are $\rho = 0.9$ for $T = 100$, and $\rho = 0.95$ for $T = 200$. (Alternative values of ρ, and unit root tests containing both intercept and trend terms were also considered. As the results obtained for these additional experiments were similar to those presented here, they have been omitted in the interests of brevity. However, the results are available from the author upon request.) Empirical rejection frequencies in all cases are calculated at the 5% nominal level of significance ($\alpha = 0.05$), with the τ_μ and $\mathrm{DF_{max}}$ tests employing critical values from Fuller [5] and Leybourne [10], respectively.

2.2. Results

The results of the size experiments are presented in Table 2.1. The results show that the τ_μ test can suffer severe size distortion across a range of values of δ and τ^*, with an empirical size of 39.3% observed for $\{\delta, \tau^*, T\} = \{0.25, 0.2, 100\}$. Although the $\mathrm{DF_{max}}$ test can also experience oversizing for large breaks early in the sample, for more moderate breaks or later breaks, it has better size properties than the t_F test. This is illustrated in Figure 2.1 where results for the rival tests are presented for $\{\delta, T\} = \{0.6, 100\}$. It should also be recognised that the values of δ considered relate to the standard deviations of the innovations, not their variances. Therefore, $\delta = 0.6$ relates to a change in variance by a factor of approximately 3, with the extreme value $\delta = 0.25$ indicating the case where $\sigma_1^2 = 16\sigma_2^2$. It can therefore be questioned how much weight should be attached to the extreme cases where δ takes such small values. (The change in variance can be considered in terms of the vari-
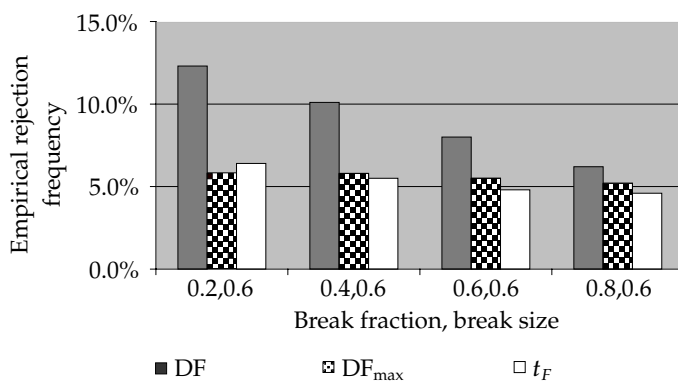
FIGURE 2.1. Empirical size for $\alpha = 0.05$ and $T = 100$.
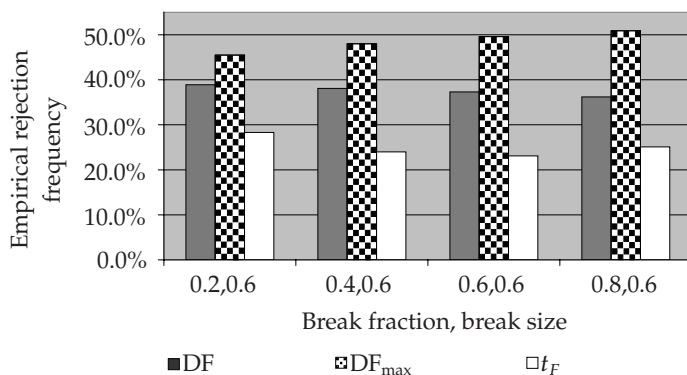
FIGURE 2.2. Power for $\alpha = 0.05$, $T = 100$, and $\rho = 0.9$.

ance ratio test. For the relatively large degrees of freedom considered here, the variance ratio test would easily reject the null of constant variances given a calculated value of 3. This indicates that a relatively large shift is being considered. A value of 16 would be viewed as extremely high.)

Considering the power results contained in Table 2.2, it can be seen that the t_F test has the lowest power of the tests in the vast majority of experiments, the exceptions being when the largest possible break occurs at the earliest possible point. When more moderate, plausible breaks in variance are considered, the DF_{max} test has a clear power advantage over the t_F test. Figure 2.2 illustrates this, displaying results for $(T, \delta, \rho) = (100, 0.6, 0.9)$, where the DF_{max} test has 61% more power than the t_F test.

TABLE 2.1. Empirical size ($\alpha = 0.05$).

τ^*		$\delta = 1.0$	$\delta = 0.8$	$\delta = 0.6$	$\delta = 0.4$	$\delta = 0.25$
	τ_μ	0.050	0.073	0.123	0.234	0.393
0.2	DF_{max}	0.050	0.051	0.058	0.080	0.132
	t_F	0.053	0.062	0.064	0.065	0.066
	τ_μ	. . .	0.068	0.101	0.165	0.239
0.4	DF_{max}	. . .	0.051	0.058	0.077	0.105
	t_F	. . .	0.054	0.055	0.058	0.059
	τ_μ	. . .	0.062	0.080	0.105	0.131
0.6	DF_{max}	. . .	0.051	0.055	0.066	0.076
	t_F	. . .	0.047	0.048	0.056	0.057
	τ_μ	. . .	0.056	0.062	0.070	0.074
0.8	DF_{max}	. . .	0.051	0.052	0.054	0.055
	t_F	. . .	0.046			

(a) $T = 100$.

τ^*		$\delta = 1.0$	$\delta = 0.8$	$\delta = 0.6$	$\delta = 0.4$	$\delta = 0.25$
	τ_μ	0.049	0.072	0.120	0.229	0.390
0.2	DF_{max}	0.050	0.051	0.058	0.082	0.138
	t_F	0.052	0.055	0.055	0.057	0.058
	τ_μ	. . .	0.066	0.099	0.162	0.237
0.4	DF_{max}	. . .	0.051	0.059	0.081	0.110
	t_F	. . .	0.053	0.052	0.052	0.050
	τ_μ	. . .	0.059	0.077	0.105	0.129
0.6	DF_{max}	. . .	0.051	0.057	0.068	0.078
	t_F	. . .	0.048	0.050	0.051	0.053
	τ_μ	. . .	0.053	0.060	0.068	0.074
0.8	DF_{max}	. . .	0.050	0.051	0.054	0.055
	t_F	. . .	0.048	0.049	0.052	0.053

(b) $T = 200$.

TABLE 2.2. Power ($\alpha = 0.05$).

τ^*		$\delta = 1.0$	$\delta = 0.8$	$\delta = 0.6$	$\delta = 0.4$	$\delta = 0.25$
	τ_μ	0.339	0.354	0.389	0.452	0.508
0.2	DF_{max}	0.512	0.487	0.455	0.428	0.408
	t_F	0.250	0.260	0.283	0.347	0.472
	τ_μ	. . .	0.352	0.381	0.422	0.448
0.4	DF_{max}	. . .	0.496	0.480	0.467	0.457
	t_F	. . .	0.243	0.240	0.287	0.395
	τ_μ	. . .	0.351	0.373	0.395	0.408
0.6	DF_{max}	. . .	0.504	0.496	0.491	0.488
	t_F	. . .	0.244	0.231	0.262	0.338
	τ_μ	. . .	0.349	0.362	0.375	0.382
0.8	DF_{max}	. . .	0.510	0.509	0.508	0.505
	t_F	. . .	0.241	0.251	0.280	0.323

(a) $\rho = 0.9$, $T = 100$.

τ^*		$\delta = 1.0$	$\delta = 0.8$	$\delta = 0.6$	$\delta = 0.4$	$\delta = 0.25$
	τ_μ	0.330	0.346	0.380	0.447	0.508
0.2	DF_{max}	0.509	0.483	0.450	0.425	0.409
	t_F	0.260	0.272	0.300	0.370	0.489
	τ_μ	. . .	0.346	0.376	0.417	0.445
0.4	DF_{max}	. . .	0.493	0.479	0.467	0.457
	t_F	. . .	0.247	0.252	0.289	0.415
	τ_μ	. . .	0.343	0.364	0.388	0.401
0.6	DF_{max}	. . .	0.501	0.495	0.490	0.485
	t_F	. . .	0.247	0.246	0.284	0.370
	τ_μ	. . .	0.340	0.353	0.369	0.378
0.8	DF_{max}	. . .	0.507	0.507	0.506	0.505
	t_F	. . .	0.260	0.273	0.292	0.333

(b) $\rho = 0.95$, $T = 200$.

3. Conclusion

In this paper, recent research on the testing of unit roots in the presence of breaks in innovation variance has been extended. The results presented show that although the t_F test of Kim et al. [9] removes size distortion in the presence of extreme variance breaks, for less extreme cases the DF_{max} test of Leybourne [10] has similar, and sometimes better, size properties. It has also been seen that the low power of the t_F test is not shared by the DF_{max} test, which has high power against near integrated alternatives over a range of plausible variance breaks. The size and power analyses therefore suggest that the DF_{max} test is of practical importance in presence of innovation variance breaks, a finding which contrasts with the suggestion of Leybourne et al. [11] for the case of breaks in level or drift.

Acknowledgment

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