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Research Article

New JLS-Factor Model versus the Standard JLS Model: A Case Study on Chinese Stock Bubbles

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In this paper, we extend the Johansen-Ledoit-Sornette (JLS) model by introducing fundamental economic factors in China (including the interest rate and deposit reserve rate) and the historical volatilities of targeted and US equity indices into the original model, which is a flexible tool to detect bubbles and predict regime changes in financial markets. We then derive a general method to incorporate these selected factors in addition to the log-periodic power law signature of herding and compare the prediction accuracy of the critical time between the original and the new JLS models (termed the JLS-factor model) by applying these two models to fit two well-known Chinese stock indices in three bubble periods. The results show that the JLS-factor model with Chinese characteristics successfully depicts the evolutions of bubbles and "antibubbles" and constructs efficient end-of-bubble signals for all bubbles in Chinese stock markets. In addition, the results of standard statistical tests demonstrate the excellent explanatory power of these additive factors and confirm that the new JLS model provides useful improvements over the standard JLS model.

1. Introduction

As our understanding of the topic deepens, people are gradually realizing that the stock market is a complex system that has many participants with different characteristics that influence each other. In addition, no stock market is completely independent of another; each connects with others to form a larger system to some extent. For this reason, the stock market shows a nonlinear mechanism in its operation process. Thus, using an equilibrium model to study the stock market is not a suitable approach.

In consideration of the limitations of classical financial theory, more and more researchers are using nonlinear dynamic systems to research financial markets in the new discipline termed "econophysics." For the bubble problem in stock markets, financial physicists combine rational expectations theory in economics with the self-organizing critical phenomenon in statistical physics to diagnose the plausible times at which the bubble burst. Among them, the most representative scholars are Sornette and Johansen and coworkers, who stated that bubbles are not characterized by an exponential increase in prices but rather by a faster-than-exponential growth in prices. They also argued that most financial crashes are the climax of the so-called log-periodic power law signatures (LPPLS) associated with speculative bubbles [1].

The predictive power of LPPLS was first discovered in acoustic emissions prior to rupture and in identifying the precursors of earthquakes [2, 3]. However, the introduction of LPPLS to predict the bursting of speculative bubbles in financial markets goes back to the pioneering works of Feigenbaum and Freund [4] and Sornette et al. [5], who independently of each other disclosed LPPLS structures prior to the crash of the S&P 500 in October 1987. Beyond that, by means of extending the renormalization group approach to the second order of perturbation, Sornette and Johansen [6] proposed the second-order LPPL "Landau" model to capture the evolution of stock prices over a longer precrash period, say seven to eight years, compared with the original LPPL model. Then, they used it to fit the Dow Jones index prior to the 1929 crash and the S&P 500 index prior to the 1987 crash.

Since then, a large body of empirical evidence supporting this proposition has been presented ([7–15] (hereafter the LPPL model is also referred to as the JLS (i.e., Johansen-Ledoit-Sornette) model)). All this research has shown that the time evolutions of the stock market index prior to the 1929, 1987, and 1998 crashes on Wall Street and the 1997 crash in Hong Kong are in very good agreement with the predictions of the LPPL model.

As for the cause of the stock market crash, Johansen and Sornette [13] argued that a large crash is mainly caused by local self-reinforcing imitation between investors and their herding behavior. Under the action of the positive price-to-price feedback mechanism, this self-reinforcing imitation process eventually leads to a bubble. Specifically, if the tendency for traders to "imitate" their "friends" increases up to the "critical" point (or critical time), many traders may place the same order (sell) at the same time, thus causing a crash [13]. However, it is also worth noting that imitation between investors and their herding behavior lead not only to speculative bubbles with accelerating overvaluations of financial markets possibly followed by crashes but also to "antibubbles" with decelerating market devaluations following all-time highs. For this, Johansen and Sornette [16] proposed a third-order Landau expansion in which demand decreases slowly with barriers that progressively reduce, leading to a power law decay of the market price accompanied by decelerating log-periodic oscillations. In addition, they documented this behavior on the Japanese Nikkei stock index from 1990 and on the gold future prices after 1980, both after their all-time highs [16]. Besides, a remarkable similarity in the behavior of the US S&P 500 index from 1996 to August 2002 and of the Japanese Nikkei index from 1985 to 1992 (11-year shift) was presented by Sornette and Zhou [17]. Several other examples have been described in the Russian stock market (Sornette et al., 1999) and in emerging and western markets [17–19]. Among them, authors have analyzed 39 world stock market indices from 2000 to the end of 2002, finding that 22 are in an antibubble regime (owing to the criterion of obtaining at least a solution in the fitting procedure, they have found no evidence of an antibubble in China during that period). However, although this third-order LPPL Landau model is suitable for describing the evolution of bubbles and antibubbles almost twelve years, some deviations may appear when using the second-order LPPL Landau model to calibrate antibubbles, implying that much higher-order Landau models are useless.

In addition to the third-order LPPL Landau model, Sornette and coworkers expanded the original LPPL model, including (a) constructing the Weierstrass-type LPPL model [20, 21], (b) extending the JLS model with second-order harmonics [17], (c) proposing the JLS-factor model in which the LPPL bubble component is augmented by fundamental economic factors [22], (d) inferring the fundamental value of the stock and crash nonlinearity from bubble calibration [23] and detecting market rebounds [24], (e) extending the JLS model to include an additional pricing factor called the "Zipf factor," which describes the diversification risk of stock market portfolio [25], (f) reducing the JLS model to a function of only three nonlinear parameters [26], (g) presenting a volatility-confined LPPL model to describe and diagnose situations when excessive public expectations of future price increases cause prices to be temporarily elevated [27], (h) introducing quantile regression to the LPPLS detection problem and defining the so-called DS LPPLS Confidence and Trust indicators that enrich considerably the diagnosis of bubbles [28] (recently, Zhang et al. [29] added two new indicators (DS LPPLS Bubble Status and End-of-Bubble) into this system), (i) employing a rigorous likelihood approach and providing interval estimates of the parameters, including the most important critical times of market regime changes [30], and (j) presenting a plausible microfounded model for the previously postulated power law finite time singular form of the crash hazard rate in the JLS model of rational expectation bubbles [31]. In addition, the research team at ETH Zurich has further developed an LPPLS-based bubble detection system, as described in Sornette et al. [32] and Zhang et al. [28].

However, although the JLS model has been extended to many different forms, the original model remains a powerful and flexible tool with which to detect financial bubbles and crashes in various markets, especially Chinese stock bubbles. Examples include the antibubble in China's stock market that started in August 2001 [33], two bubbles and subsequent market crashes in two important indices of Chinese stock markets (Shanghai Stock Exchange Composite (SSEC) index and Shenzhen Stock Exchange Component (SZSC) index) between May 2005 and July 2009 [34], two well-known Chinese stock bubbles, from August 2006 to October 2007 (bubble 1) and from October 2008 to August 2009 (bubble 2) [25], and the bubble regime that developed in Chinese stock markets in mid-2014 and started to burst in June 2015 [32].

Unfortunately, the JLS model is based on the assumption that crashes are the outcome of the interactions of market players resulting in herding behavior (i.e., an endogenous origin), while exogenous shocks (due to changes in market fundamentals) rarely play an important role, only serving as trigger factors, which is at odds with standard economy theory [35]. Although Zhou and Sornette [22] presented a general methodology under which to incorporate fundamental economic factors (e.g., interest rate, interest spread, historical volatility, implied volatility, and exchange rates) into the theory of herding to describe bubbles and antibubbles, the most surprising result is that the best model is the second-order LPPL model without any factors. Indeed, the evolution of a complex system is the result of an entangled combination of endogenous organization as well as a response to external news and exogenous shocks, especially for an equity market such as the Chinese stock market that is still immature and heavily influenced by exogenous shocks. Compared with western markets, the Chinese stock market has its own characteristics, including the following: (i) being dominated by individual investors, (ii) being highly volatile, (iii) nontradability of more than two-thirds of the shares, (iv) short sale constraints, (v) response to exogenous information such as government policies, a firm's accounting information, and stock exchange announcements, (vi) stronger imprints of herding, (vii) overspeculation, (viii) overvaluation of markets, (ix) widely taken short-term positions, (x) insider trading, and (xi) a distempered regulation system [33, 36]. These specific features make the market exhibit strong idiosyncrasies and puzzles in addition to the more common behavior of mature stock markets [37]. Therefore, in addition to the herding behavior between traders, we think that taking into account the influence of government policies and other exogenous shocks on the evolution of Chinese stock bubble may be more advisable, especially the impact from changes in monetary policy and fluctuations in international stock markets.

Given the foregoing, we generalize the standard JLS model by incorporating two fundamental economic factors in China (i.e., the interest rate and deposit reserve rate) and the historical volatility of targeted and US equity indices into the original model. We then present an ex-post analysis of what Jiang et al. [34] and Sornette et al. [32] earlier identified as being the three significant bubbles developing in major Chinese stock markets: the first bubble ran from July 2005 to October 2008, the second one ran from November 2008 to August 2009, and the third one ran from March 2014 to July 2015. We also compare the prediction accuracy of the critical time between the original and the new JLS model (i.e., the JLS-factor model). The empirical results show that the JLS-factor model with Chinese characteristics successfully diagnoses all the well-documented bubbles in Chinese stock markets. Further, by comparing the prediction accuracy of the original and new JLS models, we find that the critical time estimated by the new JLS model for Chinese stock market bubbles during 2005–2015 is closer to the actual time than the original JLS model in general, which demonstrates the excellent explanatory power of our proposed JLS-factor model. In addition, the results of different standard statistical tests show that the new JLS model is superior to the original JLS model. Moreover, the results of significance testing provide indirect evidence on the key role of fundamental economic and exogenous factors in China affecting the evolution of Chinese stock bubbles (except the 2008–2009 bubbles, which were probably punctuated by a vanishingly small change in some endogenous factors).

The remainder of this paper is structured as follows. In Section 2, we summarize the mathematical formulation of the original JLS model, while the extended JLS model is introduced in Section 3. Section 4 presents the construction of the JLS-factor model with Chinese characteristics. Section 5 describes the tests of six documented Chinese stock bubbles by using the new JLS model and the original model and compares their respective predictive power. Section 6 shows the significance test results of the original and new JLS models. Section 7 summarizes our conclusions.

2. Mathematical Formulation of the JLS Model

In this section, we recall the formation of the original JLS model, which provides a flexible framework within which to detect bubbles and predict regime changes in the price time series of a financial asset. It combines (i) the economic theory of rational expectation bubbles, (ii) behavioral finance on imitation and herding of noise traders, and (iii) the mathematical and statistical physics of bifurcations and phase transitions. The model considers the faster-than-exponential (power law with finite time singularity) increase in asset prices accompanied by accelerating oscillations as the main diagnostic of bubbles. It thus embodies a positive feedback loop of higher return anticipations competing with the negative feedback spirals of crash expectations [23].

Within the JLS framework, expected price $p(t)$ conditioned on no crash occurring is obtained as follows (see Zhou and Sornette [22], for the concrete derivation of the model):

$$E_{t_0}\left[p(t)\right] = p(t_0)\,L(t)\exp\left[\kappa\int_{t_0}^{t} h(\tau)\,d\tau\right], \qquad (1)$$

where $L(t) = \exp\{\int_{t_0}^{t}[r(\tau)+\sigma(\tau)\varphi(\tau)]d\tau\}$, $r(t)$ is the interest rate, $\sigma(t)$ is the price volatility, $\varphi(t)$ is the market price of risk of the stochastic discount factor, and $h(t)$ is the crash hazard rate, namely, the probability per unit time that the crash will happen in the next instant if it has not yet happened.

For $r(t) = \varphi(t) = 0$ and $L(t) = 1$, we have

$$E_{t_0}\left[p(t)\right] = p(t_0)\exp\left[\kappa\int_{t_0}^{t} h(\tau)\,d\tau\right]. \qquad (2)$$

Johansen et al. [14] proposed that a crash may be caused by local self-reinforcing imitation processes between noise traders that can be quantified by the theory of critical phenomena developed in the physical sciences. Hence, they assumed that the aggregate effect of noise traders can be quantified by the following dynamics of the crash hazard rate:

$$h(t) = B' x^{m-1} + C' x^{m-1}\cos\left(\omega\ln x - \phi'\right), \qquad (3)$$

where $x = |t_c - t|$, t_c is the critical time (i.e., the most probable time for the bursting of the bubble), ω is the angular log-frequency, and $\phi' \in [0,\ 2\pi]$ is an initial phase determining the unit of the time. Generalizing the definition of $t_c - t$ into $|t_c - t|$ allows for the critical time t_c to lie anywhere within the time series, which has the advantage of introducing a degree of flexibility into the search space for t_c with little additional cost [17].

The power law behavior x^{m-1} embodies the mechanisms of positive feedback at the origin of the formation of a bubble, while the cosine term on the RHS of (3) takes into account the existence of a possible hierarchical cascade of panic acceleration punctuating the course of the bubble, resulting from either a preexisting hierarchy in noise trader sizes and/or the interplay between market price impact inertia and nonlinear fundamental value investing [23].

Substituting (3) into (2) and integrating yields the LPPL equation for the price:

$$\ln\left[p(t)\right] = A + Bx^{m} + Cx^{m}\cos\left(\omega\ln x - \phi\right), \qquad (4)$$

where $A = \ln[p(t_c)]$, which gives the terminal log-price at the critical time t_c. $B = -(\kappa/m)B'$ and $C = -(\kappa/\sqrt{m^2+\omega^2})C'$, respectively, control for the amplitude of the power law acceleration and the log-periodic oscillations. The exponent m quantifies the degree of superexponential growth. ω is the angular log-frequency. ϕ is another phase different to ϕ' that contains two ingredients: information on the mechanism of the interactions between investors and a rescaling of time. The power law with exponent x^m captures the faster-than-exponential growth in the price and the term $\cos(\omega \ln x - \phi)$ describes the accelerating oscillation decorating the accelerating price. Further, although additional constraints emerge from a compilation of a significant number of historical bubbles that can be summarized as $0.1 \leq m \leq 0.9$, $6 \leq \omega \leq 13$ [26], Zhang et al. [28] found larger search ranges $m \in [0, 2]$ and $\omega \in [1, 50]$ from their research on sixteen historical bubbles.

A more general JLS model can be expressed as

$$I(t) = A + Bx^m + Cx^m \cos(\omega \ln x - \phi). \tag{5}$$

Theoretically, the order parameter $I(t)$ can be the price $p(t)$ or the logarithm of price $\ln[p(t)]$, while which one is reasonable to be the dependent variable is dependent on the following criterion. Zhou and Sornette [33] proposed that the observed price is the sum $p(t) = F(t) + M(t)$ of a fundamental price $F(t)$ and of a bubble or an antibubble $M(t)$. They had $I(t) = p(t)$ when $F(t) \ll M(t)$ and $I(t) = \ln[p(t)]$ when $F(t) \sim M(t)$. In fact, based on the rational bubble model of Johansen et al. [14] and Johansen et al. [38], if the magnitude of the crash is proportional to the price increase only associated with the contribution of the bubble, then the correct proxy is the price itself; on the contrary, if the magnitude of the crash is proportional to the price, then the correct proxy is the logarithm of the price [39]. From a theoretical view point, this is unsurprising: the rational expectation model of bubbles and crashes shows that, depending on whether the size of the crash is proportional to the price itself or that of the increase due to the bubble, either the logarithm of the price or the price itself is the correct quantity characterizing the bubble [12].

For the sake of simplicity, let us rewrite (5) in the following form:

$$y(t) = A + Bf(t) + Cg(t), \tag{6}$$

where

$$\begin{aligned} y(t) &= \ln[p(t)] \text{ or } p(t), \\ A &= \ln[p(t_0)], \\ f(t) &= |t_c - t|^m, \\ g(t) &= |t_c - t|^m \cos(\omega \ln|t_c - t| - \phi). \end{aligned} \tag{7}$$

3. Extended JLS Model

The common JLS model is the specific form when $r(t) = \varphi(t) = 0$, as mentioned above. However, $r(t)$ and $\varphi(t)$ do not equal zero in the real world. Thus, we extend the original JLS model locally. Similar to Zhou and Sornette [22], we assume that $\varphi(t)$ is a constant φ, which does not change over time. Moreover, we take the true values of $r(t)$ and $\sigma(t)$ to calibrate the model. Specifically, we specify $r(t)$ as the risk-free interest rate and employ the historical volatility of the targeted asset as a proxy for the volatility factor $\varphi(t)$. All daily data come from the iFinD database.

Through the above extension, the original JLS model can be converted into the following form:

$$y(t) = A + Bf(t) + Cg(t) + \alpha r(t) + \varphi v(t), \tag{8}$$

where

$$\begin{aligned} y(t) &= \ln[p(t)] \text{ or } p(t), \\ A &= \ln[p(t_0)], \\ f(t) &= |t_c - t|^m, \\ g(t) &= |t_c - t|^m \cos(\omega \ln|t_c - t| - \phi), \\ r(t) &= \int_{t_0}^{t} r(\tau)\, d\tau, \\ v(t) &= \int_{t_0}^{t} \sigma(\tau)\, d\tau, \\ \sigma(t) &= \sqrt{\frac{\sum_{t=1}^{n}(p_t - \overline{p})^2}{(n-1)}}, \end{aligned} \tag{9}$$

p_t is the day logarithm yield of the targeted asset, $\overline{p}$ is the average yield, $r(\tau)$ represents the risk-free interest rate, and $\sigma(\tau)$ denotes the historical volatility of the targeted asset.

However, the above model is only a local extension in view of the general JLS model. For Chinese stock markets, the influence of macroeconomic factors, national policy, and the international economic situation on the stock market must be accounted for, except for the impact of the positive feedback effect caused by investors' herding behavior. Therefore, we study the important factors that affect the volatility of Chinese stock indices in the next section and add these to the extended JLS model to construct a new JLS model (i.e., the JLS-factor model), which is suitable for China.

4. JLS-Factor Model with Chinese Characteristics

Owing to their inherent characteristics and drawbacks, Chinese stock markets are more easily affected by changes in monetary policy and fluctuations in international stock markets than mature markets. Hence, in this section, we analyze the impact of these two factors on Chinese stock market volatility and then construct a new JLS-factor model to calibrate the well-known Chinese stock bubbles.

First, the effect of monetary policy, mainly implemented by adjusting the interest rates, deposit reserve rate, and money supply, and so on, on a country's capital market has long been examined in agroscientific research globally. Theoretically, in the context of the transmission mechanism, monetary policy affects stock prices mainly through both the traditional interest rate channel [40] and the credit channel [41]. A number of empirical studies have applied different proxy variables to assess the effects of monetary policy shocks on stock market volatility, including the discount rate [42–45] (Mercer and Johnson, 1996), Federal Funds rate [46–54], interest rate [55–62] (Octavioet al., 2013), money supply [63–66], and Federal Funds futures [67–75].

Compared with foreign scholars' research, abundant works in China have assessed the relationship between the deposit reserve rate and stock market (e.g., [76–87]). All this research argues that, after changing the supply of money, the variation in the deposit reserve rate causes stock prices to change in the following four ways: (i) the effect of the market interest rate, (ii) the effect of credit scale, (iii) the effect of market structure, and (iv) the effect of stock market announcements. On the one hand, the variation in the deposit reserve rate tends to directly affect the money supply of the whole society and thus changes the capital supply to the stock market and, ultimately, the evolution of stock prices. On the other hand, as a policy signal, the adjustment in the deposit reserve rate significant affects investors' psychological expectations and thus their investment strategies and, ultimately, the evolution of stock prices. Unfortunately, early research into the relationship between swings in the deposit reserve rate and fluctuation in stock prices provided mixed results, finding no consistent relationship between these two variables and that the nature of such dynamics was unstable. From the above, a change in the deposit reserve rate is thus a factor that can affect asset prices. Therefore, we take into account its effect on the evolution of Chinese stock bubbles.

Second, as China has gradually opened up its stock market to foreign investments and cross-border listings, the comovement between the Chinese and the international stock market is increasingly strengthening. Indeed, in extreme cases such as the global financial crisis, such comovement is significantly enhanced given the deterioration of global economic fundamentals and the risk contagion among international financial markets [88, 89]. However, among all international stock markets, domestic research shows that stock price fluctuations in the United States have a more remarkable impact on that in China than others (e.g., [90–96], Hu, 2010). In particular, Zhang et al. [97] and Pan and Liu [98] found that the volatility of US stock indices can be used to predict the trend of Chinese stock prices.

Given the foregoing, we take into account the impact of the deposit reserve rate and US stock index volatility on the evolution of Chinese stock bubbles when constructing the new JLS model. Therefore, we have

$$\begin{aligned} y(t) = A + Bf(t) + Cg(t) + \alpha r(t) + \beta r_c(t) + \varphi v(t) \\ + \gamma v_a(t), \end{aligned} \tag{10}$$

where

$$\begin{aligned}
y(t) &= \ln[p(t)] \text{ or } p(t), \\
A &= \ln[p(t_0)], \\
f(t) &= |t_c - t|^m, \\
g(t) &= |t_c - t|^m \cos(\omega \ln|t_c - t| - \phi), \\
r(t) &= \int_{t_0}^{t} r(\tau)\, d\tau, \\
r_c(t) &= \int_{t_0}^{t} r_c(\tau)\, d\tau, \\
v(t) &= \int_{t_0}^{t} \sigma(\tau)\, d\tau, \\
v_a(t) &= \int_{t_0}^{t} \sigma_a(\tau)\, d\tau,
\end{aligned} \tag{11}$$

$r(\tau)$ is the risk-free interest rate, $r_c(\tau)$ represents the deposit reserve rate, and $\sigma(\tau)$ and $\sigma_a(\tau)$ are specified as the volatility of the targeted index and NASDAQ, respectively (the NASDAQ Composite Index is a barometer of market value changes in each industrial category, as it includes more than 5000 companies, which is more than any other single securities market. As a result, the NASDAQ Composite Index is more representative than the S&P 500 index and Dow Jones Industrial Average).

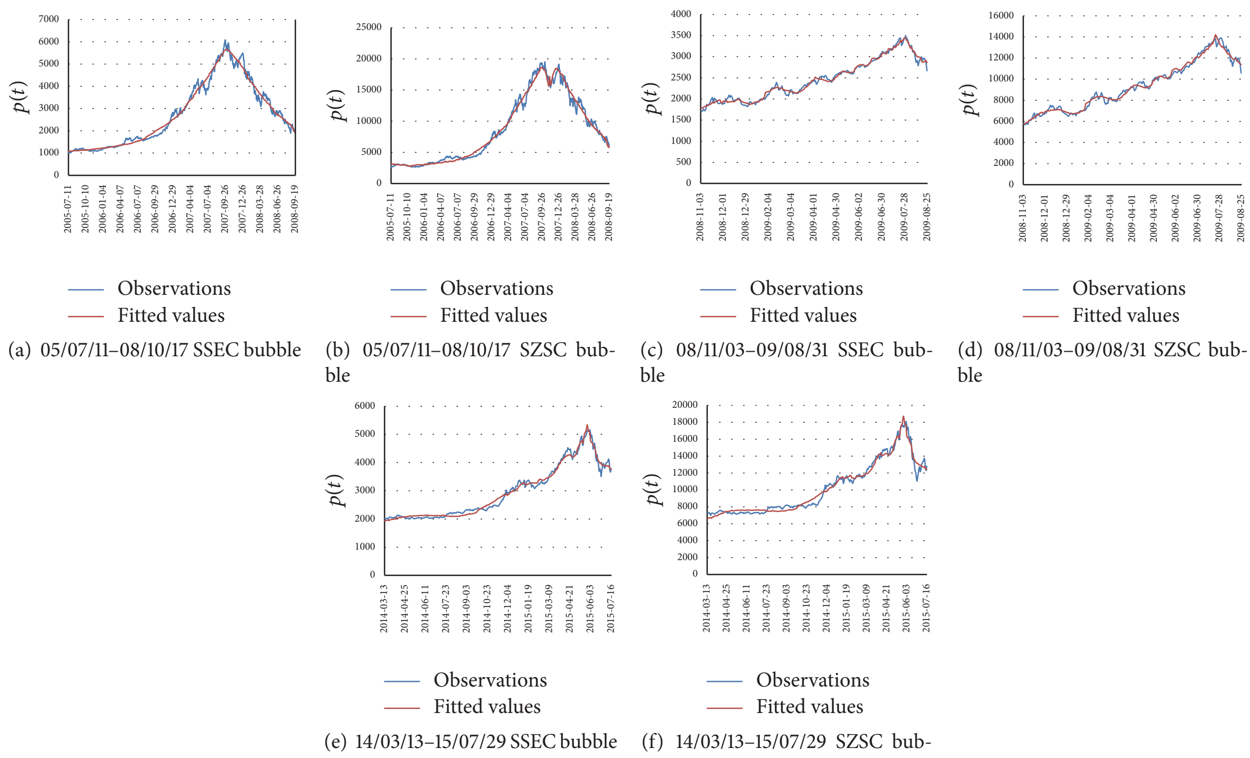
Because the specific function forms of $r(\tau)$, $r_c(\tau)$, $\sigma(\tau)$, and $\sigma_a(\tau)$ cannot be determined, we use the trapezoid scheme to integrate $r(t)$, $r_c(t)$, $v(t)$, and $v_a(t)$ in practice, following Zhou and Sornette [22]. That is, we let

$$\begin{aligned}
\int_{t_0}^{t} r(\tau)\, d\tau &\approx \sum_{\tau=t_0+1}^{t} \frac{[r(\tau-1) + r(\tau)]}{2}, \\
\int_{t_0}^{t} r_c(\tau)\, d\tau &\approx \sum_{\tau=t_0+1}^{t} \frac{[r_c(\tau-1) + r_c(\tau)]}{2}, \\
\int_{t_0}^{t} \sigma(\tau)\, d\tau &\approx \sum_{\tau=t_0+1}^{t} \frac{[\sigma(\tau-1) + \sigma(\tau)]}{2}, \\
\int_{t_0}^{t} \sigma_a(\tau)\, d\tau &\approx \sum_{\tau=t_0+1}^{t} \frac{[\sigma_a(\tau-1) + \sigma_a(\tau)]}{2}.
\end{aligned} \tag{12}$$

5. Results of the Original and New JLS Models

To visually compare the prediction accuracy of the results of the original and new JLS models, in this section, we calibrate the evolutions of two well-known Chinese stock indices (SSEC and SZSC) in three time periods, as selected by two published papers (i.e., [32, 34]) that applied the original JLS model to fit the tendency of these two indices in the corresponding periods. By observing the minimum and maximum values among the targeted indices within the specified periods, we find that the average annual growth rate is 244.55%. This finding implies that the fundamental price $F(t)$ should be much less than the bubble $M(t)$ according to Zhou and Sornette's [33] assumption. In the next step, we employ our JLS-factor model presented in (10) to fit these two indices within the same three periods with $y(t) = p(t)$.

Each calibration uses the algorithm of Universal Global Optimization provided by 1stOpt (First Optimization) software, which has been independently developed by 7D-Soft High Technology Inc. to solve any constrained or unconstrained linear and nonlinear equation(s). This software allows us to estimate all parameters (t_c, ω, m, ϕ, A, B, C, α, β,



(a) 05/07/11–08/10/17 SSEC bubble (b) 05/07/11–08/10/17 SZSC bubble (c) 08/11/03–09/08/31 SSEC bubble (d) 08/11/03–09/08/31 SZSC bubble

(e) 14/03/13–15/07/29 SSEC bubble (f) 14/03/13–15/07/29 SZSC bubble

Figure 1: Daily trajectory of the SSEC and SZSC from 2005/07/11 to 2015/07/29 by using the JLS-factor model presented in (10) with $y(t) = p(t)$. The fit of SSEC from 2005/07/11 to 2008/10/17 is illustrated in (a) as a red solid line, whose parameters are t_c = 2007/10/16, $\omega = 0.03$, $m = 1.89$, $\phi = 0.03$, $A = 4971.87$, $B = -4809410.22$, $C = 4808854.92$, $\alpha = 215.66$, $\beta = -32.46$, $\varphi = -25.11$, and $\gamma = 0.20$ with an r.m.s. of the fit residuals $\chi = 193.04$. The fit of SZSC from 2005/07/11 to 2008/10/17 is illustrated in (b) as a red solid line, whose parameters are t_c = 2007/11/26, $\omega = 1.26$, $m = 0.31$, $\phi = 3.48$, $A = 14926.14$, $B = -7159.59$, $C = 6083.23$, $\alpha = -320.05$, $\beta = 92.46$, $\varphi = 39.43$, and $\gamma = 2.73$ with an r.m.s. of the fit residuals $\chi = 643.26$. The fit of SSEC from 2008/11/03 to 2009/08/31 is illustrated in (c) as a red solid line, whose parameters are t_c = 2009/08/03, $\omega = 17.46$, $m = 0.45$, $\phi = 2.14$, $A = 3357.54$, $B = -2150.17$, $C = -122.12$, $\alpha = 74.22$, $\beta = 37.78$, $\varphi = -83.33$, and $\gamma = -0.33$ with an r.m.s. of the fit residuals $\chi = 60.49$. The fit of SZSC from 2008/11/03 to 2009/08/31 is illustrated in (d) as a red solid line, whose parameters are t_c = 2009/08/03, $\omega = 17.28$, $m = 0.53$, $\phi = 4.80$, $A = 13437.80$, $B = -10004.44$, $C = 627.84$, $\alpha = 132.05$, $\beta = 66.10$, $\varphi = -56.60$, and $\gamma = -0.25$ with an r.m.s. of the fit residuals $\chi = 282.30$. The fit of SSEC from 2014/03/13 to 2015/07/29 is illustrated in (e) as a red solid line, whose parameters are t_c = 2015/06/08, $\omega = 5.50$, $m = 0.39$, $\phi = 1.09$, $A = 5884.95$, $B = -4081.11$, $C = 316.83$, $\alpha = 105.11$, $\beta = 10.50$, $\varphi = -115.53$, and $\gamma = -0.14$ with an r.m.s. of the fit residuals $\chi = 119.37$. The fit of SZSC from 2014/03/13 to 2015/07/29 is illustrated in (f) as a red solid line, whose parameters are t_c = 2015/06/08, $\omega = 5.63$, $m = 0.34$, $\phi = 3.99$, $A = 13809.61$, $B = -15604.59$, $C = -1027.34$, $\alpha = -2.62$, $\beta = 535.60$, $\varphi = -392.55$, and $\gamma = -0.84$ with an r.m.s. of the fit residuals $\chi = 486.32$.

φ, and γ) in a given time window of analysis without inputting the initial values. The main results of our calibrations for the evolutions of the 2005–2008, 2008-2009, and 2014-2015 bubbles are illustrated in Figure 1.

As shown in Figure 1, the six fits are very close to the real trajectories of the SSEC and SZSC bubbles, which intuitively shows the superiority of our JLS-factor model. Further, we employ the 1stOpt software again to calibrate the evolutions of the SSEC and SZSC bubbles in the same periods with the original JLS model and compare the estimation accuracies of critical times between the original and new JLS models. The results are shown in Tables 1 and 2.

Tables 1 and 2 show that the estimated accuracy of the critical time by the new JLS model is in general better than that of the original JLS model, except for the 2005–2008 SZSC bubble. In particular, for the 2005–2008 and 2008-2009 SSEC bubbles and the 2008-2009 SZSC bubble, the estimation results of the new JLS model agree well with the actual time at which the bubble burst. Figure 1(b) shows that the 2005–2008 SZSC bubble peaked twice in just two and a half months, which may have been the main cause of the low estimated accuracy.

Recall the setting of $x = |t_c - t|$ above. Two potential problems are associated with this procedure if an antibubble exists during the bubble period. First, it implies that the antibubble is always associated with a bubble which, in addition, has the same t_c. Second, it implies that the bubble and antibubble are symmetric around t_c; that is, the same parameters characterize the index evolution for $t < t_c$ and for $t > t_c$[17]. However, two peaks existed in the evolution of the 2005–2008 SZSC bubble showed in Figure 1(b), and the low estimated accuracy calculated by using both the new and the original JLS models may imply two different t_c during this period: one is the critical time corresponding to the

Table 1: Prediction of the critical time of the SSEC bubbles burst for both models and their differences with the actual times of bubble burst.

Item	Estimated time interval		
	2005/7/11–2008/10/17	2008/11/3–2009/8/31	2014/3/13–2015/7/29
The actual time of bubble burst	2007/10/16 (t_c = 107.79)	2009/8/4 (t_c = 109.59)	2015/6/12 (t_c = 115.45)
The predicted critical time of the original JLS model	2007/10/26 (t_c = 107.82)	2009/7/28 (t_c = 109.57)	2015/5/11 (t_c = 115.36)
The difference between the original JLS model and actual time (days)	−10	7	30
The predicted critical time of the new JLS model	2007/10/16 (t_c = 107.79)	2009/8/3 (t_c = 109.59)	2015/6/8 (t_c = 115.43)
The difference between the new JLS model and actual time (days)	0	1	4

Value in parentheses is the predicted critical time t_c.

Table 2: Prediction of the critical time of the SZSC bubbles burst for both models and their differences with the actual times of bubble burst.

Item	Estimated time interval		
	2005/7/11–2008/10/17	2008/11/3–2009/8/31	2014/3/13–2015/7/29
The actual time of bubble burst	2007/10/31 (t_c = 107.83)	2009/8/4 (t_c = 109.59)	2015/6/12 (t_c = 115.45)
The predicted critical time of the original JLS model	2007/11/22 (t_c = 107.89)	2009/7/28 (t_c = 109.57)	2015/6/3 (t_c = 115.42)
The difference between the original JLS model and actual time (days)	−22	7	9
The predicted critical time of the new JLS model	2007/11/26 (t_c = 107.90)	2009/8/3 (t_c = 109.59)	2015/6/8 (t_c = 115.43)
The difference between the new JLS model and actual time (days)	−26	1	4

Value in parentheses is the predicted critical time t_c.

bursting of a bubble, while the other marks the inception of an antibubble. Hence, we conduct a Lomb analysis, using a parametric detrending approach, to detect the log-periodic oscillations accompanied by the 2007/11/29–2008/10/17 SZSC antibubble (here, we only employ the Lomb analysis to detect the log-periodic oscillations accompanied by an antibubble; the analysis method for a bubble is the same).

Following Zhou and Sornette [33], the assumption that a critical point at the inception of an antibubble exists can be tested by investigating two possible signatures of a critical behavior: a power law relaxation and log-periodic wobbles. Firstly, we test the power law relaxation of the 2007/11/29–2008/10/17 SZSC bubble.

The power law expression for an antibubble reads

$$I(t) = A + B\left|t - t_c\right|^m, \tag{13}$$

where t_c is an estimate of the inception of the antibubble and the order parameter $I(t)$ can be price $p(t)$ or its logarithm $\ln[p(t)]$. If $0 < m < 1$, $I(t)$ is finite; however, its first derivative $I'(t)$ is singular at t_c and B should be negative to ensure that $I(t)$ decreases.

According to the specific characteristics of the 2007/11/29–2008/10/17 SZSC bubble, we only fit it by using the power law formula (13) with $I(t) = p(t)$, whose parameters are t_c = 2008/01/10, $m = 0.74$, $A = 18732$, and $B = -15108.07$ with an r.m.s. of the fit residuals $\chi = 706.80$. According to the fitted values of m and B, we can conclude that the 2007/11/29–2008/10/17 SZSC bubble meets the signature of power law relaxation.

As for the detection of log-periodic oscillations, this is conveniently performed by removing the global trend of the index. One way is to subtract the power law fit (13) from the index and then analyze the wobbles of the obtained residuals $s(t)$ by adopting an adequate spectral analysis. Similarly, we construct the residuals $s(t)$ in the following way:

$$s(t) = \frac{[p(t) - A]}{\left|t - t_c\right|^m}, \tag{14}$$

where A, m, and t_c are obtained from the fit of the pure power law formula (13) to the data.

As implemented in Zhou and Sornette [19, 33], we also use a Lomb periodogram analysis of residuals $s(t)$ to assess the statistical significance level of the extracted log-periodicity. The Lomb periodogram of $s(t)$ is shown in Figure 2.

As shown in Figure 2, the highest peak is at $\omega = 3.91$ with height $P_n(\omega) = 3.05 \times 10^6$. This strong significant peak of the periodogram qualifies the existence of log-periodicity. More precisely, we can employ the false alarm probability P_r (false alarm probability is the chance that we falsely detect log-periodicity in a signal without true log-periodicity) to obtain the statistical significance level of the extracted log-periodicity. Under the null hypothesis of i.i.d. Gaussian fit residuals, P_r is zero [99]. However, assuming an i.i.d. structure is too restrictive as the fit residuals usually have some correlations. If the residuals have long range correlations characterized by a Hurst index H, we can use Zhou and Sornette's [100] computational method of P_r for various values of $H > 1/2$ to obtain the statistical significance

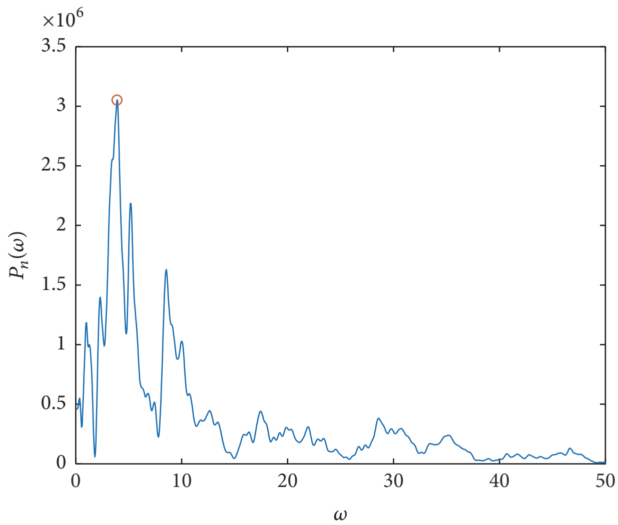


Figure 2: The Lomb periodogram analysis of $s(t)$, where $P_n(\omega)$ is a normalized Lomb power (since $P_n(\omega)$ is a normalized Lomb power, $s(t)$ and $[s(t) - \mu_s]/\sigma_s$ have identical Lomb periodogram, where μ_s and σ_s^2, resp., denote the mean and variance of this residuals $s(t)$) and ω is the log-angular frequency.

level of the extracted log-periodicity. For example, if $H = 0.6$, the false alarm probability corresponding to the observed peak $P_n(\omega) = 3.05 \times 10^6$ is $P_r < 10^{-10}$; if $H = 0.7$, it is $P_r < 10^{-8}$; if $H = 0.8$, it is $P_r < 10^{-7}$; and if $H = 0.9$, it is $P_r < 10^{-6}$. All of these mean that the statistical significance of log-periodicity is very high.

From the above, we can conclude that the 2007/11/29–2008/10/17 SZSC bubble is indeed an antibubble. Therefore, we separate the 2005–2008 SZSC bubble into two periods, namely, the 2005/11/15–2007/11/28 SZSC bubble and the 2007/11/29–2008/10/17 SZSC antibubble. Then, we fit these two bubbles by using both the new and original JLS models. The calibrations of these two SZSC bubbles are shown in Figure 3, while the comparison results for both models are shown in Table 3.

Table 3 shows that the prediction result of the 2005–2007 SZSC bubble by using the new JLS model is unsatisfactory; however, it is slightly better than the estimation result of the original JLS model. Meanwhile, the critical time estimated by using the new JLS model for the SZSC antibubble for 2007-2008 is very close to the actual time, which shows that the predictive power of the new JLS model is superior to that of the original model. These results demonstrate that the new JLS model quantifies the time evolution of Chinese stock bubbles remarkably well in terms of the price ending with a crash or a large correction at a time close to the critical time.

Moreover, the corresponding parameters for our calibrations for the evolutions of the 2005–2008, 2008-2009, and 2014-2015 bubbles are listed in Table 4 for comparison purposes.

As shown in Table 4, all the coefficients B are negative, which qualifies that these indices are in the bubble (or antibubble) regime [18]. Among the other parameter values, note that only the power law exponents m of the indices of 05/07/11–08/10/17 SSEC and 05/11/15–07/11/28 SZSC are significantly larger than 1, while the others are between 0 and 1. In the absence of log-periodic oscillations, large values of $m > 1$ imply a relatively steep upward overall acceleration of the index, while $0 < m < 1$ would mean that the overall shape of these indices shows less rapid dynamics. In addition, from the fitting results of 05/07/11–08/10/17 SSEC and 05/11/15–07/11/28 SZSC, we can find that when m are significantly larger than 1 and the angular frequency ω of the log-periodic oscillations is too small, price $p(t)$ will be compensated by a large amplitude of the power law acceleration and log-periodic oscillations; that is, the values $|B|$ and C will be larger. Finally, the effects of the risk-free interest rate, the deposit reserve rate, the volatility of the targeted index, and NASDAQ on price $p(t)$ are by and large inconsistent. This may be because the influences of these exogenous factors on the evolution of bubbles are different in different periods and the reactions of different indices to the same exogenous factors are different as the stock market is a complex system. However, it is interesting to note that this happens to indirectly reflect the instability of the Chinese stock market. Relatively speaking, the effect of the volatility of NASDAQ on price $p(t)$ is smaller than that of the others. Possible reasons for this include the hysteresis of the contagion effect and the lack of synchronicity between the economic cycles of these two countries. By contrast, it also implies that comovements between the Chinese and the US stock markets exist, although this relationship is still weak.

6. Significance Testing of the JLS-Factor Model

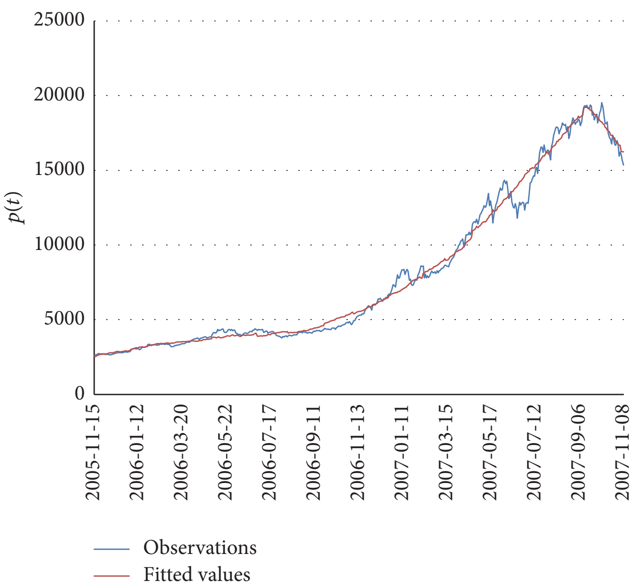
In the following, we compare the performances of these fits between the new and original JLS models by using three statistical criteria (i.e., AIC, SC, and HQC). We also test the significance level of the two exogenous factors in the new JLS model, namely, the deposit reserve rate and volatility of NASDAQ. Ideally, these tests require that the fitting residuals are i.i.d. with Gaussian distributions. In reality, the residuals have remaining dependence structures at small scales. As a consequence, the standard statistical significance of the above tests cannot be read from Gaussian statistics tables. Nevertheless, these tests provide useful diagnostics to gauge the relative (rather than absolute) performance of competing models and are thus instructive to identify models [22]. The test results are shown in Tables 5–7.

These tables show that the new JLS model performs better than the original JLS model in all cases according to both the adjusted R-square values and the three test statistics. In addition, according to the significant degree of corresponding coefficients of the deposit reserve rate and volatility of NASDAQ, we find that although the influence of these two factors on the evolution of the 2008-2009 SSEC and SZSC bubbles is not significant, the fitting effect of the whole model is improved after joining these two variables. This finding implies that Chinese stock bubbles are due not only to the exogenous influence of the People's Bank of China and volatility of international stock indices but also to the endogenous self-organization of the markets resulting

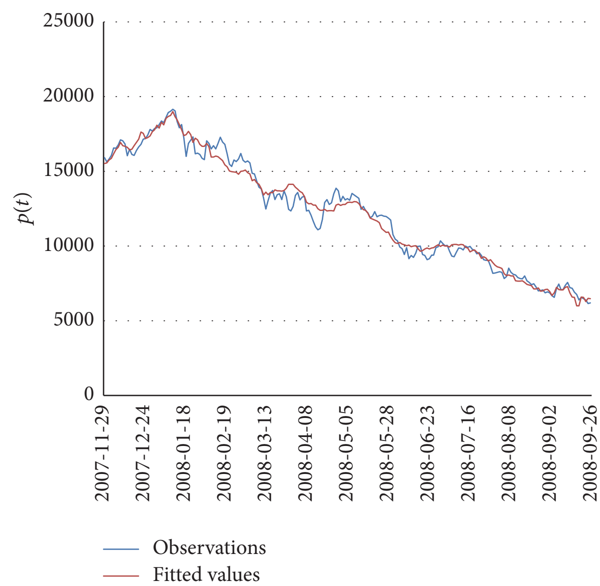
Table 3: Prediction of the critical time of the SZSC bubbles burst for both models and their differences with the actual times of bubble burst.

Item	Estimated time interval	
	2005/11/15–2007/11/28	2007/11/29–2008/10/17
The actual time of bubble burst	2007/10/31 (t_c = 107.83)	2008/1/14 (t_c = 108.04)
The predicted critical time of the original JLS model	2007/12/13 (t_c = 107.95)	2008/1/11 (t_c = 108.03)
The difference between the original JLS model and actual time (days)	−43	3
The predicted critical time of the new JLS model	2007/10/8 (t_c = 107.76)	2008/1/15 (t_c = 108.04)
The difference between the new JLS model and actual time (days)	23	−1

Value in parentheses is the predicted critical time t_c.



(a) 05/11/15–07/11/28 SZSC bubble



(b) 07/11/29–08/10/17 SZSC bubble

Figure 3: Daily trajectory of the SZSC from 2005/11/15 to 2008/10/17 by using the JLS-factor model presented in (10) with $y(t) = p(t)$. The fit of SZSC from 2005/11/15 to 2007/11/28 is illustrated in (a) as a red solid line, whose parameters are t_c = 2007/10/08, $\omega = 0.07$, $m = 1.86$, $\phi = 0.07$, $A = 20506.38$, $B = -4784674.93$, $C = 4780598.98$, $\alpha = -183.50$, $\beta = -296.39$, $\varphi = 118.94$, and $\gamma = 1.87$ with an r.m.s. of the fit residuals $\chi = 523.35$. The fit of SZSC from 2007/11/29 to 2008/10/17 is illustrated in (b) as a red solid line, whose parameters are t_c = 2008/01/15, $\omega = 14.85$, $m = 0.53$, $\phi = 2.82$, $A = 12998.06$, $B = -12652.93$, $C = 1164.16$, $\alpha = 861.36$, $\beta = -275.90$, $\varphi = 101.22$, and $\gamma = 4.89$ with an r.m.s. of the fit residuals $\chi = 561.68$.

from positive feedback between herding investors. For the 2008-2009 Chinese stock bubbles, as in Jiang et al.'s [34] analysis, the regime change in these bubbles occurred in the absence of any significant modification of the economic and financial conditions or any visible driving force. Here, a vanishingly small change in some of the control parameters may have led to a macroscopic bifurcation or phase transition [34]. This means that universal LPPLS sufficiently reflect the fundamental tendency of investors to speculate and herd. Further, the change in the deposit reserve rate and volatility of NASDAQ can dramatically affect the movements of Chinese stock bubbles in the remaining periods in different ways.

7. Conclusion and Discussion

We introduced a new JLS model that combines fundamental economic factors in China (including the interest rate and deposit reserve rate) and the historical volatilities of targeted indices and US equity indices with the original model. The new JLS model not only keeps the dynamic characteristics of a bubble caused by positive feedback but also considers exogenous shocks that trigger the bursting of bubbles. Further, we analyzed in detail six financial bubbles in Chinese stock markets by calibrating the JLS-factor model (see (10)) to two important Chinese stock indices (SSEC and SZSC) from July 2005 to July 2015. We then compared the prediction accuracy of the critical time fitted by the new JLS model with that of the original model. The results of this comparison and the indirect significance tests indicate that all Chinese stock bubbles are a combination of speculative herding behavior and policy-induced reactions as well as international stock index volatility. These results confirm the sensible explanation and superiority of our proposed JLS model.

Table 4: Parameters of the fits of the indices indicated in the first column are calculated by the JLS-factor model (10), where t_c is the most probable time for the bursting of bubble (or the inception of an antibubble), $\omega \in [0.01,\ 40]$ is the angular log-frequency [24], $m \in [0,\ 2]$ quantifies the degree of superexponential growth, $\phi \in [0,\ 2\pi]$ is phase, A gives the terminal price at the critical time t_c, B and C, respectively, control for the amplitude of the power law acceleration and the log-periodic oscillations, α, β, φ, and γ, respectively, measure the effects of the risk-free interest rate, the deposit reserve rate, the volatility of the targeted index, and NASDAQ on the price $p(t)$, and χ denotes the root-mean-square (r.m.s.).

Bubble (antibubble)	t_c	ω	m	ϕ	10^3A	10^4B	10^3C	α	β	φ	γ	χ
05/07/11–08/10/17 SSEC	07/10/16	0.03	1.89	0.03	4.97	−480.94	4808.85	215.66	−32.46	−25.11	0.20	193.04
05/07/11–08/10/17 SZSC	07/11/26	1.26	0.31	3.48	14.93	−0.72	6.08	−320.05	92.46	39.43	2.73	643.26
08/11/03–09/08/31 SSEC	09/08/03	17.46	0.45	2.14	3.36	−0.21	−0.12	74.22	37.78	−83.33	−0.33	60.49
08/11/03–09/08/31 SZSC	09/08/03	17.28	0.53	4.80	13.44	−1.00	0.63	132.05	66.10	−56.60	−0.25	282.30
14/03/13–15/07/29 SSEC	15/06/08	5.50	0.39	1.09	5.88	−0.41	0.32	105.11	10.50	−115.53	−0.14	119.37
14/03/13–15/07/29 SZSC	15/06/08	5.63	0.34	3.99	13.81	−1.56	−1.03	−2.62	535.60	−392.55	−0.84	486.32
05/11/15–07/11/28 SZSC	07/10/08	0.07	1.86	0.07	20.51	−478.47	4780.60	−183.50	−296.39	118.94	1.87	523.35
07/11/29–08/10/17 SZSC	08/01/15	14.85	0.53	2.82	12.00	−1.27	1.16	861.36	−275.90	101.22	4.89	561.68

Table 5: The significance test results of the new and the original JLS model for the SSEC bubbles.

Independent variable	2005/7/11–2008/10/17		2008/11/3–2009/8/31		2014/3/13–2015/7/29	
	JLS model	New JLS model	JLS model	New JLS model	JLS model	New JLS model
A	5731.247	4992.469	2825.911	4653.301	4792.062	7704.706
	$(280.536)^{***}$	$(91.959)^{***}$	$(61.353)^{***}$	$(24.645)^{***}$	$(212.031)^{***}$	$(48.030)^{***}$
B	−442310.7	−441176.2	−880.458	−3444.690	−2954.052	−5275.828
	$(-104.248)^{***}$	$(-62.898)^{***}$	$(-10.370)^{***}$	$(-15.337)^{***}$	$(-88.287)^{***}$	$(-40.295)^{***}$
C	441796.9	440839.6	−0.518	−154.749	−362.186	−365.562
	$(104.014)^{***}$	$(62.763)^{***}$	$(-48.883)^{***}$	$(-12.484)^{***}$	$(-17.492)^{***}$	$(-19.400)^{***}$
α		8.616		−1.487		−13.058
		$(11.445)^{***}$		(−0.852)		$(-4.771)^{***}$
β		−1.437		−0.203		2.568
		$(-6.260)^{***}$		(−0.724)		$(6.028)^{***}$
φ		−2.214		−0.439		−2.669
		$(-3.491)^{***}$		(−0.257)		$(-2.855)^{***}$
γ		−5.284		0.650		−22.477
		$(-4.733)^{***}$		(0.693)		$(-9.467)^{***}$
N	796	796	340	340	340	340
$\overline{R}^2$	0.978	0.983	0.913	0.984	0.961	0.986
F-statistic	17320.70^{***}	7531.207^{***}	1784.438^{***}	2089.920^{***}	4146.165^{***}	3995.439^{***}
AIC	13.593	13.337	14.674	11.066	13.273	12.249
SC	13.610	13.378	14.708	11.180	13.307	12.328
HQC	13.599	13.353	14.688	11.112	13.286	12.280

The regression coefficients, adjusted R-squares ($\overline{R}^2$), and three statistical criteria are presented. The t-statistics are reported in the parentheses. ***Significance at the 1% confidence level.

Table 6: The significance test results of the new and the original JLS model for the SZSC bubbles.

Independent variable	2005/7/11–2008/10/17		2008/11/3–2009/8/31		2014/3/13–2015/7/29	
	JLS model	New JLS model	JLS model	New JLS model	JLS model	New JLS model
A	18490.62	20234.73	2827.723	4643.542	17060.47	28286.88
	$(79.333)^{***}$	$(65.072)^{***}$	$(61.104)^{***}$	$(24.763)^{***}$	$(184.250)^{***}$	$(49.029)^{***}$
B	−7103.479	−8099.557	−879.179	−3427.315	−10429.75	−19848.04
	$(-23.023)^{***}$	$(-29.957)^{***}$	$(-10.349)^{***}$	$(-15.398)^{***}$	$(-78.527)^{***}$	$(-41.031)^{***}$
C	6244.976	6196.875	−0.519	−154.527	−1126.786	−1288.108
	$(52.350)^{***}$	$(57.204)^{***}$	$(-48.751)^{***}$	$(-12.533)^{***}$	$(-14.849)^{***}$	$(-17.401)^{***}$
α		3.127		−1.639		50.255
		(1.095)		(−0.941)		$(4.384)^{***}$
β		1.877		−0.192		−5.513
		$(2.234)^{*}$		(−0.686)		$(-3.137)^{**}$
φ		5.347		−0.349		14.983
		$(3.076)^{**}$		(−0.204)		$(3.927)^{***}$
γ		−39.379		0.685		−98.284
		$(-11.916)^{***}$		(0.734)		$(-9.908)^{***}$
N	796	796	340	340	340	340
$\overline{R}^2$	0.978	0.987	0.913	0.984	0.951	0.979
F-statistic	17954.89^{***}	10329.40^{***}	1782.521^{***}	2089.289^{***}	3271.277^{***}	2673.850^{***}
AIC	16.174	15.643	14.675	11.067	15.902	15.046
SC	16.192	15.684	14.709	11.180	15.936	15.125
HQC	16.181	15.658	14.689	11.113	15.916	15.078

The regression coefficients, adjusted R-squares ($\overline{R}^2$), and three statistical criteria are presented. The t-statistics are reported in the parentheses. *Significance at the 10% confidence level. **Significance at the 5% confidence level. ***Significance at the 1% confidence level.

Table 7: The significance test results of the new and the original JLS model for the 2005–2008 SZSC bubbles.

Independent variable	2005/11/15–2007/11/28		2007/11/29–2008/10/17	
	JLS model	New JLS model	JLS model	New JLS model
A	17692.97 (260.833)***	41967.41 (30.219)***	15512.45 (72.230)***	19841.75 (130.481)***
B	−39857.89 (−115.809)***	−98196.21 (−32.662)***	−10840.88 (−20.378)***	−40897.70 (−21.170)***
C	−36775.25 (−105.480)***	−89665.52 (−33.281)***	−2.800 (−54.535)***	1885.201 (7.814)***
α		290.262 (20.157)***		−140.944 (−14.366)***
β		−90.608 (−20.270)***		23.000 (13.351)***
φ		8.009 (2.782)***		107.469 (15.265)***
γ		7.62 (1.419)		−17.028 (−1.587)
N	495	495	340	340
$\overline{R}^2$	0.983	0.993	0.904	0.980
F-statistic	14598.80***	10925.42***	1594.129***	1731.150***
AIC	15.899	15.108	18.096	15.380
SC	15.925	15.167	18.130	15.490
HQC	15.909	15.131	18.109	15.424

The regression coefficients, adjusted R-squares ($\overline{R}^2$), and three statistical criteria are presented. The t-statistics are reported in the parentheses. *** Significance at the 1% confidence level.

However, it is interesting to find that there is no obvious reason to believe that there is any critical difference between SSEC and SZSC, while the crucial fitting parameters are very different during the bubble time 2005–2008. In reality, the evolutions of the standardized 2005–2008 SSEC bubble and the standardized 2005–2008 SZSC bubble are similar and their correlation coefficient is as high as 0.99. However, these few differences might lead to significantly different fitting results.

The word "critical" is used in science with different meanings. Sornette and Johansen [101] used it in the context of the critical phenomena studied in statistical physics in connection with phase transitions. Here, however, it describes a system at the border between order and disorder, which is characterized by an extremely large susceptibility to external factors and a strong correlation between different parts of the system. Examples of such systems are liquids and magnets, where the system will progressively become orderly under small external changes. In particular, this helps address the question of what is/are the cause(s) of bubbles and crashes. The crucial insight is that a system made of competing investors subjected to the myriad of influences, both exogenous news and endogenous interactions and reflexivity, can develop into endogenously self-organized self-reinforcing regimes that would qualify as bubbles; moreover, crashes occur as a global self-organized transition. The implication of modeling a market crash as a bifurcation is to solve the question of what makes a crash: in the framework of bifurcation theory (or phase transitions), sudden shifts in behavior arise from small changes in circumstances, with qualitative changes in the nature of the solutions that can occur abruptly when the parameters change smoothly. That is, a minor change of circumstances, interaction strength, or heterogeneity may lead to a sudden and dramatic change, such as during an earthquake and a financial crash. Note that, according to this "critical" point of view, the specific manner by which prices collapse is not the most important problem: a crash occurs because the market has entered an unstable phase and any small disturbance or process may have triggered the existence of this instability [102]. For example, think of a ruler held up vertically on your finger: this unstable position will lead eventually to its collapse as a result of a small (or an absence of adequate) motion of your hand or due to any tiny whiff of air. The collapse is fundamentally due to the unstable position; the instantaneous cause of the collapse is secondary [1].

From the above, we know that if the bubble state is unstable, a small disturbance will trigger the bursting of the bubble. Hence, although there are few differences between the 2005–2008 SSEC bubble and the 2005–2008 SZSC bubble, as long as the trigger factors (exogenous or endogenous) show small differences, the burst time of the bubble will be strikingly different, as will the results of the other JLS model parameters. Thus, the crucial fitting parameters are very different during the bubble time of 2005–2008.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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