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Research Article

Energy Efficient Power Allocation for OFDM-Based Cognitive Radio Systems with Partial Intersystem CSI

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This paper investigates energy efficient power allocation for orthogonal frequency division multiplexing- (OFDM-) based cognitive radio (CR) systems with partial intersystem channel state information (CSI) available. The goal is to maximize energy efficiency (EE) while ensuring the minimum rate of secondary user (SU) and keeping the average interference power (AIP) introduced to primary user (PU) within a target probability level. We propose a suboptimal algorithm to solve this optimization problem based on classic water-filling (WF) technique. Moreover, we first address the relation between EE and water level. In order to reduce complexity, a simplified algorithm with closed-form solution is also proposed. Numerical results are provided to corroborate our theoretical analysis and to demonstrate the effectiveness of the proposed schemes.

1. Introduction

Cognitive radio (CR) [1, 2], which provides a flexible platform that improves the spectral efficiency (SE), has become a promising technology to tackle the problem of spectrum scarcity. In an underlay CR system, a secondary user (SU) can be allowed access to the licensed spectrum band only if the resulting interference to the primary user (PU) is within a tolerable threshold [2].

Orthogonal frequency division multiplexing (OFDM) has been regarded as a potential multicarrier modulation technique for CR systems due to its inherent resistance to multipath fading and the flexibility in resource allocation [3]. In OFDM-based CR systems, the interference power constraint to PU can be divided into two categories: average interference power (AIP) constraint over all subcarriers and peak interference power (PIP) constraint for each subcarrier. From the perspective of SE, the AIP constraint is more practical than the PIP constraint since it is more flexible for power allocation [4]. In this paper, we focus on the AIP constraint.

In OFDM-based CR systems, power allocation plays an important role in SE improvement while protecting PU from disturbance of SU. Previous works on dynamic power allocation for OFDM-based CR systems mainly focused on spectrum sensing, spectrum sharing, and SE optimization [5–9]. However, the methods to maximize SE may not be suitable to the future green communications. Energy efficiency (EE) [10–14], which is defined as the number of transmission bits per unit energy, can be used to evaluate the performance. Hence, it is necessary to improve the energy utilization as efficiently as possible.

Several recent works have been done to consider EE in CR systems. In [11], both the optimal transmission duration and energy efficient power allocation were investigated to maximize EE. Energy efficient power allocation and spectrum sharing in heterogeneous cognitive radio networks with femtocells were studied in [12]. However, neither of them considers the quality-of-service (QoS) of SU. In [13], a method named as water-filling factor aided search (WFAS) was proposed to maximize EE under multiple constraints with perfect channel state information (CSI) at

SU-transmitter (Tx). However, compared to intrasystem CSI, it would be difficult for the SU-Tx to obtain the perfect intersystem CSI due to its lack of cooperation and limited feedback overhead. The global ϵ-optimal power allocation solution is achieved based on bisection search technique, whose accuracy of the solution is dependent on the number of iterations. The high computational complexity also incurs additional energy consumption. In [14], we have developed low complexity algorithm to maximize the system EE for multiuser OFDM-based cognitive radio with PIP constraint. If we focus on the AIP constraint, these algorithms cannot be applied directly. Motivated by the above discussions, it is necessary to study the energy efficient design under the AIP constraint.

In this paper, we propose an energy efficient power allocation algorithm for OFDM-based CR systems with partial intersystem CSI. It can be regarded as an improved version of the schemes proposed in [13]. By analyzing the relationship between water level and EE, a closed-form power allocation solution with low complexity and exact optimality is also derived.

The remainder of this paper is organized as follows. Section 2 introduces an OFDM-based CR system model and formulates the corresponding power allocation problem. In Section 3, a bisection search aided energy efficient power allocation algorithm is described. To reduce complexity, a simple method with closed-form expression is developed to maximize EE. In Sections 4 and 5, simulation results and conclusions are given, respectively.

2. System Model and Problem Formulation

2.1. System Model. Consider an OFDM-based CR system, where a SU coexists with a PU over the same spectrum band. The spectrum band is equally divided into N subchannels with bandwidth B for each. Let $\mathcal{N} \triangleq \{1, \ldots, n, \ldots, N\}$ denote the set of these subchannels. Define h_n^{ss} as the channel response between SU-Tx and SU-receiver (Rx) on the nth subcarrier. In this model, we assume that perfect intrasystem CSI, h_n^{ss}, is available at the SU-Tx. However, in a practical system, it is difficult or even infeasible to perfectly obtain h_n^{sp} between SU-Tx and PU-Rx due to the lack of cooperation. It is reasonable to assume that only the statistics distribution of h_n^{sp} is known. To protect the communication links of licensed users, the probability that the AIP exceeds a certain threshold should be kept within a target level. Using the uncorrelated fading channel model [15], h_n^{ss} and h_n^{sp} are independent Gaussian distributed random variables with $\mathcal{CN}(0, \lambda_1)$ and $\mathcal{CN}(0, \lambda_2)$, respectively.

2.2. Problem Formulation. During each transmission, the interference caused by PU-Tx may degrade the performance of SU-Rx. If there are a large number of subchannels, this interference introduced into SU-Rx can be approximated as additive white Gaussian noise (AWGN) according to central limit theorem [6]. Let p_n be the transmit power on the nth subcarrier; the data rate is given by

$$r_n = B \log_2 \left(1 + g_n p_n\right), \tag{1}$$

where $g_n = |h_n^{ss}|^2/\sigma^2$ is the channel gain to noise ratio (CNR) on the nth subcarrier and σ^2 denotes the noise which includes thermal noise and interference.

Consequently, the overall throughput R and transmit power P are

$$R = \sum_{n \in \mathcal{N}} r_n, \tag{2}$$

$$P = \sum_{n \in \mathcal{N}} p_n, \tag{3}$$

respectively.

Referring to [10], the circuit power consumption that is incurred by signal processing and active circuit blocks, such as digital to analog converter (DAC), can be modeled as a linear function of throughput:

$$P_c = P_s + \beta R, \tag{4}$$

where P_s is the static circuit power and β is the dynamic circuit power per unit data rate.

Since the total transmit power is limited, we have

$$P \leq \widehat{P}, \tag{5}$$

where $\widehat{P}$ is the maximum allowable transmit power.

In order to guarantee the user experience, the transmission rate should be restricted by

$$R \geq \check{R}, \tag{6}$$

where $\check{R}$ is the minimum rate requirement related to the type of traffic.

To protect the activity of PU, the power allocation problem must include a AIP outage constraint. Thus, we have

$$P_o = \Pr\left[\frac{1}{N}\sum_{n \in \mathcal{N}} \left|h_n^{sp}\right|^2 p_n \geq I_{\text{th}}\right] \leq \epsilon, \tag{7}$$

where I_{th} is the predefined threshold and ϵ is the target level.

Our objective is to determine the transmit power p_n ($\forall n \in \mathcal{N}$) such that the EE is maximized under the data rate constraint and AIP outage constraint. Therefore, the optimization problem P1 considered in this paper can be formulated as follows:

$$\text{P1}: \max_{p_n} \quad U = \frac{R}{\alpha P + P_c} \tag{8a}$$

$$\text{s.t.} \quad P \leq \widehat{P}, \tag{8b}$$

$$R \geq \check{R}, \tag{8c}$$

$$P_o = \Pr\left[\frac{1}{N}\sum_{n \in \mathcal{N}} \left|h_n^{sp}\right|^2 p_n \geq I_{\text{th}}\right] \leq \epsilon, \tag{8d}$$

where α is a constant related to the efficiency of power amplifier. It is possible that problem in (8a)–(8d) does not have any feasible solution, when $\check{R}$ cannot be obtained subject to the constraints (5) and (7). In this case, the SU-Tx may have to decrease the minimum rate requirement to make the solution feasible.

3. Energy Efficient Design

In the following, we first introduce two auxiliary optimization problems and then propose an optimal energy efficient power allocation algorithm to achieve the maximum EE performance.

Define a rate maximization (Max R) problem P2 as

$$\mathrm{P2}: \max_{p_n} \quad R \tag{9a}$$

$$\text{s.t.} \quad P \le \widehat{P}, \tag{9b}$$

$$P_o = \Pr\left[\frac{1}{N}\sum_{n\in\mathcal{N}} |h_n^{sp}|^2 p_n \ge I_{\text{th}}\right] \le \epsilon. \tag{9c}$$

Since the overall data rate is an increasing function of transmit power P, it is shown that P2 has a unique solution for all $P \ge 0$.

The other conventional optimization problem is transmit power minimization (Min P) problem P3 subject to data rate constraint and AIP outage constraint. It can be described as follows:

$$\mathrm{P3}: \min_{p_n} \quad P \tag{10a}$$

$$\text{s.t.} \quad R \ge \check{R}, \tag{10b}$$

$$P_o = \Pr\left[\frac{1}{N}\sum_{n\in\mathcal{N}} |h_n^{sp}|^2 p_n \ge I_{\text{th}}\right] \le \epsilon. \tag{10c}$$

Referring to [5], in a practical system with a sufficiently large number of subchannels, random variable $X = \sum_n |h_n^{sp}|^2 p_n$ can be modeled as a normal distributed variable based on central limit theorem, with mean m_o and variance σ_o^2

$$m_o = \sum_{n\in\mathcal{N}} p_n \lambda_2, \tag{11}$$

$$\sigma_o^2 = \sum_{n\in\mathcal{N}} (p_n\lambda_2)^2, \tag{12}$$

respectively.

Then the equivalent constraint of (7) can be expressed as

$$P_o = \frac{1}{2}\operatorname{erfc}\left(\frac{NI_{\text{th}} - m_o}{\sqrt{2}\sigma_o}\right) \le \epsilon, \tag{13}$$

where $\operatorname{erfc}(x) = (2/\sqrt{\pi})\int_x^\infty e^{-t^2} dt$.

If the transmit power P is given, the objective of problem P1 is equivalent to problem P2, and the existing water-filling power allocation scheme can be used [16]. However, besides adapting the power loading on all subcarriers, the overall transmit power can also be adapted according to the CSI and AIP level to maximize the EE. Hence, the existing power allocation schemes cannot be applied directly.

Without loss of generality, assume that $g_1 \ge g_2 \ge \cdots \ge g_N$. It is noticeable that the exact power allocation solution for P2 is too difficult to address. However, if we drop the AIP constraint from P2, it can be solved by the water-filling algorithm and the power distribution is

$$p_n = \left[\mu - \frac{1}{g_n}\right]^+, \tag{14}$$

for all $n \in \mathcal{N}$, where $[x]^+ = \max(x, 0)$ and μ is the water level.

Lemma 1. *The P_o is a monotonic increasing function of water level μ, for $\mu \ge 1/g_1$, and the maximum allowable water level μ_o, for $P_o \le \epsilon$, is*

$$\mu_o = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \tag{15}$$

where $a = (2d^2\overline{N} - \overline{N}^2)\lambda_2^2$, $b = 2NI_{th}\overline{N}\lambda_2 + (2\overline{N} - 4d^2)G_1\lambda_2^2$, $c = 2G_2\lambda_2^2 d^2 - (NI_{th} + \lambda_2 G_1)^2$, $G_1 = \sum_{n=1}^{\overline{N}}(1/g_n)$, $G_2 = \sum_{n=1}^{\overline{N}}(1/g_n^2)$, $d = \operatorname{erfc}^{-1}(2\epsilon)$, $\operatorname{erfc}^{-1}(\cdot)$ is the inverse function of $\operatorname{erfc}(\cdot)$, and $\overline{N}$ is the number of subchannels with transmit power $p_n > 0$.

Proof. Please see Appendix A. □

In order to protect PU, the maximum allowable water level is restricted which may lead the power of some subcarriers equal to zero for special channel realizations. Here we present an efficient algorithm to solve μ_o. See Algorithm 1.

Step 1. Initialization, $\mu_o \leftarrow 0$, $\overline{N} \leftarrow 1$, $\mathcal{S} \leftarrow \varnothing$.
Step 2. Calculate μ_o according to (15).
Step 3. Update, $\mathcal{S} \leftarrow \{n \mid \mu_o > 1/g_n, \forall n \in \mathcal{N}\}$
Step 4. If $|\mathcal{S}| = \overline{N}$, then terminate the algorithm, else set $\overline{N} \leftarrow |\mathcal{S}|$ and go to Step 2.

Algorithm 1: Solving μ_o.

Lemma 2. *For problem P2 without AIP constraint, the optimal water level $\widehat{\mu}$ to maximize system rate under the total transmit power $\widehat{P}$ is*

$$\widehat{\mu} = \min_{1\le M\le N} \frac{1}{M}\left(\widehat{P} + \sum_{n=1}^{M}\frac{1}{g_n}\right). \tag{16}$$

Proof. Please see Appendix B. □

Lemma 3. *For problem P3 without AIP constraint, the optimal water level $\check{\mu}$ to meet the minimum rate requirement $\check{R}$ is*

$$\check{\mu} = \min_{1\le M\le N} 2^{(\check{R}/BM)}\left(\prod_{n=1}^{M} g_n\right)^{-1/M}. \tag{17}$$

Proof. Please see Appendix C. □

With the help of Lemmas 1 to 3, the optimal water levels $\widehat{\mu}^*$ and $\check{\mu}^*$ for problems P2 and P3 are

$$\widehat{\mu}^* = \min(\widehat{\mu}, \mu_o), \tag{18}$$

$$\check{\mu}^* = \min(\check{\mu}, \mu_o), \tag{19}$$

respectively.

Step 1. Initialization, $\mu^* \leftarrow 0$.
Step 2. Calculate $\check{\mu}^*$ and $\widehat{\mu}^*$ according to (18) and (19).
Step 3. If $\widehat{\mu}^* \leq \check{\mu}^*$, then $\mu_{\text{mid}} \leftarrow \widehat{\mu}^*$ and go to Step 7; else set $\mu_{\text{low}} \leftarrow \check{\mu}^*$ and $\mu_{\text{high}} \leftarrow \widehat{\mu}^*$, respectively.
Step 4. Set $\mu_{\text{mid}} \leftarrow 1/2\left(\mu_{\text{low}} + \mu_{\text{high}}\right)$.
Step 5. If its first derivative with respect to water level $\mathrm{d}U/\mathrm{d}\mu|_{\mu=\mu_{\text{mid}}} > 0$, then $\mu_{\text{low}} \leftarrow \mu_{\text{mid}}$, else $\mu_{\text{high}} \leftarrow \mu_{\text{mid}}$.
Step 6. Repeat Step 4 until $\mu_{\text{high}} - \mu_{\text{low}} \leq \xi$ where ξ is a small positive constant to control the convergence accuracy.
Step 7. Finish, the optimal solution is $\mu^* \leftarrow \mu_{\text{mid}}$.

Algorithm 2: Bisection search aided algorithm (BSAA).

Since the system rate R is a strictly increasing function of transmit power P, it is shown that R has a unique value, for all $\mu \geq 1/g_1$. Hence, the equivalent problem of P1 can be given as

$$\text{P4}: \max_{\mu} U(\mu) = \frac{R}{\alpha P + P_c}, \tag{20a}$$

$$\text{s.t.} \quad \check{\mu}^* \leq \mu \leq \widehat{\mu}^*. \tag{20b}$$

Lemma 4. *The system EE U is a quasiconcave function of water level μ for $\mu \geq 1/g_1$.*

Proof. Please see Appendix D. □

According to Lemma 4, a unique global optimal water level $\widetilde{\mu}$ always exists without any constraint in P4. Hence, the optimal water level μ^* is

$$\mu^* = \min\left(\widehat{\mu}^*, \max\left(\widetilde{\mu}, \check{\mu}^*\right)\right). \tag{21}$$

3.1. Bisection Search Aided Algorithm (BSAA). The remaining thing for P1 is to address μ^* and it can be solved by bisection search technique. Here we list the power allocation method in Algorithm 2.

Our goal aims to maximize the system EE under the constraints on transmit power, the minimum rate requirement, and AIP outage. The water-filling based algorithm is proposed to obtain the optimal solution. The key work of Algorithm 2 is to determine the sign of $\mathrm{d}U/\mathrm{d}\mu$. It can be determined by calculating the value of $U(\mu + \Delta\mu) - U(\mu)$, where $\Delta\mu$ is an infinitely small positive constant.

3.2. Simplified Energy Efficient Power Allocation (SiPA). Although the suboptimal power allocation solution is desirable, the cost to pay is the computational complexity in iterative search. It takes at most $\lceil \log_2((\widehat{\mu}^* - \check{\mu}^*)/\xi) \rceil$ iterations to convergence, where $\lceil x \rceil$ denotes the smallest integer not less than x. Thus, this straightforward approach is clearly not practical, and we need a low complexity approach to solve this problem.

Since EE U is a quasiconcave function of water level μ, the optimal unconstrained water level always exists. Differentiating (20a) with respect to μ and setting the derivative to zero, we can obtain

$$\frac{\partial U}{\partial \mu} = F\left(1 + \frac{P_s}{\alpha \widetilde{N} \mu} - \frac{G_3}{\mu} - \ln\left(\mu G_4\right)\right) = 0, \tag{22}$$

or

$$\frac{P_s G_4}{\alpha \widetilde{N} e} - \frac{G_3 G_4}{e} = \frac{\mu G_4}{e} \ln\left(\frac{\mu G_4}{e}\right), \tag{23}$$

where e is the base of the natural logarithm, F is a nonzero value independent of μ, and G_3 and G_4 are defined as

$$G_3 = \frac{1}{\widetilde{N}} \sum_{n=1}^{\widetilde{N}} \frac{1}{g_n}, \tag{24}$$

$$G_4 = \left(\prod_{n=1}^{\widetilde{N}} g_n\right)^{1/\widetilde{N}}, \tag{25}$$

respectively. Let $X = \ln(\mu G_4/e)$ and $Y = ((P_s G_4)/(\alpha \widetilde{N} e)) - ((G_3 G_4)/e)$; then (23) can be expressed as

$$X e^X = Y. \tag{26}$$

Its solution is $X = W_0(Y)$, where $W_0(\cdot)$ denotes the real branch of the Lambert function [13]. Substituting this solution into (26), the global optimal water level $\widetilde{\mu}$ without any constraint is

$$\widetilde{\mu} = \frac{e}{G_4} \exp\left[W_0\left(\frac{P_s G_4}{\alpha \widetilde{N} e} - \frac{G_3 G_4}{e}\right)\right]. \tag{27}$$

According to Lemma 4, the optimal water level μ^* corresponding to P1 can be obtained by substituting $\widetilde{\mu}$ into (21).

4. Numerical Results

Simulations have been performed to evaluate the performance of our proposed algorithms. The channel gains $|h_n^{ss}|^2$ and $|h_n^{sp}|^2$ ($n \in \mathcal{N}$) are independent chi-square distributed random variables. Simulation parameters are set as follows: $B = 15$ kHz, $N = 512$, $\lambda = \lambda_1/\lambda_2$ (we fix $\lambda_2 = 1$ and vary λ_1), $\widehat{P} = 3$ W, $\sigma^2 = 0.01$ W, $I_{\text{th}} = \rho\sigma^2$, $\epsilon = 0.05$, $P_s = 0.8$ W, $\alpha = 2$, $\check{R} = 2.3$ Mbps, and $\beta = 0.2$ W/Mbps. The simulation curves are obtained by averaging over 1000 channel realizations.

In Figure 1, we compare the average EE of various power allocation algorithms with $\rho = 0.5$. Compared with other algorithms, our proposed algorithms can achieve significant performance gain in terms of EE. The BSAA and SiPA have the same performance, and both of them always perform better than the simplified WFAS algorithm in [13] at the cost

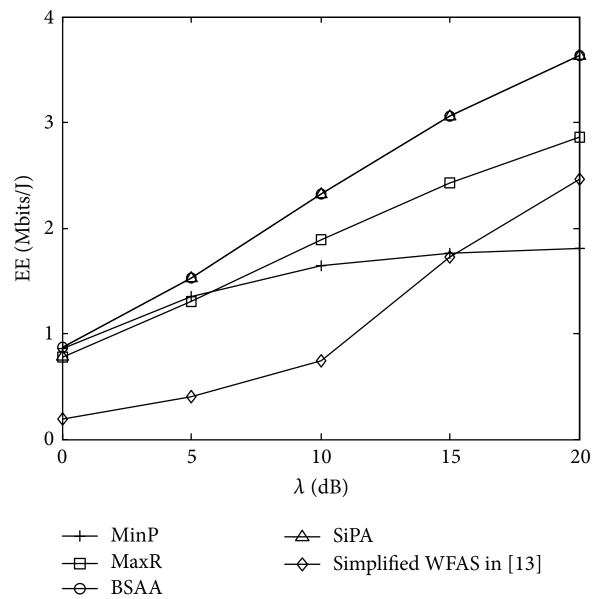

Figure 1: System EE versus average CNR, $\rho = 0.5$.

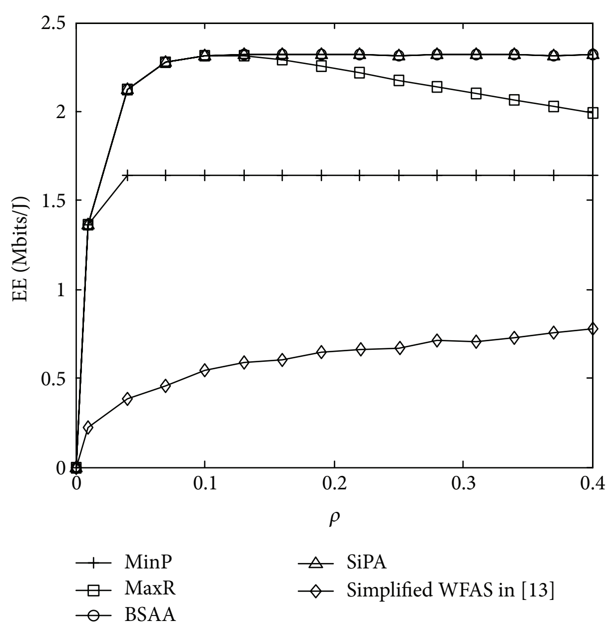

Figure 2: System EE versus average CNR, $\lambda = 10$ dB.

of serious interference to PU. In the low λ region, the transmit power dominates the total power consumption while in the high λ region the circuit power does for the energy efficient design. However, the gap between proposed algorithms and conventional ones becomes larger as λ keeps increasing.

In Figure 2, the average EE dramatically increases as ρ increases in the low region. However, when ρ is greater than a saturation point, the EE does not further increase for energy efficient power allocation. In addition, it can be observed that all schemes may have to operate exactly at the maximum allowable transmit power to meet the minimum rate requirement when ρ is much smaller than the threshold.

5. Conclusion

In this paper, we considered the energy efficient design for OFDM-based CR systems under the AIP outage probability constraint and minimum rate requirement constraint. Due to the loose cooperation, the intersystem CSI is partially known to the SU-Tx. To solve this problem, we proposed a suboptimal algorithm based on bisection search technique. Furthermore, in order to reduce the computational complexity, we also derived a closed-form solution for simplified algorithm without performance loss. We demonstrate the superior EE performance of the proposed algorithms by extensive simulations.

Appendices

A. Proof of Lemma 1

From (11), (12), and (14), (13) can be written as

$$P_o = \frac{1}{2}\operatorname{erfc}\left(\frac{NI_{\text{th}} - \lambda_2 \sum_{n=1}^{\widetilde{N}} \left(\mu - (1/g_n)\right)}{\lambda_2 \sqrt{2\sum_{n=1}^{\widetilde{N}} \left(\mu - (1/g_n)\right)^2}}\right) \le \epsilon, \quad \text{(A.1)}$$

where $\mu > 1/g_n$, for $n = 1, 2, \ldots, \widetilde{N}$, and $\mu \le 1/g_n$, for $n = \widetilde{N} + 1, \ldots, N$.

It can be easily proved that P_o is a monotonic increasing function of μ. Let μ_o be the maximum allowable power level corresponding to AIP constraint; we have

$$P_o = \frac{1}{2}\operatorname{erfc}\left(\frac{NI_{\text{th}} - \lambda_2 \sum_{n=1}^{\widetilde{N}} \left(\mu_o - (1/g_n)\right)}{\lambda_2 \sqrt{2\sum_{n=1}^{\widetilde{N}} \left(\mu_o - (1/g_n)\right)^2}}\right) = \epsilon. \quad \text{(A.2)}$$

After some mathematical manipulation, a single equation with the variable μ_o can be derived as

$$a{\mu_o}^2 + b\mu_o + c = 0, \quad \text{(A.3)}$$

where $a = (2d^2\overline{N} - \overline{N}^2)\lambda_2^2$, $b = 2NI_{\text{th}}\overline{N}\lambda_2 + (2\overline{N} - 4d^2)G_1\lambda_2^2$, $c = 2G_2\lambda_2^2 d^2 - (NI_{\text{th}} + \lambda_2 G_1)^2$, $G_1 = \sum_{n=1}^{\overline{N}}(1/g_n)$, $G_2 = \sum_{n=1}^{\overline{N}}(1/g_n^2)$, $d = \operatorname{erfc}^{-1}(2\epsilon)$, and $\operatorname{erfc}^{-1}(\cdot)$ is the inverse function of $\operatorname{erfc}(\cdot)$. When solving the above equation, the maximum allowable power level corresponding to AIP constraint is

$$\mu_o = \frac{-b + \sqrt{b^2 - 4ac}}{2a}. \quad \text{(A.4)}$$

Here Lemma 1 is proved.

B. Proof of Lemma 2

For a given allowable transmit power $\widehat{P}$, the optimal power allocation can be achieved by classical water-filling method. The true power level is assumed to be $\widehat{\mu}$; we have

$$\widehat{P} = \sum_{n=1}^{N} p_n = \sum_{n=1}^{M} \left(\widehat{\mu} - \frac{1}{g_n}\right) = M\widehat{\mu} - \sum_{n=1}^{M} \frac{1}{g_n}, \tag{B.1}$$

or

$$\widehat{\mu} = \frac{1}{M}\left(\widehat{P} + \sum_{n=1}^{M} \frac{1}{g_n}\right), \tag{B.2}$$

where $M \le N$, $p_n = \widehat{\mu} - (1/g_n) > 0$, $n = 1, 2, \ldots, M$, and $\widehat{\mu} \le (1/g_n)$, $n = M+1, \ldots, N$.

For arbitrary $1 \le M_1 < M < M_2 \le N$, we have $\mu_i = (1/M_i)(\widehat{P} + \sum_{n=1}^{M_i}(1/g_n))$, $i = 1, 2$. Because $\widehat{\mu} > (1/g_n)$, $n = 1, \ldots, M$, and $\widehat{\mu} \le (1/g_n)$, $n = M+1, \ldots, N$ in (B.2), we derive the results that $\mu_1 > \widehat{\mu}$ and $\mu_2 > \widehat{\mu}$ as follows:

$$\begin{aligned}
\mu_1 - \widehat{\mu} &= \frac{1}{M_1}\left(M\widehat{\mu} - \sum_{n=1}^{M} \frac{1}{g_n} + \sum_{n=1}^{M_1} \frac{1}{g_n}\right) - \widehat{\mu} \\
&= \frac{1}{M_1}\left[\left(M - M_1\right)\widehat{\mu} - \sum_{n=M_1+1}^{M} \frac{1}{g_n}\right] \\
&= \frac{1}{M_1}\sum_{n=M_1+1}^{M}\left(\widehat{\mu} - \frac{1}{g_n}\right) \\
&> 0, \\
\mu_2 - \widehat{\mu} &= \frac{1}{M_2}\left(M\widehat{\mu} - \sum_{n=1}^{M} \frac{1}{g_n} + \sum_{n=1}^{M_2} \frac{1}{g_n}\right) - \widehat{\mu} \\
&= \frac{1}{M_2}\left[-\left(M_2 - M\right)\widehat{\mu} + \sum_{n=M+1}^{M_2} \frac{1}{g_n}\right] \\
&= -\frac{1}{M_2}\sum_{n=M+1}^{M_2}\left(\widehat{\mu} - \frac{1}{g_n}\right) \\
&\ge 0.
\end{aligned} \tag{B.3}$$

Hence the true power level $\widehat{\mu}$ corresponding to the maximum allowable transmit power $\widehat{P}$ is

$$\widehat{\mu} = \min_{1 \le M \le N} \frac{1}{M}\left(\widehat{P} + \sum_{n=1}^{M} \frac{1}{g_n}\right). \tag{B.4}$$

Here Lemma 2 is proved.

C. Proof of Lemma 3

Let $\check{\mu}$ be the optimal water level corresponding to the minimum rate requirement $\check{R}$ without AIP constraint; we have

$$\begin{aligned}
\check{R} &= B\sum_{n=1}^{N} \log_2\left(1 + g_n p_n\right) \\
&= B\sum_{n=1}^{M} \log_2\left(1 + \left(\check{\mu} - \frac{1}{g_n}\right) g_n\right) \\
&= B\log_2\left(\check{\mu}^M \prod_{n=1}^{M} g_n\right),
\end{aligned} \tag{C.1}$$

or

$$\check{\mu} = 2^{(\check{R}/BM)}\left(\prod_{n=1}^{M} g_n\right)^{-1/M}, \tag{C.2}$$

where $M \le N$, $p_n = \check{\mu} - (1/g_n) > 0$, $n = 1, 2, \ldots, M$, and $\check{\mu} \le (1/g_n)$, $n = M+1, \ldots, N$.

For arbitrary $1 \le M_1 < M < M_2 \le N$, we have $\mu_i = 2^{(\check{R}/BM_i)}(\prod_{n=1}^{M_i} g_n)^{-1/M_i}$, $i = 1, 2$. It can be seen that $\check{\mu} g_n > 1$, $n = 1, \ldots, M$ and $\check{\mu} g_n \le 1$, $n = M+1, \ldots, N$. We derive the results that $\mu_1 > \check{\mu}$ and $\mu_2 > \check{\mu}$ as follows:

$$\begin{aligned}
\frac{\mu_1}{\check{\mu}} &= \frac{2^{(\check{R}/BM_1)}\left(\prod_{n=1}^{M_1} g_n\right)^{-1/M_1}}{\check{\mu}} \\
&= \frac{\left(\check{\mu}^M \prod_{n=1}^{M} g_n \prod_{n=1}^{M_1} 1/g_n\right)^{1/M_1}}{\check{\mu}} \\
&= \left(\prod_{n=M_1+1}^{M} \check{\mu} g_n\right)^{1/M_1} \\
&> 1, \\
\frac{\mu_2}{\check{\mu}} &= \frac{2^{(\check{R}/BM_2)}\left(\prod_{n=1}^{M_2} g_n\right)^{-1/M_2}}{\check{\mu}} \\
&= \frac{\left(\check{\mu}^M \prod_{n=1}^{M} g_n \prod_{n=1}^{M_2} 1/g_n\right)^{1/M_2}}{\check{\mu}} \\
&= \left(\prod_{n=M+1}^{M_2} \frac{1}{\check{\mu} g_n}\right)^{1/M_2} \\
&\ge 1.
\end{aligned} \tag{C.3}$$

Since $\mu_i > 0$ $(i = 1, 2)$, the optimal power level $\check{\mu}$ that satisfies $R = \check{R}$ without the AIP constraint is

$$\check{\mu} = \min_{1 \le M \le N} 2^{(\check{R}/BM)}\left(\prod_{n=1}^{M} g_n\right)^{-1/M}. \tag{C.4}$$

Here Lemma 3 is proved.

D. Proof of Lemma 4

From problem (8a), the objective of EE optimization is modeled as

$$\max_{p_n} U(P) = \frac{1}{((\alpha P + P_s)/R(P)) + \beta}, \quad \text{(D.1)}$$

where P is the total transmit power and $R(p)$ is the corresponding data rate.

Since $\alpha > 0$, $R > 0$, $P > 0$, and $P_s > 0$, the EE U is a strictly monotone decreasing function of $\alpha P + P_s/R$. Then the equivalent optimization problem is reformulated as

$$\max_{p_n} \overline{U}(P) = \frac{R(P)}{\alpha P + P_s}. \quad \text{(D.2)}$$

For arbitrary $\theta \in (0, 1)$, since $R(P)$ is concave function of P, we have

$$\begin{aligned} \overline{U}(\theta x_1 + (1-\theta) x_2) &\geq \frac{\theta R(x_1) + (1-\theta) R(x_2)}{\theta(\alpha x_1 + P_s) + (1-\theta)(\alpha x_2 + P_s)} \\ &\geq \min\left(\overline{U}(x_1), \overline{U}(x_2)\right). \end{aligned} \quad \text{(D.3)}$$

According to [13, 14], we can conclude that U is quasiconcave, for $P \geq 0$. Since P is a monotonic increasing function of water level, for $\mu > (1/\max g_n)$. Hence, EE is a quasiconcave function of water level.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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