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Research Article

The H_∞ Control for Bilinear Systems with Poisson Jumps

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This paper discusses the state feedback H_∞ control problem for a class of bilinear stochastic systems driven by both Brownian motion and Poisson jumps. By completing square method, we obtain the H_∞ control by solutions of the corresponding Hamilton-Jacobi equations (HJE). By the tensor power series method, we also shift such HJEs into a kind of Riccati equations, and the H_∞ control is represented with the form of tensor power series.

1. Introduction

The main purpose of H_∞ control design is to find the law to efficiently eliminate the effect of the disturbance [1, 2]. Theoretically, study of H_∞ control first starts from the deterministic linear systems, and the derivation of the state-space formulation of the standard H_∞ control leads to a breakthrough, which can be found in the paper [3]. In recent years, stochastic H_∞ control systems, such as Markovian jump systems [4–6], H_∞ Gaussian control design [7], and Itô differential systems [8–13], have received a great deal of attention. However, up to now, most of the work on stochastic H_∞ control is confined to Itô type or Markovian jump systems. Yet, there are still many systems which contain Poisson jumps in economics and natural science. In 1970s, Boel and Varaiya [14] and Rishel [15] considered the optimal control problem with random Poisson jumps, and many basic results have been made. From then on, many scholars and economists also study the system and its applications; for further reference, we refer to [16–20] and their references. But those results mostly concentrate on optimal control and its application in financial market or corresponding theories. Of course, such model still can be disturbed by exogenous disturbance and its robustness is also an important problem. The objective of this paper is to develop an H_∞-type theory over infinite time horizon for the disturbance attenuation of stochastic bilinear systems with Poisson jumps by dynamic state feedback.

Generally, the key of H_∞ control design is to solve a general Hamilton-Jacobi equation (HJE). However, up to now, there is still no effective algorithm to solve such a general HJE. In order to solve the HJE given in this paper, we extend a tensor power series approach which is used in [21] and also give the simulation of the trajectory of output z under H_∞ control. This paper will follow along the lines of [22] to study the stochastic H_∞ control with infinite horizons and finite horizon for a class of nonlinear stochastic differential systems with Poisson jumps. The paper is organized as follows.

In Section 2, we review Itô's theories about the system driven by Brownian motion and Poisson jumps. In Section 3, we obtain the H_∞ by solving the HJE which is proved by the completing square method. In Section 4, we discuss the problem of finite horizon H_∞ control with jumps, and using the tensor power series approach, we discuss the approximating H_∞ control given in the paper. For convenience, we adopt the following notation.

$\mathcal{S}_n(\mathcal{R})$ denotes the set of all real $n\times n$ symmetric matrices; A' is the transpose of the corresponding matrix A; $A > 0$ $(A \geq 0)$ is the positive definite (semidefinite) matrix A; I is the identity matrix; $\mathbb{E}\xi$ is the expectation of random variable ξ; $\|x\|$ is the Euclidean norm of vector $x \in \mathcal{R}^{n_x}$ and n_x is the dimension of x; $\mathcal{L}^2([0,T],\mathcal{R}^{n_y})$ is the set of n_y-dimensional stochastic process y defined on interval $[0,T]$ (T can take ∞), taking values in $\mathscr{R}^{n_y}$, with norm

$$\|y\|_{\mathscr{L}^2([0,T],\mathscr{R}^{n_y})} = \left(E\int_0^T \|y(t)\|^2 dt\right)^{1/2} < \infty; \quad (1)$$

$C^{1,2}(\mathscr{R}_+, \mathscr{R}^{n_x})$ is the class of function $V(t,x)$ twice continuously differential with respect to $x \in \mathscr{R}^{n_x}$ and once continuously differential with respect to t; $\langle x, y\rangle$ is the inner product of two vectors $x, y \in \mathscr{R}^n$.

2. Preliminaries

For a given complete probability space $(\Omega, \mathscr{F}, \mathbb{P})$, let W_t and μ be the Brownian motion and the Poisson random measure, respectively, which are mutually independent:

(i) a 1-dimensional standard Brownian motion $\{W_t\}_{t\geq 0}$;

(ii) a Poisson random measure μ on $\mathbb{R}_+ \times E$, where $E \subset \mathbb{R}^l$ is a nonempty open set equipped with its Borel field $\mathscr{B}(E)$, with the compensator $\widehat{\mu}(de, dt) = \lambda(de)dt$, such that $\{\widetilde{\mu}((0,t]\times A) = (\mu - \widehat{\mu})((0,t]\times A)\}_{t\geq 0}$ is a martingale for all $A \in \mathscr{B}(E)$ satisfying $\lambda(A) < \infty$. Here λ is an arbitrarily given σ-finite Lévy measure on $(E, \mathscr{B}(E))$, that is, a measure on $(E, \mathscr{B}(E))$ with the property that $\int_E (1 \wedge |e|^2)\lambda(de) < \infty$. We also let

$$\mathscr{F}_t = \sigma\left[\int\int_{(0,s]\times A} \widetilde{\mu}(de, ds) : s \leq t, A \in \mathscr{B}(E)\right] \vee \sigma \times [W_s : s \leq t] \vee \mathscr{N}, \quad (2)$$

where $\mathscr{N}$ denotes the totality of P-null sets.

In order to discuss the systems driven by Brownian motion and Poisson jumps, we first review the theorem about Itô's formula for such stochastic processes.

Theorem 1. *Let M_t be a square integral continuous martingale; A_t is a continuous adapted process with finite variance. $\gamma(s,e)$ is locally square integral due to μ and P; $x(t)$ satisfies the following Itô type stochastic process:*

$$x(t) = x(0) + M_t + A_t + \int_0^t \int_E \gamma(s,e)\, \widetilde{\mu}(de, ds). \quad (3)$$

Then for $F(t,x) \in C^{1,2}(\mathscr{R}_+, \mathscr{R}^{n_x})$, we have (see [23] Chapter I, §3, Theorem 11)

$$\begin{aligned} dF(t,x_t) &= F_t(t,x_t)\,dt + F_x(t,x_t)\,d(M_t + A_t) \\ &\quad + \frac{1}{2}F_{xx}(t,x_t)\,d\langle M\rangle_t + \int_E [F(t, x_t + \gamma(t,e)) - F(t,x_t) \\ &\quad - F_x(t,x_t)\gamma(t,e)]\,\lambda(de)\,dt \\ &\quad + \int_E [F(t, x_t + \gamma(t,e)) - F(t,x_t)]\,\widetilde{\mu}(de, dt), \end{aligned} \quad (4)$$

where $\langle M\rangle$ denotes the predictable compensator of martingale M.

In the paper, for convenience, x_t is shorten as x. Furthermore, for $F \in C^{1,2}(\mathscr{R}_+, \mathscr{R}^{n_x})$, if using Itô formula to $F(t,x_t)$ and integrating from s to t $(0 \leq s < t)$, then taking expectation with both sides

$$\begin{aligned} &\mathbb{E}F(t,x_t) - \mathbb{E}F(s,x_s) \\ &\quad = \int_s^t \mathbb{E}F_t(r,x_r)\,dr \\ &\quad + \frac{1}{2}\int_s^t \mathbb{E}\left[\sigma(r,x_r)' F_{xx}(r,x_r)\,\sigma(r,x_r)\right] dr \\ &\quad + \int_s^t \mathbb{E}\left\{\int_E [F(r, x_r + \gamma(r, x_{r-}, e)) \right. \\ &\quad - F(r,x_r) - \langle F_x(r,x_r), \gamma(r,e)\rangle] \\ &\quad \left. \times \lambda(de)\right\} dr, \end{aligned} \quad (5)$$

we can see that $\mathbb{E}F(t,x_t)$ is continuous with respect to time t. Since we mainly use the results of expectations of some well functions on x_t and those expectations are continuous with respect to time t, so, for briefness, in the rest of this paper the sign x_{t-} under integration $\int_E$ is also shortened as x.

3. The H_∞ Control for Bilinear Systems with Jumps

We consider the following bilinear system driven by Poisson jumps:

$$\begin{aligned} dx &= (Ax + Bxu + Kv)\,dt + CxdW + \int_E G(e)\,x\widetilde{\mu}(de, dt), \\ z &= \begin{bmatrix} Mx \\ u \end{bmatrix}, \end{aligned} \quad (6)$$

where $v \in \mathscr{L}^2([0,T], \mathscr{R}^{n_d})$ represents the exogenous disturbance, A, B, and C are constant $n_x \times n_x$ matrices, $K \in \mathscr{R}K \in \mathscr{R}^{n_x \times n_d}$, and $G(e) \in \mathscr{R}^{n_x \times n_x}$ only depends on e. If there exists an $u_T^* \in \mathscr{L}^2([0,T], \mathscr{R}^{n_u})$ such that for any given $\gamma > 0$ and all $v \in \mathscr{L}^2([0,T], \mathscr{R}^{n_d})$, $x(0) = 0$, the closed-loop system satisfies

$$\|z\|_{\mathscr{L}^2([0,T],\mathscr{R}^{n_z})} \leq \gamma \|v\|_{\mathscr{L}^2([0,T],\mathscr{R}^{n_d})}, \quad (7)$$

we call u_T^* the H_∞ control of (6).

Theorem 2. *Suppose there exists a nonnegative solution $V \in C^{1,2}([0,T],\mathscr{R}^{n_x})$ to the HJE*

$$\begin{aligned}\mathscr{H}_T^1(V_T(t,x)) := {}& \frac{\partial V_T}{\partial t} + \frac{\partial V_T}{\partial x}' Ax + \frac{1}{2}x'M'Mx \\ &+ \frac{1}{2\gamma^2}\frac{\partial V_T}{\partial x}' KK' \frac{\partial V_T}{\partial x} \\ &+ \frac{1}{2}x'C'\frac{\partial^2 V_T}{\partial x^2}Cx - \frac{1}{2}\frac{\partial V_T}{\partial x}' Bxx'B'\frac{\partial V_T}{\partial x} \\ &+ \int_E \Big[V_T(t,x+G(e)x) - V_T(t,x) \\ &- \frac{\partial V}{\partial x}' G(e)x\Big]\lambda(de) = 0, \\ V_T(T,x) = 0, \quad V_T(t,0) = 0, \quad & \forall (t,x) \in [0,T]\times\mathscr{R}^{n_x}. \end{aligned} \tag{8}$$

Then $u_T^* = -x'B'(\partial V_T/\partial x)$ is an H_∞ control for system (6).

Proof. Applying Itô's formula to $V(t,x)$, we have

$$\begin{aligned} V(T,x_T) - V(0,0) ={}& \int_0^T \Big\{V(t,x) + V_x'(t,x)(Ax + Bxu + Kv) \\ &+ \frac{1}{2}x'C'V_{xx}Cx \\ &+ \int_E [V(t,x+G(e)x) - V(t,x) \\ &- V_x(t,x)G(e)x]\,\lambda(de)\Big\}dt \\ &+ \int_0^T V_x'(t,x)CxdW_t \\ &+ \int_0^T [V(t,x+G(e)x) - V(t,x)]\,\tilde{\mu}(de,dt). \end{aligned} \tag{9}$$

Taking expectation with both sides and applying $V(T,x)=0$ and $V(t,0)=0$, we obtain

$$\begin{aligned} 0 = \int_0^T \mathbb{E}\Big\{&V(t,x) + V_x'(t,x)(Ax + Bxu + Kv) \\ &+ \frac{1}{2}x'C'V_{xx}Cx \\ &+ \int_E [V(t,x+G(e)x) - V(t,x) \\ &- V_x(t,x)G(e)x]\,\lambda(de) \\ &+ x_t'M'Mx + u_t'u - \gamma^2 v_t'v\Big\}dt \\ &- \|z\|^2_{\mathscr{L}^2([0,T],\mathscr{R}^{n_z})} + \gamma^2\|v\|^2_{\mathscr{L}^2([0,T],\mathscr{R}^{n_d})}. \end{aligned} \tag{10}$$

Completing square for u and v, respectively, we have

$$\begin{aligned} 0 = {}& \int_0^T \mathbb{E}\left\{\mathscr{H}_T^1(V(t,x)) + \|u - u_T^*\|^2 - \frac{1}{\gamma^2}\|v - v_T^*\|^2\right\}dt \\ &- \|z\|^2_{\mathscr{L}^2([0,T],\mathscr{R}^{n_z})} + \gamma^2\|v\|^2_{\mathscr{L}^2([0,T],\mathscr{R}^{n_d})}, \end{aligned} \tag{11}$$

where

$$u_T^* = -x'B'\frac{\partial V_T}{\partial x}, \qquad v_T^* = \frac{1}{\gamma^2}\left(K'\frac{\partial V_T}{\partial x} + K'\frac{\partial^2 V_T}{\partial x^2}Cx\right). \tag{12}$$

By HJE (8) and let $u = u_T^*$, we have

$$\begin{aligned} &\|z\|^2_{\mathscr{L}^2([0,T],\mathscr{R}^{n_z})} - \gamma^2\|v\|^2_{\mathscr{L}^2([0,T],\mathscr{R}^{n_d})} \\ &\quad = -\frac{1}{\gamma^2}\int_0^T \mathbb{E}\left\{\|v - v_T^*\|^2\right\}dt. \end{aligned} \tag{13}$$

So, the following inequality is true:

$$\|z\|^2_{\mathscr{L}^2([0,T],\mathscr{R}^{n_z})} \le \gamma^2\|v\|^2_{\mathscr{L}^2([0,T],\mathscr{R}^{n_d})}. \tag{14}$$

This proves that u_T^* is an H_∞ control for system (6). □

Remark 3. From the proof of Theorems 2 and (13), we can see that (u_T^*, v_T^*) given by (12) is a saddle point for the following stochastic game problem:

$$\min_{u\in\mathscr{L}^2([0,T],\mathscr{R}^{n_u})}\ \max_{d\in\mathscr{L}^2([0,T],\mathscr{R}^{n_d})} \mathbb{E}\int_0^T \left(\|z\|^2 - \gamma^2\|d\|^2\right)dt. \tag{15}$$

4. The Tensor Power Series Representation of H_∞ Control

Generally speaking, it is very hard to solve HJE (8). Now we use an approximation algorithm which is called tensor power series approach to solve a special case of HJE (8). In what follows, suppose $V_T(t,x)$ satisfying (8) has the following form:

$$V_T(t,x) = \sum_{i=1}^{\infty}\left\langle \otimes_i x, P_i(t)\otimes_i x\right\rangle, \tag{16}$$

where $P_i(t)$, $i \ge 1$, are symmetrically and continuously differential matrix-valued functions on $[0,T]$, $\otimes$ is the Kronecker product of matrix (or vectors), and $\otimes_i x = x\otimes\cdots\otimes x$ is i times Kronecker product of x.

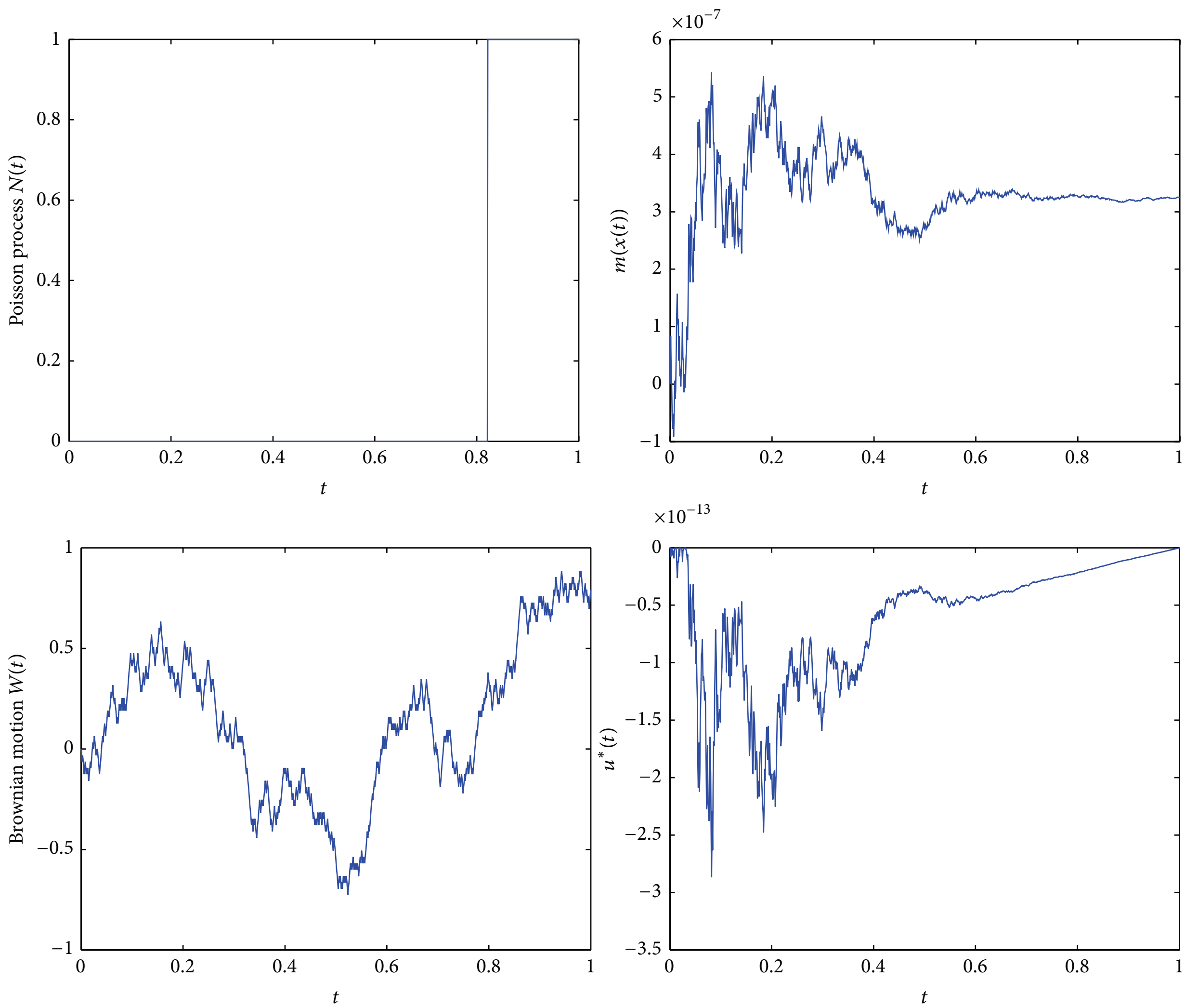

Figure 1: Tracking performance of Example 10.

Theorem 4. *For given $\gamma > 0$, suppose $P_i(t)$ $(i = 1, 2, \ldots)$ satisfy the following Riccati equations:*

$$\begin{aligned}&\dot{P}_1 + A^{(1)}P_1 + P_1A^{(1)'} + \frac{1}{2}M'M + \frac{2}{\gamma^2}\left(Q^{(1)}P_1\right)\otimes\left(P_1Q^{(1)'}\right)\\&\quad + R^{(1)}(P_1) + \int_E \Big[\left(I_{n_x} + G(e)\right)'P_1\left(I_{n_x} + G(e)\right) - P_1\\&\qquad - G^{(1)}(e)P_1 - P_1G^{(1)'}(e)\Big]\lambda(de) = 0,\\&\dot{P}_i + A^{(i)}P_i + P_iA^{(i)'} + \frac{2}{\gamma^2}\sum_{r+j=i+1}\left(K^{(r)}P_r\right)\otimes\left(P_jK^{(j)'}\right)\\&\quad + R^{(i)}(P_i) - 2\sum_{r+j=i}\left(B^{(r)}P_r\right)\otimes\left(P_jB^{(j)'}\right)\\&\quad + \int_E \Big[\left(\otimes_i\left(I_{n_x} + G(e)\right)\right)'P_i\left(\otimes_i\left(I_{n_x} + G(e)\right)\right)\\&\qquad - P_i - G^{(i)}(e)P_i - P_iG^{(i)'}(e)\Big]\lambda(de) = 0,\end{aligned}$$

$$P_i(T) = 0, \quad i = 1, 2, \ldots. \tag{17}$$

Then the H_∞ control u_T^* for system (6) can be given by

$$u_T^* = -2\sum_{i=1}^{\infty}\left\langle \otimes_i x, \left(B^{(i)}P_i\right)\otimes_i x\right\rangle, \tag{18}$$

where $B^{(i)} = \sum_{j=1}^{n_x} B_j' \otimes D^{(i,j)}$ and $D^{(i,j)}$ is given by following Lemma 6.

In order to prove Theorem 4, we need the following lemmas, and Lemmas 5–8 are given without proofs.

Lemma 5. *For any $x \in \mathscr{R}^{n_x}$, $y \in \mathscr{R}^{n_y}$, $u \in \mathscr{R}^{n_u}$, $v \in \mathscr{R}^{n_v}$, $P \in \mathscr{R}^{n_x\times n_y}$, and $Q \in \mathscr{R}^{n_u\times n_v}$ we have*

$$\langle x, Py\rangle\langle u, Qv\rangle = \langle x\otimes u, (P\otimes Q)(y\otimes v)\rangle. \tag{19}$$

Lemma 6. *For any matrix $P \in \mathcal{S}_{n_x^i}(\mathcal{R})$, $K \in \mathcal{R}^{n_k}$, and integer i, we have*

$$\frac{\partial \langle \otimes_i x, P\otimes_i x\rangle'}{\partial x} K = 2\left\langle \otimes_{i-1}x, \left(K^{(i)}P\right)\otimes_i x\right\rangle, \tag{20}$$

where $K^{(i)} = \sum_{j=1}^{n_x} k_j D^{(i,j)}$, $D^{(i,j)} = \sum_{l=1}^{i} D_l^{(i,j)}$, and $D_l^{(i,j)} = I_{n_x^{l-1}} \otimes e_j' \otimes I_{n_x^{i-l}}$.

Lemma 7. *Let $V_T(t,x) = \sum_{i=1}^{\infty}\langle \otimes_i x, P_i(t)\otimes_i x\rangle$ exist. We have*

$$\begin{aligned}&\frac{\partial V_T}{\partial x}' KK' \frac{\partial V_T}{\partial x}\\ &\quad = 4\sum_{m=1}^{\infty}\left\langle \otimes_m x, \sum_{i+j=m+1}\left(K^{(i)}P_i\right)\otimes\left(P_j K^{(j)'}\right)\otimes_m x\right\rangle.\end{aligned} \tag{21}$$

Lemma 8. *For any matrix $P \in \mathcal{S}_{n_x^i}(\mathcal{R})$, $x \in \mathcal{R}^{n_x}$, and integer i, we have*

$$\frac{\partial \left\langle \otimes_i x, P\otimes_i x\right\rangle'}{\partial x} Ax = 2\left\langle \otimes_i x, \left(A^{(i)}P\right)\otimes_i x\right\rangle, \tag{22}$$

where $A^{(i)} = \sum_{j=1}^{n_x} A_j' \otimes D^{(i,j)}$, and A_j is the jth row vector of matrix A.

Lemma 9. *For any matrix $P \in \mathcal{S}_{n_x^i}(\mathcal{R})$ and integer i, we have*

$$x'C'\frac{\partial^2 \left\langle \otimes_i x, P\otimes_i x\right\rangle}{\partial x^2} Cx = 2\left\langle \otimes_i x, R^{(i)}(P)\otimes_i x\right\rangle, \tag{23}$$

where $R^{(i)}(P) = C^{(i)}PC^{(i)'} + Q^{(i)}P$, $Q^{(i)} = \sum_{s=1}^{n_x}\sum_{t=1}^{n_x} C_s' \otimes C_t' \otimes (D^{(i-1,s)}D^{(i,t)})$, C_s is the sth row vector of matrix C, and $C^{(i)}$ is determined as $A^{(i)}$ in Lemma 8.

Proof. Let $K = Cx = (k_1, k_2, \ldots, k_{n_x})'$; then we have

$$\begin{aligned}
&x'C'\frac{\partial^2 \left\langle \otimes_i x, P\otimes_i x\right\rangle}{\partial x^2}Cx\\
&= \sum_{s=1}^{n_x}\sum_{t=1}^{n_x}\frac{\partial^2 \left\langle \otimes_i x, P\otimes_i x\right\rangle}{\partial x_s \partial x_t}k_s k_t\\
&= 2\sum_{s=1}^{n_x}\sum_{t=1}^{n_x}k_s k_t \frac{\partial}{\partial x_s}\left\langle \otimes_{i-1}x, \sum_{l=1}^{i}D_l^{(i,t)}P\right\rangle\\
&= 2\sum_{s=1}^{n_x}\sum_{t=1}^{n_x}k_s k_t\left[\sum_{m=1}^{i-1}\left\langle \otimes_{m-1}x\otimes e_s\otimes\otimes_{i-m-1}x, \sum_{l=1}^{i}D_l^{(i,t)}P\otimes_i x\right\rangle\right.\\
&\quad\left. + \sum_{m=1}^{i}\left\langle \otimes_{i-1}x, \sum_{l=1}^{i}D_l^{(i,t)}P\otimes_{m-1}x\otimes e_s\otimes\otimes_{i-m}x\right\rangle\right]\\
&= 2\sum_{s=1}^{n_x}\sum_{t=1}^{n_x}k_s k_t\left[\left\langle \otimes_{i-2}x, \sum_{m=1}^{i-1}D_m^{i-1,s}\sum_{l=1}^{i}D_l^{(i,t)}P\otimes_i x\right\rangle\right.\\
&\quad\left. + \left\langle \otimes_{i-1}x, \sum_{m=1}^{i}\sum_{l=1}^{i}D_l^{(i,t)}PD_m^{(i,t)'}\otimes_{i-1}\right\rangle\right]\\
&= 2\sum_{s=1}^{n_x}\sum_{t=1}^{n_x}\left[\left\langle x, C_s'\right\rangle\left\langle x, C_t'\right\rangle\right.\\
&\quad\times\left\langle \otimes_{i-2}x, \sum_{m=1}^{i-1}D_m^{i-1,s}\sum_{l=1}^{i}D_l^{(i,t)}P\otimes_i x\right\rangle\\
&\quad + \left\langle x, C_s'\right\rangle\left\langle \otimes_{i-1}x, \sum_{m=1}^{i}\sum_{l=1}^{i}D_l^{(i,t)}PD_m^{(i,t)'}\otimes_{i-1}\right\rangle\\
&\quad\left.\times\left\langle C_t', x\right\rangle\right].
\end{aligned} \tag{24}$$

By Lemma 5,

$$\begin{aligned}
&x'C'\frac{\partial^2 \left\langle \otimes_i x, P\otimes_i x\right\rangle}{\partial x^2}Cx\\
&= 2\sum_{s=1}^{n_x}\sum_{t=1}^{n_x}\left[\left\langle \otimes_i x, C_s'\otimes C_t'\otimes\sum_{m=1}^{i-1}D_m^{i-1,s}\right.\right.\\
&\quad\left.\times\sum_{l=1}^{i}D_l^{(i,t)}P\otimes_i x\right\rangle\\
&\quad + \left\langle \otimes_i x, \left(C_s'\otimes\sum_{m=1}^{i}\sum_{l=1}^{i}D_l^{(i,t)}PD_m^{(i,t)'}\right.\right.\\
&\quad\left.\left.\left.\otimes C_t\right)\otimes_i x\right\rangle\right].
\end{aligned} \tag{25}$$

So we can obtain (23). □

Proof of Theorem 4. Applying Lemmas 6–9, we have

$$\begin{aligned}V_T(t, x + G(e)x) = \sum_{i=1}^{\infty}&\left\langle \otimes_i x, \left(\otimes_i\left(I_{n_x} + G(e)\right)\right)' P_i(t)\right.\\ &\left.\times\left(\otimes_i\left(I_{n_x} + G(e)\right)\right)\otimes_i x\right\rangle,\end{aligned} \tag{26}$$

$$\frac{\partial V_T}{\partial t} = \sum_{i=1}^{\infty}\left\langle \otimes_i x, \dot{P}_i \otimes_i x\right\rangle, \tag{27}$$

$$\frac{\partial V_T}{\partial x}' Ax = \sum_{i=1}^{\infty} \left\langle \otimes_i x, \left(A^{(i)} P_i\right) \otimes_i x \right\rangle + \sum_{i=1}^{\infty} \left\langle \otimes_i x, \left(P_i A^{(i)'}\right) \otimes_i x \right\rangle, \tag{28}$$

$$\frac{\partial V}{\partial x}' G(e)\, x = \sum_{i=1}^{\infty} \left\langle \otimes_i x, \left(D^{(i)}(e)\, P_i\right) \otimes_i x \right\rangle + \sum_{i=1}^{\infty} \left\langle \otimes_i x, \left(P_i D^{(i)'}\right) \otimes_i x \right\rangle, \tag{29}$$

$$\frac{\partial V_T}{\partial x}' Bxx'B' \frac{\partial V_T}{\partial x} = 4 \sum_{m=2}^{\infty} \left\langle \otimes_m x, \sum_{i+j=m} \left(B^{(i)} P_i\right) \otimes \left(P_j B^{(j)'}\right) \otimes_m x \right\rangle, \tag{30}$$

$$x'C' \frac{\partial^2 V_T}{\partial x^2} Cx = 2 \sum_{i=1}^{\infty} \left\langle \otimes_i x, R^{(i)}(P_i) \otimes_i x \right\rangle. \tag{31}$$

Substituting (26)–(31) and (21) into (8) with terminal conditions $P_i(T) = 0$ ($i = 1, 2, \ldots$), we can prove that $V_T(t, x)$ satisfies HJE (8). By Theorem 2, the H_∞ control for system (6) can be given as

$$u_T^* = -x'B' \frac{\partial V_T}{\partial x}. \tag{32}$$

Similar to (29), we prove that the H_∞ control u_T^* can be represented with the form of (18). □

By Theorem 4, we can obtain the approximation of H_∞ control for system (6).

Now we apply the result of tensor power approach to an example.

Example 10. Consider the system (6) with coefficients

$$A = \begin{bmatrix} 0.04 & 0.02 \\ 0.02 & 0.04 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.02 & 0.02 \\ 0.02 & 0.02 \end{bmatrix},$$
$$K = \begin{bmatrix} 0.02 \\ 0.02 \end{bmatrix}, \qquad C = \begin{bmatrix} -0.02 & 0.04 \\ 0.02 & 0.02 \end{bmatrix}, \tag{33}$$
$$G = \begin{bmatrix} 0.02 & -0.02 \\ 0.02 & 0.02 \end{bmatrix}, \qquad M = \begin{bmatrix} -0.04 & 0.02 \end{bmatrix}.$$

$N(t)$ is Poisson measure with parameter $\lambda = 2$; $W(t)$ is 1-dimensional Brownian motion and $\gamma = 1$. Here the approximation of u_T^* is given by

$$u^* = -2 \sum_{i=1}^{6} \left\langle \otimes_i x, \left(B^{(i)} P_i\right) \otimes_i x \right\rangle, \tag{34}$$

and Figure 1 is the simulation of $m(x) = Mx$ and H_∞ control u^*, where u^* is the approximation of u_T^* of system (6). For the theories of simulation, we will discuss them in another paper. Here we only give the results of simulation.

5. Concluding Remarks

We have discussed the state feedback H_∞ control for a class of bilinear stochastic system where both Brownian motion and Poisson process are present. In order to solve the HJE given in the paper, we also discuss the method of tensor power series approach.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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