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Positive solutions to nonlinear first-order impulsive dynamic equations on time scales

Wen Guan*

*Correspondence:
mathguanw@163.com
Department of Applied Mathematics, Lanzhou University of Technology, Lanzhou, Gansu 730050, People's Republic of China

Abstract

By using the classical fixed point theorem for operators on a cone, in this paper, some results of single and multiple positive solutions to a class of nonlinear first-order periodic boundary value problems of impulsive dynamic equations on time scales are obtained. It is worth noticing that the nonlinearity f and the pulse function in this paper are not positive.

MSC: 39A10; 34B15

Keywords: time scale; periodic boundary value problem; fixed point; impulsive dynamic equation

1 Introduction

The theory of dynamic equations on time scales has been a new important mathematical branch [1–3] since it was initiated by Hilger [4]. At the same time, the boundary value problems of impulsive dynamic equations on time scales have received considerable attention [5–21] since the theory of impulsive differential equations is much richer than the corresponding theory of differential equations without impulse effects [22–24].

In [18], by using the Guo-Krasnoselskii fixed point theorem, when the nonlinearity f and the pulse function are positive, Wang considered the existence of one or two positive solutions to the following PBVPs of impulsive dynamic equations on time scales:

$$\begin{cases} x^{\triangle}(t) + p(t)x(\sigma(t)) = f(t, x(\sigma(t))), & t \in J := [0, T]_{\mathbb{T}}, t \neq t_k, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), & k = 1, 2, \ldots, m, \\ x(0) = x(\sigma(T)). \end{cases} \tag{1.1}$$

In [20], by using the Schaefer fixed point theorem, Wang and Weng obtained the existence of at least one solution to the problem (1.1).

Motivated by the results mentioned above, in this paper, we shall obtain the existence of single and multiplicity positive solutions to the problem (1.1) where $\mathbb{T}$ is an arbitrary time scale, $T > 0$ is fixed, $0, T \in \mathbb{T}$, $f \in C(J \times [0, \infty), \mathbb{R})$, $I_k \in C([0, \infty), \mathbb{R})$, $p : [0, T]_{\mathbb{T}} \to (0, \infty)$ is right-dense continuous, $t_k \in (0, T)_{\mathbb{T}}$, $0 < t_1 < \cdots < t_m < T$, and for each $k = 1, 2, \ldots, m$, $x(t_k^+) = \lim_{h \to 0^+} x(t_k + h)$ and $x(t_k^-) = \lim_{h \to 0^-} x(t_k + h)$ which represent the right and left limits of $x(t)$ at $t = t_k$. The main tool used in this paper is the classical fixed point theorem for operators on a cone.

It is worth noticing that: (i) The nonlinearity f and the pulse function in this paper are not positive, so our hypotheses on the nonlinearity f and the pulse function are weaker than condition of [18]. (ii) For the case $I_k(x) \equiv 0$, $k = 1, 2, \dots, m$, the problem (1.1) reduces to the problem studied by [25, 26]; for the case $p(t) = 0$, the problem (1.1) reduces to the problem studied by [12, 19]; for the case $p(t) = 0$ and $\mathbb{T} = \mathbb{Z}$, the problem (1.1) reduces to the problem studied by [27]. This paper's ideas come from [28].

Throughout this work, we assume the knowledge of time scales and time-scale notation, first introduced by Hilger [4]. For more information on time scales, please see the texts by Bohner and Peterson [2, 3].

In the remainder of this section, we state the following fixed point theorem [29].

Theorem 1.1 [29] *Let X be a Banach space and K is a cone in X. Assume Ω_1, Ω_2 are open subsets of X with $0 \in \Omega_1$, $\overline{\Omega}_1 \subset \Omega_2$. Let*

$$\Phi : K \cap (\overline{\Omega}_2 \setminus \Omega_1) \to K$$

be a continuous and completely continuous operator such that

(i) $\|\Phi x\| \le \|x\|$ *for* $x \in K \cap \partial\Omega_1$;

(ii) *there exists* $e \in K \setminus \{0\}$ *such that* $x \ne \Phi x + \lambda e$ *for* $x \in K \cap \partial\Omega_2$ *and* $\lambda > 0$.

Then Φ has a fixed point in $K \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

Remark 1.1 In Theorem 1.1, if (i) and (ii) are replaced by

(i) $\|\Phi x\| \le \|x\|$ for $x \in K \cap \partial\Omega_2$;

(ii) there exists $e \in K \setminus \{0\}$ such that $x \ne \Phi x + \lambda e$ for $x \in K \cap \partial\Omega_1$ and $\lambda > 0$,

then Φ has also a fixed point in $K \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

2 Main results

Throughout the rest of this paper, we always assume that the points of impulse t_k are right-dense for each $k = 1, 2, \dots, m$.

We define

$$PC = \bigl\{x \in \bigl[0, \sigma(T)\bigr]_{\mathbb{T}} \to \mathbb{R} : x_k \in C(J_k, \mathbb{R}), k = 0, 1, 2, \dots, m, \text{and there exist} \\ x\bigl(t_k^+\bigr) \text{ and } x\bigl(t_k^-\bigr) \text{ with } x\bigl(t_k^-\bigr) = x(t_k), k = 1, 2, \dots, m\bigr\},$$

where x_k is the restriction of x to $J_k = (t_k, t_{k+1}]_{\mathbb{T}} \subset (0, \sigma(T)]_{\mathbb{T}}$, $k = 1, 2, \dots, m$, and $J_0 = [0, t_1]_{\mathbb{T}}$, $t_{m+1} = \sigma(T)$.

Let

$$X = \bigl\{x : x \in PC, x(0) = x\bigl(\sigma(T)\bigr)\bigr\}$$

with the norm $\|x\| = \sup_{t \in [0, \sigma(T)]_{\mathbb{T}}} |x(t)|$, then X is a Banach space.

Lemma 2.1 [18] *Suppose $h : [0, T]_{\mathbb{T}} \to \mathbb{R}$ is rd-continuous, then x is a solution of*

$$x(t) = \int_0^{\sigma(T)} G(t, s)h(s)\triangle s + \sum_{k=1}^{m} G(t, t_k)I_k\bigl(x(t_k)\bigr), \quad t \in \bigl[0, \sigma(T)\bigr]_{\mathbb{T}},$$

where

$$G(t,s)=\begin{cases}\frac{e_p(s,t)e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1}, & 0\le s\le t\le\sigma(T),\\ \frac{e_p(s,t)}{e_p(\sigma(T),0)-1}, & 0\le t<s\le\sigma(T),\end{cases}$$

if and only if x is a solution of the boundary value problem

$$\begin{cases}x^{\triangle}(t)+p(t)x(\sigma(t))=h(t), & t\in J, t\neq t_k,\\ x(t_k^+)-x(t_k^-)=I_k(x(t_k^-)), & k=1,2,\dots,m,\\ x(0)=x(\sigma(T)).\end{cases}$$

Lemma 2.2 *Let $G(t,s)$ be defined as in Lemma* 2.1, *then*

$$A=\frac{1}{e_p(\sigma(T),0)-1}\le G(t,s)\le\frac{e_p(\sigma(T),0)}{e_p(\sigma(T),0)-1}=B\quad\textit{for all } t,s\in\bigl[0,\sigma(T)\bigr]_{\mathbb{T}}.$$

Remark 2.1 Let $G(t,s)$ be defined as in Lemma 2.1, then $\int_0^{\sigma(T)}G(t,s)p(s)\triangle s=1$.

Let

$$K=\bigl\{x\in X:x(t)\ge\delta\|x\|,t\in\bigl[0,\sigma(T)\bigr]_{\mathbb{T}}\bigr\},$$

where $\delta=\frac{A}{B}\in(0,1)$. It is not difficult to verify that K is a cone in X.

For convenience, we denote

$$\begin{aligned}
f^0&=\lim_{x\to0^+}\sup\max_{t\in[0,T]_{\mathbb{T}}}\frac{f(t,x)}{p(t)x}, &\qquad I^0(k)&=\lim_{x\to0^+}\sup\frac{I_k(x)}{x};\\
f_0&=\lim_{x\to0^+}\inf\min_{t\in[0,T]_{\mathbb{T}}}\frac{f(t,x)}{p(t)x}, & I_0(k)&=\lim_{x\to0^+}\inf\frac{I_k(x)}{x};\\
f^\infty&=\lim_{x\to\infty}\sup\max_{t\in[0,T]_{\mathbb{T}}}\frac{f(t,x)}{p(t)x}, & I^\infty(k)&=\lim_{x\to\infty}\sup\frac{I_k(x)}{x};\\
f_\infty&=\lim_{x\to\infty}\inf\min_{t\in[0,T]_{\mathbb{T}}}\frac{f(t,x)}{p(t)x}, & I_\infty(k)&=\lim_{x\to\infty}\inf\frac{I_k(x)}{x};\\
f^u&=\sup_{x\in[\delta u,u]}\max_{t\in[0,T]_{\mathbb{T}}}\frac{f(t,x)}{p(t)x}, & I^u(k)&=\sup_{x\in[\delta u,u]}\frac{I_k(x)}{x};\\
f_u&=\inf_{x\in[\delta u,u]}\min_{t\in[0,T]_{\mathbb{T}}}\frac{f(t,x)}{p(t)x}, & I_u(k)&=\inf_{x\in[\delta u,u]}\frac{I_k(x)}{x}.
\end{aligned}$$

Now we state our main results.

Theorem 2.1 *Suppose there exist* $0<\alpha<\beta$ *such that*

$$f(t,x)\ge0\quad\textit{and}\quad I_k(x)\ge0,\quad t\in[0,T]_{\mathbb{T}},x\in[\delta\alpha,\beta].$$

Then the problem (1.1) *has at least one positive solution if one of the following two conditions holds*:

(i) $f_\alpha+A\sum_{k=1}^m I_\alpha(k)\ge1, f^\beta+B\sum_{k=1}^m I^\beta(k)\le1$;

(ii) $f^\alpha+B\sum_{k=1}^m I^\alpha(k)\le1, f_\beta+A\sum_{k=1}^m I_\beta(k)\ge1$.

Proof We define an operator $\Phi : K \to X$ by

$$\Phi(x)(t) = \int_0^{\sigma(T)} G(t,s)f\big(s,x\big(\sigma(s)\big)\big)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k\big(u(t_k)\big), \quad t \in \big[0,\sigma(T)\big]_{\mathbb{T}}.$$

It is obvious that fixed points of Φ are solutions of the problem (1.1). Similar to [18], $\Phi : K \to X$ is completely continuous.

Define the open sets

$$\Omega_1 = \big\{x \in X : \|x\| < \alpha\big\},$$
$$\Omega_2 = \big\{x \in X : \|x\| < \beta\big\}.$$

Firstly, we claim that $\Phi : K \cap (\overline{\Omega}_2 \setminus \Omega_1) \to K$.

In fact, for any $x \in K \cap (\overline{\Omega}_2 \setminus \Omega_1)$, we have $\delta\alpha \le x \le \beta$, by Lemma 2.2

$$\|\Phi x\| \le B\left[\int_0^{\sigma(T)} f\big(s,x\big(\sigma(s)\big)\big)\triangle s + \sum_{k=1}^{m} I_k\big(x(t_k)\big)\right]$$

and

$$\begin{aligned}(\Phi x)(t) &= \int_0^{\sigma(T)} G(t,s)h_x(s)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k\big(x(t_k)\big)\\ &\ge A\left[\int_0^{\sigma(T)} f\big(s,x\big(\sigma(s)\big)\big)\triangle s + \sum_{k=1}^{m} I_k\big(x(t_k)\big)\right].\end{aligned}$$

So

$$(\Phi x)(t) \ge \frac{A}{B}\|\Phi x\| = \delta\|\Phi x\|, \quad \textit{i.e.}, \Phi x \in K.$$

Therefore, $\Phi : K \cap (\overline{\Omega}_2 \setminus \Omega_1) \to K$.

Secondly, we prove the result provided condition (i) holds.

Since

$$f(t,x) \ge f_\alpha p(t)x, \qquad I_k(x) \ge I_\alpha(k)x, \quad k = 1,2,\ldots,m, x \in [\delta\alpha,\alpha].$$

Let $e \equiv 1$, then $e \in K$. We assert that

$$x \ne \Phi x + \lambda e \quad \text{for } x \in K \cap \partial\Omega_1 \text{ and } \lambda > 0. \tag{2.1}$$

If not, there would exist $x_0 \in K \cap \partial\Omega_1$ and $\lambda_0 > 0$ such that $x_0 = \Phi x_0 + \lambda_0 e$.

Since $x_0 \in K \cap \partial\Omega_1$, then $\delta\alpha = \delta\|x_0\| \le x_0(t) \le \alpha$. Let $\mu = \min_{t\in[0,\sigma(T)]_{\mathbb{T}}} x_0(t)$, then for any $t \in [0,\sigma(T)]_{\mathbb{T}}$, by the first inequality of (i) we have

$$\begin{aligned}x_0(t) &= (\Phi x_0)(t) + \lambda_0\\ &= \int_0^{\sigma(T)} G(t,s)f\big(s,x_0\big(\sigma(s)\big)\big)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k\big(x_0(t_k)\big) + \lambda_0\end{aligned}$$

$$
\begin{aligned}
&\geq \int_0^{\sigma(T)} G(t,s) f_\alpha p(s) x_0\big(\sigma(s)\big)\triangle s + A\sum_{k=1}^m I_\alpha(k)x(t_k) + \lambda_0 \\
&\geq \mu\left[f_\alpha + A\sum_{k=1}^m I_\alpha(k)\right] + \lambda_0 \\
&\geq \mu + \lambda_0.
\end{aligned}
$$

This implies that $\mu \geq \mu + \lambda_0$, and this is a contradiction. Therefore (2.1) holds.

On the other hand, by using the second inequality of (i), we have

$$
f(t,x) \leq \left[1 - B\sum_{k=1}^m I^\beta(k)\right] p(t)x, \quad t \in [0,T]_{\mathbb{T}}, x \in [\delta\beta, \beta].
$$

We assert that

$$
\|\Phi x\| \leq \|x\| \quad \text{for } x \in K \cap \partial\Omega_2. \tag{2.2}
$$

In fact, for any $x \in K \cap \partial\Omega_2$, then $\delta\beta = \delta\|x\| \leq x(t) \leq \beta$, we have

$$
\begin{aligned}
(\Phi x)(t) &= \int_0^{\sigma(T)} G(t,s) f\big(s, x\big(\sigma(s)\big)\big)\triangle s + \sum_{k=1}^m G(t,t_k) I_k\big(x(t_k)\big) \\
&\leq \left(1 - B\sum_{k=1}^m I^\beta(k)\right)\int_0^{\sigma(T)} G(t,s)p(s)x\big(\sigma(s)\big)\triangle s + B\sum_{k=1}^m I^\beta(k)x(t_k) \\
&\leq \left[\left(1 - B\sum_{k=1}^m I^\beta(k)\right)\int_0^{\sigma(T)} G(t,s)p(s)\triangle s + B\sum_{k=1}^m I^\beta(k)\right]\|x\| \\
&= \|x\|.
\end{aligned}
$$

Therefore, $\|\Phi x\| \leq \|x\|$.

It follows from Remark 1.1, (2.1), and (2.2) that Φ has a fixed point $x \in K \cap (\overline{\Omega}_2 \setminus \Omega_1)$.

In a similar way, we can prove the result by Theorem 1.1 if condition (ii) holds. □

Theorem 2.2 *Suppose there exist* $0 < \alpha < \rho < \beta$ *such that*

$$
f(t,x) \geq 0 \quad \textit{and} \quad I_k(x) \geq 0, \quad t \in [0,T]_{\mathbb{T}}, x \in [\delta\alpha, \beta].
$$

Then the problem (1.1) *has at least two positive solution if one of the following two conditions holds*:

(i) $f_\alpha + A\sum_{k=1}^m I_\alpha(k) \geq 1, f^\rho + B\sum_{k=1}^m I^\rho(k) < 1, f_\beta + A\sum_{k=1}^m I_\beta(k) \geq 1$;

(ii) $f^\alpha + B\sum_{k=1}^m I^\alpha(k) \leq 1, f_\rho + A\sum_{k=1}^m I_\rho(k) > 1, f^\beta + B\sum_{k=1}^m I^\beta(k) \leq 1$.

Proof We only prove the result when condition (i) holds. In a similar way we can obtain the result if condition (ii) holds.

Define Ω_1, Ω_2 as in Theorem 2.1 and define

$$
\Omega_3 = \big\{x \in X : \|x\| < \rho\big\}.
$$

Similar to the proof of Theorem 2.1, we can prove that

$$x \neq \Phi x + \lambda e \quad \text{for } x \in K \cap \partial\Omega_1 \text{ and } \lambda > 0, \tag{2.3}$$

$$x \neq \Phi x + \lambda e \quad \text{for } x \in K \cap \partial\Omega_2 \text{ and } \lambda > 0, \tag{2.4}$$

where $e \equiv 1 \in K$, and

$$\|\Phi x\| < \|x\| \quad \text{for } x \in K \cap \partial\Omega_3. \tag{2.5}$$

Thus we can obtain the existence of two positive solutions x_1 and x_2 by using Theorem 1.1 and Remark 1.1, respectively. It is easy to see that $\alpha \leq \|x_1\| < \rho < \|x_2\| \leq \beta$. □

Theorem 2.3 *Suppose that there exist positive* $0 < \alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \cdots < \alpha_n < \beta_n$ *such that*

$$f(t,x) \geq 0 \quad \textit{and} \quad I_k(x) \geq 0, \quad t \in [0,T]_{\mathbb{T}}, x \in [\delta\alpha_1, \beta_n].$$

Then the problem (1.1) *has at least n positive solutions* x_i $(1 \leq i \leq n)$ *satisfying* $\alpha_i \leq \|x_i\| \leq \beta_i$, $1 \leq i \leq n$, *if one of the following two conditions holds*:
(i) $f_{\alpha_i} + A\sum_{k=1}^m I_{\alpha_i}(k) \geq 1, f^{\beta_i} + B\sum_{k=1}^m I^{\beta_i}(k) \leq 1$;
(ii) $f^{\alpha_i} + B\sum_{k=1}^m I^{\alpha_i}(k) \leq 1, f_{\beta_i} + A\sum_{k=1}^m I_{\beta_i}(k) \geq 1$.

Remark 2.2 In Theorem 2.3, assume (i) and (ii) are replaced by
(i) $f_{\alpha_i} + A\sum_{k=1}^m I_{\alpha_i}(k) > 1, f^{\beta_i} + B\sum_{k=1}^m I^{\beta_i}(k) < 1$;
(ii) $f^{\alpha_i} + B\sum_{k=1}^m I^{\alpha_i}(k) < 1, f_{\beta_i} + A\sum_{k=1}^m I_{\beta_i}(k) > 1$.
Then the problem (1.1) has at least $2n - 1$ positive solutions.

3 Application

In this section, we are going to apply our main existence results obtained in Section 2 to some illustrating examples.

Theorem 3.1 *Suppose that* $f(t,x) \geq 0$ *and* $I_k(x) \geq 0$, $t \in [0,T]_{\mathbb{T}}$, $x \in \mathbb{R}^+ = [0,\infty)$. *Then the problem* (1.1) *has at least one positive solution if one of the following two conditions holds*:
(i) $f_0 + A\sum_{k=1}^m I_0(k) > 1, f^\infty + B\sum_{k=1}^m I^\infty(k) < 1$;
(ii) $f^0 + B\sum_{k=1}^m I^0(k) < 1, f_\infty + A\sum_{k=1}^m I_\infty(k) > 1$.

Proof It is a direct consequence of Theorem 2.1 taking α small enough and β large enough, respectively. □

In particular, we have the following results, the main results of [18].

Corollary 3.1 *Suppose that* $f(t,x) \geq 0$ *and* $I_k(x) \geq 0$, $t \in [0,T]_{\mathbb{T}}$, $x \in \mathbb{R}^+$. *Then the problem* (1.1) *has at least one positive solution if one of the following two conditions holds*:
(i) $f_0 = \infty$ *or* $\sum_{k=1}^m I_0(k) = \infty, f^\infty = 0, \sum_{k=1}^m I^\infty(k) = 0$;
(ii) $f_\infty = \infty$ *or* $I_\infty(k) = \infty, f^0 = 0, \sum_{k=1}^m I^0(k) = 0$.

Example 3.1 We consider the following problem on $\mathbb{T}$:

$$\begin{cases} x^{\triangle}(t) + p(t)x(\sigma(t)) = (x(\sigma(t)))^a + (x(\sigma(t)))^b, & t \in J, t \neq t_k, \\ x(t_k^+) - x(t_k^-) = (x(t_k^-))^{a'} + (x(t_k^-))^{b'}, & k = 1, 2, \ldots, m, \\ x(0) = x(\sigma(T)), \end{cases} \tag{3.1}$$

where $p : [0, T]_{\mathbb{T}} \to (0, \infty)$ is right-dense continuous, $0 < a, b < 1$, $0 < a', b' < 1$ or $a, b > 1$, $a', b' > 1$.

Then it is easy to see that

$$f_0 = \infty, \qquad \sum_{k=1}^{m} I_0(k) = \infty, \qquad f^{\infty} = 0, \qquad \sum_{k=1}^{m} I^{\infty}(k) = 0$$

for $0 < a, b < 1, 0 < a', b' < 1$,

$$f_{\infty} = \infty, \qquad I_{\infty}(k) = \infty, \qquad f^0 = 0, \qquad \sum_{k=1}^{m} I^0(k) = 0$$

for $a, b > 1, a', b' > 1$.

Therefore, by Corollary 3.1, it follows that the problem (3.1) has at least one positive solution.

Theorem 3.2 *Suppose that* $f(t,x) \geq 0$ *and* $I_k(x) \geq 0$, $t \in [0, T]_{\mathbb{T}}$, $x \in \mathbb{R}^+$. *Then the problem* (1.1) *has at least two positive solutions if one of the following two conditions holds*:

(i) $f_0 + A \sum_{k=1}^{m} I_0(k) > 1, f_{\infty} + A \sum_{k=1}^{m} I_{\infty}(k) > 1$ *and there exists a* $\rho > 0$ *such that* $\delta\rho \leq x \leq \rho$ *implies*

$$f(t,x)/p(t)x + B \sum_{k=1}^{m} I^{\rho}(k) < 1, \quad t \in [0, T]_{\mathbb{T}};$$

(ii) $f^0 + B \sum_{k=1}^{m} I^0(k) < 1, f^{\infty} + B \sum_{k=1}^{m} I^{\infty}(k) < 1$ *and there exists a* $\rho > 0$ *such that* $\delta\rho \leq x \leq \rho$ *implies*

$$f(t,x)/p(t)x + A \sum_{k=1}^{m} I_{\rho}(k) > 1, \quad t \in [0, T]_{\mathbb{T}}.$$

Corollary 3.2 *Suppose that* $f(t,x) \geq 0$ *and* $I_k(x) \geq 0$, $t \in [0, T]_{\mathbb{T}}$, $x \in \mathbb{R}^+$. *Then the problem* (1.1) *has at least two positive solutions if one of the following two conditions holds*:

(i) $f_0 = \infty$ *or* $\sum_{k=1}^{m} I_0(k) = \infty, f_{\infty} = \infty$ *or* $I_{\infty}(k) = \infty$, *and there exists a* $\rho > 0$ *such that* $\delta\rho \leq x \leq \rho$ *implies*

$$f(t,x)/p(t)x + B \sum_{k=1}^{m} I^{\rho}(k) < 1, \quad t \in [0, T]_{\mathbb{T}};$$

(ii) $f^{\infty} = 0$, $\sum_{k=1}^{m} I^{\infty}(k) = 0, f^0 = 0$, $\sum_{k=1}^{m} I^0(k) = 0$, *and there exists a* $\rho > 0$ *such that* $\delta\rho \leq x \leq \rho$ *implies*

$$f(t,x)/p(t)x + A \sum_{k=1}^{m} I_{\rho}(k) > 1, \quad t \in [0, T]_{\mathbb{T}}.$$

Example 3.2 We consider the following problem on $\mathbb{T}$:

$$\begin{cases} x^{\triangle}(t) + p(t)x(\sigma(t)) = q(t)[(x(\sigma(t)))^a + (x(\sigma(t)))^b], & t \in J, t \neq t_k, \\ x(t_k^+) - x(t_k^-) = a_k x(t_k^-), & k = 1, 2, \ldots, m, \\ x(0) = x(\sigma(T)), \end{cases} \tag{3.2}$$

where $0 < a < 1 < b$, $p : [0, T]_{\mathbb{T}} \to (0, \infty)$ is right-dense continuous and $q : [0, T]_{\mathbb{T}} \to (0, \infty)$ is continuous such that

$$\max_{t \in [0,T]_{\mathbb{T}}} \frac{q(t)}{p(t)} < \delta\bigl(1 - mBa^+\bigr) \sup_{x \in (0,\infty)} \frac{x}{x^a + x^b}$$

holds, where $a^+ = \max\{a_1, a_2, \ldots, a_m\}$.

In fact, $f(t, x) = q(t)(x^a + x^b)$, it is easy to see that

$$f_0 = \lim_{x \to 0^+} \inf \frac{f(t,x)}{x} = \infty, \qquad f_\infty = \lim_{x \to \infty} \inf \frac{f(t,x)}{x} = \infty, \quad \text{uniformly for } t \in [0, T]_{\mathbb{T}}.$$

Set

$$F(x) = \frac{x}{x^a + x^b}, \quad x > 0,$$

then $F(0+) = F(\infty) = 0$, so there exists a $\rho > 0$ such that

$$F(\rho) = \sup_{x \in (0,\infty)} F(x).$$

So, for $\delta\rho \le x \le \rho$, we have

$$\begin{aligned} f(t,x)/p(t)x + B\sum_{k=1}^{m} I^{\rho}(k) &= \frac{q(t)}{p(t)}\left(\frac{x^a + x^b}{x}\right) + mBa^+ \\ &\le \max_{t \in [0,T]_{\mathbb{T}}} \frac{q(t)}{p(t)}\left(\frac{\rho^a + \rho^b}{\delta\rho}\right) + mBa^+ < 1. \end{aligned}$$

Therefore, together with Corollary 3.2, it follows that the problem (3.2) has at least two positive solutions.

Competing interests

The author declares that she has no competing interests.

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