Hindawi
Advances in Mathematical Physics
Volume 2018, Article ID 3545083, 9 pages
https://doi.org/10.1155/2018/3545083

Research Article

Fractional-Order Sliding Mode Synchronization for Fractional-Order Chaotic Systems

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Received 5 October 2017; Revised 19 December 2017; Accepted 20 December 2017; Published 17 January 2018

Academic Editor: Christos Volos

Some sufficient conditions, which are valid for stability check of fractional-order nonlinear systems, are given in this paper. Based on these results, the synchronization of two fractional-order chaotic systems is investigated. A novel fractional-order sliding surface, which is composed of a synchronization error and its fractional-order integral, is introduced. The asymptotical stability of the synchronization error dynamical system can be guaranteed by the proposed fractional-order sliding mode controller. Finally, two numerical examples are given to show the feasibility of the proposed methods.

1. Introduction

In the past two decades, synchronization of chaotic systems (CSs) has received more and more attention, and a lot of interesting works have been done, which have potential application values in secret communications, signal processing, and complex systems [1–9]. Recently, control and synchronization of fractional-order chaotic systems (FOCSs), which can be seen as a generalization of the integer-order CSs, have been studied extensively. A lot of controllers have been implemented such as active control [10], feedback control [11], sliding mode control [12, 13], adaptive control, [14, 15], and adaptive fuzzy control [8, 9, 16].

It is well known that sliding mode control (SMC) is a very effective control method to cope with system uncertainties and external disturbances [17–27]. Consequently, it has been used to synchronize FOCSs. For example, a novel FOCS and its SMC have been studied in [28]; SMC of a 3D FOCS using a fractional-order switching type controller is investigated in [29]. Using a hierarchical fuzzy neural network, [30] proposed a new adaptive SMC method for the synchronization of uncertain FOCSs. On the other hand, it is well known that, in stability analysis of nonlinear systems, quadratic Lyapunov functions are most commonly used. However, [31, 32] show that it is not realistic to use quadratic Lyapunov functions in the stability analysis of fractional-order nonlinear systems due to the complicated infinite series produced by differentiating the squared Lyapunov function with fractional order. It should be mentioned that, in most aforementioned works, the stability analysis is given based on fractional Lyapunov methods. How to establish some stability analysis methods according to the model of FOCSs is a meaningful work.

In control theory, stability analysis is an essential aspect. With respect to fractional-order linear systems, the stability condition was firstly investigated in [33]. Then, using LMI, some sufficient conditions are given in [34]. The related results on the stability analysis of fractional-order nonlinear systems can be seen in [35–41] and the references therein. It should be pointed out that the stability criterion for fractional-order nonlinear systems requires further study. Thus, proposing some new stability criterion for FOCSs is necessary. In this paper, we will give two sufficient conditions for the stability of a class of FOCSs. Based on these theorems, a fractional-order SMC will be given. The contributions of this paper are concluded as follows: (1) two sufficient conditions are proposed to check the stability of the fractional-order nonlinear system and (2) a novel fractional-order SMC is given, and the stability of the closed-loop system is proven rigorously.

2. Preliminaries

In this section, we will give some properties of fractional calculus. The qth fractional-order integral is expressed as [42]

$$\mathbb{D}^q f(t) = \frac{1}{\Gamma(q)} \int_0^t (t-\tau)^{q-1} f(\tau)\, d\tau. \quad (1)$$

The Caputo fractional derivative is given by

$$\mathbb{D}^q f(t) = \frac{1}{\Gamma(n-q)} \int_0^t (t-\tau)^{n-q-1} f^{(n)}(\tau)\, d\tau, \quad (2)$$

where q is the fractional order satisfying $n-1 \le q < n$.

The Laplace transform of Caputo fractional derivative is given as [42]

$$\int_0^\infty e^{-st} \mathbb{D}^q x(t)\, dt = s^q F(s) - \sum_{k=0}^{n-1} s^{q-k-1} x^{(k)}(0), \quad (3)$$

where $F(s) = \mathscr{L}\{f(t)\}$. In the next section, we will use the following results.

The Mittag-Leffler function is given by

$$E_{\beta_1,\beta_2}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\beta_1 k + \beta_2)}, \quad (4)$$

where $\beta_1, \beta_2 > 0$ and $z \in C$. The Laplace transform of (4) is

$$\mathscr{L}\left\{t^{\beta_2-1} E_{\beta_1,\beta_2}\left(-at^{\beta_1}\right)\right\} = \frac{s^{\beta_1-\beta_2}}{s^{\beta_1}+a}. \quad (5)$$

Lemma 1 (see [42]). *Let $A \in R^{n\times n}$, $0 < \alpha \le 1$, β be an arbitrary real number, and $b > 0$ be a real constant; then,*

$$E_{\alpha,\beta}(A) \le \frac{b}{1+\|A\|}, \quad (6)$$

where $\mu \le |\arg(\mathrm{eig}(A))| \le \pi$ with $\mu \in R$ satisfying $\pi\alpha/2 < \mu <$ $\min\{\pi, \pi\alpha\}$.

Lemma 2 (see [43]). *Let $t \in [0, T]$ and*

$$x(t) \le h(t) + \int_0^t k(\tau)\, x(\tau)\, d\tau, \quad (7)$$

where $k(t) \ge 0$. Then, one has

$$x(t) \le h(t) + \int_0^t k(\tau)\, h(\tau) \exp\left[\left[\int_\tau^t k(u)\, du\right] d\tau.\right. \quad (8)$$

Lemma 3 (see [42, 44]). *Let $0 < \alpha < 2$. β is a complex number, and μ is a real number. If*

$$\frac{\pi\alpha}{2} < \mu < \min\{\pi, \pi\alpha\}, \quad (9)$$

then, for an arbitrary integer $n \ge 1$, the following expansion holds:

$$E_{\alpha,\beta}(z) = -\sum_{j=1}^{n} \frac{1}{\Gamma(\beta-\alpha j)\, z^j} + \circ\left(\frac{1}{|z|^{n+1}}\right). \quad (10)$$

3. Main Results

3.1. Some Sufficient Conditions for the Stability Analysis of Fractional-Order Systems. Consider a class of fractional-order systems described by

$$\mathbb{D}^q x_j(t) = c_j x_j(t) + \sum_{i=1}^{n} a_{ij} f_i(x_i(t)), \quad (11)$$

or equivalently

$$\mathbb{D}^q x(t) = Cx(t) + Af(x(t)), \quad (12)$$

where $j = 1, 2, \ldots, n$, $0 < q < 1$, and $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector; $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T \in \mathbb{R}^n$ represents a smooth nonlinear function, $A = \{a_{ij}\}$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, n$, and $C = \mathrm{diag}(c_i)$ are two matrices. Then, we have the following results.

Theorem 4. *If $c_i < 0$ and the nonlinear function is bounded, that is, there exists a constant $m_i > 0$ such that*

$$|f_i(x_i(t))| \le m_i, \quad (13)$$

then there exist two positive constants t_0 and M such that

$$|x_i(t)| \le M \quad (14)$$

for all $t > t_0$.

Proof. It follows from (11) that

$$X_i(s) = \frac{s^{q-1}}{s^q - c_i} x_i(0) + \frac{1}{s^q - c_i} \sum_{j=1}^{n} a_{ij} \mathscr{L}\left(f_j\left(x_j(t)\right)\right). \quad (15)$$

Using (5), one solves (15) as

$$\begin{aligned} x_i(t) &= x_i(0)\, E_{q,1}(c_i t^q) + \sum_{j=1}^{n} a_{ij} \\ &\cdot \int_0^t (t-\tau)^{q-1} E_{q,q}\left(c_i (t-\tau)^q\right) f_j\left(x_j(\tau)\right) d\tau. \end{aligned} \quad (16)$$

Thus, according to (13), one has

$$\begin{aligned} &|x_i(t)| \\ &\le |x_i(0)|\, E_{q,1}(c_i t^q) \\ &+ \sum_{j=1}^{n} |a_{ij}|\, m_j \int_0^t (t-\tau)^{q-1} E_{q,q}\left(c_i (t-\tau)^q\right) d\tau. \end{aligned} \quad (17)$$

Noting that the Laplace transform of a Mittag-Leffler function is

$$\int_0^t \tau^{\beta-1} E_{\beta_1,\beta_2}\left(-k\tau^{\beta_1}\right) d\tau = t^{\beta_2} E_{\beta_1,\beta_2+1}\left(-kt^{\beta_1}\right), \quad (18)$$

then one has

$$\left|x_i(t)\right| \le \left|x_i(0)\right| E_{q,1}\left(c_i t^q\right) + A_i t^q E_{q,q+1}\left(c_i t^q\right), \tag{19}$$

where $A_i = \sum_{j=1}^{n} |a_{ij}| m_j$ is a positive constant.

It follows from Lemma 3 that

$$A_i t^q E_{q,q+1}\left(-c_i t^q\right) \le \frac{A_i}{-c_i}. \tag{20}$$

Consequently, for large enough time t, one has

$$\lim_{t\to\infty} \left|x_i(t)\right| \le M, \tag{21}$$

where $M = \max_{1\le i\le n}\{A_i/c_i\}$. This ends the proof of Theorem 4. □

It should be pointed out that Theorem 4 can only drive $x_i(t)$ to a small region of zero. To discuss the asymptotic stability, one needs the following assumptions.

Assumption 5. The equilibrium point of system (11) is the origin.

Assumption 6. $f(x(t))$ is a Lipshitz continuous function; that is, the following inequality holds:

$$\left\| f(x(t)) - f(y(t)) \right\| \le l \left\| x(t) - y(t) \right\|, \tag{22}$$

where $l > 0$ is a Lipshitz constant.

Remark 7. It should be mentioned that Assumptions 5 and 6 are reasonable. In fact, every equilibrium point of system (11) can be moved to the origin by some linear transformations. In many FOCSs, the nonlinear functions are smooth and Lipshitz continuous, for example, fractional-order Lorenz system, fractional-order Chen system, fractional-order Lü system, fractional-order financial system, and fractional Volta system [45].

Theorem 8. *Consider system (12). Under Assumption 6, if $(lb/c)\|A\| < q$, where $c = \max_{1\le i\le n} - c_i$, then the asymptotical stability of system (12) can be guaranteed.*

Proof. Suppose that $x(t), y(t) \in R^n$ are two arbitrary solutions of (12). Denote $e(t) = x(t) - y(t)$; then, one has

$$\mathbb{D}^q e(t) = Ce(t) + A\left(f(x(t)) - f(y(t))\right). \tag{23}$$

It follows from (23) that

$$\begin{aligned} s^q E(s) = {} & s^{q-1} e(0) + CE(s) \\ & + A\mathscr{L}\left\{f(x(t)) - f(y(t))\right\}, \end{aligned} \tag{24}$$

where $E(s) = \mathscr{L}\{e(t)\}$.

After some straightforward manipulators, one has

$$\begin{aligned} E(s) = {} & \left(Is^q - C\right)^{-1} \\ & \cdot \left(s^{q-1} e(0) + A\mathscr{L}\left\{f(x(t)) - f(y(t))\right\}\right). \end{aligned} \tag{25}$$

Solving (25) yields

$$\begin{aligned} e(t) = {} & E_{q,1}\left(Ct^q\right) e(0) + A\int_0^t (t-\tau)^{q-1} \\ & \cdot E_{q,q}\left(C(t-\tau)^q\right) \mathscr{L}\left\{f(x(t)) - f(y(t))\right\} d\tau. \end{aligned} \tag{26}$$

According to Assumption 6 and Lemma 1, one can find a constant $b > 0$ such that

$$\begin{aligned} \|e(t)\| \le {} & \frac{b\|e(0)\|}{1+\|C\| t^q} \\ & + lb\|A\| \int_0^t \frac{(t-\tau)^{q-1}}{1+\|C\|(t-\tau)^q} \|e(\tau)\| d\tau. \end{aligned} \tag{27}$$

Using Lemma 2, one has

$$\begin{aligned} \|x(t)\| & \le \frac{b\|e(0)\|}{1+\|C\|t^q} + \int_0^t \frac{lb\|A\|(t-\tau)^{\alpha-1}\|e(0)\|}{\left(1+\|C\|(t-\tau)^q\right)\left(1+\|C\|\tau^q\right)} \exp\left(\int_\tau^t \frac{lb\|A\|(t-u)^{q-1}}{1+\|C\|(t-u)^q} du\right) d\tau \\ & = \frac{b\|e(0)\|}{1+\|C\|t^q} + \int_0^t \frac{bl\|A\|(t-\tau)^{q-1}\|e(0)\|}{\left(1+\|C\|\tau^q\right)\left(1+\|C\|(t-\tau)^q\right)^{1-b/q\|C\|}} d\tau \\ & \le \frac{b\|e(0)\|}{1+\|C\|t^q} + lb\|A\|\|e(0)\|\|C\|^{lb\|A\|/q\|C\|-2} \int_0^t (t-\tau)^{lb\|A\|/\|C\|-1} \tau^{-q} d\tau \\ & = \frac{b\|e(0)\|}{1+\|C\|t^q} + lb\|A\|\|e(0)\|\|C\|^{lb\|A\|/q\|C\|-2} \frac{\Gamma\left(lb\|A\|/\|C\|\right)\Gamma(1-q)}{\Gamma\left(1+lb\|A\|/\|C\|-q\right)} t^{lb\|A\|/\|C\|-q}. \end{aligned} \tag{28}$$

Noting that $(lb/c)\|A\| < q$, where $c = \|C\| = \max_{1\le i\le n} - c_i$, then according to (28) one has

$$\lim_{t\to\infty} \|e(t)\| = 0, \tag{29}$$

which completes the proof. □

3.2. Synchronization Controller Design. The master and slave FOCSs are defined, respectively, as

$$\mathbb{D}^q \zeta(t) = P\zeta(t) + Q\hbar(\zeta(t)), \tag{30}$$

$$\mathbb{D}^q \hat{\zeta}(t) = P\hat{\zeta}(t) + Q\hbar\left(\hat{\zeta}(t)\right) + Gu(t), \tag{31}$$

where $\zeta(t), \hat{\zeta}(t) \in \mathbb{R}^n$ are the state vectors of the master FOCS and slave FOCS, respectively, $P, Q, G \in \mathbb{R}^{n\times n}$ are three constant matrices, G is a positive definite control gain matrix, and $u(t) \in \mathbb{R}^n$ represents the control input.

Define the synchronization error $\varepsilon(t) = \zeta(t) - \hat{\zeta}(t)$. The objective of this section is to design a proper control input $u(t)$ such that $\varepsilon(t)$ converges to zero eventually. To proceed, let us give the following assumption first.

Assumption 9. $\hbar$ is a Lipshitz continuous function; that is, the following inequality holds:

$$\left\|\hbar(\zeta(t)) - \hbar\left(\hat{\zeta}(t)\right)\right\| \le a_0 \left\|\zeta(t) - \hat{\zeta}(t)\right\|, \qquad (32)$$

where $a_0 > 0$ is a constant.

To meet the synchronization object, let us construct the following fractional-order sliding mode surface:

$$\begin{aligned} s(t) &= \Lambda\varepsilon(t) \\ &\quad - \frac{1}{\Gamma(q)}\Lambda(P+K)\int_0^t (t-\tau)^{(q-1)}\varepsilon(\tau)\,d\tau, \end{aligned} \qquad (33)$$

where $\Lambda, K \in \mathbb{R}^{n\times n}$ are two design matrices. Then, it follows from (30), (31), and (33) that

$$\begin{aligned} \mathbb{D}^q s(t) &= \Lambda P\varepsilon(t) + \Lambda Q\left(\hbar(\zeta(t)) - \hbar\left(\hat{\zeta}(t)\right)\right) - Gu(t) \\ &\quad - \Lambda(P+K)\varepsilon(t) \\ &= \Lambda Q\left(\hbar(\zeta(t)) - \hbar\left(\hat{\zeta}(t)\right)\right) - Gu(t) \\ &\quad - \Lambda K\varepsilon(t). \end{aligned} \qquad (34)$$

Consequently, let $\mathbb{D}^q s(t) = 0$; the control input can be given as

$$u(t) = G^{-1}\Lambda Q\left(\hbar(\zeta(t)) - \hbar\left(\hat{\zeta}(t)\right)\right) - G^{-1}\Lambda K\varepsilon(t). \qquad (35)$$

Now, we can give the following results.

Theorem 10. *Consider the master FOCS (30) and the slave FOCS (31) under Assumption 9. Suppose that the sliding surface is given by (33) and the control input is designed as (35). If the design matrices satisfy $P - \Lambda K < 0$ and $a_0\|Q - \Lambda Q\| \le \underline{\lambda} q$, where $\underline{\lambda}$ is the smallest eigenvalue of $\Lambda K - P$, then one can conclude that the synchronization error converges to the origin asymptotically.*

Proof. It follows from (30) and (31) that

$$\mathbb{D}^q\varepsilon(t) = P\varepsilon(t) + Q\hbar\left(\hbar(\zeta(t)) - \hbar\left(\hat{\zeta}(t)\right)\right) - Gu(t). \qquad (36)$$

Substituting (35) into (36) yields

$$\begin{aligned} \mathbb{D}^q\varepsilon(t) &= (P - \Lambda K)\varepsilon(t) \\ &\quad + (Q - \Lambda Q)\hbar\left(\hbar(\zeta(t)) - \hbar\left(\hat{\zeta}(t)\right)\right). \end{aligned} \qquad (37)$$

Noting that $P - \Lambda K < 0$ and $a_0\|Q - \Lambda Q\| \le \underline{\lambda} q$, it follows from (37), Assumption 9, and Theorem 8 that $\lim_{t\to\infty}\varepsilon(t) = 0$. This completes the proof of Theorem 10. □

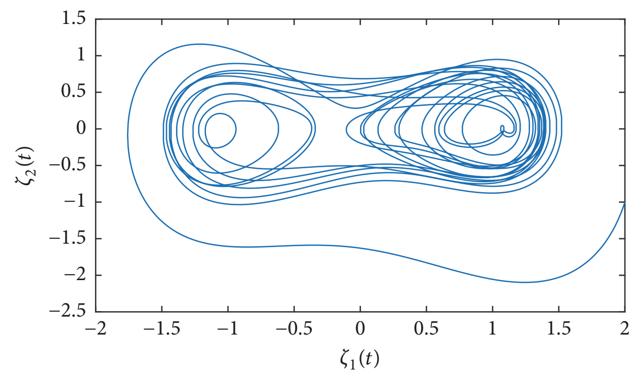

Figure 1: Phase attractor of FOCS (38).

4. Simulation Results

In this section, two examples will be given to show the effectiveness of the proposed method.

4.1. Synchronizing Two 2D Fractional-Order Duffing Systems. The fractional-order Duffing system is described by [46]

$$\begin{aligned} \mathbb{D}^q\zeta_1(t) &= \zeta_2(t), \\ \mathbb{D}^q\zeta_2(t) &= \zeta_1(t) - \zeta_1^3(t) - 0.15\zeta_2(t) + 0.3\cos(t). \end{aligned} \qquad (38)$$

The Jacobian matrix of system (38) for the equilibrium point $E_1^* = (\zeta_1^*, \zeta_2^*)$ is

$$J_{E^*} = \begin{bmatrix} 0 & 1 \\ 1 - 3\zeta_1^* & -0.15 \end{bmatrix}. \qquad (39)$$

It is easy to know that system (38) has three equilibria: $E_1 = (1.0729, 0)$, $E_2 = (-0.9062, 0)$, and $E_3 = (-0.1667, 0)$. For equilibrium E_1, we get the eigenvalues $\lambda_1 \approx -0.0750 + 1.4876i$ and $\lambda_2 \approx -0.0750 - 1.4876i$. For equilibrium E_2, the eigenvalues are $\lambda_1 \approx 1.8548$ and $\lambda_2 \approx -2.0048$. For equilibrium E_3, we obtain the eigenvalues $\lambda_1 \approx 1.1521$ and $\lambda_2 \approx -1.3021$. According to these eigenvalues, we can conclude that a minimal commensurate order to obtain the chaotic behavior of system (38) is [45]

$$q > \frac{2}{\pi}\arctan\left(\frac{1.4876}{0.0750}\right) = 0.9679. \qquad (40)$$

Under the initial conditions $\zeta_1(0) = 2$ and $\zeta_2(0) = -1$ and the fractional order $q = 0.98$, FOCS (38) shows a chaotic behavior, which is depicted in Figure 1.

According to (30) and (38), it is easy to know that

$$\begin{aligned} P &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \\ Q &= \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \\ \hbar(t) &= \begin{bmatrix} 0 \\ -\zeta_1^3(t) - 0.15\zeta_2(t) + 0.3\cos(t) \end{bmatrix}. \end{aligned} \qquad (41)$$

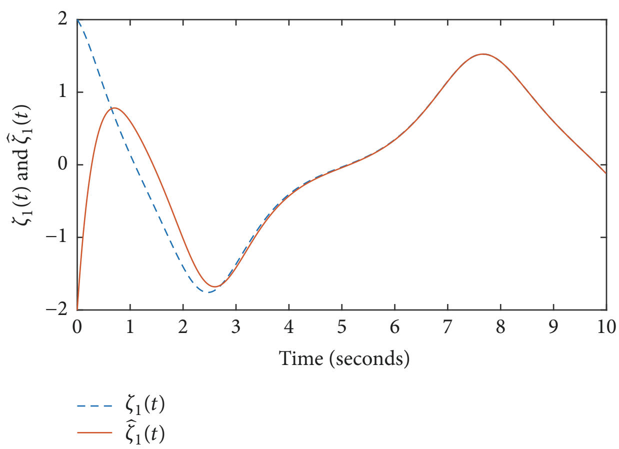

FIGURE 2: Synchronization between $\zeta_1(t)$ and $\hat{\zeta}_1(t)$.

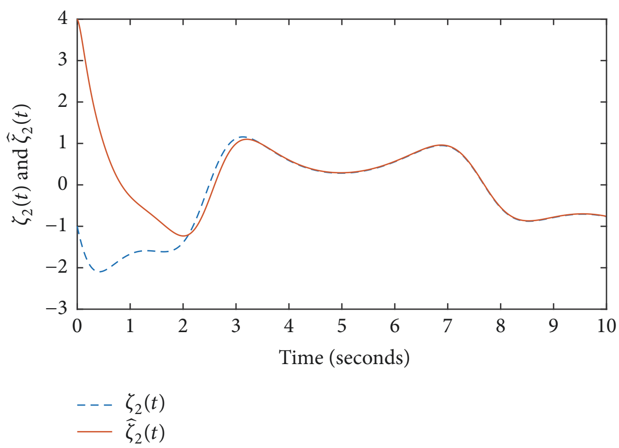

FIGURE 3: Synchronization between $\zeta_2(t)$ and $\hat{\zeta}_2(t)$.

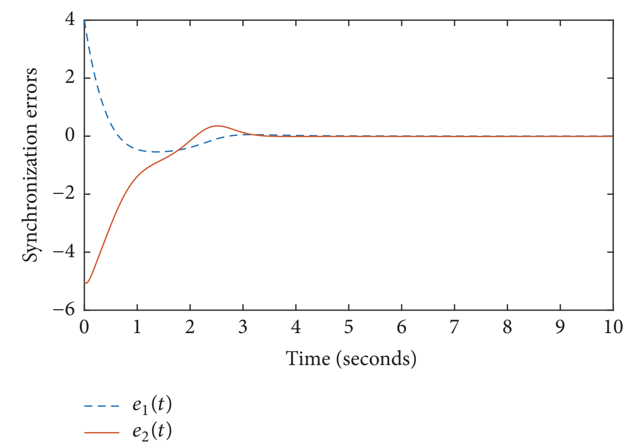

FIGURE 4: Time response of synchronization errors $e_1(t)$ and $e_2(t)$.

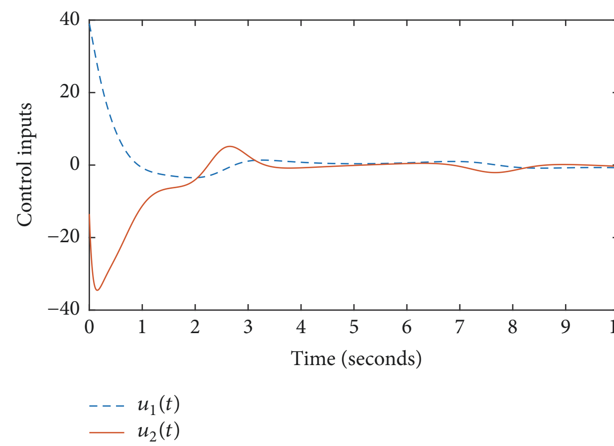

FIGURE 5: Time response of control inputs $u_1(t)$ and $u_2(t)$.

Since system (38) is a chaotic system, then we know that both signals $\zeta_1(t)$ and $\zeta_2(t)$ are bounded (from Figure 1, one knows that $|\zeta_1(t)| < 2$ and $|\zeta_2(t)| < 2$). Thus, Assumption 9 is satisfied with $a_0 = 3$.

In the simulation, the initial condition for the slave FOCS is $\hat{\zeta}_1(0) = -2$ and $\hat{\zeta}_2(0) = 4$. Suppose that $G = \text{diag}(1, 1)$. The design matrices are chosen as

$$K = \begin{bmatrix} -73 & 34 \\ 52 & -12 \end{bmatrix},$$
$$\Lambda = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}. \tag{42}$$

Thus, we have that $\|Q - \Lambda Q\| = 0.728$, $\underline{\lambda} = 3.1$, and the two conditions $P - \Lambda K < 0$ and $a_0\|Q - \Lambda Q\| \leq \underline{\lambda} q$ in Theorem 10 are satisfied.

The simulation results are presented in Figures 2–5. The results where the state variables of the slave FOCS track the master system's states are presented in Figures 2 and 3. The time response of the synchronization errors is depicted in Figure 4. From these pictures, we can see that the synchronization controller works well, and the synchronization errors converge to the origin fast. From (35), we know that the synchronization control input is a continuous function. The smoothness of the control input is given in Figure 5, from which we can see that the proposed controller has small fluctuation.

4.2. Synchronizing Two 3D Fractional-Order Chaotic Neural Networks. Let us consider the following fractional-order chaotic neural networks expressed by [15]

$$\begin{aligned}
\mathbb{D}^q \zeta_1(t) &= -\zeta_1(t) + 2\tanh(\zeta_1(t)) - 1.2\tanh(\zeta_2(t)), \\
\mathbb{D}^q \zeta_2(t) &= -\zeta_2(t) + 2\tanh(\zeta_1(t)) + 1.71\tanh(\zeta_2(t)) \\
&\quad + 1.15\tanh(\zeta_3(t)), \\
\mathbb{D}^q \zeta_3(t) &= -\zeta_3(t) - 4.75\tanh(\zeta_1(t)) \\
&\quad + 1.10\tanh(\zeta_3(t)).
\end{aligned} \tag{43}$$

Suppose that $q = 0.95$ and the initial condition is $\zeta_1(0) = -0.3$, $\zeta_2(0) = 0.4$, and $\zeta_3(0) = 0.3$. The dynamical behavior of FOCS (43) is given in Figure 6.

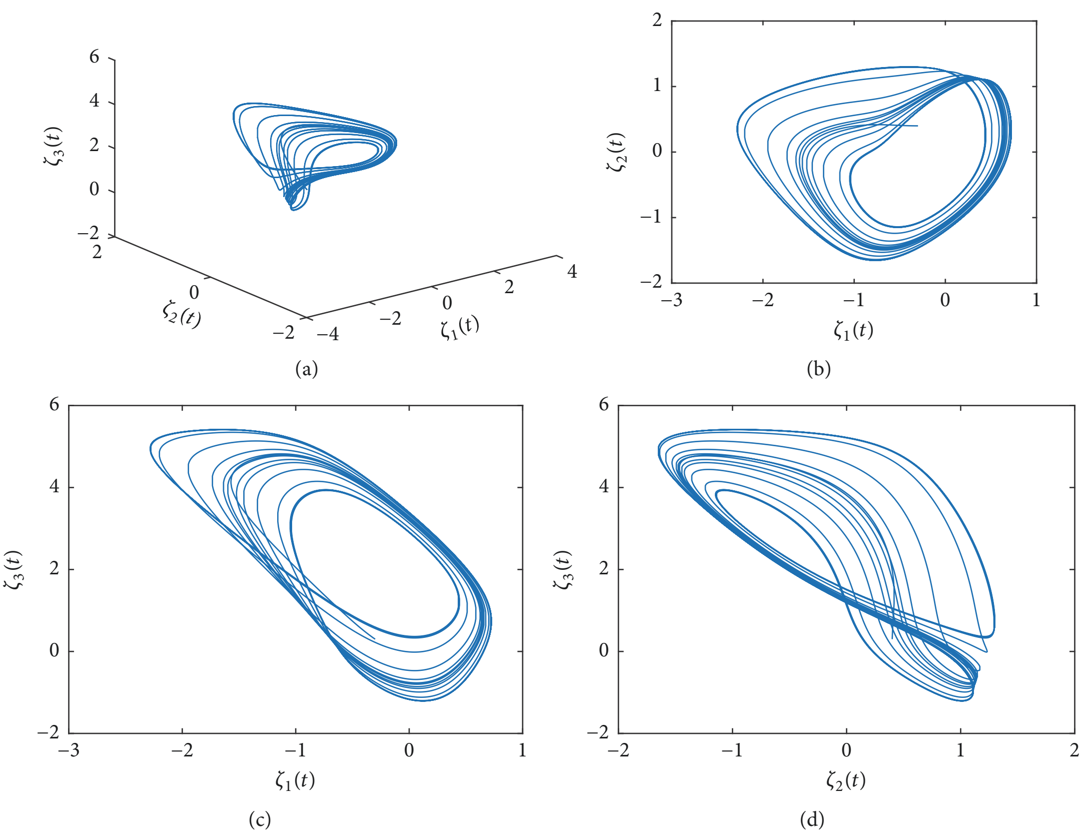

FIGURE 6: Dynamical behavior of system (43) in (a) 3D space, (b) $\zeta_1(t) - \zeta_2(t)$ plane, (c) $\zeta_1(t) - \zeta_3(t)$ plane, and (d) $\zeta_2(t) - \zeta_3(t)$ plane.

It is easy to know in the master chaotic system (43) that $P = \mathrm{diag}(-1, -1, -1)$, $\hbar(t) = [\tanh(\zeta_1(t)), \tanh(\zeta_2(t)), \tanh(\zeta_3(t))]^T$,

$$Q = \begin{bmatrix} 2.00 & -1.20 & 0.00 \\ 2.00 & 1.71 & 1.15 \\ -4.75 & 0.00 & 1.10 \end{bmatrix}. \quad (44)$$

Thus, we have $\|Q\| = 1.235$ and $\hbar(t)$ satisfy the Lipshitz condition. The Lipshitz constant a_0 can be chosen as 1.

The initial condition of the slave FOCS is $\hat{\zeta}_1(0) = 3.2$, $\hat{\zeta}_2(0) = -4$, and $\hat{\zeta}_3(0) = -3.5$. Let $G = \mathrm{diag}(1, 1, 1)$. The design matrices are chosen as

$$K = \begin{bmatrix} 1.2484 & -0.1401 & 0.0127 \\ 0.0127 & 1.1210 & -1.1019 \\ -0.1146 & -0.0892 & 0.9172 \end{bmatrix},$$
$$\Lambda = \begin{bmatrix} 0.8 & 0.1 & 0 \\ 0 & 0.9 & 0.1 \\ 0.1 & 0.1 & 1.1 \end{bmatrix}. \quad (45)$$

Thus, we know that $\|Q - \Lambda Q\| = 0.7115$, $\underline{\lambda} = 2$, and the two conditions $P - \Lambda K < 0$ and $a_0\|Q - \Lambda Q\| \leq \underline{\lambda} q$ in Theorem 10 are satisfied.

The simulation results are given in Figures 7 and 8. Just like the results in Figures 2–5, we know that good synchronization performance has been obtained.

5. Conclusion

In this paper, two stability criteria for fractional-order nonlinear systems are given. Based on these theorems, the synchronization of two identical FOCSs is addressed. A fractional-order sliding surface, which contains a fractional-order integral of the synchronization errors, is given. The proposed controller can guarantee the asymptotical stability of the closed-loop systems. However, in the controller design, we need to know the exact value of the Lipchitz constant. How to reduce this condition is one of our future research directions.

Conflicts of Interest

The author does not have a direct financial relation with any commercial identity mentioned in this paper that might lead to conflicts of interest.

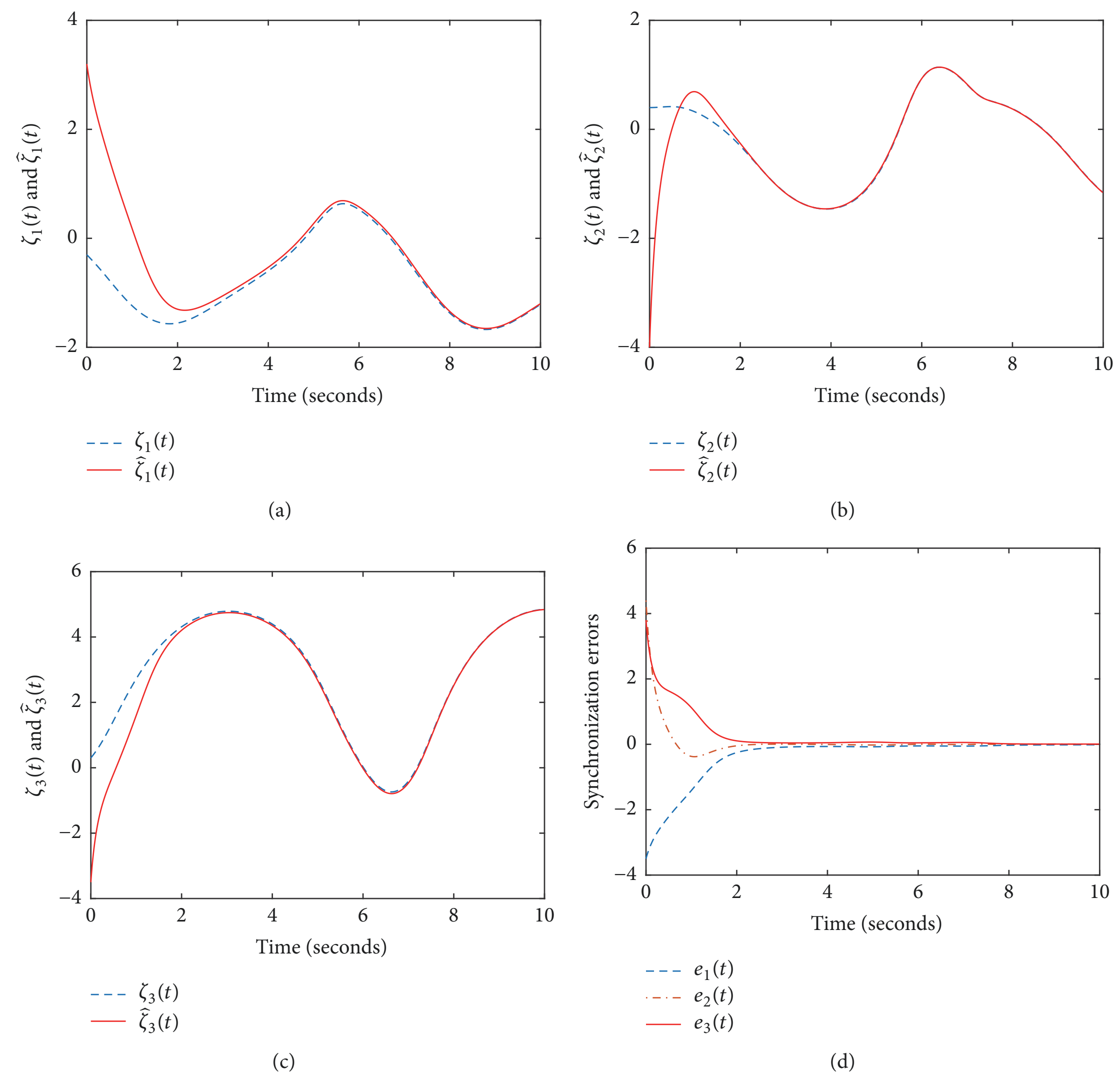

FIGURE 7: Simulation results in (a) synchronization between $\zeta_1(t)$ and $\widehat{\zeta}_1(t)$, (b) synchronization between $\zeta_2(t)$ and $\widehat{\zeta}_2(t)$, (c) synchronization between $\zeta_3(t)$ and $\widehat{\zeta}_3(t)$, and (d) synchronization errors $e_1(t)$, $e_2(t)$, and $e_3(t)$.

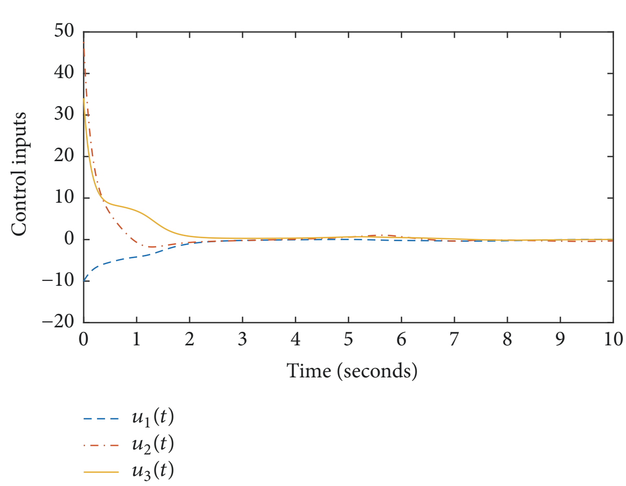

FIGURE 8: Time response of control inputs $u_1(t)$, $u_2(t)$, and $u_3(t)$.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant no. 11302184) and the Young and Middle-Aged Teacher Education and Science Research Foundation of Fujian Province of China (Grant no. JAT170423).

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