# Channel Estimation and Information Symbol Detection for DS-UWB Communication Systems 

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Received 16 December 2013; Accepted 14 February 2014; Published 27 March 2014
Academic Editor: Xiaojie Su
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#### Abstract

The UWB channel estimation and multiuser detection problem are investigated. The information symbol and channel parameter are considered as unknown variables. The multiuser detector and UWB channel estimator are designed jointly. For symbol detection, the one-step predictor of channel parameter is used and the estimation error is treated as a multiplicative noise; then a Riccati equation and a Lyapunov equation will be needed. If the transmitted symbols are uncorrelated and identically distributed random variables with zero mean and unit variance, only a Riccati equation needs to be solved. For UWB channel estimation, the one-step predictor of information symbol is used and the estimation error is also considered as a multiplicative noise. The solutions to the above two problems are obtained by solving a couple of Riccati equations together with two Lyapunov equations.


## 1. Introduction

In the communications literature, a number of different algorithms have been proposed for channel estimation problems with accurate models [1-19]. In [1], a subspace-based estimation algorithm is developed. The algorithms in $[2,3]$ are based on the maximum likelihood estimation method. Due to the performance benefits of the Kalman algorithms, many works have focused on the Kalman-filter-based channel estimation algorithms [13-15]. These algorithms require a state-space model for the random process to be estimated.

As for the UWB channel, many different types of channel models have been proposed. In general, the UWB propagation channel models are characterized by a dense multipath propagation and the clustering phenomenon and can be classified as deterministic and statistical [17, 18]. Deterministic models apply an electromagnetic simulation tool to obtain exact propagation characteristics for a specified geometry. Statistical models are normally less complex than the deterministic models and can provide sufficiently accurate channel information. In [20], three channel models were considered, namely, the Rayleigh tap delay line model, the $\Delta$-K model, and the Saleh-Valenzuela (S-V) model. The comparisons showed that the S-V model gives the best fit to the measured
channel characteristics. This double exponential channel model is commonly used for UWB realistic indoor channel, and the channel impulse response is given by

$$
\begin{equation*}
\mathscr{H}(t)=\sum_{l_{c}=0}^{L_{c}-1} \sum_{l_{r}=0}^{L_{r}-1} \alpha_{l_{c} l_{r}} \delta\left(t-T_{l_{c}}-\tau_{l_{c} l_{r}}\right), \tag{1}
\end{equation*}
$$

where
(i) $\left\{\alpha_{l_{c} l_{r}}\right\}$ are the multipath gain coefficients, $l_{c}$ refers to the cluster, and $l_{r}$ refers to the rays in one cluster;
(ii) $T_{l_{c}}$ is the delay of the $l_{c}$ th cluster which is defined as the TOA of the first arriving multipath component within the $l_{c}$ th cluster;
(iii) $\tau_{l_{c} l_{r}}$ is the delay of the $l_{r}$ th multipath component relative to the $l_{c}$ th cluster arrival time $T_{l_{c}}$.

The clustering channel model relies on two classes of the parameters, namely, intercluster and intracluster parameters, which characterize the cluster and multipath component, respectively. In the above model, $\left\{L_{c}, T_{l_{c}}\right\}$ and $\left\{T_{r}, \tau_{l_{c} l_{r}}, \alpha_{l_{c} l_{r}}\right\}$ are classified as the intercluster and intracluster parameters, respectively. The distributions of the cluster arrive time $T_{l_{c}}$
and the ray arrive time $\tau_{l_{c} l_{r}}$ can be given by two Poisson processes:

$$
\begin{gather*}
p\left(T_{l_{c}} \mid T_{l_{c}-1}\right)=\Lambda \exp \left[-\Lambda\left(T_{l_{c}}-T_{l_{c}-1}\right)\right], \quad l_{c}>0, \\
p\left(\tau_{l_{c} l_{r}} \mid \tau_{l_{c} l_{r}-1}\right)=\lambda \exp \left[-\lambda\left(\tau_{l_{c} l_{r}}-\tau_{l_{c} l_{r}-1}\right)\right], \tag{2}
\end{gather*}
$$

where $\Lambda$ and $\lambda$ are mean cluster arrival rate and mean ray arrival rate, respectively. The channel coefficients are defined as follows:

$$
\begin{equation*}
\alpha_{l o l_{r}}=p_{l c l_{r}} \beta_{l o l_{r}} \tag{3}
\end{equation*}
$$

where $p_{l_{c} l_{r}}$ is equiprobable to $\pm 1$ to account for signal inversion due to reflection; $\beta_{l, l_{r}}$ correspond to the fading associated with the $l_{c}$ th ray of the $l_{r}$ th cluster, which can be modeled as a log-normal distribution:

$$
\begin{equation*}
20 \log 10\left(\beta_{l_{c} l_{r}}\right) \propto \operatorname{Normal}\left(\mu_{c} l_{r}, \sigma^{2}\right) \tag{4}
\end{equation*}
$$

where $\mu_{c_{c} l_{r}}$ is given by

$$
\begin{equation*}
\mu_{c o l}=\frac{10 \ln \left(\Omega_{0}\right)-10 T_{l_{c}} / \Gamma-10 \tau_{c, l_{r}} / \gamma}{\ln (10)}-\frac{\sigma^{2} \ln (10)}{20} \tag{5}
\end{equation*}
$$

where $\Omega_{0}$ is the mean power of the first path of the first cluster. The behavior of the averaged power delay profile is

$$
\begin{equation*}
E\left[\left.\left|\beta_{l o l}\right|_{r}\right|^{2}\right]=\Omega_{0} e^{-T_{l c} / \Gamma} e^{-T_{l, l r} / \gamma}, \tag{6}
\end{equation*}
$$

which reflect the exponential decay of each cluster.
With mapping, the above two-dimensional channel model can be reduced to a one-dimensional channel model:

$$
\begin{equation*}
\mathscr{H}(t)=\sum_{l=0}^{L} \alpha^{l} \delta\left(t-\tau_{l}\right), \tag{7}
\end{equation*}
$$

where $l=l_{c} L_{c}+l_{r}$ and $L=L_{c} L_{r}-1$ are the number of the resolvable multipath components; $\tau_{l}=T_{l_{c}}+\tau_{l_{c} l_{r}}$ is the delay of the $l$ th path relative to the first path; $\alpha^{l}=\alpha_{c c l_{r}}$ is the fading coefficient of path $l$.

After mapping, the one-dimensional model can be dealt with by using some conventional channel estimation algorithm that is used for narrowband systems, such as maximum likelihood approach and least mean square approach. In this paper, we will pursue a Kalman-filter-based approach with information symbols unknown.

## 2. UWB System Model

In this paper, we consider a binary DS-CDMA UWB communication system with $K$ multiple access users. The transmitted baseband signal of the $k$ th user is given by $[15,16]$

$$
\begin{equation*}
x_{k}(t)=\sqrt{A_{k}} \sum_{n=-\infty}^{\infty} b_{k}(n) s_{k}\left(t-n T_{s}\right), \tag{8}
\end{equation*}
$$

where $A_{k}$ is the transmitted bit energy, $T_{s}$ is symbol duration, $b_{k}(n)$ is the modulated information symbol of the $k$ th user and
is chosen randomly from the set $\{-1,+1\}$, and $s_{k}(t)$ represents the transmitted waveform and has the form

$$
\begin{equation*}
s_{k}(t)=\sum_{i=0}^{N} \widetilde{c}_{k}(i) \psi\left(t-i T_{c}\right), \tag{9}
\end{equation*}
$$

where $N$ is the spreading gain, $\widetilde{c}_{k}(i)$ is the spreading code of the $k$ th user with period $N$, and $\psi(t)$ is the real transmitted monocycle waveform shape in the time interval $0 \leq t \leq T_{c}$, that is, $\psi(t)=0$ if $t \notin\left[0, T_{c}\right]$, and has energy $(1 / N)$.

Note that the channel coefficient $\alpha^{l}$ in (7) is fading with respect to time $t$; the channel impulse response for the $k$ th user can be described by

$$
\begin{equation*}
\mathscr{H}_{k}(t)=\sum_{l=0}^{L} \alpha_{k}^{l}(t) \delta\left(t-\tau_{k, l}\right) \tag{10}
\end{equation*}
$$

where $\tau_{k, l}$ is the time delay for the $l$ th path of user $k$. Then the received signal component from the $k$ th user can be represented as

$$
\begin{align*}
& y_{k}(t)=x_{k}(t) * \mathscr{H}_{k}(t) \\
& =\sqrt{A_{k}} \sum_{n=-\infty}^{\infty} b_{k}(n) \sum_{i=0}^{N-1} \widetilde{c}_{k}(i) \\
& \quad \times \sum_{j=0}^{L} \alpha_{k}^{l}(t) \psi\left(t-n T_{s}-i T_{c}-\tau_{k, j}\right)  \tag{11}\\
& =\sqrt{A_{k}} \sum_{n=-\infty}^{\infty} b_{k}(n) \\
& \quad \times \sum_{i=0}^{N-1} \widetilde{c}_{k}(i) g_{k}\left(t, t-n T_{s}-i T_{c}\right),
\end{align*}
$$

where $*$ denotes the convolution, and

$$
\begin{equation*}
g_{k}(t, \tau) \triangleq \sum_{j=0}^{L} \alpha_{k}^{l}(t) \psi\left(\tau-\tau_{k, j}\right) . \tag{12}
\end{equation*}
$$

The total received signal at the receiver is the superposition of the signals of the $K$ users, given by

$$
\begin{equation*}
r(t)=\sum_{k=0}^{K} y_{k}(t)+v(t), \tag{13}
\end{equation*}
$$

where $v(t)$ is a white Gaussian noise with zero mean. The discrete-time signal is generated by sampling the output of a pulse-matched filter (PMF) at the monocycle rate (as shown in Figure 1) and given by [13]

$$
\begin{aligned}
& y_{k}(n N+j) \\
& \quad=\int_{n T_{s}+j T_{p}}^{n T_{s}(j+1) T_{p}} y_{k}(t) \psi\left(t-n T_{s}-j T_{p}\right) d t
\end{aligned}
$$



Figure 1: Received discrete-time signal.

In this paper the multipath delay $\left\{\tau_{k, j}\right\}$ of the UWB channel is assumed to be an integral multiple of the monocycle length $T_{p}$; then the above equation can be rewritten as

$$
\begin{aligned}
& y_{k}(n N+j) \\
& \qquad \sqrt{A_{k}} \sum_{m=-\infty}^{\infty} b_{k}(m) \\
& \times \int_{0}^{T_{p}} \sum_{i=0}^{N_{c}-1} \widetilde{c}_{k}(i) \\
& \times \sum_{l=0}^{L} \alpha_{k}^{l}\left(t+n T_{s}+j T_{p}\right) \\
& \\
& \times \psi\left(t+(n-m) T_{s}\right. \\
& \\
& \left.\quad+\left(j-i N_{p}-l\right) T_{c}\right)
\end{aligned}
$$

$$
\times \psi(t) d t
$$

$$
\begin{align*}
& =\int_{n T_{s}+j T_{p}}^{n T_{s}+(j+1) T_{p}} \sum_{m=-\infty}^{\infty} \sqrt{A_{k}} b_{k}(m) \\
& \times \sum_{i=0}^{N_{c}-1} \widetilde{c}_{k}(i) g_{k}\left(t, t-m T_{s}-i T_{c}\right) \\
& \times \psi\left(t-n T_{s}-j T_{p}\right) d t \\
& =\sqrt{A_{k}} \sum_{m=-\infty}^{\infty} b_{k}(m) \\
& \times \int_{0}^{T_{p}} \sum_{i=0}^{N_{c}-1} \widetilde{c}_{k}(i) g_{k} \\
& \times\left(t+n T_{s}+j T_{p}, t+(n-m) T_{s}\right. \\
& \left.+\left(j-i N_{p}\right) T_{p}\right) \psi(t) d t . \tag{14}
\end{align*}
$$

$$
\begin{align*}
& =b_{k}(n) \sum_{l=0}^{L} c_{k}\left(\left\lfloor\frac{j-l}{N_{p}}\right\rfloor\right) \\
& \times \int_{0}^{T_{p}} \alpha_{k}^{l}\left(t+n T_{s}+j T_{p}\right) d t \\
& +b_{k}(n-1) \sum_{l=0}^{L} c_{k}\left(\left\lfloor\frac{N+j-l}{N_{p}}\right\rfloor\right) \\
& \quad \times \int_{0}^{T_{p}} \alpha_{k}^{l}\left(t+n T_{s}+j T_{p}\right) d t \\
& =b_{k}(n) \sum_{l=0}^{L} c_{k}\left(\left\lfloor\frac{j-l}{N_{p}}\right\rfloor\right) h_{k}^{l}(n, j) \\
&  \tag{15}\\
& \quad+b_{k}(n-1) \sum_{l=0}^{L} c_{k}\left(\left\lfloor\frac{N+j-l}{N_{p}}\right\rfloor\right) h_{k}^{l}(n, j),
\end{align*}
$$

where

$$
\begin{align*}
h_{k}^{l}(n, j) & =\int_{0}^{T_{p}} \alpha_{k}^{l}\left(t+n T_{s}+j T_{c}\right) d t \\
c_{k}(i) & = \begin{cases}\frac{\sqrt{A_{k}}}{N} \widetilde{c}_{k}(i), & 0 \leq i \leq N-1 ; \\
0, & \text { otherwise }\end{cases} \tag{16}
\end{align*}
$$

Further assuming that $h_{k}^{l}(n, j)$ is invariant during a symbol interval and using $h_{k}^{l}(n)$ to denote the channel parameter in the $n$th symbol, then we have

$$
\begin{align*}
y_{k}(n N+j)= & b_{k}(n) \sum_{l=0}^{L} c_{k}\left(\left\lfloor\frac{j-l}{N_{p}}\right\rfloor\right) h_{k}^{l}(n) \\
& +b_{k}(n-1) \sum_{l=0}^{L} c_{k}\left(\left\lfloor\frac{N+j-l}{N_{p}}\right\rfloor\right) h_{k}^{l}(n) \tag{17}
\end{align*}
$$

By collecting $N$ successive samples, the channel output from the $k$ th user at the $n$th symbol can be expressed as

$$
\begin{align*}
\mathbf{y}_{k}(n) & =\left[\begin{array}{llll}
y_{k}(n N) & y_{k}(n N+1) & \cdots & y_{k}(n N+N-1)
\end{array}\right]^{T} \\
& =b_{k}(n) C_{k}^{0} \mathbf{h}_{k}(n)+b_{k}(n-1) C_{k}^{1} \mathbf{h}_{k}(n), \tag{18}
\end{align*}
$$

where $C_{k}^{0}$ and $C_{k}^{1}$ are the signature sequence matrices with dimension $N \times(L+1)$ and have the forms

$$
C_{k}^{0}=\left[\begin{array}{cccc}
c_{k}(0) & 0 & \cdots & 0  \tag{19}\\
c_{k}\left(\left\lfloor\frac{l}{N_{p}}\right\rfloor\right) & c_{k}(0) & \cdots & 0 \\
c_{k}\left(\left\lfloor\frac{2}{N_{p}}\right\rfloor\right) & c_{k}\left(\left\lfloor\frac{1}{N_{p}}\right\rfloor\right) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
c_{k}\left(\left\lfloor\frac{N-1}{N_{p}}\right\rfloor\right) & c_{k}\left(\left\lfloor\frac{N-2}{N_{p}}\right\rfloor\right) & \cdots & c_{k}\left(\left\lfloor\frac{N-L-1}{N_{p}}\right\rfloor\right)
\end{array}\right],
$$

and $\mathbf{h}_{k}(n)$ is the parameter collection of all multipath components

$$
\mathbf{h}_{k}(n)=\left[\begin{array}{llll}
h_{k}^{0}(n) & h_{k}^{1}(n) & \cdots & h_{k}^{L}(n) \tag{20}
\end{array}\right]^{T}
$$

The total received discrete-time signal of all users can be given by

$$
\begin{align*}
\mathbf{r}(n) & =\left[\begin{array}{llll}
r(n N) & r(n N+1) & \cdots & r(n N+N-1)
\end{array}\right]^{T} \\
& =\sum_{k=1}^{K} \mathbf{y}_{k}(n)+\mathbf{v}(n) \tag{21}
\end{align*}
$$

where

$$
\begin{align*}
\mathbf{v}(n) & =\left[\begin{array}{llll}
v(n N) & v(n N+1) & \cdots & v(n N+N-1)
\end{array}\right]^{T}, \\
v(n N+j) & =\int_{n T_{s}+j T_{c}}^{n T_{s}+(j+1) T_{c}} v(t) \psi\left(t-n T_{s}-j T_{c}\right) d t . \tag{22}
\end{align*}
$$

Considering that the channel parameters $\left\{h_{k}^{l}(n)\right\}$ and the user information symbols $\left\{b_{k}(n)\right\}$ are unknown, in this paper, we pursue a joint design method for user detection and channel parameter estimation.

The problems investigated in this paper can be stated as follows.

Problem I. Given the received signal sequence $\{\mathbf{r}(s)\}_{s=0}^{n}$, with $\left\{\mathbf{h}_{k}(s), k=1, \ldots, K\right\}_{s=0}^{n}$ not exactly known, find an optimal symbol detector $\left\{\widehat{b}_{k}(n \mid n), k=1, \ldots, K\right\}$ by using a priori estimate $\left\{\widehat{\mathbf{h}}_{k}(s \mid s-1), k=1, \ldots, K\right\}_{s=0}^{n}$ of channel parameter which is recursively calculated in Problem-II.

Problem II. Given the received signal sequence $\{\mathbf{r}(s)\}_{s=0}^{n}$, with the information symbol $\left\{b_{k}(s), k=1, \ldots, K\right\}_{s=0}^{n}$ not exactly known, find a channel estimator $\left\{\widehat{\mathbf{h}}_{k}(n \mid n), k=1, \ldots, K\right\}$ by using a priori estimate $\left\{\widehat{b}_{k}(s \mid s-1), k=1, \ldots, K\right\}_{s=0}^{n}$ which is recursively calculated in Problem-I.

Remark 1. In this section, we have adopted a signal model for DS-CDMA communication systems similar to that in [15, 16]. Different from [15], in (7) the information symbol matrix is unknown which will be detected together with the channel parameter. In most relevant works, the information symbol is considered known for channel estimation [14], or channel parameter is known for user detection [12] and only few works investigate a joint estimation scheme considering both of the aforementioned unknown variables [13]. This paper will propose a Kalman-filter-based joint design method for multiuser detection and channel estimation.

Remark 2. The above two problems cannot be solved separately, because the channel parameters and information
symbols are not exactly known. Different from [13], in this paper the symbol detector and channel estimator are designed simultaneously. The solutions of the detector and channel estimator will be obtained via solving coupled Riccati equations together with two Lyapunov equations.

## 3. Multiuser Detector

In this section, a first-order state-space model is applied to symbol detection for the proposed UWB system, where the channel parameter is not exactly known. Then we can
employ the Kalman filter to estimate all users' symbols simultaneously. In view of (7), for multiuser detection the total received discrete-time signal $\mathbf{r}(n)$ can be expressed as

$$
\begin{align*}
\mathbf{r}(n) & =[r(n N) r(n N+1) \cdots r(n N+N-1)]^{T} \\
& =\sum_{k=1}^{K} \mathbf{y}_{k}(n)+\mathbf{v}(n)  \tag{23}\\
& =C H(n) \mathbf{b}(n)+\mathbf{v}(n)
\end{align*}
$$

where

$$
\begin{align*}
& C=\left[\begin{array}{llllllll}
C_{1}^{0} & C_{2}^{0} & \cdots & C_{K}^{0} & C_{1}^{1} & C_{2}^{1} & \cdots & C_{K}^{1}
\end{array}\right], \\
& H(n)=\operatorname{diag}\left\{\mathbf{h}_{1}(n), \mathbf{h}_{2}(n), \ldots, \mathbf{h}_{K}(n), \mathbf{h}_{1}(n), \mathbf{h}_{2}(n), \cdots, \mathbf{h}_{K}(n)\right\},  \tag{24}\\
& \mathbf{b}(n)=\left[\begin{array}{lllllll}
b_{1}(n) & b_{2}(n) & \cdots & b_{K}(n) & b_{1}(n-1) & b_{2}(n-1) & \cdots
\end{array} b_{K}(n-1)\right]^{T} \text {. }
\end{align*}
$$

Note the symbol vector $\mathbf{b}(n)$ defined in (23); the first-order non-Gaussian Markov transition model is defined as

$$
\begin{equation*}
\mathbf{b}(n+1)=\Phi \mathbf{b}(n)+\mathbf{w}(n) \tag{25}
\end{equation*}
$$

where

$$
\begin{gather*}
\Phi=\left[\begin{array}{cc}
0_{K, K} & 0_{K, K} \\
I_{K} & 0_{K, K}
\end{array}\right],  \tag{26}\\
\mathbf{w}(n)=\left[\begin{array}{lll}
b_{1}(n+1) & \cdots & b_{K}(n+1) \\
0_{1, K}
\end{array}\right]^{T}
\end{gather*}
$$

where $0_{m, n}$ denotes the $m \times n$ all-zero matrix, and $I_{m}$ is the $m \times m$ identity matrix; the noise vector $\mathbf{w}(n)$ is white with zero mean and covariance matrix

$$
\begin{equation*}
Q_{w}(n)=E\left\{\mathbf{w}(n) \mathbf{w}^{T}(n)\right\} \tag{27}
\end{equation*}
$$

For the convenience of discussion, we first give the following definitions.

Definition 3. For a given symbol $n$, let $\widehat{\xi}(n \mid n-1)$ denote the optimal estimation of $\xi(n)$, which is the projection of $\xi(n)$ onto the linear space

$$
\begin{equation*}
\mathscr{L}\{\mathbf{r}(0) \cdots \mathbf{r}(n-1)\} \tag{28}
\end{equation*}
$$

Definition 4. For multiuser detection with unknown UWB channel parameters, define

$$
\begin{equation*}
\mathbf{e}_{b}(n) \triangleq \mathbf{r}(n)-\widehat{\mathbf{r}}(n \mid n-1) \tag{29}
\end{equation*}
$$

For UWB channel estimation with unknown information symbols, define

$$
\begin{equation*}
\mathbf{e}_{h}(n) \triangleq \mathbf{r}(n)-\widehat{\mathbf{r}}(n \mid n-1) \tag{30}
\end{equation*}
$$

where $\widehat{\mathbf{r}}(n \mid n-1)$ is defined as in Definition 3.

As in the standard Kalman filtering, we define the onestep prediction error covariance matrix of the information symbol and channel parameter as

$$
\begin{align*}
& P_{b}(n) \triangleq E\left\{\widetilde{\mathbf{b}}(n \mid n-1) \widetilde{\mathbf{b}}^{T}(n \mid n-1)\right\} \\
& P_{h}(n) \triangleq E\left\{\widetilde{\mathbf{h}}(n \mid n-1) \widetilde{\mathbf{h}}^{T}(n \mid n-1)\right\} \tag{31}
\end{align*}
$$

where

$$
\begin{align*}
\widetilde{\mathbf{b}}(n \mid n-1) & \triangleq \mathbf{b}(n)-\widehat{\mathbf{b}}(n \mid n-1)  \tag{32}\\
\widetilde{\mathbf{h}}(n \mid n-1) & \triangleq \mathbf{h}(n)-\widehat{\mathbf{h}}(n \mid n-1)
\end{align*}
$$

where $\mathbf{h}(n)$ is the stack of channel parameters of all users

$$
\mathbf{h}(n)=\left[\begin{array}{llll}
\mathbf{h}_{1}^{T}(n) & \mathbf{h}_{2}^{T}(n) & \cdots & \mathbf{h}_{K}^{T}(n) \tag{33}
\end{array}\right]^{T}
$$

and $\widehat{\mathbf{b}}(n \mid n-1)$ and $\widehat{\mathbf{h}}(n \mid n-1)$ are defined as in Definition 3.
Note that the elements of UWB channel parameter matrix are unknown. In this section, we will use the one-step prediction $\widehat{H}(n \mid n-1)$ instead of $H(n)$ and consider the estimation error $\widetilde{H}(n \mid n-1)$ as a multiplicative noise for symbol detection. The optimal detector is given according to the following theorem.

Theorem 5. Consider the discrete-time state-space signal model (23) and (25); when the channel parameter matrix $H(n)$ is unknown, the information symbol detector is given by

$$
\begin{align*}
\widehat{\mathbf{b}}(n \mid n)= & {\left[I_{2 K}-K_{b}(n) C \widehat{H}(n \mid n-1)\right] \widehat{\mathbf{b}}(n \mid n-1) }  \tag{34}\\
& +K_{b}(n) \mathbf{r}(n)
\end{align*}
$$

where $\widehat{H}(n \mid n-1)$ is the one-step prediction of UWB channel parameter obtained from the next section, and $K_{b}(n)$ is the detector gain matrix

$$
\begin{equation*}
K_{b}(n)=P_{b}(n) \widehat{H}^{T}(n \mid n-1) C^{T}\left[Q_{e}^{b}(n)\right]^{-1} \tag{35}
\end{equation*}
$$

where $Q_{e}^{b}(n)$ is the covariance matrix of innovation $\mathbf{e}_{b}(n)$

$$
\begin{align*}
Q_{e}^{b}(n)= & C \widehat{H}(n \mid n-1) P_{b}(n) \widehat{H}^{T}(n \mid n-1) C^{T} \\
& +C\left(\left[\frac{P_{h}(n) \mid P_{h}(n)}{P_{h}(n) \mid P_{h}(n)}\right] \circ\left[\prod_{b}(n) \otimes I_{L+1}\right]\right) C^{T} \\
& +Q_{v}(n), \tag{36}
\end{align*}
$$

where $\circ$ denotes the Hadamard product and $\otimes$ is the Kronecker product. $\Pi_{b}(n)$ satisfies the following Lyapunov equation:

$$
\begin{equation*}
\prod_{b}(n+1)=\Phi \prod_{b}(n) \Phi^{T}+Q_{w}(n), \tag{37}
\end{equation*}
$$

where $P_{b}(n)$ is the symbol estimation error covariance matrix and satisfies the following Riccati equation:

$$
\begin{equation*}
P_{b}(n+1)=\Phi P_{b}(n) \Phi-\Phi K_{b}(n) Q_{e}^{b}(n) K_{b}^{T}(n) \Phi+Q_{w}(n) \tag{38}
\end{equation*}
$$

where $P_{h}(n)$ is the UWB channel parameter estimation error covariance matrix which will be calculated recursively in the next section. The one-step prediction of the information symbol is given by

$$
\begin{equation*}
\widehat{\mathbf{b}}(n+1 \mid n)=\Phi \widehat{\mathbf{b}}(n \mid n), \tag{39}
\end{equation*}
$$

which will be used for channel estimator design.
Proof. From Definition 3 we know that the a priori estimate $\widehat{\mathbf{r}}(n \mid n-1)$ is the projection of $\mathbf{r}(n)$ onto the linear space $\mathscr{L}\{\mathbf{r}(0), \ldots, \mathbf{r}(n-1)\}$ and consider the channel parameter matrix $H(n)$ as an unknown variable; then we have

$$
\begin{align*}
\widehat{\mathbf{r}}(n \mid n-1) & =\operatorname{Proj}\{\mathbf{r}(n) \mid \mathbf{r}(0), \ldots, \mathbf{r}(n-1)\}  \tag{40}\\
& =C \widehat{H}(n \mid n-1) \widehat{\mathbf{b}}(n \mid n-1)
\end{align*}
$$

In view of (7) and Definition 4, we obtain

$$
\begin{align*}
\mathbf{e}_{b}(n)= & \mathbf{r}(n)-\widehat{\mathbf{r}}(n \mid n-1) \\
= & C H(n) \mathbf{b}(n)-C \widehat{H}(n \mid n-1) \widehat{\mathbf{b}}(n \mid n-1)+\mathbf{v}(n) \\
= & C \widehat{H}(n \mid n-1) \widetilde{\mathbf{b}}(n \mid n-1) \\
& +C \widetilde{H}(n \mid n-1) \mathbf{b}(n)+\mathbf{v}(n) . \tag{41}
\end{align*}
$$

It is apparent that $\mathbf{e}_{b}(n)$ is with zero mean and $E\left\{\mathbf{e}_{b}(s) \mathbf{e}_{b}(j)\right\}=$ 0 if $s \neq j$. The stochastic process $\left\{\mathbf{e}_{b}(s)\right\}_{s=0}^{n}$ is termed as the innovation sequence associated with the received signal sequence. The covariance matrix of $\mathbf{e}_{b}(n)$, denoted as $Q_{e}^{b}(n)$, is calculated as follows:

$$
\begin{aligned}
Q_{e}^{b}(n) & \triangleq\langle\mathbf{e}(n), \mathbf{e}(n)\rangle \\
& =\langle C \widehat{H}(n \mid n-1) \widetilde{\mathbf{b}}(n \mid n-1)
\end{aligned}
$$

$$
\begin{align*}
& +C \widetilde{H}(n \mid n-1) \mathbf{b}(n)+\mathbf{v}(n), \\
& C \widehat{H}(n \mid n-1) \widetilde{\mathbf{b}}(n \mid n-1) \\
& +C \widetilde{H}(n \mid n-1) \mathbf{b}(n)+\mathbf{v}(n)\rangle \\
& =C \widehat{H}(n \mid n-1) P_{b}(n) \widehat{H}^{T}(n \mid n-1) C^{T} \\
& +C\langle\widetilde{H}(n \mid n-1) \mathbf{b}(n), \\
& \quad \widetilde{H}(n \mid n-1) \mathbf{b}(n)\rangle C^{T}+Q_{v}(n), \tag{42}
\end{align*}
$$

where $\langle$,$\rangle denotes the inner product, and$

$$
\begin{align*}
& \langle\widetilde{H}(n \mid n-1) \mathbf{b}(n), \widetilde{H}(n \mid n-1) \mathbf{b}(n)\rangle \\
& =\underset{\{\widetilde{H}, \mathbf{b}\}}{E}\left\{\widetilde{H}(n \mid n-1) \mathbf{b}(n) \mathbf{b}^{T}(n) \widetilde{H}^{T}(n \mid n-1)\right\} \\
& =\underset{\{\widetilde{\mathbf{h}}, b\}}{E}\left\{\left[\begin{array}{c}
\widetilde{\mathbf{h}}_{1}(n \mid n-1) b_{1}(n) \\
\vdots \\
\widetilde{\mathbf{h}}_{K}(n \mid n-1) b_{K}(n) \\
\widetilde{\mathbf{h}}_{1}(n \mid n-1) b_{1}(n-1) \\
\vdots \\
\widetilde{\mathbf{h}}_{K}(n \mid n-1) b_{K}(n-1)
\end{array}\right]\right. \\
& \left.\times\left[\begin{array}{c}
\widetilde{\mathbf{h}}_{1}(n n-1) b_{1}(n) \\
\vdots \\
\widetilde{\mathbf{h}}_{K}(n n-1) b_{K}(n) \\
\widetilde{\mathbf{h}}_{1}(n n-1) b_{1}(n-1) \\
\vdots \\
\widetilde{\mathbf{h}}_{K}(n n-1) b_{K}(n-1)
\end{array}\right]^{T}\right\}  \tag{43}\\
& =\underset{\{\widetilde{\mathbf{h}}, b\}}{E}\left[\begin{array}{l|l}
M_{11}(n) & M_{12}(n) \\
\hline M_{12}^{T}(n) & M_{22}(n)
\end{array}\right] \\
& =\left[\begin{array}{c|c}
P_{h}(n) & P_{h}(n) \\
\hline P_{h}(n) & P_{h}(n)
\end{array}\right] \circ\left[\prod_{b}(n) \otimes I_{L+1}\right],
\end{align*}
$$

where $\circ$ denotes the Hadamard product and $\otimes$ is the Kronecker product; $\Pi_{b}(n)$ denotes the inner product of symbol vector $\mathbf{b}(n)$, given by

$$
\begin{equation*}
\prod_{b}(n) \triangleq\langle\mathbf{b}(n), \mathbf{b}(n)\rangle \tag{44}
\end{equation*}
$$

and satisfies the following Lyapunov equation:

$$
\begin{equation*}
\prod_{b}(n+1)=\Phi \prod_{b}(n+1) \Phi^{T}+Q_{w}(n) \tag{45}
\end{equation*}
$$

where $P_{h}(n)$ is the parameter estimation error covariance matrix which will be calculated recursively in the next section. In the third step $\left\{M_{i j}(n), i, j=1,2\right\}$ are as follows:

$$
\begin{gather*}
M_{11}(n)=\left[\begin{array}{ccccc}
\widetilde{\mathbf{h}}_{1} b_{1}^{2}(n) \widetilde{\mathbf{h}}_{1}^{T} & \widetilde{\mathbf{h}}_{1} b_{1}(n) b_{2}(n) \widetilde{\mathbf{h}}_{2}^{T} & \cdots & \widetilde{\mathbf{h}}_{1} b_{1}(n) b_{K}(n) \widetilde{\mathbf{h}}_{K}^{T} \\
\widetilde{\mathbf{h}}_{2} b_{2}(n) b_{1}(n) \widetilde{\mathbf{h}}_{1}^{T} & \widetilde{\mathbf{h}}_{2} b_{2}^{2}(n) \widetilde{\mathbf{h}}_{2}^{T} & \cdots & \widetilde{\mathbf{h}}_{2} b_{2}(n) b_{K}(n) \widetilde{\mathbf{h}}_{K}^{T} \\
\vdots & \vdots & \ddots & \vdots \\
\widetilde{\mathbf{h}}_{K} b_{K}(n) b_{1}(n) \widetilde{\mathbf{h}}_{1}^{T} & \widetilde{\mathbf{h}}_{K} b_{K}(n) b_{2}(n) \widetilde{\mathbf{h}}_{2}^{T} & \cdots & \widetilde{\mathbf{h}}_{K} b_{K}^{2}(n) \widetilde{\mathbf{h}}_{K}^{T}
\end{array}\right], \\
M_{12}(n)=\left[\begin{array}{cccc}
\widetilde{\mathbf{h}}_{1} b_{1}(n) b_{1}(n-1) \widetilde{\mathbf{h}}_{1}^{T} & \widetilde{\mathbf{h}}_{1} b_{1}(n) b_{2}(n-1) \widetilde{\mathbf{h}}_{2}^{T} & \cdots & \widetilde{\mathbf{h}}_{1} b_{1}(n) b_{K}(n-1) \widetilde{\mathbf{h}}_{K}^{T} \\
\widetilde{\mathbf{h}}_{2} b_{2}(n) b_{1}(n-1) \widetilde{\mathbf{h}}_{1}^{T} & \widetilde{\mathbf{h}}_{2} b_{2}(n) b_{2}(n-1) \widetilde{\mathbf{h}}_{2}^{T} & \cdots & \widetilde{\mathbf{h}}_{2} b_{2}(n) b_{K}(n-1) \widetilde{\mathbf{h}}_{K}^{T} \\
\vdots & \vdots & \ddots & \vdots \\
\widetilde{\mathbf{h}}_{K} b_{K}(n) b_{1}(n-1) \widetilde{\mathbf{h}}_{1}^{T} & \widetilde{\mathbf{h}}_{K} b_{K}(n) b_{2}(n-1) \widetilde{\mathbf{h}}_{2}^{T} & \cdots & \widetilde{\mathbf{h}}_{K} b_{K}(n) b_{K}(n-1) \widetilde{\mathbf{h}}_{K}^{T}
\end{array}\right], \\
M_{22}(n)=\left[\begin{array}{cccc}
\widetilde{\mathbf{h}}_{1} b_{1}^{2}(n-1) \widetilde{\mathbf{h}}_{1}^{T} & \widetilde{\mathbf{h}}_{1} b_{1}(n-1) b_{2}(n-1) \widetilde{\mathbf{h}}_{2}^{T} & \cdots & \widetilde{\mathbf{h}}_{1} b_{1}(n-1) b_{K}(n-1) \widetilde{\mathbf{h}}_{K}^{T} \\
\widetilde{\mathbf{h}}_{2} b_{2}(n-1) b_{1}(n-1) \widetilde{\mathbf{h}}_{1}^{T} & \widetilde{\mathbf{h}}_{2} b_{2}^{2}(n-1) \widetilde{\mathbf{h}}_{2}^{T} & \cdots & \widetilde{\mathbf{h}}_{2} b_{2}(n-1) b_{K}(n-1) \widetilde{\mathbf{h}}_{K}^{T} \\
\vdots & \vdots & \ddots & \vdots \\
\widetilde{\mathbf{h}}_{K} b_{K}(n-1) b_{1}(n-1) \widetilde{\mathbf{h}}_{1}^{T} & \widetilde{\mathbf{h}}_{K} b_{K}(n-1) b_{2}(n-1) \widetilde{\mathbf{h}}_{2}^{T} & \cdots & \widetilde{\mathbf{h}}_{K} b_{K}^{2}(n-1) \widetilde{\mathbf{h}}_{K}^{T}
\end{array}\right], \tag{46}
\end{gather*}
$$

where, for the convenience, $\left\{\widetilde{\mathbf{h}}_{i}(n \mid n-1), i=1, \ldots, K\right\}$ have been replaced by $\left\{\widetilde{\mathbf{h}}_{i}\right\}$ without confusion. Then substituting (43) into (42), we can get (36).

In terms of the definition of projection, we know that

$$
\begin{equation*}
\widehat{\mathbf{b}}(n+1 \mid n)=\operatorname{Proj}\{\mathbf{b}(n+1) \mid \mathbf{r}(0), \ldots, \mathbf{r}(n)\} \tag{47}
\end{equation*}
$$

From the linear estimation theory, the linear space spanned by the innovation sequence $\left\{\mathbf{e}_{b}(s)\right\}_{s=0}^{n}$ contains the same information as the one spanned by the received signal sequence $\{\mathbf{r}(s)\}_{s=0}^{n}$; that is,

$$
\begin{equation*}
\mathscr{L}\{\mathbf{r}(0), \ldots, \mathbf{r}(n)\}=\mathscr{L}\left\{\mathbf{e}_{b}(0), \ldots, \mathbf{e}_{b}(n)\right\} \tag{48}
\end{equation*}
$$

Then the projection in (48) can be rewritten as

$$
\begin{align*}
\widehat{\mathbf{b}}(n+1 \mid n)= & \operatorname{Proj}\left\{\mathbf{b}(n+1) \mid \mathbf{e}_{b}(0), \ldots, \mathbf{e}_{b}(n)\right\} \\
= & \operatorname{Proj}\left\{\Phi \mathbf{b}(n)+\mathbf{w}(n) \mid \mathbf{e}_{b}(0), \ldots, \mathbf{e}_{b}(n-1)\right\} \\
& +\operatorname{Proj}\left\{\Phi \mathbf{b}(n)+\mathbf{w}(n) \mid \mathbf{e}_{b}(n)\right\} \\
= & \Phi \widehat{\mathbf{b}}(n \mid n-1)+\Phi K_{b}(n) \mathbf{e}_{b}(n), \tag{49}
\end{align*}
$$

where $K_{b}(n)$ is the parameter of the projection of $\mathbf{b}(n)$ onto $\mathbf{e}_{b}(n)$, which yields the stationary point of the following error Gramian matrix:

$$
\begin{equation*}
\left\langle\mathbf{b}(n)-K_{b}(n) \mathbf{e}_{b}(n), \mathbf{b}(n)-K_{b}(n) \mathbf{e}_{b}(n)\right\rangle \tag{50}
\end{equation*}
$$

and satisfies

$$
\begin{align*}
K_{b}(n) Q_{e}^{b}(n)= & \left\langle\mathbf{b}(n), e_{b}(n)\right\rangle \\
= & \langle\mathbf{b}(n), C \widehat{H}(n \mid n-1) \widetilde{\mathbf{b}}(n \mid n-1) \\
& +C \widetilde{H}(n \mid n-1) \mathbf{b}(n)+\mathbf{v}(n)\rangle  \tag{51}\\
= & P_{b}(n) \widehat{H}^{T}(n \mid n-1) C^{T} .
\end{align*}
$$

Thus we have

$$
\begin{equation*}
K_{b}(n)=P_{b}(n) \widehat{H}^{T}(n \mid n-1) C^{T}\left[Q_{e}^{b}(n)\right]^{-1} \tag{52}
\end{equation*}
$$

In view of (52) and (53), it is apparent that we have

$$
\begin{equation*}
P_{b}(n+1)+\Phi K_{b}(n) Q_{e}^{b}(n) K_{b}^{T}(n) \Phi=\Phi P_{b}(n) \Phi+Q_{w}(n), \tag{53}
\end{equation*}
$$

which is (38).
In view of (49), the projection in (48) can be further given by

$$
\begin{align*}
\widehat{\mathbf{b}}(n+1 \mid n)= & \Phi \widehat{\mathbf{b}}(n \mid n-1)+\Phi K_{b}(n) \mathbf{e}_{b}(n) \\
= & \Phi\left[I_{2 K}-K_{b}(n) C \widehat{H}(n \mid n-1)\right] \\
& \times \widehat{\mathbf{b}}(n \mid n-1)+\Phi K_{b}(n) \mathbf{r}(n)  \tag{54}\\
= & \Phi \widehat{\mathbf{b}}(n \mid n),
\end{align*}
$$

where we have defined detector as

$$
\begin{align*}
\widehat{\mathbf{b}}(n \mid n) \triangleq & {\left[I_{2 K}-K_{b}(n) C \widehat{H}(n \mid n-1)\right] }  \tag{55}\\
& \times \widehat{\mathbf{b}}(n \mid n-1)+K_{b}(n) \mathbf{r}(n),
\end{align*}
$$

which is (34).

Remark 6. In Theorem 5, the UWB channel estimation error $\widetilde{H}(n \mid n-1)$ is considered as a multiplicative noise which is in matrix form, and the transmitted symbols may be colored and cross-correlated for different users. Their statistic characteristics are represented in the Lyapunov equation in (37). If the transmitted symbols are uncorrelated and identically distributed random variables with zero mean and unit variance, then the above result is equivalent to that proposed in [13] where the channel estimation error is considered as an additive noise.

Corollary 7. If one assumes the transmitted symbols are uncorrelated and identically distributed random variables with zero mean and unit variance, that is, $E\left\{\mathbf{b}(n) \mathbf{b}^{T}(n)\right\}=I_{2 K}$, then the information symbol detector is given by

$$
\begin{align*}
\widehat{\mathbf{b}}(n \mid n)= & {\left[I_{2 K}-K_{b}(n) C \widehat{H}(n \mid n-1)\right] }  \tag{56}\\
& \times \widehat{\mathbf{b}}(n \mid n-1)+K_{b}(n) \mathbf{r}(n),
\end{align*}
$$

where

$$
\begin{align*}
K_{b}(n)= & P_{b}(n) \widehat{H}^{T}(n \mid n-1) C^{T}\left[Q_{e}^{b}(n)\right]^{-1} \\
Q_{e}^{b}(n)= & C \widehat{H}(n \mid n-1) P_{b}(n) \widehat{H}^{T}(n \mid n-1) C^{T} \\
& +C\left(\left[\begin{array}{c|c}
P_{h}(n) & 0 \\
\hline 0 & P_{h}(n)
\end{array}\right] \circ\left[I_{2 K} \otimes J_{L+1}\right]\right) C^{T} \\
& +Q_{v}(n), \\
P_{b}(n+1)= & \Phi P_{b}(n) \Phi-\Phi K_{b}(n) Q_{e}^{b}(n) K_{b}^{T}(n) \Phi+Q_{w}(n), \tag{57}
\end{align*}
$$

with $J_{L+1}$ being the all-one matrix with dimension $(L+1) \times(L+$ 1) and $Q_{w}(n)=\left[\begin{array}{cc}I_{K} & 0_{K, K} \\ 0_{K, K} & 0_{K, K}\end{array}\right]$.

Proof. The proof is straightforward. Note that the transmitted symbols are uncorrelated and identically distributed random variables with zero mean and unit variance; then we can easily obtain that $\Pi_{b}(n)=I_{2 K}$ and $E_{\{\tilde{\mathbf{h}}, b\}}\left\{M_{12}\right\}=0$. It is apparent that $Q_{w}(n)=\left[\begin{array}{cc}I_{K} & 0_{K, K} \\ 0_{K, K} & 0_{K, K}\end{array}\right]$.

## 4. Channel Estimator

For UWB channel estimation, the received discrete-time signal in (23) can be reexpressed as

$$
\begin{align*}
\mathbf{r}(n) & =\left[\begin{array}{llll}
r(n N) & r(n N+1) & \cdots & r(n N+N-1)
\end{array}\right]^{T} \\
& =\sum_{k=1}^{K} \mathbf{y}_{k}(n)+\mathbf{v}(n)  \tag{58}\\
& =C B(n) \mathbf{h}(n)+\mathbf{v}(n),
\end{align*}
$$

where $B(n)$ is the information symbol matrix and is defined as

$$
\begin{gather*}
B(n)=[\bar{B}(n) \bar{B}(n-1)]^{T}  \tag{59}\\
\bar{B}(n)=\operatorname{diag}\left\{b_{1}(n) I_{L+1}, b_{2}(n) I_{L+1}, \ldots, b_{K}(n) I_{L+1}\right\}
\end{gather*}
$$

and $\mathbf{h}(n)$ is as shown in (33), which can be modeled by using a first-order autoregressive (AR) model as

$$
\begin{equation*}
\mathbf{h}(n+1)=\Gamma \mathbf{h}(n)+\mathbf{u}(n), \tag{60}
\end{equation*}
$$

where $\mathbf{u}(n)$ is a white random variable with zero mean and covariance matrix $Q_{u}(n)$, and $\Gamma$ is the channel correlation matrix, given by

$$
\begin{equation*}
\Gamma=\operatorname{diag}\left\{a_{1}^{0}, \ldots, a_{1}^{L}, \ldots, a_{K}^{0}, \ldots, a_{K}^{L}\right\} \tag{61}
\end{equation*}
$$

where the scalar factor $\left\{a_{k}^{l}, k=1, \ldots, K, l=0, \ldots, L\right\}$ denotes the state transition coefficient of the $k$ th user in the $l$ th path. The above AR model for the channel parameter is only an approximation to the actual statistics of these random processes.

Similar to multiuser detection, for the UWB channel estimation the symbol matrix is treated as an unknown variable and uses the one-step prediction $\widehat{B}(n \mid n-1)$ instead of $B(n)$ and considers the estimation error $\widetilde{B}(n \mid n-1)$ as a multiplicative noise for channel estimation. The optimal estimation is given according to the following theorem.

Theorem 8. Consider the discrete-time state-space signal model (60) and (62); when the information symbol matrix $B(n)$ is unknown, the channel estimator is given by

$$
\begin{align*}
\widehat{\mathbf{h}}(n \mid n)= & {\left[I_{K(L+1)}-K_{h}(n) C \widehat{B}(n \mid n-1)\right] \widehat{\mathbf{h}}(n \mid n-1) } \\
& +K_{h}(n) \mathbf{r}(n), \tag{62}
\end{align*}
$$

where $\widehat{B}(n \mid n-1)$ is the one-step prediction of information symbol obtained from the previous section, and $K_{h}(n)$ is the estimator gain matrix:

$$
\begin{equation*}
K_{h}(n)=P_{h}(n) \widehat{B}^{T}(n \mid n-1) C^{T}\left[Q_{e}^{h}(n)\right]^{-1} \tag{63}
\end{equation*}
$$

where $Q_{e}^{h}(n)$ is the covariance matrix of innovation $\mathbf{e}_{h}(n)$ :

$$
\begin{align*}
Q_{e}^{h}(n)= & C \widehat{B}(n \mid n-1) P_{h}(n) \widehat{B}^{T}(n \mid n-1) C^{T} \\
& +C\left(\left[\begin{array}{|}
\Pi_{h}(n) & \Pi_{h}(n) \\
\hline \Pi_{h}(n) \mid \Pi_{h}(n)
\end{array}\right] \circ\left[P_{b}(n) \otimes I_{L+1}\right]\right) C^{T} \\
& +Q_{v}(n) \tag{64}
\end{align*}
$$

where $P_{b}(n)$ is the information symbol estimation error covariance matrix which is obtained in the previous section, and $\Pi_{h}(n)$ satisfies the following Lyapunov equation:

$$
\begin{equation*}
\prod_{h}(n+1)=\Gamma \prod_{h}(n) \Gamma^{T}+Q_{u}(n) \tag{65}
\end{equation*}
$$

where $P_{b}(n)$ is the channel estimation error covariance matrix and satisfies the following Riccati equation:

$$
\begin{equation*}
P_{h}(n+1)=\Phi P_{h}(n) \Phi-\Gamma K_{h}(n) Q_{e}^{h}(n) K_{h}^{T}(n) \Gamma+Q_{u}(n) . \tag{66}
\end{equation*}
$$

The one-step prediction of the information symbol is given by

$$
\begin{equation*}
\widehat{\mathbf{h}}(n+1 \mid n)=\Gamma \widehat{\mathbf{h}}(n \mid n) \tag{67}
\end{equation*}
$$

which will be used for the design of the user detector.


Figure 2: The proposed algorithm structure.

Proof. Consider the information symbol matrix $B(n)$ as an unknown variable; then we have

$$
\begin{equation*}
\widehat{\mathbf{r}}(n \mid n-1)=C \widehat{B}(n \mid n-1) \widehat{\mathbf{h}}(n \mid n-1) \tag{68}
\end{equation*}
$$

In view of (7) and Definition 4, we obtain

$$
\begin{align*}
\mathbf{e}_{h}(n)= & C \widehat{B}(n \mid n-1) \widetilde{\mathbf{h}}(n \mid n-1)  \tag{69}\\
& +C \widetilde{B}(n \mid n-1) \mathbf{h}(n)+\mathbf{v}(n)
\end{align*}
$$

It is apparent that $\mathbf{e}_{h}(n)$ is with zero mean and $E\left\{\mathbf{e}_{h}(s) \mathbf{e}_{h}(j)\right\}=$ 0 if $s \neq j$. The covariance matrix of $\mathbf{e}_{h}(n)$ is denoted as $Q_{e}^{h}(n)$. The following proof of this theorem is very similar to that of Theorem 5, so we omit it here.

Remark 9. Different from [7, 10], for UWB channel estimation, the users' symbols are also considered as unknown variables in this paper. The one-step prediction of symbol matrix is used, and the estimation error is treated as a multiplicative noise in matrix form. The detector and channel estimator are designed jointly and cannot be solved separately. The algorithm structure is as shown in Figure 2.

## 5. Conclusions

The information symbol and channel parameter are considered as unknown variables in this paper. The multiuser detector and UWB channel estimator are designed jointly. For symbol detection, the one-step predictor of channel parameter is used and the estimation error is treated as a multiplicative noise; then a Riccati equation and a Lyapunov equation will be needed. If the transmitted symbols are uncorrelated and identically distributed random variables with zero mean and unit variance, only a Riccati equation needs to be solved. For UWB channel estimation, the one-step predictor of information symbol is used and the estimation error is also considered as a multiplicative noise. The solutions to the above two problems are obtained by solving a couple of Riccati equations together with two Lyapunov equations.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding to the publication of this paper.

## Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant nos. 61203029, 61104050, and 61273124), the Natural Science Foundation of Shandong Province (Grant no. ZR2011FQ020), the Research Fund for the Doctoral Program of Higher Education of China (Grant no. 20120131120058), and the Scientific Research Foundation for Outstanding Young Scientists of Shandong Province (Grant no. BS2013DX008).

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