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Relativistic Model of Hamiltonian Renormalization for Bound States and Scattering Amplitudes

Abstract We test the renormalization group procedure for effective particles on a model of fermion–scalar interaction based on the Yukawa theory. The model is obtained by truncating the Yukawa theory to just two Fock sectors in the Dirac front form of Hamiltonian dynamics, a fermion, and a fermion and a boson, for the purpose of simple analytic calculation that exhibits steps of the procedure.

1 Introduction

The renormalization group procedure for effective particles (RGPEP) [1] is a tool developed for describing bound-states in QCD [2]. It has been shown that the RGPEP passes the test of producing asymptotic freedom in the front form Hamiltonian of QCD in third-order calculations in expansion in powers of the bare coupling constant [3,4]. Similar calculations for the quark–gluon coupling constant are yet to be done. However, the lowest order required for studying nontrivial effects of nonabelian gauge group of QCD in bound-state dynamics is fourth. In this article, we present a simple Hamiltonian model stemming from Yukawa field theory [5,6], in which we apply the RGPEP in order to verify its utility in fourth-order calculations and dynamics of bound states. The model simplicity allows us to illustrate the properties of an effective theory by a precise example, including bound states. Our analytic results in the simple model are helpful in organizing our thinking about fourth-order derivation of effective QCD, which is needed in calculations of gluon dynamics in heavy-quarkonia, *cf.* [7]. The model we study has been studied before and solved non-perturbatively by Głazek and Perry [5]. Their results were reproduced up to fourth order by Masłowski and Więckowski using similarity renormalization group procedure [6]. Our analysis differs by using the RGPEP with a new generator, which is known to apply well in third-order derivation of effective QCD and can be used in fourth-order. In the next sections we present the model, renormalize it, and study the effective fermion–boson coupling constant.

2 Model Theory

The construction of the model Hamiltonian starts with the Lagrangian of Yukawa theory,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\mu^2\phi^2 + \bar{\psi}(i\slashed{\partial} - m)\psi - g\phi\bar{\psi}\psi, \tag{1}$$

where ψ is a fermion field and ϕ is a scalar field. From the Lagrangian, we obtain the Euler–Lagrange equations and the stress–energy tensor density $\mathcal{T}^{\mu\nu}$. In the front-form (FF) of dynamics [8], the hypersurface on which

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we quantize the theory is defined by setting $x^+ = x^0 + x^3 = 0$. Variables $x^- = x^0 - x^3$, $x^\perp = (x^1, x^2)$ constitute the FF "spatial" directions. Only half of the components of the fermion field are independent. These are $\psi_+ = \Lambda_+\psi$, where $\Lambda_\pm = \gamma^0\gamma^\pm/2$ are projection matrices. The part $\psi_- = \Lambda_-\psi$ dynamically depends on ψ_+ and ϕ. The FF energy density is

$$\mathcal{T}^{+-} = -\frac{1}{2}(\partial^\perp\phi)^2 + \frac{1}{2}\mu^2\phi^2 + \psi_+^\dagger(i\alpha^\perp\partial^\perp + \beta m + g\beta\phi)\frac{1}{i\partial^+}(i\alpha^\perp\partial^\perp + \beta m + g\beta\phi)\psi_+, \quad (2)$$

where $1/i\partial^+$ is a result of solving the constraint equation for ψ_-. The quantum Hamiltonian is obtained by replacing the classical fields ψ_+ and ϕ by quantum operators,

$$\psi_+(x) = \Lambda_+ \sum_\sigma \int [p] \left[u_\sigma(p)b_{p\sigma}e^{-ipx} + v_\sigma(p)d_{p\sigma}^\dagger e^{ipx} \right]\Big|_{x^+=0}, \quad (3)$$

$$\phi(x) = \int [k] \left[a_k e^{-ikx} + a_k^\dagger e^{ikx} \right]\Big|_{x^+=0}, \quad (4)$$

where the integration measure is $[k] = \theta(k^+)dk^+d^2k^\perp/16\pi^3k^+$ and the creation and annihilation operators obey canonical commutation, or anticommutation, relations

$$\left[a_p, a_{p'}^\dagger\right] = 2(2\pi)^3 p^+ \delta(p^+ - p'^+)\delta^{(2)}(p^\perp - p'^\perp), \quad (5)$$

$$\left\{b_{p\sigma}, b_{p'\sigma'}^\dagger\right\} = 2(2\pi)^3 p^+ \delta(p^+ - p'^+)\delta^{(2)}(p^\perp - p'^\perp)\delta_{\sigma\sigma'}. \quad (6)$$

We omit the anticommutation relations for antiparticles.

The quantum canonical Hamiltonian, defined as integral of normal ordered product of fields given in Eq. (2), is ill-defined, because it leads to divergent integrals in loop corrections to energies of states. In fact, any estimate of the order of magnitude of the interaction energy gives infinity [9]. Therefore, to properly define the quantum Hamiltonian we need to regulate and renormalize it.

To simplify the renormalization problem we enormously simplify the theory by restricting the Hilbert space of states to one fermion, $|1\rangle = b_{p_1\sigma_1}^\dagger|0\rangle$, and one fermion and one boson, $|2\rangle = b_{p_2\sigma_2}^\dagger a_{k_2}^\dagger|0\rangle$, where $|0\rangle$ is the vacuum state. In such truncated space only three interaction terms are active: creation of boson from a fermion, its Hermitian conjugation, and the so-called seagull term [see Eq. (8) below]. The truncated Hamiltonian still leads to divergences. We are interested in the elements of the RGPEP that are intact in the truncated model.

We regularize the interaction terms by restricting the invariant mass squared of the particles in the ingoing and outgoing states by Λ^2. The regulating function is

$$\theta_2^\Lambda = \theta\left[\Lambda^2 - \mathcal{M}^2(x, \kappa)\right], \quad (7)$$

where θ is the Heaviside theta step function and $\mathcal{M}^2(x, \kappa)$ is the invariant mass squared of the particles in a state $|2\rangle$. The invariant mass squared is a function of relative momenta $x = k_2^+/\mathcal{P}^+$ and $\kappa^\perp = k_2^\perp - x\mathcal{P}^\perp$, where $\mathcal{P}^\mu = k_2^\mu + p_2^\mu$. The model Hamiltonian is

$$H_{\text{model}} = \int_1 p_1^- |1\rangle\langle 1| + \int_2 \left(p_2^- + k_2^-\right)|2\rangle\langle 2| + g\int_{21} \theta_2^\Lambda \, \tilde{\delta} \, \bar{u}_{\sigma_2}(p_2)u_{\sigma_1}(p_1)|2\rangle\langle 1| + H.c.$$

$$+ g^2 \int_{22'} \theta_2^\Lambda\theta_{2'}^\Lambda \, \tilde{\delta} \, \bar{u}_{\sigma_2}(p_2)\frac{\gamma^+}{2\mathcal{P}^+}u_{\sigma_2'}\left(p_2'\right)|2\rangle\langle 2'| + X_\Lambda, \quad (8)$$

where $\tilde{\delta}$ denotes the three-momentum conservation Dirac δ-functions and integral symbols contain integrals over momentum variables as well as sums over spins. X_Λ contains any counterterms that are needed for the effective theory not to depend on the cutoff parameter Λ.

3 The RGPEP

Regularized theory is well-defined in the sense that the solutions to the Hamiltonian eigenvalue problem exist for finite Λ. Nevertheless, the eigenvalues and eigenvectors depend badly on the cutoff parameter Λ. In other words, they depend on the number of momentum scales one has to sum over [9]. To eliminate dependence of physical quantities on Λ we introduce the concept of effective particle. Detailed presentation of the RGPEP can be found in Ref. [1].

The effective particle is defined as the state created by the effective creation operator. The latter is produced by a unitary rotation operator $\mathcal{U}_t$ from the initial, bare particle operator:

$$q_t = \mathcal{U}_t\, q_0\, \mathcal{U}_t^\dagger, \tag{9}$$

where q_0 denotes any of the initial creation and annihilation operators $b_{p\sigma}$, $b_{p\sigma}^\dagger$, a_k, $a_k^\dagger$, and q_t denotes their effective counterparts. t is a scale parameter whose fourth root has interpretation of the size of effective particles. It can assume positive values. The FF vacuum state, $|0\rangle$, is annihilated by annihilation operators irrespective of the value of t. Any state in our truncated Fock space can be constructed using any one of the operator bases: the canonical one at $t = 0$ or effective ones at any value of $t > 0$. In particular, the basis states are related to each other in the following way,

$$|1\rangle_t = b_{t\,p_1\sigma_1}^\dagger|0\rangle = \mathcal{U}_t|1\rangle, \quad |2\rangle_t = b_{t\,p_2\sigma_2}^\dagger a_{t\,k_2}^\dagger|0\rangle = \mathcal{U}_t|2\rangle. \tag{10}$$

The Hamiltonian of the theory can be expressed with use of either q_0 or q_t,

$$H_t(q_t) = H_0(q_0), \tag{11}$$

where $H_0(q_0)$ means that the Hamiltonian is expressed using operators q_0 and coefficients in front of their products are the ones in the initial theory, while $H_t(q_t)$ is the same Hamiltonian expressed using operators q_t and the coefficients in front of them are functions of t. For technical reasons, we introduce also $H_t(q_0) \equiv \mathcal{H}_t$. In the model,

$$\mathcal{H}_t = \int_{21} \left[\tilde{\delta}\, \widetilde{\mathcal{H}}_t(2;1)\, |2\rangle\langle 1| + H.c. \right] + \int_{22'} \tilde{\delta}\, \widetilde{\mathcal{H}}_t(2;2')\, |2\rangle\langle 2'| + \int_{11'} \tilde{\delta}\, \widetilde{\mathcal{H}}_t(1;1')|1\rangle\langle 1'|. \tag{12}$$

The unitary rotation $\mathcal{U}_t$ and the family of Hamiltonians $\mathcal{H}_t$ are defined through the RGPEP evolution equation

$$\frac{d}{dt}\mathcal{H}_t = \left[[\mathcal{H}_f, \mathcal{H}_{Pt}], \mathcal{H}_t \right], \tag{13}$$

with the initial condition

$$\mathcal{H}_0 = H_0(q_0) = H_{\text{model}}. \tag{14}$$

$\mathcal{H}_f = H_{\text{model}}|_{g=0}$ is the free part of $\mathcal{H}_t$ and $\mathcal{H}_{Pt}$ is the same as $\mathcal{H}_t$ except that every term is multiplied by the square of sum of $+$ momenta of the ingoing particles. Equation (13) implies $\mathcal{U}_t = T \exp\left(-\int_0^t d\tau [\mathcal{H}_f, \mathcal{H}_{P\tau}] \right)$, where T denotes ordering in τ. The double commutator structure of Eq. (13) ensures that the effective particles do not interact unless the difference in free invariant masses between ingoing and outgoing states in the interaction vertex is smaller than $\lambda = t^{-1/4}$. In the lowest order (in g) effective vertex,

$$\widetilde{\mathcal{H}}_t(2;1) = \theta_2^\Lambda\, g\, e^{-t(\mathcal{M}_2^2 - m^2)^2} \bar{u}_{\sigma_2}(p_2)u_{\sigma_1}(p_1) + O(g^3). \tag{15}$$

The form factor $e^{-t(\mathcal{M}_2^2 - m^2)^2}$ falls exponentially with the free invariant mass of fermion–boson state effectively preventing the interaction from happening when $\mathcal{M}_2^2 - m^2 \gg t^{-1/2}$. Similar form factor, $e^{-t(\mathcal{M}_2^2 - \mathcal{M}_{2'}^2)^2}$, is present in the $2' \to 2$ vertex.

Because the effective interactions are suppressed by the form factors, we expect that in the effective theory no dependence on $\Lambda \to \infty$ should arise, not just in observables but in all Hamiltonian matrix elements between states of finite kinematical quantum numbers. The initial theory and the effective one are equivalent, and we evaluate the latter from the former. When the former leads to divergences, we have to adjust it so that the effective theory is free from divergences. This is achieved by introducing counterterms in the initial theory. We impose the following prescription for the counterterms in the initial theory:

1. Propose the initial theory.
2. Calculate the effective theory.
3. If any matrix element of the effective theory Hamiltonian is divergent when $\Lambda \to \infty$, then add appropriate counterterm to the initial theory (unique up to the finite part in the Λ-dependent functions), which cancels the divergence.
4. Constrain finite parts of counterterms by available kinematical symmetry requirements.
5. Repeat steps 2–4 until the matrix elements of effective Hamiltonians are free from divergences and obey kinematical symmetries.

Once the counterterms in the initial theory are found, one can solve the dynamical problems in the finite effective theories and adjust the finite parts of the counterterms to data. The last step corresponds to expressing bare constants in terms of observables in perturbative calculations of observables. One can choose freely the RGPEP scale parameter t of an effective theory used to adjust the finite free parameters. Effective theories with different t are related by the RGPEP Eq. (13). One can simplify the description of phenomena of interest by choosing t, which is analogous to procedures known in literature [10].

4 Calculation of Counterterms

In the first order of perturbative expansion in powers of g, the effective Hamiltonian acquires a form factor, see Eq. (15). In the second order, we have effective $2' \to 2$ vertex, which does not require counterterm. We also have the effective mass term,

$$\widetilde{\mathcal{H}}_{t2}(1; 1) = \widetilde{\mathcal{H}}_{02}(1; 1) + \int \frac{dx d^2\kappa \, \theta_2^\Lambda}{16\pi^3 x(1-x)} \frac{e^{-2t(\mathcal{M}^2 - m^2)^2} - 1}{\mathcal{M}^2(x, \kappa) - m^2} \sum_{\sigma_2} \bar{u}_{\sigma_1}(p_1) u_{\sigma_2}(p_2) \bar{u}_{\sigma_2}(p_2) u_{\sigma_1}(p_1), \quad (16)$$

where $\widetilde{\mathcal{H}}_{02}(1; 1)$ is the counterterm to be determined.

The part of the integrand multiplied by $e^{-2t(\mathcal{M}^2 - m^2)^2}$ falls off to zero quickly for large transverse momentum κ. Therefore, that part depends very little on Λ when $\Lambda \to \infty$. However, the numerator of the integrand contains also 1, which is subtracted from $e^{-2t(\mathcal{M}^2 - m^2)^2}$. This 1 gives a part of integral, which diverges when $\Lambda \to \infty$. The product of spinors $\bar{u}_1 u_2 \bar{u}_2 u_1$ behaves like κ^2 for large relative transverse momentum, so does $\mathcal{M}^2$ in the denominator. Therefore, the integration over κ gives the leading term of order Λ^2, which is badly divergent for $\Lambda \to \infty$. To cancel this divergence the counterterm is defined as a term of the same operator structure with a coefficient that cancels the diverging number. Hence,

$$\widetilde{\mathcal{H}}_{02}(1; 1) = \omega^2 + 2m^2(\alpha + \beta) + \widetilde{\mathcal{H}}_{02}^{\text{finite}}, \tag{17}$$

where

$$\omega^2 = \int \frac{dx d^2\kappa \, \theta_2^\Lambda}{16\pi^3 x(1-x)} (1-x), \tag{18}$$

$$\alpha = \int \frac{dx d^2\kappa \, \theta_2^\Lambda}{16\pi^3 x(1-x)} \frac{1-x}{\mathcal{M}^2(x, \kappa) - m^2}, \tag{19}$$

$$\beta = \int \frac{dx d^2\kappa \, \theta_2^\Lambda}{16\pi^3 x(1-x)} \frac{1}{\mathcal{M}^2(x, \kappa) - m^2}, \tag{20}$$

and $\widetilde{\mathcal{H}}_{02}^{\text{finite}}$ is the finite part, on which we concentrate in the next paragraph. Division of the counterterm into ω^2, which is quadratically divergent and α and β, which are logarithmically divergent, is dictated by utility of these symbols in the higher order calculations.

To fix the finite part of the mass counterterm we need a physical condition. We demand that the physical fermion state is a solution of the effective Hamiltonian eigenproblem for some t with the FF energy eigenvalue fulfilling the relativistic dispersion relation,

$$H_t |\mathcal{P}\sigma\rangle_{\text{phys},t} = \frac{m_{\text{phys}}^2 + \mathcal{P}^{\perp 2}}{\mathcal{P}^+} |\mathcal{P}\sigma\rangle_{\text{phys},t}, \tag{21}$$

where m_{phys} is the physical mass of the fermion and

$$|\mathcal{P}\sigma\rangle_{\text{phys},t} = \int_1 \mathcal{P}^+\tilde{\delta}\, c^\sigma_{\sigma_1 t}|1\rangle_t + \int_2 \mathcal{P}^+\tilde{\delta}\, \phi^\sigma_{\sigma_2 t}(x,\kappa)|2\rangle_t. \tag{22}$$

We solve Eq. (21) up to the second order in the expansion in powers of g and find that the solution exists if the following constraints are fulfilled,

$$m = m_{\text{phys}}, \quad \widetilde{\mathcal{H}}^{\text{finite}}_{02} = 0. \tag{23}$$

Moreover, the effective wavefunction of the physical fermion is

$$\phi^\sigma_{\sigma_2 t}(x,\kappa) = g\frac{e^{-t(\mathcal{M}_2^2-m^2)^2}}{m^2-\mathcal{M}_2^2}\sum_{\sigma'}\bar{u}_{\sigma_2}(p_2)u_{\sigma'}(\mathcal{P})c^\sigma_{\sigma't} + O(g^3). \tag{24}$$

This result reveals that as we increase t, the contribution of the two-particle component to the physical fermion decreases because of the form factor. In other words, the bigger the size parameter t the more similar the effective fermion to the physical one.

In the third order in g, the effective theory contains only fermion $\to$ fermion–boson vertex (and its Hermitian conjugate). The counterterm, which secures finiteness of the effective third order vertex when $\Lambda \to \infty$ is

$$\widetilde{\mathcal{H}}_{03}(2;1) = m(\alpha+\beta+A)\bar{u}_{\sigma_2}(p_2)\frac{\gamma^+}{2\mathcal{P}^+}u_{\sigma_1}(p_1) + \frac{\alpha+B}{2}\bar{u}_{\sigma_2}(p_2)u_{\sigma_1}(p_1), \tag{25}$$

where A and B are finite parts of the logarithmically divergent functions in front of the two different spinor structures of the counterterm. The counterterm divergence can be absorbed into parameters of the initial theory. The first term on the right hand side of Eq. (25) shifts the mass of the fermion in the fermion sector by $\delta m = g^2 m(\alpha+\beta+A)$. The second term shifts the coupling constant. The fact that the fermion mass may be different in different Fock sectors is a feature of the Tamm–Dancoff-truncated theories [5,11]. The $1 \to 2$ vertex with counterterm divergences absorbed in the initial theory parameters, is

$$\widetilde{\mathcal{H}}_0(2;1) = \theta_2^\Lambda\, g_\Lambda\, \bar{u}_m(p_2,\sigma_2)u_{m_\Lambda}(p_1,\sigma_1), \tag{26}$$

where u_m means a spinor with mass m, u_{m_Λ} is a spinor with mass m_Λ and

$$g_\Lambda = g + (\alpha+B)g^3 + \cdots, \quad m_\Lambda = m + g^2 m(\alpha+\beta+A) + \cdots. \tag{27}$$

For the effective third-order vertex, see Sect. 5.

The fourth-order calculation of the effective Hamiltonian $\mathcal{H}_t$ reveals the form of the fourth-order counterterms. They are

$$\widetilde{\mathcal{H}}_{04}(2;2') = (\alpha+C)\bar{u}_{\sigma_2}(p_2)\frac{\gamma^+}{2\mathcal{P}^+}u_{\sigma_{2'}}(p_{2'}) \tag{28}$$

for the the seagull interaction vertex and

$$\widetilde{\mathcal{H}}_{04}(1,1') = \delta_{\sigma_1\sigma_{1'}}\left[(\alpha+B)\omega^2 + 2m^2(\alpha+B+A)(\alpha+\beta) + m^2(\alpha+\beta)^2\right] + \widetilde{\mathcal{H}}^{\text{finite}}_{04} \tag{29}$$

for the fermion mass term. C and $\widetilde{\mathcal{H}}^{\text{finite}}_{04}$ are finite parts of the counterterms.

To fix the finite part of the mass counterterm we again use Eq. (21). Solving it to fourth order in g gives us

$$\widetilde{\mathcal{H}}^{\text{finite}}_{04} = 0. \tag{30}$$

The RGPEP respects 7 kinematical symmetries. In order to secure the full Poincare symmetry in the model, we simply demand that the symmetries corresponding to the dynamical symmetry generators are directly visible in the fermion–boson $\to$ fermion–boson scattering amplitude. The fourth-order contribution to the T matrix is

$$g^4\,\theta_{2_o}^\Lambda\theta_{2_i}^\Lambda\,\bar{u}_{\sigma_{2o}}(p_{2o})\left[\Gamma_1(\mathcal{P}^2)\slashed{\mathcal{P}} + \Gamma_2(\mathcal{P}^2)m + \Gamma_3(\mathcal{P}^2)\frac{\gamma^+}{2\mathcal{P}^+}\right]u_{\sigma_{2i}}(p_{2i}), \tag{31}$$

where $\mathcal{P}$ is the total four-momentum, evaluated using the physical mass parameters for the boson and fermion. Hence, $\mathcal{P}^2$ is the physical invariant mass squared of the incoming or outgoing particles. Γ_1, Γ_2 and Γ_3 are finite. In particular,

$$\Gamma_3(\mathcal{P}^2) = B - C - \frac{2m^2}{\mathcal{P}^2 - m^2} A. \tag{32}$$

The only term breaking the Lorentz covariance of the scattering amplitude is the one multiplied by Γ_3. Therefore, we demand that $\Gamma_3(\mathcal{P}^2) = 0$. The counterterms securing Lorentz covariance of the scattering amplitude are obtained without introducing functions of momenta by setting

$$A = 0, \quad B = C. \tag{33}$$

B remains unspecified, which allows one to freely choose at which scale coupling constant g is defined.

5 Running of the Effective Hamiltonian Coupling Constant

Beside the prescription for the counterterms, RGPEP provides the family of equivalent effective theories numbered with parameter t. For example, the effective fermion–boson–fermion vertex is

$$\widetilde{\mathcal{H}}_t(2; 1) = \theta_2^{\Lambda} \, \tilde{g}_t \left(\mathcal{M}_2^2 \right) \, e^{-t(\mathcal{M}_2^2 - m^2)^2} \cdot \bar{u}_{\sigma_2}(p_2) \left[1 + \delta m_t \left(\mathcal{M}_2^2 \right) \frac{\gamma^+}{2\mathcal{P}^+} \right] u_{\sigma_1}(p_1), \tag{34}$$

where $\tilde{g}_t(\mathcal{M}_2^2) = g + B_t(\mathcal{M}_2^2)g^3$. The quantities $B_t(\mathcal{M}_2^2)$ and $\delta m_t(\mathcal{M}_2^2)$ are finite when $\Lambda \to \infty$ but quite complicated. We do not write them explicitly. The square bracket above is similar to the one present in the initial theory, which is interpreted as shifting the mass of the fermion in the fermion sector by δm, cf. Eq. (27). In the effective theory, however, δm depends on the free invariant mass $\mathcal{M}_2$ of the outgoing fermion–boson state. For t much smaller than m^{-4} this dependence is negligible.

Almost every element of Eq. (34) depends on the invariant mass $\mathcal{M}_2$. Therefore, to clarify the picture, we define effective coupling constant $g_t = \tilde{g}_t(m^2)$ and rewrite the effective vertex,

$$\widetilde{\mathcal{H}}_t(2; 1) = \theta_2^{\Lambda} \, g_t \, f_t \left(\mathcal{M}_2^2 \right) \cdot \bar{u}_{\sigma_2}(p_2) \left[1 + \delta m_t(\mathcal{M}_2^2) \frac{\gamma^+}{2\mathcal{P}^+} \right] u_{\sigma_1}(p_1), \tag{35}$$

where f_t is a new form factor, which contains second order corrections to the exponential form factor of Eq. (15). In this form, one can interpret the effective vertex. First of all, its strength is characterized by the effective coupling constant g_t, which for $t \ll m^{-4}$ depends on t in the following way,

$$g_t = g_{t_0} + \frac{g_{t_0}^3}{32\pi^2} \log \frac{\lambda}{\lambda_0} + \cdots, \quad \lambda \gg m, \tag{36}$$

where $\lambda = t^{-1/4}$ and $\lambda_0 = t_0^{-1/4}$. It is noteworthy that the bare coupling, cf. Eq. (27), in the initial Hamiltonian depends in the same way on Λ for $\Lambda \gg m$. This finding is a manifestation of the fact that, in the effective theory, the finite-width vertex form factors assume the role analogous to the regulating role played by the Heaviside θ-functions with cutoff parameter Λ in the initial theory. Therefore, in practice, one can omit θ_2^{Λ} in the effective-theory Eq. (35). Moreover, higher-order calculations introduce further corrections to the vertex form factors and parameters like δm, because we have freedom of choosing t while in exact calculations no observable depends on t.

6 Concluding Remarks

The counterterms found in the model Hamiltonian with use of the RGPEP in its most recent form agree with the ones found previously using similarity [6]. We also provide the lowest order effective wave function of the physical fermion and an example of the Hamiltonian running coupling constant in the effective theory. An interesting further study of the model would be an exploration of nonperturbative solutions to the RGPEP evolution Eq. (13).

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