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Divergence-free vector fields with inverse shadowing

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Abstract

We show that if a divergence-free vector field has the C^1-stably orbital inverse shadowing property with respect to the class of continuous methods $\mathcal{T}_d$, then the vector field is Anosov. The results extend the work of Bessa and Rocha (J. Differ. Equ. 250:3960-3966, 2011).

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1 Introduction

The notion of inverse shadowing property is a dual notion of the shadowing property. It was studied by [1–7]. In fact, Pilyugin [7] showed that every structurally stable diffeomorphism has the inverse shadowing property with respect to the class of continuous methods. In [3], Lee proved that if a diffeomorphism has the C^1-stably inverse shadowing property with respect to the class of continuous methods $\mathcal{T}_d$, then the diffeomorphism is structurally stable. For vector fields, Lee and Lee [5] introduced the notion of inverse shadowing for flows and showed that every expansive flow with the shadowing property has the inverse shadowing property with respect to the class of continuous methods. Lee *et al.* [6] showed that the C^1-interior of the set of vector fields with the orbital shadowing property with respect to the class $\mathcal{T}_d$ coincides with the set of structurally stable vector fields. From the facts, Lee [4] showed that if a volume-preserving diffeomorphism has the C^1-stably inverse shadowing property with respect to the class $\mathcal{T}_d$, then the diffeomorphism is Anosov. Moreover, if a volume-preserving diffeomorphism has the C^1-stably orbital inverse shadowing property with respect to the class $\mathcal{T}_d$, then the diffeomorphism is Anosov. In this spirit, we study divergence-free vector fields with the inverse, orbital inverse shadowing property with respect to the class $\mathcal{T}_d$.

Let us be more detailed. Let M be a closed, connected, and C^∞ Riemannian manifold endowed with a volume form μ, and let μ denote the Lebesgue measure associated to it. Given a C^r ($r \geq 1$) vector field $X: M \to TM$, the solution of the equation $x' = X(x)$ gives rise to a C^r flow, X^t; on the other hand, given a C^r flow, we can define a C^{r-1} vector field by considering $X(x) = \frac{dX^t(x)}{dt}\big|_{t=0}$. We say that X is *divergence-free* if its divergence is equal to zero. Note that, by the Liouville formula, a flow X^t is volume-preserving if and only if the corresponding vector field X is divergence-free. Let $\mathfrak{X}^r_\mu(M)$ denote the space of C^r divergence-free vector fields, and we consider the usual C^r Whitney topology on

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this space. Let $X \in \mathfrak{X}_\mu^1(M)$ for any $\delta > 0$ and $T > 0$. We say that a (δ, T)-pseudo orbit of $X \in \mathfrak{X}_\mu^1(M)$ if

$$d\big(X^{t_i}(x_i), x_{i+1}\big) < \delta,$$

for any $t_i \geq T$, $i \in \mathbb{Z}$. A mapping $\Psi : \mathbb{R} \times M \to M$ is called a (δ, T)-*method* for X if for any $x \in M$, the map $\Psi_x : \mathbb{R} \to M$ defined by

$$\Psi_x(t) = \Psi(t, x), \quad t \in \mathbb{R},$$

is a (δ, T)-pseudo orbit of X. Ψ is said to be *complete* if $\Psi(0, x) = x$ for any $x \in M$. Then a (δ, T)-method for X can be considered a family of (δ, T)-pseudo orbits of X. A method Ψ for X is said to be *continuous* if the map $\Psi' : M \to M^\mathbb{R}$, given by $\Psi'(x)(t) = \Psi(t, x)$ for $x \in M$ and $t \in \mathbb{R}$, is continuous under the compact open topology on $M^\mathbb{R}$, where $M^\mathbb{R}$ denotes the set of all functions from $\mathbb{R}$ to M. The set of all complete $(\delta, 1)$-methods [resp. complete continuous $(\delta, 1)$-methods] for $X \in \mathfrak{X}_\mu^1(M)$ will be denoted by $\mathcal{T}_a(\delta, X)$ [resp. $\mathcal{T}_c(\delta, X)$]. We can see that if Y is another vector field which is sufficiently close to X in the C^0 topology, then it induces a complete continuous method for X. Let $\mathcal{T}_h(\delta, X)$ [resp. $\mathcal{T}_d(\delta, X]$ be the set of all complete continuous $(\delta, 1)$ methods for X which are induced by C^1 vector fields Y with $d_0(X, Y) < \delta$ [resp. $d_1(X, Y) < \delta$], where d_0 is the C^0 metric on $\mathfrak{X}_\mu^1(M)$ such that

$$d_0(X, Y) = \sup_{x \in M}\big\{\big\|X(x) - Y(x)\big\|\big\},$$

and d_1 is the C^1 metric on $\mathfrak{X}_\mu^1(M)$ such that

$$d_1(X, Y) = d_0(X, Y) + \sup_{x \in M}\big\|DX(x) - DY(x)\big\|.$$

We say that a vector field X has the *inverse shadowing property* with respect to the class $\mathcal{T}_\alpha$ ($\alpha = a, c, h, d$) if for any $\epsilon > 0$, there is $\delta > 0$ such that for any (δ, T)-method $\Psi \in \mathcal{T}_\alpha(\delta, X)$ and any point $x \in M$, there are $y \in M$ and an increasing homeomorphism $h : \mathbb{R} \to \mathbb{R}$ with $h(0) = 0$ such that

$$d\big(X^{h(t)}(x), \Psi(t, y)\big) < \epsilon, \quad t \in \mathbb{R}.$$

We denote by $\mathcal{IS}_{\mu,\alpha}(M)$ the set of divergence-free vector fields on M with the inverse shadowing property with respect to the class $\mathcal{T}_\alpha$, where $\alpha = a, c, h, d$. Let int $\mathcal{IS}_{\mu,\alpha}(M)$ be the C^1-interior of the set of divergence-free vector fields on M with the inverse shadowing property with respect to the class $\mathcal{T}_\alpha$, where $\alpha = a, c, h, d$.

We say that $X \in \mathfrak{X}_\mu^1(M)$ has the C^1-stably inverse shadowing property with respect to the class $\mathcal{T}_\alpha$ if there is a C^1-neighborhood $\mathcal{U}(X) \subset \mathfrak{X}_\mu^1(M)$ of X such that for any $Y \in \mathcal{U}(X)$, Y has the inverse shadowing property with respect to the class $\mathcal{T}_d$, where $\alpha = a, c, h, d$.

Remark 1.1 Let $X \in \mathfrak{X}_\mu^1(M)$. Then $\mathcal{IS}_{\mu,a}(M) \subset \mathcal{IS}_{\mu,c}(M) \subset \mathcal{IS}_{\mu,h}(M) \subset \mathcal{IS}_{\mu,d}(M)$, where $\alpha = a, c, h, d$.

We say that $X \in \mathfrak{X}_\mu^1(M)$ has the orbital inverse shadowing property with respect to the class $\mathcal{T}_d$ if for any $\epsilon > 0$, there is $\delta > 0$ such that for any $(\delta, 1)$-method $\Psi \in \mathcal{T}_d(\delta, X)$ and any point $x \in M$, there is $y \in M$ such that

$$d_H\big(\overline{\text{Orb}_X(x)}, \overline{\text{Orb}(y, \Psi)}\big) < \epsilon,$$

where $\text{Orb}(y, \Psi) = \{\Psi(t, y) : t \in \mathbb{R}\}$. Note that if X has the inverse shadowing property with respect to the class $\mathcal{T}_d$, then X has the orbital shadowing property with respect to the class $\mathcal{T}_d$. But the converse is not true. Indeed, an irrational rotation map does not have the inverse shadowing property. And the map has the orbital shadowing property. Let int $\mathcal{OIS}_{\mu,\alpha}(M)$ be the C^1-interior of the set of divergence-free vector fields on M with the orbital inverse shadowing property with respect to the class $\mathcal{T}_\alpha$, where $\alpha = a, c, h, d$.

Let Λ be a closed X_t-invariant set. We say that Λ is *hyperbolic* if there are constants $C > 0$, $\lambda > 0$ and a continuous splitting $T_\Lambda M = E^s \oplus \langle X(x) \rangle \oplus E^u$ such that

$$\big\| DX^t|_{E^s(x)} \big\| \leq Ce^{-\lambda t} \quad \text{and} \quad \big\| DX^{-t}|_{E^u(x)} \big\| \leq Ce^{-\lambda t}$$

for any $x \in \Lambda$ and $t > 0$. If $\Lambda = M$, then X is called *Anosov*.

Given a vector field X, we denote by $PO(X)$ the set of closed orbits of X and by $Sing(X)$ the set of *singularities* of X, *i.e.*, those points $x \in M$ such that $X(x) = \vec{0}$. Denote by $Crit(X) = Sing(X) \cup PO(X)$. Let $R := M \setminus Sing(X)$ be the set of *regular* points. We assume that the exponential map $\exp_p : T_p M(1) \to M$ is well defined for all $p \in M$, where $T_p M(1) = \{v \in T_p M : \|v\| \leq 1\}$. Given $x \in R$, we consider its normal bundle $N_x = \langle X(x) \rangle^\perp \subset T_x M$, and let $N_x(r)$ be the r-ball in N_x. Let $\mathcal{N}_{x,r} = \exp_x(N_x(r))$. For any $x \in R$ and $t \in \mathbb{R}$, there are $r > 0$ and a C^1 map $\tau : \mathcal{N}_{x,r} \to \mathbb{R}$ with $\tau(x) = t$ such that $X^{\tau(y)}(y) \in \mathcal{N}_{X^t(x),1}$ for any $y \in \mathcal{N}_{x,r}$. We say $\tau(y)$ is the *first return time* of y. Then we define the *Poincarè map f* by

$$f : \mathcal{N}_{x,r} \to \mathcal{N}_{X^t(x),1},$$

$$y \mapsto f(y) = X^{\tau(y)}(y).$$

Let $\mathcal{N} = \bigcup_{x \in R} \mathcal{N}_x$ be the normal bundle based on R. We can define the associated *linear Poincaré flow* by

$$P_X^t(x) := \Pi_{X^t(x)} \circ DX^t(x),$$

where $\Pi_{X^t(x)} : T_{X^t(x)} M \to N_{X^t(x)}$ is the projection along the direction of $X(X^t(x))$. We say that a vector field $X \in \mathfrak{X}_\mu^1(M)$ is *topologically stable* if for any $\epsilon > 0$, there is $\delta > 0$ such that for any $Y \in \mathfrak{X}_\mu^1(M)$ with $d_0(X, Y) < \delta$, there is a semi-conjugacy (h, τ) from Y to X satisfying $d(h(x), x) < \epsilon$ for all $x \in M$.

In [8], the authors proved that if a vector field is in the C^1-interior of the set of topologically stable vector fields (not divergence-free vector fields), then X satisfies Axiom A and the strong transversality condition. Robinson [9] proved that if a vector field satisfies Axiom A and the strong transversality condition, then the vector field is structurally stable. For divergence-free vector fields, Bessa and Rocha [10] showed that if a divergence-free vector field is in the C^1-interior of the set of topological stable vector fields $X \in \mathfrak{X}_\mu^1(M)$, then X is Anosov.

Remark 1.2 We have the following implication: topological stability $\Rightarrow$ inverse shadowing property with respect to the continuous method $\mathcal{T}_d$ $\Rightarrow$ orbital inverse shadowing property with respect to the continuous method $\mathcal{T}_d$.

From the above remark, we know that our result is a slight generalization of the main theorem in [10]. In this paper, we omit the phrase 'with respect to the class $\mathcal{T}_d$' for simplicity. So, we say that X has the inverse, orbital inverse shadowing property means that X has the inverse, orbital inverse shadowing property with respect to the class $\mathcal{T}_d$. Note that if $X \in \text{int}\,\mathcal{IS}^1_{\mu,\alpha}(M)$ or $X \in \text{int}\,\mathcal{OIS}^1_{\mu,\alpha}(M)$, then it means that X has the C^1-stably inverse, orbital inverse shadowing property with respect to the class $\mathcal{T}_\alpha$, $\alpha = a, c, h, d$. The following is the main result of this paper.

Theorem 1.3 *Let $X \in \mathfrak{X}^1_\mu(M)$. Then*

$$\text{int}\,\mathcal{IS}^1_{\mu,d}(M) = \text{int}\,\mathcal{OIS}^1_\mu(M) = \mathcal{A}^1_\mu(M),$$

where $\mathcal{A}^1_\mu(M)$ is the set of divergence-free Anosov vector fields.

2 Proof of Theorem 1.3

Let M be as before, and let $X \in \mathfrak{X}^1_\mu(M)$. The perturbations (Lemma 2.1) for volume-preserving vector fields allows to realize them as perturbations of a fixed volume-preserving flow. Fix $X \in \mathfrak{X}^1_\mu(M)$ and $\tau > 0$. A *one-parameter area-preserving linear family* $\{A_t\}_{t \in \mathbb{R}}$ associated to $\{X^t(p); t \in [0, \tau]\}$ is defined as follows:

- $A_t : N_p \to N_p$ is a linear map for all $t \in \mathbb{R}$,
- $A_t = id$ for all $t \le 0$ and $A_t = A_\tau$ for all $t \ge \tau$,
- $A_t \in SL(n, \mathbb{R})$, and
- the family A_t is C^∞ on the parameter t.

The following result, proved in [11, Lemma 3.2], is now stated for $X \in \mathfrak{X}^1_\mu(M)$ instead of $X \in \mathfrak{X}^4_\mu(M)$ because of the improved *smooth C^1 pasting lemma* proved in [12, Lemma 5.2].

Lemma 2.1 *Given $\epsilon > 0$ and a vector field $X \in \mathfrak{X}^1_\mu(M)$, there exists $\xi_0 = \xi_0(\epsilon, X)$ such that for all $\tau \in [1, 2]$, for any periodic point p of period greater than 2, for any sufficiently small flowbox $\mathcal{T}$ of $\{X_t(p); t \in [0, \tau]\}$ and for any one-parameter linear family $\{A_t\}_{t \in [0,\tau]}$ such that $\|A'_t A_t^{-1}\| < \xi_0$ for all $t \in [0, \tau]$, there exists $Y \in \mathfrak{X}^1_\mu(M)$ satisfying the following properties:*

(a) *Y is ϵ-C^1-close to X;*
(b) *$Y^t(p) = X^t(p)$ for all $t \in \mathbb{R}$;*
(c) *$P^\tau_Y(p) = P^\tau_X(p) \circ A_\tau$, and*
(d) *$Y|_{\mathcal{T}^c} \equiv X|_{\mathcal{T}^c}$.*

Remark 2.2 Let $X \in \mathfrak{X}^1_\mu(M)$. By Zuppa's theorem [13], we can find Y C^1-close to X such that $Y \in \mathfrak{X}^\infty_\mu(M)$, $Y^\pi(p) = p$ and $P^\pi_Y(p)$ has an eigenvalue λ with $|\lambda| = 1$.

A divergence-free vector field X is a *divergence-free star vector field* if there exists a C^1-neighborhood $\mathcal{U}(X)$ of X in $\mathfrak{X}^1_\mu(M)$ such that if $Y \in \mathcal{U}(X)$, then every point in $Crit(Y)$ is hyperbolic. The set of divergence-free star vector fields is denoted by $\mathcal{G}^1_\mu(M)$. Then we get the following theorem.

Theorem 2.3 [14, Theorem 1.1] *If $X \in \mathcal{G}_\mu^1(M)$, then $Sing(X) = \emptyset$ and X is Anosov.*

Thus, to prove Theorem 1.3, it is enough to show that if X has the inverse shadowing property, or the orbital inverse shadowing property, then $X \in \mathcal{G}_\mu^1(M)$.

Proposition 2.4 *Let $X \in \mathrm{int}\,\mathcal{IS}_{\mu,d}^1(M)$. Then $X \in \mathcal{G}_\mu^1(M)$.*

Proof Suppose that $X \in \mathfrak{X}_\mu^1(M)$ has the C^1-stably inverse shadowing property. Then there is a C^1-neighborhood $\mathcal{U}(X)$ of X such that for any $Y \in \mathcal{U}(X)$, Y has the inverse shadowing property. Let $p \in \gamma \in PO(X)$ with $X^\pi(p) = p$ and U_p be a small neighborhood of $p \in M$. We will derive a contradiction. Assume that there is an eigenvalue λ of $P_X^\pi(p)$ such that $|\lambda| = 1$. By Remark 2.2, there is $Y \in \mathcal{U}(X)$ such that $Y \in \mathfrak{X}_\mu^\infty(M)$, $Y_Y^\pi(p) = p$ and $P_Y^\pi(p)$ has an eigenvalue λ with $|\lambda| = 1$. Using Moser's theorem (see [15]), there is a smooth conservative change of coordinates $\varphi_p : U_p \to T_p M$ such that $\varphi_p(p) = \vec{0}$. Let $f : \varphi_p^{-1}(N_p) \to \mathcal{N}_p$ be the Poincarè map associated to Y^t, and let $\mathcal{V}$ be a C^1-neighborhood of f. Here, $\mathcal{N}_p$ is the Poincarè section through p. By Lemma 2.1, we can find a small flowbox $\mathcal{T}$ of $Y^{[0,t_0]}$, $0 < t_0 < \pi$, and there are $Z \in \mathcal{U}_0(Y) \subset \mathcal{U}(X), g \in \mathcal{V}$ and $\alpha > 0$ such that

(a) $Z^t = Y^t$ for all $t \in \mathbb{R}$,

(b) $P_Z^{t_0}(p) = P_Y^{t_0}(p)$,

(c) $Z|_{\mathcal{T}^c} = Y|_{\mathcal{T}^c}$,

(d) $g(x) = \varphi_p^{-1} \circ P_Y^\pi(p) \circ \varphi_p(x)$ for all $x \in B_\alpha(p) \cap \varphi_p^{-1}(N_p)$, and

(e) $g(x) = f(x)$ for all $x \notin B_{4\alpha}(p) \cap \varphi_p^{-1}(N_p)$.

Then $P_Z^\pi(p)$ has an eigenvalue λ with $|\lambda| = 1$. For $0 < \epsilon < \alpha/8$, let $0 < \delta < \epsilon$ be as in the definition of the inverse shadowing property of Z^t.

Take a linear map $A_t : N_p \to N_p$ for all $t \in \mathbb{R}$ such that if $\|P_Z^\pi(p) \circ A_{t_0} - P_Z^\pi(p)\| < \delta_0$, then $d_1(Z, W) < \delta$, and $P_Z^\pi(p) \circ A_{t_0}$ is a hyperbolic linear Poincarè flow. Set $P_W^t(p) = P_Z^\pi(p) \circ A_{t_0}$. Then $W^t \in \mathcal{T}_d(Z)$, and $p \in \gamma$ is a periodic point of W^t. Since $Z \in \mathcal{U}(X)$ for any $x \in M$, there exist $y \in M$ and an increasing homeomorphism $h : \mathbb{R} \to \mathbb{R}$ with $h(0) = 0$ such that $d(Z^{h(t)}(x), W^t(y)) < \epsilon$ for all $t \in \mathbb{R}$.

First, we assume that $\lambda = 1$ (the other case is similar). Then we can choose a vector v associated to λ such that $\|v\| < \alpha/4$. Since $\varphi_p^{-1}(v) \in \varphi_p^{-1}(N_p) \setminus \{p\}$,

$$g\big(\varphi_p^{-1}(v)\big) = \varphi_p^{-1} \circ P_Y^\pi(p) \circ \varphi_p\big(\varphi_p^{-1}(p)\big) = \varphi_p^{-1} \circ P_Y^\pi(p)(v) = \varphi_p^{-1}(v).$$

Then we can take $z \in \varphi_p^{-1}(N_p)$ such that

$$p \notin B_\epsilon\big\{g_1^i(z) : i \in \mathbb{Z}\big\} \subset B_{\alpha/2}(p) \cap \varphi_p^{-1}(N_p),$$

where g_1 is the Poincarè map associated to W^t. Since $P_W^t(p)$ is a hyperbolic linear Poincarè flow, we can see that

$$d\big(g^i(p), g_1^i(z)\big) > \epsilon$$

for some $i \in \mathbb{Z}$. Thus

$$d\big(Z^{h(t)}(p), W^t(z)\big) > \epsilon$$

for some $t \in \mathbb{R}$. This is a contradiction by the fact that Z has the inverse shadowing property.

Finally, we assume that λ is complex. By [10, Lemma 3.2], there is $Z \in \mathcal{U}(X)$ such that $P_Z^\pi(p)$ is a rational rotation. Then there is $l > 0$ such that $P_Z^{l+\pi}(p)$ is the identity. As in the previous argument, we get a contradiction. □

Proposition 2.5 *Let $X \in \operatorname{int} \mathcal{OIS}_{\mu,d}^1(M)$. Then $X \in \mathcal{G}_\mu^1(M)$.*

Proof Suppose that X has the C^1-stably orbital inverse shadowing property. Then there is a C^1-neighborhood $\mathcal{U}(X)$ of X such that for any $Y \in \mathcal{U}(X)$, Y has the orbital inverse shadowing property. Let $p \in \gamma \in PO(X)$ with $X^\pi(p) = p$ and U_p be a small neighborhood of $p \in M$. We will derive a contradiction. Assume that there is an eigenvalue λ of $P_X^\pi(p)$ such that $|\lambda| = 1$. As in the proof of Proposition 2.4, we get a hyperbolic linear Poincarè flow $P_W^t(p)$, and $W^t \in \mathcal{T}_d(Z)$, and $p \in \gamma$ is a periodic point of W^t. Since $Z \in \mathcal{U}(X)$ for any $x \in M$, there exist $y \in M$ such that

$$d_H\left(\overline{\mathcal{O}(x, Z^t)}, \overline{\mathcal{O}(y, W^t)}\right) < \epsilon$$

for all $t \in \mathbb{R}$. Take $t' = \min\{|t| : W^t(y) \in \varphi_p^{-1}(N_p)\}$, and let $w = W^{t'}(y) \in \varphi_p^{-1}(N_p)$.

If $\lambda \in \mathbb{R}$ or $\lambda \in \mathbb{C}$, then as in the proof of Lemma 2.4, we get a contradiction. Indeed, since $P_W^t(p)$ is a hyperbolic linear Poincarè flow, there is $j > 0$ such that $g_1^j(w) \notin B_{\alpha/4}(p) \cap \varphi_p^{-1}(N_p)$. Thus

$$d_H\left(\overline{\mathcal{O}(x, Z^t)}, \overline{\mathcal{O}(w, W^t)}\right) > \epsilon$$

for some $t \in \mathbb{R}$. Since $Z \in \mathcal{U}(X)$, this is a contradiction. □

End of the proof of Theorem 1.3 By Proposition 2.4 and Proposition 2.5, we have $X \in \mathcal{G}_\mu^1(M)$. Thus by Theorem 2.3, we get $Sing(X) = \emptyset$ and X is Anosov. □

From the result of [10], we get the following corollary.

Corollary 2.6 *Let $X \in \mathfrak{X}_\mu^1(M)$. Then*

$$\operatorname{int} \mathcal{TS}_\mu^1(M) = \operatorname{int} \mathcal{IS}_{\mu,d}^1(M) = \operatorname{int} \mathcal{OIS}_\mu^1(M) = \operatorname{int} \mathcal{A}_\mu^1(M),$$

where $\operatorname{int} \mathcal{TS}_\mu^1(M)$ is the C^1-interior of the set of divergence-free vector fields on M which are topologically stable.

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