Research Article

Cross-Efficiency Evaluation Method with Compete-Cooperate Matrix

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Cross-efficiency evaluation method is an effective and widespread adopted data envelopment analysis (DEA) method with self-assessment and peer-assessment to evaluate and rank decision making units (DMUs). Extant aggressive, benevolent, and neutral cross-efficiency methods are used to evaluate DMUs with competitive, cooperative, and nontendentious relationships, respectively. In this paper, a symmetric (nonsymmetric) compete-cooperate matrix is introduced into aggressive and benevolent cross-efficiency methods and compete-cooperate cross-efficiency method is proposed to evaluate DMUs with diverse (relative) relationships. Deviation maximization method is applied to determine the final weights of cross-evaluation to enhance the differentiation ability of cross-efficiency evaluation method. Numerical demonstration is provided to illustrate the reasonability and practicability of the proposed method.

1. Introduction

Data envelopment analysis (DEA) is a nonparametric programming method for evaluating the relative efficiencies of a group of decision making units (DMUs) with multiple inputs and outputs. Since Charnes et al. [1] proposed the CCR model in 1978 and Banker et al. [2] proposed the BCC model in 1984, DEA is widely used in various fields. The traditional DEA models, including the CCR and BCC model, are based on self-assessment system; the obtained input and output weights of evaluated DMUs take the aim at maximizing their own efficiency, which will cause problems in three aspects. (1) The traditional DEA models can only distinguish the efficient and inefficient DMUs but cannot rank the merits and with a lower degree of differentiation on CCR-efficient DMUs. (2) The obtained efficiency weights are only beneficial to the single DMU, which is easy to exaggerate its own advantages in some inputs and outputs angles, but circumvent its disadvantages in other input and output angles, resulting in lip-deep efficient phenomena. (3) Each DMU selects its own favorable weighting scheme, lacking comparability among DMUs.

In response to these problems, scholars have proposed a number of improvements [3–5]; the typical methods include cross-efficiency evaluation method [6], public-weight method [7], superefficient DEA method [8], and other DEA methods, wherein the cross-efficiency evaluation method has been applied repeatedly, which is proposed by Sexton et al. [9] in 1986, as an expansion and improvement of traditional DEA model. The essence of the method is the introduction of peer-assessment system, using self-assessment and peer-assessment system to evaluate the efficiencies of DMUs. The cross-efficiency method is possible to get complete ranks and comparable evaluated scores, which has a higher degree of differentiation on CCR-efficient DMUs. Therefore, this method has been widespread and widely used to deal with specific problems in academic fields [10–13].

But the cross-efficiency method may fall into a predicament of multiple solutions, so many scholars have been keen on the improvement of the traditional cross-efficiency model. The typical treatments to avoid the problem include Doyle and Green's [14] aggressive and benevolent cross-efficiency evaluation methods, which introduce secondary objective functions to cross-efficiency evaluation method and can select the optimal weights to minimize and maximize the sum of the outputs of other DMUs, respectively. Later Wang and Chin [15], based on the aggressive and benevolent

methods, propose the neutral DEA model, which has effectively reduced the number of zero outputs' weights. Wu et al. [16] introduce secondary goals in cross-efficiency evaluation to avoid multiple solutions, they propose a novel model to determine the final cross-efficiency and optimize the ranking order, they indicate that pursuing the best ranking is more important than maximizing the individual score, and this model is able to draw the best ranking. Jahanshahloo et al. [17] also introduce secondary goals to cross-efficiency and select symmetric weights and propose the symmetric weight assignment technique (SWAT) method to effectively select weights from multiple optimal solutions. Wu et al. [18] propose a weight-balanced DEA method to deal with the nonunique cross-efficiency scores resulting from the presence of alternate optima in traditional DEA models. This method can effectively lessen the differences in weighted data and reduce the zero weights. Wang et al. [19] introduce a virtual ideal DMU (IDMU) and a virtual antideal DMU (ADMU) to cross-efficiency evaluation method, propose several new DEA models, and result in neutral cross-evaluation scores, which enhance the theory and methodology of cross-efficiency evaluation method and can be more neutral and logical. Wang et al. [20] introduce neutral input and output weights for each DMU, replace the aggressive or benevolent ones, thus minimize the virtual disparity in cross-efficiency evaluation, and reduce the number of zero weights.

In this paper, we mainly aim for the improvement on the practicality and application of cross-efficiency evaluation method. The benevolent, aggressive, and neutral cross-efficiency evaluation methods suppose that the relationships of DMUs are absolutely partnership, competitive, and nontendentious, respectively. But in practical applications, the following two situations generally exist. (1) The relationships of evaluated DMUs are complex; they not only involve partnership relationships but also involve competitive relationships. (2) The relationship of a pair of DMUs is relativity. A DMU regards another DMU as friend and partner, while another DMU regards the DMUs as enemies and rivals. Focusing on the two situations, we introduce a compete-cooperate matrix into aggressive and benevolent cross-efficiency methods and build compete-cooperate cross-efficiency model. Our method can effectively evaluate the efficiencies of DMUs which has complex relationships compared to extant cross-efficiency methods. In addition, extant cross-efficiency methods obtain the final scores of DMUs by calculating the average of self-assessment scores and peer-assessment scores. This method sets all DMUs on equal status, with lower degree of differentiation on self-assessment and peer-assessment. In this paper, we apply the deviation maximization method [21] to calculate the weights of each model, which give different evaluated scores with different importance and can effectively widen the gap of scores of DMUs.

The paper is arranged as follows. Section 2 introduces the CCR model, aggressive model, benevolent model, and neutral model. Section 3 introduces the proposed compete-cooperate cross-efficiency model and the deviation maximization method. Section 4 provides a numerical example. Section 5 finally shows the conclusion.

2. Traditional Cross-Efficiency Models

Let there be n DMUs, where DMU_j ($j = 1, 2, \ldots, n$) uses m kind of resources to produce s kind of outputs. The input and output vectors can be denoted as $x_j = (x_{1j}, x_{2j}, \ldots, x_{mj})$ and $y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})$. Then, for a given DMU_d ($1 \leq d \leq n$), its efficiency score of E_{dd} can be determined by the CCR model as follows:

$$
\begin{aligned}
\max \quad & E_{dd} = \sum_{r=1}^{s} \mu_{rd} y_{rd} \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_{id} x_{id} = 1 \\
& \sum_{i=1}^{m} v_{id} x_{ij} - \sum_{r=1}^{s} \mu_{rd} y_{rj} \geq 0, \quad j = 1, 2, \ldots, n \\
& \mu_{rd} \geq 0, \ v_{id} \geq 0, \ r = 1, 2, \ldots, s; \ i = 1, 2, \ldots, m,
\end{aligned}
\tag{1}
$$

where v_{id} ($i = 1, 2, \ldots, m$) and μ_{rd} ($r = 1, 2, \ldots, s$) are the weights assigned to x_{ij} and y_{ij}, respectively. Let E_{dd}^* be the CCR-efficiency score of DMU_d and reflect its self-assessment and let $v_{1d}^*, \ldots, v_{md}^*, \mu_{1d}^*, \ldots, \mu_{sd}^*$ be the optimal solutions to model (1). Then the cross-efficiency model where we can obtain the cross-efficiency scores of all the DMUs with self-assessment score and peer-assessment scores can be presented as

$$
\begin{aligned}
\theta_{jd}^* &= \frac{\sum_{r=1}^{s} \mu_{rj}^* y_{rd}}{\sum_{i=1}^{m} v_{rj}^* x_{id}}, \quad j = 1, \ldots, n; \ j \neq d \\
\overline{\theta}_d^* &= \frac{1}{n} \left(\sum_{j=1, j \neq d}^{n} \theta_{jd}^* + E_{dd}^* \right).
\end{aligned}
\tag{2}
$$

Using the average cross-efficiency scores, we can compare and rank all the DMUs. However, the cross-efficiency scores may not be unique because of the existence of alternate optimal weights, which reduce the usefulness of the cross-efficiency evaluation method. To resolve the problem, the most representative and most applied model and the aggressive and benevolent cross-efficiency models are proposed by Doyle and Green, which can be shown as follows:

$$
\begin{aligned}
\min \text{ or } \max \quad & \sum_{r=1}^{s} \left(\mu_{rd} \sum_{j=1, j \neq d}^{n} y_{rj} \right) \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_{id} \left(\sum_{j=1, j \neq d}^{n} x_{ij} \right) = 1 \\
& \sum_{r=1}^{s} \mu_{rd} y_{rd} - E_{dd}^* \sum_{i=1}^{m} v_{id} x_{id} = 0 \\
& \sum_{i=1}^{m} v_{id} x_{id} - \sum_{r=1}^{s} \mu_{rd} y_{rd} \geq 0
\end{aligned}
$$

$$j = 1, 2, \ldots, n; \ j \neq d$$

$$\mu_{rd} \geq 0, \ v_{id} \geq 0,$$

$$r = 1, 2, \ldots, s; \ i = 1, 2, \ldots, m,$$

$$(3)$$

where E_{dd}^* is the CCR-efficiency score of DMU_d obtained from model (1). The aggressive efficiency model, with a min-objective function in model (3), is given to minimize the other DMUs' cross-efficiency on the promise of unchanged CCR-efficiency value, and the benevolent efficiency model, with a max-objective function, is given to maximize the cross-efficiency of other DMUs. Then Wang and Chin, based on aggressive and benevolent models, proposed a neutral DEA model for cross-efficiency evaluation; the model is presented as

$$\max \quad \delta = \min_{r \in \{1, \ldots, s\}} \left(\frac{\mu_{rd} y_{rd}}{\sum_{i=1}^{m} v_{id} x_{id}} \right)$$

$$\text{s.t.} \quad \sum_{r=1}^{s} \mu_{rd} y_{rj} - \sum_{i=1}^{m} v_{rd} x_{ij} \leq 1$$

$$\sum_{r=1}^{s} \mu_{rd} y_{rd} - E_{dd} \sum_{i=1}^{m} v_{id} x_{id} = 0 \qquad (4)$$

$$j = 1, 2, \ldots, n; \ j \neq d$$

$$\mu_{rd} \geq 0, \ r = 1, 2, \ldots, s$$

$$v_{id} \geq 0, \ i = 1, 2, \ldots, m,$$

where $\mu_{rd} y_{rd} / \sum_{i=1}^{m} v_{id} x_{id}$ is the efficiency of the DMU_d of the rth output. Compared with the aggressive and benevolent methods, there is no difficulty for DMUs to make a subjective choice and determine the input and output weights just from their own perspective in neutral DEA method.

3. Compete-Cooperate Cross-Efficiency Model

3.1. Compete-Cooperate Matrix and Compete-Cooperate Cross-Efficiency Model. In actual application of cross-efficiency method, we often encounter the following two situations. (1) The relationships of DMUs are complex; there not only exist partner related DMUs, but also involve competitive related DMUs. (2) The relationship between two DMUs is relativity. Some DMU_j $(j = 1, 2, \ldots, n)$ reckons DMU_g $(g = 1, 2, \ldots, n, \ g \neq j)$ as a cooperative partner, while DMU_g $(g = 1, 2, \ldots, n, \ g \neq j)$ reckons DMU_j $(j = 1, 2, \ldots, n)$ as a competitor. In these cases, we cannot simply use aggressive or benevolent cross-efficiency model to calculate the values of DMUs.

In this paper, we introduce a cross-efficiency model with compete-cooperate matrix to resolve these problems. First of all, we should build the compete-cooperate matrix. For the first situation, we argue that if two DMUs are cooperative partners, set the coefficient of the matrix as 1. However, if the relationship between two DMUs is competition, then set the coefficient of the matrix as −1. The coefficient of self-assessment will be 0. For the second situation, we argue

that if DMU_j $(j = 1, 2, \ldots, n)$ reckons DMU_g $(g = 1, 2, \ldots, n, \ g \neq j)$ as a cooperative friend, set the coefficient of the matrix as 1. If DMU_g $(g = 1, 2, \ldots, n, \ g \neq j)$ reckons DMU_j $(j = 1, 2, \ldots, n)$ as a competitor, set the coefficient of the matrix as −1 and set the coefficient of self-assessment as 0. For the first situation, the compete-cooperate matrix is a symmetric matrix and for the second situation, the matrix is a nonsymmetric matrix. Then the compete-cooperate cross-efficiency model is built as follows:

$$\max \quad \sum_{r=1}^{s} \mu_{rd} \left(\sum_{j=1}^{n} g_{rj} y_{rj} \right)$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_{id} \sum_{j=1}^{n} x_{ij} = 1$$

$$\sum_{i=1}^{m} v_{id} x_{id} - \sum_{r=1}^{s} \mu_{rd} y_{rd} \geq 0 \qquad (5)$$

$$\sum_{r=1}^{s} \mu_{rd} y_{rd} - E_{dd}^* \sum_{i=1}^{m} v_{id} x_{id} = 0$$

$$j = 1, 2, \ldots, n$$

$$\mu_{rd} \geq 0, \ v_{id} \geq 0,$$

$$r = 1, 2, \ldots, s; \ i = 1, 2, \ldots, m,$$

where g_{rj} is the compete-cooperate relationship matrix, determined by the relationship of DMUs, and E_{dd}^* is the CCR-efficiency value of DMU_d obtained from model (1). Obviously, the compete-cooperate cross-efficiency model can effectively evaluate the efficiencies of DMUs with complex and relative relationship. It is the biggest advantage of the compete-cooperate cross-efficiency model.

3.2. Deviation Maximization Method. Extant cross-efficiency evaluation methods, like aggressive, benevolent, and neutral methods, generally seek the final cross-efficiency scores by calculating the average after determining the self- and peer-assessment scores of each DMU; then each of the evaluated scores participates in the evaluation on the equal weights. Generally speaking, when we evaluate the m index of n DMUs, we usually want to widen the gap of DMUs' efficiency values, in order to pull the grade, facilitate the sorting, and enhance the ability of differentiation. So we need to choose the best weight coefficient index to widen the gap of the efficiency values of the DMUs. Assuming $\omega = (\omega_1, \omega_2, \ldots, \omega_m)^T$ is the weight coefficient vector and $x_i = (x_{i1}, x_{i2}, \ldots, x_{im})^T$ is m vectors of the i evaluation object, the scoring matrix

$$A = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}. \qquad (6)$$

TABLE 1: Description of the inputs and outputs of 30 provinces.

	Indices	Description	Unit	Average	Median
Inputs	x_1	R&D expenditure	10^9 yuan	343.22	258.38
	x_2	R&D personnel FTE	10^4 man	10.82	8.05
	x_3	Amount of technical inflow contract	10^9 yuan	187.58	122.45
Outputs	y_1	Number of accepted domestic patents	Piece	38115.53	20025
	y_2	Output value for new products	10^9 yuan	852.30	201.94
	y_3	Amount of contract deals in technical markets	10^9 yuan	194.70	63.54

TABLE 2: Stage of economic development division (unit: yuan).

Stage of economic development	Primary	Industrialization I	Industrialization II	Industrialization III	Industrialization IV	Advanced
Per capita income limit	4742.39	9471.364	18949.44	36684.77	71048.65	113683.2
Per capita income ceiling	9471.364	18949.44	36684.77	71048.65	113683.2	170524.8

Then the final scores of n DMUs can be presented as

$$Y = A\omega, \tag{7}$$

where $y = (y_1, y_2, \ldots, y_n)^T$ is the final score vector of n DMUs and y_i is the efficiency value of DMU_j $(j = 1, 2, \ldots, n)$; in order to widen the gap between DMUs' efficiency values, we need to make the variance of efficiency values as large as possible, which can be presented as

$$\max s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \overline{y})^2. \tag{8}$$

Put (7) into (8), and normalize the raw data; the following equation will be obtained:

$$ns^2 = \omega^T A^T A\omega = \omega^T H\omega, \tag{9}$$

where $H = A^T A$ is a real symmetric matrix. ω is the weight vector; so $\omega^T \omega = 1$. Therefore, the way to make the variance can be described as

$$
\begin{aligned}
\max \quad & \omega^T H\omega \\
\text{s.t.} \quad & \omega^T \omega = 1 \\
& \omega > 0,
\end{aligned}
\tag{10}
$$

where ω is the eigenvector for the maximum eigenvalue of H, and (10) gets its maximum value. Then normalize ω to obtain the optimal weight coefficient vector. Consider $\omega^* = (\omega_1^*, \omega_2^*, \ldots, \omega_n^*)^T$. In this paper, the deviation maximization method is applied to obtain the final weight factor for aggressive, benevolent, neutral, and proposed compete-cooperate cross-efficiency methods.

4. An Illustrative Example

In this section, we use a specific example to illustrate our method. The example aims at evaluating the technology

innovation efficiency of domestic 31 provinces (DMUs). Each province has to be evaluated in terms of three inputs (x_1, x_2, x_3) and three outputs (y_1, y_2, y_3). Because of the incomplete data of the Tibet Autonomous Region, we choose the remaining 30 provinces as DMUs. We use the data of the 30 provinces in 2012, obtained from *China Statistical Yearbook on Science and Technology* 2013 and economy prediction system. Table 1 shows the specific description of the inputs and outputs of 30 provinces.

In this paper, we only choose the first situation as an example, since the calculating process of the two situations is only different in the data of compete-cooperate matrix g_{rj}. We apply the CCR-efficiency method and the aggressive, benevolent, and neutral cross-efficiency methods and report their scores in this section to be compared with the proposed compete-cooperate cross-efficiency model for some necessary analysis. First of all, we need to give certain values to the compete-cooperate matrix g_{rj}. This paper, according to regional GDP in 2012, divides GDP amount of 30 provinces into 6 stages; Table 2 shows the specific stage of each section. We argue that the relationship of two provinces whose GDP amount belongs to the same interval is partnership; set the coefficient of the matrix as 1. The relationship of two provinces whose GDP amount belongs to different intervals is competition; set the coefficient of the matrix as −1. Set coefficient of self-assessment as 0.

In this paper, we apply the deviation maximization method to get the optimal weights. Table 3 shows the weights for aggressive, benevolent, neutral, and proposed compete-cooperate cross-efficiency models. Table 4 shows the final efficiency scores and ranks of the 30 provinces in CCR, aggressive, benevolent, neutral, and proposed DEA models.

It can be seen from the example scores and ranks that the traditional CCR-efficiency model has lower differentiation on CCR-efficient DMUs; there are a number of CCR-efficient provinces, including Zhejiang, Anhui, Beijing, Guangdong, and Jiangsu provinces, while using the cross-efficiency evaluation method did not get such a result. Therefore,

TABLE 3: Optimal weights of aggressive, benevolent, neutral, and proposed models.

Benevolent		Aggressive		Neutral		Compete-cooperate	
0.0529	0.0207	0.0499	0.0234	0.0449	0.0264	0.0480	0.0226
0.0087	0.0316	0.0331	0.0297	0.0146	0.0315	0.0330	0.0298
0.0203	0.0271	0.0186	0.0241	0.0176	0.0353	0.0186	0.0245
0.0199	0.0539	0.0183	0.0441	0.0188	0.0488	0.0184	0.0444
0.0289	0.0119	0.0270	0.0141	0.0266	0.0077	0.0274	0.0135
0.0249	0.0200	0.0234	0.0203	0.0241	0.0407	0.0236	0.0203
0.0211	0.0448	0.0201	0.0377	0.0203	0.0514	0.0199	0.0388
0.0488	0.0400	0.0521	0.0373	0.0472	0.0362	0.0519	0.0377
0.0383	0.0392	0.0342	0.0406	0.0376	0.0428	0.0345	0.0399
0.0539	0.0360	0.0607	0.0361	0.0445	0.0353	0.0609	0.0365
0.0552	0.0477	0.0607	0.0406	0.0382	0.0470	0.0591	0.0407
0.0452	0.0452	0.0444	0.0448	0.0428	0.0381	0.0437	0.0454
0.0366	0.0377	0.0348	0.0407	0.0307	0.0397	0.0333	0.0420
0.0267	0.0148	0.0276	0.0151	0.0251	0.0183	0.0276	0.0151
0.0231	0.0248	0.0236	0.0229	0.0365	0.0312	0.0254	0.0234

TABLE 4: Scores and ranks of the 30 provinces of the 5 DEA models.

Province	CCR	Cross-efficiency methods			Proposed
		Benevolent	Aggressive	Neutral	
Beijing	1.0000 (1)	0.9416 (2)	0.7782 (3)	0.9327 (2)	0.8874 (3)
Tianjin	0.8812 (6)	0.5220 (12)	0.4479 (13)	0.5261 (12)	0.4920 (13)
Hebei	0.3069 (27)	0.2814 (25)	0.2495 (25)	0.2794 (25)	0.2656 (25)
Shanxi	0.2826 (29)	0.2268 (28)	0.1883 (28)	0.2180 (27)	0.2022 (27)
Inner Mongolia	0.4851 (22)	0.3209 (23)	0.2529 (24)	0.2906 (24)	0.2756 (24)
Liaoning	0.5161 (21)	0.3615 (22)	0.3014 (21)	0.3542 (21)	0.3279 (21)
Jilin	0.3683 (24)	0.2996 (24)	0.2619 (23)	0.2965 (23)	0.2853 (23)
Heilongjiang	0.8715 (7)	0.7100 (6)	0.6060 (6)	0.7008 (6)	0.6705 (6)
Shanghai	0.6940 (14)	0.5823 (9)	0.4968 (11)	0.5880 (9)	0.5523 (10)
Jiangsu	1.0000 (1)	0.9727 (1)	0.8931 (1)	0.9697 (1)	0.9264 (1)
Zhejiang	1.0000 (1)	0.9344 (3)	0.8648 (2)	0.9175 (3)	0.8886 (2)
Anhui	1.0000 (1)	0.7753 (4)	0.7222 (4)	0.7805 (4)	0.7686 (4)
Fujian	0.7333 (12)	0.4688 (15)	0.3962 (15)	0.4403 (16)	0.4054 (16)
Jiangxi	0.4506 (23)	0.4109 (19)	0.3554 (18)	0.4072 (19)	0.3844 (17)
Shandong	0.8549 (8)	0.5422 (10)	0.5489 (8)	0.5666 (10)	0.5720 (9)
Henan	0.7968 (11)	0.4887 (14)	0.4813 (12)	0.4980 (14)	0.5096 (12)
Hubei	0.5486 (20)	0.4540 (16)	0.3871 (16)	0.4544 (15)	0.4309 (15)
Hunan	0.8088 (10)	0.5188 (13)	0.5135 (10)	0.5329 (11)	0.5404 (11)
Guangdong	1.0000 (1)	0.7737 (5)	0.7203 (5)	0.7688 (5)	0.7407 (5)
Guangxi	0.2911 (28)	0.2352 (26)	0.2232 (26)	0.2319 (26)	0.2285 (26)
Hainan	0.3165 (26)	0.1747 (29)	0.1283 (29)	0.1463 (29)	0.1295 (29)
Chongqing	0.6921 (15)	0.4354 (18)	0.3542 (19)	0.4100 (18)	0.3682 (19)
Sichuan	0.6977 (13)	0.6131 (8)	0.5539 (7)	0.6175 (8)	0.5899 (8)
Guizhou	0.6456 (18)	0.4534 (17)	0.3698 (17)	0.4192 (17)	0.3830 (18)
Yunnan	0.5494 (19)	0.4026 (20)	0.3217 (20)	0.3770 (20)	0.3491 (20)
Shanxi	0.8268 (9)	0.6435 (7)	0.5300 (9)	0.6463 (7)	0.6132 (7)
Gansu	0.6540 (17)	0.5332 (11)	0.4226 (14)	0.5093 (13)	0.4782 (14)
Qinghai	0.6727 (16)	0.3920 (21)	0.2994 (22)	0.3367 (22)	0.3208 (22)
Ningxia	0.2377 (30)	0.1548 (30)	0.1209 (30)	0.1390 (30)	0.1259 (30)
Xinjiang	0.3669 (25)	0.2335 (27)	0.1884 (27)	0.2160 (28)	0.1953 (28)

TABLE 5: The results of Spearman correlation analysis for the five models.

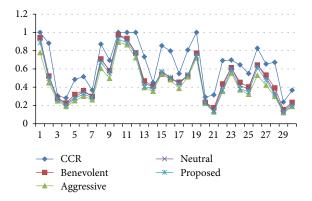| Correlation coefficient	CCR	Benevolent	Aggressive	Neutral	Proposed
CCR	1.000	0.939	0.943	0.936	0.935
Benevolent	0.939	1.000	0.990	0.996	0.992
Aggressive	0.943	0.990	1.000	0.994	0.997
Neutral	0.936	0.996	0.994	1.000	0.996
Proposed	0.935	0.992	0.997	0.996	1.000

FIGURE 1: Efficiency scores of the five models.

the traditional CCR method which relies solely on self-assessment system has certain drawbacks compared with cross-efficiency evaluation method.

Table 4 shows the scores and ranks of 30 provinces from CCR, aggressive, benevolent, neutral, and proposed compete-cooperate DEA methods, which are different in some provinces, but there are still the same parts. Compared with CCR model, the 5 CCR-efficient provinces are top-ranked in proposed model, but there are also some provinces, such as Tianjin, which is ranked 6 in CCR model but ranked 13 in proposed model. Thus, the result of the analysis of proposed model is more flexible. Compared with aggressive, benevolent, and neutral cross-efficiency methods, provinces top-ranked from top 1 to top 6 are the same, including Jiangsu, Zhejiang, Beijing, Anhui, Guangdong, and Heilongjiang province, but there are still subtle differences on the ranking of individual provinces, such as Shanghai and Jiangxi province.

In this paper, Spearman method is used to analyze the correlation among CCR model, aggressive, benevolent, and neutral cross-efficiency model, and proposed compete-cooperate cross-efficiency method. Table 5 shows the results of Spearman correlation analysis for the five models. Correlation coefficients r_s are all between 0.9 and 1, which shows that the efficiency scores and ranks of the five methods are significantly correlated and highly consistent, but relatively speaking, the results of proposed method and aggressive method are highly consistent.

Figure 1 shows the efficiency scores of CCR, aggressive, benevolent, neutral, and proposed compete-cooperate DEA methods. It can be seen that the efficiency scores evaluated by traditional CCR model, benevolent model, and neutral model can be higher than other methods' scores; efficiency

scores calculated by aggressive cross-efficiency method can be lower than other methods' scores. However, the scores of the 30 provinces calculated by proposed method are lower than CCR, benevolent, and neutral methods but higher than aggressive model. Although the extant cross-efficiency models, use self-assessment and peer-assessment system, can avoid the problem of appearing multiple CCR-efficient provinces in CCR model, complete competitive relationships in the aggressive model cause the lower scores, complete friendly relationships in the benevolent model cause the higher scores. The relationships of the provinces are non-tendentious in neutral model, while the proposed compete-cooperate cross-efficiency method takes the relationship of cooperation and competition of the provinces into account and produces more rational evaluated scores and more reliable ranks.

5. Conclusions

Cross-efficiency evaluation method is a method for assessing and evaluating the efficiency scores of DMUs, with self-assessment and peer-assessment system, avoiding low degree of differentiation of CCR-efficient DMUs in traditional CCR model. However, the aggressive, benevolent, and neutral cross-efficiency evaluation methods can only resolve the problem of DMUs with competitive, partnership, and non-tendentious relationships, respectively. But in practice, the relationships of DMUs are not absolute; there may exist two situations: (1) Some DMUs are cooperative partnership, but others are competitive relationship. (2) There exists relative relationship between two or several DMUs. Namely, some DMU_1 regards DMU_2 as a partner, while DMU_2 regards DMU_1 as a competitor. For these cases, this paper introduced a compete-cooperate matrix into aggressive and benevolent cross-efficiency models, built compete-cooperate cross-efficiency model, and applied deviation maximization method to obtain the final weight factor for cross-efficiency evaluation methods to widen the gap between the efficiency values of the DMUs. Furthermore, we use an example of technological innovation efficiency evaluation of the 30 provinces to analyze and interpret the proposed model. The results showed that proposed compete-cooperate cross-efficiency model has significant consistency with CCR, aggressive, benevolent, and neutral model; it can effectively evaluate the efficiency of DMUs with complex and relative relationship issues and enhance the stability and practicality of cross-efficiency evaluation. Future work may focus on determining the degree of authority of DMUs, namely, the weight of each DMU.

Conflict of Interests

Acknowledgment

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