Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2015, Article ID 241536, 12 pages
http://dx.doi.org/10.1155/2015/241536

Research Article

An Optimal Decision Model of Production-Inventory with MTS and ATO Hybrid Model Considering Uncertain Demand

Dezhi Zhang,[1] Xialian Li,[1] Xiamiao Li,[1] Shuangyan Li,[2] and Qi Qian[1]

[1] *School of Traffic & Transportation Engineering, Central South University, Changsha, Hunan 410075, China*
[2] *Forestry Engineering Doctoral Research Center, Central South University of Forestry and Technology, Changsha, Hunan 410004, China*

Correspondence should be addressed to Shuangyan Li; lishuangyan585@shou.com

Received 28 January 2015; Accepted 4 May 2015

Academic Editor: Anders Eriksson

This paper presents an optimization decision model for a production system that comprises the hybrid make-to-stock/assemble-to-order (MTS/ATO) organization mode with demand uncertainty, which can be described as a two-stage decision model. In the first decision stage (i.e., before acquiring the actual demand information of the customer), we have studied the optimal quantities of the finished products and components, while in the second stage (i.e., after acquiring the actual demand information of the customer), we have made the optimal decision on the assignment of components to satisfy the remaining demand. The optimal conditions on production and inventory decision are deduced, as well as the bounds of the total procurement quantity of the components in the ATO phase and final products generated in the MTS phase. Finally, an example is given to illustrate the above optimal model. The findings are shown as follows: the hybrid MTS and ATO production system reduces uncertain demand risk by arranging MTS phase and ATO phase reasonably and improves the expected profit of manufacturer; applying the strategy of component commonality can reduce the total inventory level, as well as the risk induced by the lower accurate demand forecasting.

1. Introduction

Over recent years, assembly manufacturing enterprises have faced fierce competition in the market as a result of individual and diverse needs of customers as well as delivery uncertainty among suppliers. To address these challenges, these enterprises have to apply the hybrid operator mode that comprises the make-to-stock/assemble-to-order (MTS/ATO) production organization mode.

This hybrid mode has two features. First, before the actual demand is observed, the assembly manufacturer procures a certain quantity of each component required for assembling the final product. This quantity is determined based on the forecasted demand and assembly capacity. A certain quantity of the final product may have to be assembled in advance. Second, after confirming the actual customer demand, the manufacturer may need to assemble more final products to fulfil the needs of customers as much as possible [1, 2].

The following managerial questions often arise in MTS and ATO production systems. How can a manufacturer make a reasonable production decision to evade uncertainty demand as much as possible? How can a manufacturer rationally allocate the limited components in the inventory for assembling additional final products?

Several studies have investigated the optimal decision model of inventory-production for assembly systems. These studies can be classified into the following aspects.

(1) Hybrid Production Mode in Assembly Systems. Many studies have aimed to investigate the inventory and related issues of ATO systems. Song and Zipkin [3] presented a general formulation of ATO systems and provided a comprehensive survey of recent literature on ATO systems. Moon and Choi [4] investigated the hybrid MTO and make-in-advance production mode. Tsubone et al. [5] investigated the production-planning system for a combination of MTS and MTO products. Kalantari et al. [6] developed a decision support system for order acceptance/rejection in hybrid MTS/MTO production systems. Soman et al. [7] reviewed the combined MTO/MTS production situations and developed

a comprehensive hierarchical planning framework. Perona et al. [8] studied the optimisation problem of inventory management and customer order decoupling point under various customer service and demand levels.

Eynan and Rosenblatt [9] investigated the multiperiod hybrid policy problem in a multiproduct environment with component commonality, which comprised the assemble-in-advance (AIA) and assemble-to-order (ATO) policies.

(2) Production-Planning and Inventory Control Problem of Assembly Manufacturing. With regard to inventory control, Fu et al. [1] investigated the inventory and production-planning problem for a contract manufacturer who anticipates an order of a single product with an uncertain quantity. An optimal inventory and production decision model was developed based on profit maximisation. Benjaafar and Elhafsi [10] studied the optimal production and inventory control of an ATO system with multicomponent and multiclass customer-based Markov decision process. They found that the optimal inventory allocation for each component was a rationing policy with different rationing levels across various demand classes. Pang et al. [11] addressed an inventory rationing problem in a lost sales MTS production system with batch ordering and multiple demand classes. Pal et al. [12] investigated the integration of all stakeholders in a supply chain and established a multiechelon production inventory model to determine the optimal ordering lot size.

(3) Common Component Allocation Policy. Component commonality has been widely recognized as a key factor in achieving product variety at a low cost. Song and Zhao [13] found that the value of commonality depended significantly on component costs, lead times, and dynamic allocation rules. Therefore, assembly manufacturing systems must determine the optimal base-stock policy and component allocation process when making decisions.

Hillier [14] presented a component replenishing model using the (Q, r) strategy in an ATO situation and found that the benefit of order splitting was larger than that of risk pooling. Ma et al. [15] studied the dynamic process of production time and component procurement lead time as well as their effects on commonality and postponement. They also built a primary inventory level model according to multiperiod, multistage assembly, multiproduct, and stochastic demand. Mohebbi and Choobineh [16] focused on the common components of a two-level BOM (Bill of Material, BOM) under demand uncertainty and its effect on an ATO system. They found that commonality would bring considerable benefits when demand and supply were both uncertain. Hsu et al. [17] developed an optimisation model to determine the costs and optimal stocking quantities of the components of an ATO product with an uncertain demand according to their delivery lead times. Xiao et al. [18] proposed a model for optimising the inventory and production decisions for a single-period ATO system that produced two types of final products under the ATO environment. Each type of product was used to fulfil a customer order, and these two products had a common component. Bernstein et al. [19] developed an optimal decision model for allocating the common components of a set of related products according to the observed demands. They also proposed a collection of allocation mechanisms that involved varying degrees of demand aggregation and a scheme under which all demands were observed prior to making the allocation decisions for each demand.

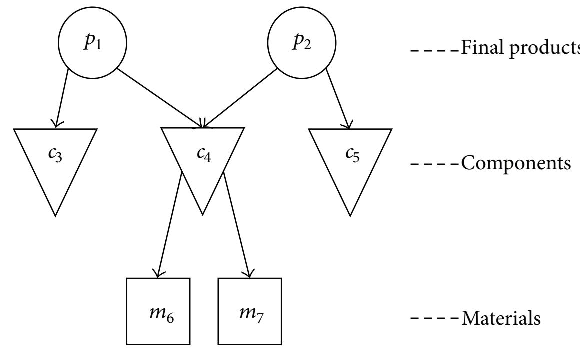


Figure 1: BOM of products 1 and 2.

The recent work of Xiao et al. [2] is closely related to our research problem. They proposed a profit maximisation model for a single-product, single-period ATO system with an uncertain assembly capacity, established the structural properties of the optimal solutions, and identified the sufficient and necessary condition under which the AIA strategy should be adopted.

The preceding analysis shows us that the inventory-production optimal decision issue on the MTS/ATO hybrid production mode has been rarely investigated in the existing literature.

Our model differs from that of Xiao et al. [2, 18] in several aspects. First, we investigate the optimal stocking and production decisions under the MTS/ATO hybrid operator mode. Second, we consider a single-period two-product system that shares common components, which is more complex than a single-product ATO system. Third, we analyse the bounds and properties for the total procurement quantity of components that are acquired at the beginning of the period and the amount of final products that are generated in the MTS phase.

The rest of this paper is organised as follows. Section 2 describes the regional logistics network components and decision-making behaviours. Section 3 provides the optimal decision model and corresponding algorithms. Section 4 presents the bounds and properties for some decision variables. Section 5 concludes the paper.

2. Decision Problem Description

This paper considers an optimal production and inventory decision method for a hybrid MTS/ATO production system with uncertain demand. As shown in Figure 1, we assume that assembly manufacturers produce two final products and their BOM in a product family. To satisfy the required service level of customers and control a rational inventory level by considering the uncertainty of customer demands,

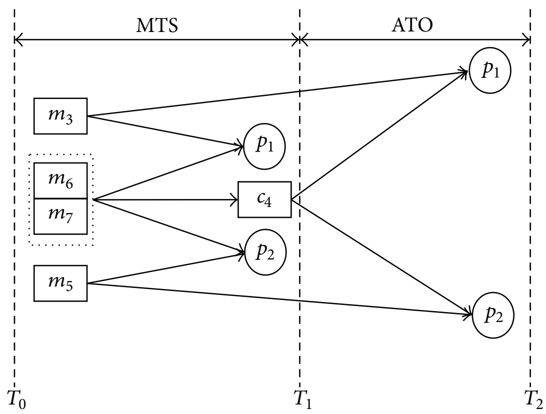


Figure 2: Production procedure under the MTS/ATO hybrid production mode.

the manufacturer procures a certain quantity of components that are required for assembling the final product according to the forecasted demand and assembly capacity. Meanwhile, a certain quantity of final products may need to be assembled in advance (i.e., before the actual demand is observed). The aforementioned decision process is further described as follows.

As shown in Figure 2, the sequence of management decisions under the hybrid MTS/ATO system is presented as follows.

At time T_0, the manufacturer procures a certain quantity of components and prepares production plans according to the forecasted demand to maximise the expected profit. The advanced production of two types of products (a_1, a_2) and the stock of components (q_3, q_4, q_5) are determined.

Phase T_0–T_1: this phase is the so-called MTS phase in which the manufacturer assembles a certain quantity of two types of products in advance. A certain quantity of extra components must be procured to meet the delivery demand of the final product, but the final assembly can be finished after receiving the orders of customers.

At time T_1, the customer demand information is confirmed. The manufacturer makes decisions on whether and how much additional assembly is needed, where (y_1^A, y_2^A) is determined.

Phase T_1–T_2: this phase is the so-called ATO phase, in which the manufacturer satisfies the demand of customers using the on-hand products according to the actual demand information. A demand gap is observed when the actual demand of customers is greater than the amount of products assembled in advance. An additional assembly is also arranged based on the inventory of components to fulfil the customer demand as much as possible.

At time T_2, T_2 is the order delivery time as required by the customer.

3. Optimal Decision Model

3.1. Notations. The definitions and notations are denoted as follows:

p_i: unit revenue of the final product i $(i = 1, 2)$;

s_i: quantity of the final product i $(i = 1, 2)$;

r_i: unit production cost of product i by the MTS mode $(i = 1, 2)$;

m_i: unit cost of assembling the components into the end product i $(i = 1, 2)$;

c_i: unit cost of component j $(j = 3, 4, 5)$, especially for intermediate component 4, c_4 that includes the procurement cost of components 6 and 7 as well as the assembly cost of common intermediate component 4;

u_i: unit penalty cost of the unsatisfied demand for product i $(i = 1, 2)$;

w_i: unit salvage price for final product i $(i = 1, 2)$;

v_i: unit salvage price for component i $(i = 3, 4, 5)$, $w_1 < v_3 + v_4$, $w_2 < v_4 + v_5$;

D_i: demand for product i $(i = 1, 2)$.

Decision Variables. Consider

y_i^A: quantity of product i that is assembled in the ATO phase $(i = 1, 2)$;

a_i: quantity of product i that is manufactured in the MTS phase $(i = 1, 2)$;

q_j: quantity of component j that is acquired in the MTS phase $(j = 3, 4, 5)$.

3.2. Assumptions. To facilitate the presentation of the essential ideas without loss of generality, this study makes the following basic assumptions:

*A*1: the actual demands for the two types of final products are irrelevant and determined at the same time.

*A*2: given that the purchase lead time is too long for the assembly of the corresponding final products after confirming the actual demand information of the customer, the manufacturer makes the procurement decision at time T_0. Meanwhile, the inventories of products 1 and 2 are both zero at time T_0.

*A*3: without loss of generality, the unit profit of final product 1 is greater than that of final product 2, which satisfies the following expressions:

$$\begin{aligned} p_1 - r_1 + u_1 &> p_2 - r_2 + u_2, \\ p_1 - m_1 - c_3 - c_4 + u_1 &> p_2 - m_2 - c_4 - c_5 + u_2. \end{aligned} \tag{1}$$

*A*4: the cost of assembling units in advance in the MTS phase is lower than that for the normal assembly of products in the ATO phrase, which satisfies the following expressions:

$$\begin{aligned} p_1 - r_1 &> p_1 - m_1 - c_3 - c_4, \\ p_2 - r_2 &> p_2 - m_2 - c_4 - c_5. \end{aligned} \tag{2}$$

*A*5: compared with product demand, production capacity is infinite and product storage is allowed.

*A*6: the salvage value of the unused components is smaller than the original procurement cost. Similarly, the salvage value of the redundant final product is smaller than the revenue of such products.

3.3. Problem Formulation and Optimality Properties. We formulate our optimisation problem in a backward order. At time T_1, the manufacturer observes the realized demand for products 1 and 2 (i.e., D_1 and D_2), which are partially satisfied by on-hand products directly. The remaining portion of the demand is fulfilled by the postponed inventory as much as possible. We determine the optimal assembly decisions (y_1^{A*}, y_2^{A*}) according to the on-hand products, component inventory, and actual demands, which comprise the decision optimisation problem in the ATO phase (denoted as P_2). We also determine the optimal stock of primary products (a_1^*, a_2^*) and components that are needed for assembling the final products (q_3^*, q_4^*, q_5^*) to maximise the expected profit, which is the decision optimisation problem in the MTS phase (denoted as P_1).

Therefore, the decision optimisation problem (P_2) can be formulated as follows:

$$\begin{aligned} T\left(y_1^A, y_2^A\right) = \max \Big\{ & [p_1 s_1 + p_2 s_2] - \left[\left(m_1 y_1^A + m_2 y_2^A\right) \right. \\ & \left. + u_1 (D_1 - s_1)^+ + u_2 (D_2 - s_2)^+\right] \\ & + \left[w_1 (a_1 - D_1)^+ + w_2 (a_2 - D_2)^+ + v_3 \left(q_3 - y_1^A\right)^+ \right. \\ & \left. + v_4 \left(q_4 - y_1^A - y_2^A\right)^+ + v_5 \left(q_5 - y_2^A\right)^+\right]\Big\} \end{aligned} \tag{3}$$

subject to

$$y_1^A \le k_1 \tag{4}$$

$$y_2^A \le k_2 \tag{5}$$

$$y_1^A + y_2^A \le q_4 \tag{6}$$

$$\begin{aligned} y_1^A &\ge 0, \\ y_2^A &\ge 0, \end{aligned} \tag{7}$$

where $k_1 = \min\{D_1 - a_1, q_3\}$, $k_2 = \min\{D_2 - a_2, q_5\}$, $s_1 = \min\{D_1, a_1 + y_1^A\}$, and $s_2 = \min\{D_2, a_2 + y_2^A\}$.

The objective function (3) represents the maximisation of the expected total profit, which comprises three parts. The first part denotes the total revenue of final products 1 and 2, the second part represents the total and penalty cost for the unsatisfied demand for products, and the third part denotes the total salvage benefit of the redundant final products and components. Equation (4) shows the added-assembly decision of product 1 in the ATO phase y_1^A, which is constrained by the quantity of component 3 and demand fulfilment shortage $(D_1, a_1)^+$. Similarly, (5) shows the added-assembly decision of product 2 in the ATO phase y_2^A, which is constrained by the quantity of component 3 and demand fulfilment shortage $(D_2, a_2)^+$. Equation (6) implies that the total quantities of added products 1 and 2 must not exceed the available inventory of component 4. Equation (7) shows that the quantity of products 1 and 2 is nonnegative.

We always use the on-hand final products that are generated in the MTS phase to satisfy the demands of customers. In other words, we assemble additional products only when the amount of preassembled products in phase 1 does not satisfy the actual demand of customers. The added-assembly quantities of the two products in the ATO phrase are constrained by the limited inventory of components and the demand fulfilment shortage.

With regard to the product structure, the quantities of exclusive components do not need to exceed those of common components. Meanwhile, the sum of exclusive components must be larger than the volume of common components or the common components will be left over, which does not comply with our objective (see [20])

$$q_4 \ge \max\{q_3, q_5\}, \tag{8}$$

$$q_4 \le q_3 + q_5. \tag{9}$$

Therefore, the added-assembly quantities of products 1 and 2 satisfy (8) and (9), respectively. Consider

$$y_1^A = \min\{D_1 - a_1, q_3\}, \tag{10}$$

$$y_2^A = \min\left\{(D_2 - a_2)^+, q_5, q_4 - y_1^A\right\}. \tag{11}$$

We also investigate the optimal extra production decisions in the ATO phase. After confirming the demand information of the customers, the manufacturer must determine whether or not additional assembly is necessary and how to allocate the inventory of common component (c_4) between the two products according to their demand gap and inventory constraints.

After observing the actual demand for products 1 and 2, we have 10 possible cases that are denoted as Ω_i ($i = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$) as shown in Figure 3.

We now discuss the additional-assembly decisions in each area by allocating common components according to product priority. The optimal combinations of the assembly quantities and actual sales quantities of products 1 and 2 are analysed as follows:

(1) In domain $\Omega_1 = \{a_1 \ge D_1, a_2 \ge D_2\}$, the manufacturer can supply the customer with the preassembled products. Meanwhile, the amounts of unused products 1 and 2 are denoted as $(a_1 - D_1)$, $(a_2 - D_2)$, respectively. Therefore, $y_1^A = 0$, $y_2^A = 0$, and $s_1 = D_1$, $s_2 = D_2$.

(2) In domain $\Omega_2 = \{a_1 \le D_1 \le a_1 + q_3, a_2 \ge D_2\}$, the actual demand of customers for product 1 is larger than the amount of preassembled products in the MTS phase but is fulfilled by the amount of added-assembled products in the ATO phrase. The demand for product 2 is less than the stock in the MTS phase. Therefore, $y_1^A = D_1 - a_1$, $y_2^A = 0$, $s_1 = D_1$, $s_2 = D_2$.

(3) In domain $\Omega_3 = \{D_1 \le a_1, a_2 \le D_2 \le a_2 + q_5\}$, the demand quantities of product 2 are larger than its preassembled quantity in the MTS phase but are fulfilled completely by the quantity of added-assembled products in the ATO phrase. The demand for product 1 is less than the stock in the MTS phase. Therefore, $y_1^A = 0$, $y_2^A = D_2 - a_2$; $s_1 = D_1$, $s_2 = D_2$.

(4) In domain $\Omega_4 = \{a_1 \le D_1 \le a_1 + q_3, a_2 \le D_2 \le a_2 + q_5, D_1 + D_2 \le a_1 + a_2 + q_4\}$, the actual demand of customers for products 1 and 2 is larger than the preassembled quantity of these products in the MTS phase but is fulfilled completely by the added-assembled quantity of these products in the ATO phase. Therefore, $y_1^A = D_1 - a_1$, $y_2^A = D_2 - a_2$; $s_1 = D_1$, $s_2 = D_2$.

(5) In domain $\Omega_5 = \{D_1 \le a_1, D_2 \ge a_2 + q_5\}$, the actual demand of customers for product 2 is larger than the amount of preassembled products in the MTS phase, while the demand gap is even larger than the amount of exclusive component 5. Therefore, the actual demand of some customers is not satisfied. In this situation, the manufacturer focuses on the assembly of product 2 to generate a higher profit. Therefore, $y_1^A = 0$, $y_2^A = q_5$; $s_1 = D_1$, $s_2 = a_2 + q_5$.

(6) In domain $\Omega_6 = \{a_1 \le D_1 \le a_1 + q_4 - q_5, D_2 \ge a_2 + q_5\}$, both the demands for products 1 and 2 are larger than the amount of preassembled products in the MTS phase. The manufacturer assembles additional quantities of product 1 because $q_4 - (D_1 - a_1) \ge q_4 - (q_4 - q_5) = q_5$. In this case, the amount of component 5 becomes the bottleneck constraint for assembling product 2. Therefore, $y_1^A = D_1 - a_1$, $y_2^A = q_5$; $s_1 = D_1$, $s_2 = a_2 + q_5$.

(7) In domain $\Omega_7 = \{a_1 + q_4 - q_5 \le D_1 \le a_1 + q_3, D_1 + D_2 \ge a_1 + a_2 + q_4\}$, the demands for products 1 and 2 not only are larger than that for the preassembled products in the MTS phase, but also exceed the limit of common component 4. Given the product priority, the manufacturer arranges for the additional assembly of product 1 and uses the leftover components to assemble product 2. Meanwhile, given that $D_2 - a_2 \ge (a_1 + a_2 + q_4 - D_1) - a_2 = a_1 + q_4 - D_1$, we assume that $y_1^A = D_1 - a_1$, $y_2^A = q_4 - D_1 + a_1$; $s_1 = D_1$, $s_2 = a_1 + a_2 + q_4 - D_1$. As shown in Figure 3, when $B^{(7)}$ is the demand point (D_1, D_2), the actual quantity of sales is at point $U^{(7)}$.

(8) In domain $\Omega_8 = \{D_1 \ge a_1 + q_3, D_2 \ge a_2 + q_4 - q_3\}$, the demand for either product 1 or 2 is unsatisfied. All of the on-hand inventories for each product are used and all of the stocked components are allocated to satisfy the demand gaps. Given the product priority, the manufacturer arranges for the additional assembly of product 1 and uses the leftover components to assemble product 2. Given that $D_2 - a_2 \ge (a_2 + q_4 - q_3) - a_2 = q_4 - q_3$, we assume that $y_1^A = q_3$, $y_2^A = q_4 - q_3$; $s_1 = a_1 + q_3$, $s_2 = a_2 + q_4 - q_3$. As shown in Figure 3, when $B^{(8)}$ is the demand point (D_1, D_2), the actual sales volume is at point $U^{(8)}$.

(9) In domain $\Omega_9 = \{D_1 \ge a_1 + q_3, a_2 + q_4 - q_3 \ge D_2 \ge a_2\}$, both the demands for products 1 and 2 are larger than their stock in the MTS phase, and the manufacturer is faced with the problem of how to distribute the common components. Given the priority strategy, the manufacturer arranges the additional assembly for product 1 and then uses the leftover components to assemble product 2. As a result, the additional assembly volume of product 1 is equivalent to the exclusive component volume of the same product. Given that $D_2 - a_2 \le q_4 - q_3$, we assume that $y_1^A = q_3$, $y_2^A = D_2 - a_2$; $s_1 = a_1 + q_3$, $s_2 = D_2$.

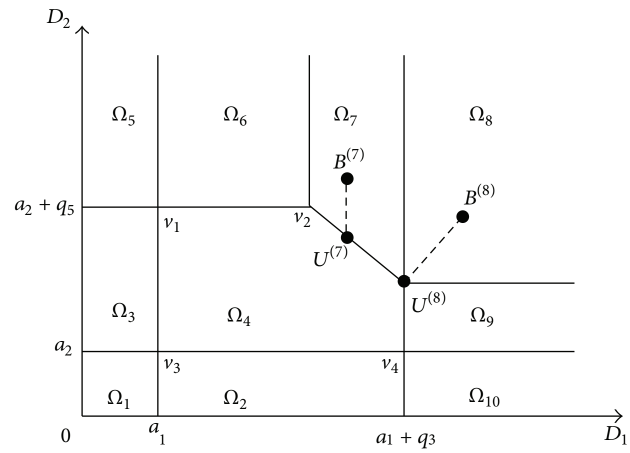


FIGURE 3: Demand space in the MTS/ATO production system.

(10) In domain $\Omega_{10} = \{D_1 \ge a_1 + q_3, D_2 \le a_2\}$, the demand for product 1 is larger than the stock of this product in the MTS phase. The demand gap is larger than the volume of exclusive component 3, which becomes the bottleneck constraint for the assembly volume of product 1. In this case, the manufacturer concentrates on the assembly of product 1 to generate more profit. Therefore, $y_1^A = q_3$, $y_2^A = 0$; $s_1 = a_1 + q_3$, $s_2 = D_2$.

From the preceding analysis, under the strategy of commonality with product priority, the additional quantity of assembled products and the actual selling amount of both products 1 and 2 are shown in Table 1.

For the nonidentical profits of the two types of final products, the optimal solution for the ATO phrase can be easily obtained in the form of a greedy algorithm. Obviously, we are interested in making optimal decisions for AIA and components inventory. By considering all possible combinations of future demand and component inventory, the manufacturer determines how many final products must be assembled in advance and how many extra inventories of each component must be prepared at time T_0 to maximise the expected profit. The optimisation problem in the first-stage decision optimal model (p_1) is formulated as follows:

$$Z(a_1, a_2, q_3, q_4, q_5) = E\left(T\left(y_1^A, y_2^A\right)\right) - r_1 a_1 - r_2 a_2 - c_3 q_3 - c_4 q_4 - c_5 q_5 \tag{12}$$

Table 1: Additional assembly arrangements and actual sales of commonality strategy with product priority in separate demand areas.

Demand area	Product 1 additional assembly volume	Product 2 additional assembly volume	Product 1 actual sales	Product 2 actual sales
$D_1, D_2 \in \Omega_1$	0	0	D_1	D_2
$D_1, D_2 \in \Omega_2$	$D_1 - a_1$	0	D_1	D_2
$D_1, D_2 \in \Omega_3$	0	$D_2 - a_2$	D_1	D_2
$D_1, D_2 \in \Omega_4$	$D_1 - a_1$	$D_2 - a_2$	D_1	D_2
$D_1, D_2 \in \Omega_5$	0	q_5	D_1	$a_2 + q_5$
$D_1, D_2 \in \Omega_6$	$D_1 - a_1$	q_5	D_1	$a_2 + q_5$
$D_1, D_2 \in \Omega_7$	$D_1 - a_1$	$a_1 + q_4 - D_1$	D_1	$a_1 + a_2 + q_4 - D_1$
$D_1, D_2 \in \Omega_8$	q_3	$q_4 - q_3$	$a_1 + q_3$	$a_2 + q_4 - q_3$
$D_1, D_2 \in \Omega_9$	q_3	$D_2 - a_2$	$a_1 + q_3$	D_2
$D_1, D_2 \in \Omega_{10}$	q_3	0	$a_1 + q_3$	D_2

subject to

$$\begin{aligned} a_1 &\geq 0, \\ a_2 &\geq 0, \\ q_3 &\geq 0, \\ q_4 &\geq 0, \\ q_5 &\geq 0, \end{aligned} \tag{13}$$

where objective function (10) represents the maximized expected profit that is equivalent to the profit of the preassembled products (sales revenue minus assembly cost and shortage cost) minus the production cost in the MTS phase and total component cost of phase 1. Equation (11) shows that, at time T_0, the amount of products 1 and 2 that are assembled in advance is nonnegative. Equation (12) shows that the amount of components that are purchased in the MTS phase is nonnegative.

Proposition 1. *The expected revenue function $Z(a_1, a_2, q_3, q_4, q_5)$ is jointly concave in $Q = (a_1, a_2, q_3, q_4, q_5)$. The proof of Proposition 1 is given in the appendix.*

Given that the objective function equation (12) is concave and that the constraint functions are convex, this nonlinear programming model is considered as a convex optimisation problem. Therefore, based on the first-order necessary conditions of the Kuhn-Tucker sufficiency theorem, the globally optimal solution is the stable value of $Z^*(a_1^*, a_2^*, q_3^*, q_4^*, q_5^*)$.

Beginning with the quantity of final product 1 that is produced in the MTS phase, a_1, the following equation is obtained:

$$\begin{aligned} a_1 &\geq 0, \\ a_2 &\geq 0, \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{\partial Z}{\partial a_1} &= (p_1 + u_1 - v_3 - m_1 - p_2 - u_2 + v_5 + m_2) \\ &\quad \cdot \Pr(D_1 > a_1 + q_3) + (p_2 + u_2 - v_4 - v_5 - m_2) \\ &\quad \cdot \Pr(a_1 + q_3 < D_1, D_2 < a_2 + q_4 - q_3) + (p_2 + u_2 \\ &\quad - v_4 - v_5 - m_2) \Pr(\min(D_1, a_1 + q_3) \\ &\quad + \min(D_2, a_2 + q_5) > a_1 + a_2 + q_4) + (m_1 + v_3 \\ &\quad + v_4 - w_1) \Pr(D_1 > a_1) + w_1 - r_1. \end{aligned} \tag{15a}$$

Likewise, the expression for the derivative with respect to the quantity of final product 2 that is produced in the MTS phase (a_2) is expressed as follows:

$$\begin{aligned} \frac{\partial Z}{\partial a_2} &= (p_2 + u_2 - v_4 - v_5 - m_2) \Pr(D_1 < a_1 + q_4 \\ &\quad - q_3, D_2 > a_2 + q_5) + (p_2 + u_2 - v_4 - v_5 - m_2) \\ &\quad \cdot \Pr(\min(D_1, a_1 + q_3) + \min(D_2, a_2 + q_5) > a_1 \\ &\quad + a_2 + q_4) + (m_2 + v_4 + v_5 - w_2) \Pr(D_2 > a_2) \\ &\quad + w_2 - r_2. \end{aligned} \tag{15b}$$

The expression for the derivative with respect to the quantity of component 3 that is acquired in the MTS phase (q_3) is expressed as follows:

$$\begin{aligned} \frac{\partial Z}{\partial q_3} &= [(p_1 + u_1 - v_3 - v_4 - m_1) - (p_2 + u_2 - v_4 \\ &\quad - v_5 - m_2)] \Pr(D_1 > a_1 + q_3) + (p_2 + u_2 - v_4 \\ &\quad - v_5 - m_2) \Pr(D_1 > a_1 + q_3, D_2 \leq a_2 + q_4 - q_3) \\ &\quad + v_3 - c_3, \end{aligned} \tag{15c}$$

$$\begin{aligned} \frac{\partial Z}{\partial q_4} &= (p_2 + u_2 - v_4 - v_5 - m_2) \Pr(\min(D_1, a_1 + q_3) \\ &\quad + \min(D_2, a_2 + q_5) > a_1 + a_2 + q_4). \end{aligned} \tag{15d}$$

The expression for the derivative with respect to the quantity of component 5 that is acquired in the MTS phase (q_5) is expressed as follows:

$$\frac{\partial Z}{\partial q_5} = (p_2 + u_2 - v_4 - v_5 - m_2) \cdot \Pr(D_1 \le a_1 + q_4 - q_5, D_2 > a_2 + q_5) + v_5 - c_5. \tag{15e}$$

By equating and then rearranging (15a), (15b), (15c), (15d), and (15e) to zero, we obtain the following:

$$\frac{p_1 + u_1 - r_1}{p_1 + u_1 - w_1} = \Pr(D_1 < a_1) + \frac{p_1 + u_1 - v_3 - v_4 - m_1}{p_1 + u_1 - w_1} \Pr(a_1 + q_3 > D_1 > a_1) - \frac{p_2 + u_2 - v_4 - v_5 - m_2}{p_1 + u_1 - w_1} \Pr(a_1 + q_3 > D_1 > a_1 + q_4 - q_5, D_1 + D_2 < a_1 + a_2 + q_4), \tag{16a}$$

$$\frac{m_2 + v_4 + v_5 - r_2}{m_2 + v_4 + v_5 - w_2} = \Pr(D_2 < a_2) - \frac{p_2 + u_2 - v_4 - v_5 - m_2}{m_2 + v_4 + v_5 - w_2} \Pr(D_1 < a_1 + q_4 - q_5, D_2 > a_2 + q_5) - \frac{p_2 + u_2 - v_4 - v_5 - m_2}{m_2 + v_4 + v_5 - w_2} \cdot \Pr(\min(D_1, a_1 + q_3) + \min(D_2, a_2 + q_5) > a_1 + a_2 + q_4), \tag{16b}$$

$$\frac{p_1 + u_1 - c_3 - v_4 - m_1}{p_1 + u_1 - v_3 - v_4 - m_1} = \Pr(D_1 < a_1 + q_3) + \frac{p_2 + u_2 - v_4 - v_5 - m_2}{p_1 + u_1 - v_3 - v_4 - m_1} \Pr(D_1 > a_1 + q_3, D_2 > a_2 + q_4 - q_3), \tag{16c}$$

$$\frac{c_4 - v_4}{p_2 + u_2 - v_4 - v_5 - m_2} = \Pr(\min(a_2, D_2) + \min(a_1, D_2) > a_1 + a_2 + q_4), \tag{16d}$$

$$\frac{p_2 + u_2 - c_4 - c_5 - m_2}{p_1 + u_1 - v_3 - v_4 - m_1} = \Pr(D_2 < a_2 + q_5) - \Pr(a_2 + q_5 > D_2 > a_2 + q_4 - q_3, D_1 + D_2 > a_1 + a_2 + q_4). \tag{16e}$$

Relations (8) and (9) as well as (16a), (16b), (16c), (16d), and (16e) comprise the set of optimality conditions.

4. Bounds and Properties

We determine the bounds for the quantity of the two kinds of products that are produced before confirming the demands of customers and the total amount of each component that is acquired at the beginning of the period.

We can rewrite (16a) as follows:

$$\frac{p_1 + u_1 - r_1}{p_1 + u_1 - w_1} = \Pr(D_1 < a_1) + \frac{p_1 + u_1 - v_3 - v_4 - m_1}{p_1 + u_1 - w_1} \Pr(a_1 < D_1 < a_1 + q_3) - \frac{p_2 + u_2 - v_4 - v_5 - m_2}{p_1 + u_1 - w_1} \Pr(a_1 + q_4 - q_5 < D_1 < a_1 + q_3, D_1 + D_2 < a_1 + a_2 + q_4). \tag{17}$$

Equation (17) can be regarded as the optimality condition of the simple newsvendor problem for final product 1 in the MTS phase, with two additional terms on the right-hand side. The second term on the right-hand side adjusts the production quantity downwards when the amount of final products insufficiently covers the demand for product 1 and when the demand gap is satisfied by ATO. The third term adjusts the production quantity upwards when the amount of final products insufficiently covers the demand for product 1, when the remaining demand for product 1 is satisfied by ATO, and when the demand for product 2 is satisfied by the sum of final products and leftover components.

Given that $\Pr(D_1 < a_1)$ is nondecreasing in a_1, $p_1 + u_1 - v_3 - v_4 - m_1 > p_2 + u_2 - v_4 - v_5 - m_2$, and $\Pr(a_1 + q_3 > D_1 > a_1) \ge \Pr(a_1 + q_3 > D_1 > a_1 + q_4 - q_5, D_1 + D_2 < a_1 + a_2 + q_4)$, the second and third parts on the left-hand side of (17) are removed to obtain the nonnegative upper bound ${a_1}^u$ of a_1 for the sum of these two terms, which is expressed as follows:

$$\Pr(D_1 < {a_1}^u) = \frac{p_1 + u_1 - r_1}{p_1 + u_1 - w_1}. \tag{18}$$

The salvage value of a final product must not exceed the sum of the salvage value that is required for assembling the ordered product because the manufacturer prefers to reserve components than final products with high demand uncertainty. Therefore, the quantity of final products that are produced in the MTS phase is maintained at a low level.

Equation (16a) can be rewritten as follows:

$$\frac{m_1 + v_3 + v_4 - r_1}{m_1 + v_3 + v_4 - w_1} = \Pr(D_1 < a_1) - \frac{p_1 + u_1 - v_3 - m_1 - p_2 - u_2 + v_5 + m_2}{m_1 + v_3 + v_4 - w_1} \Pr(D_1 > a_1 + q_3) - \frac{p_2 + u_2 - v_4 - v_5 - m_2}{m_1 + v_3 + v_4 - w_1} \Pr(D_1 > a_1 + q_3, D_2 < a_2 + q_4 - q_3) - (p_2 + u_2 - v_4 - v_5 - m_2) \Pr(\min(D_1, a_1 + q_3) + \min(D_2, a_2 + q_5) > a_1 + a_2 + q_4). \tag{19}$$

The second, third, and fourth parts on the left-hand side of (19) are non-positive. By removing these parts, we can obtain the lower bound for a_1, which can be expressed as follows:

$$\Pr\left(D_1 < {a_1}^l\right) = \frac{m_1 + v_3 + v_4 - r_1}{m_1 + v_3 + v_4 - w_1}. \tag{20}$$

When the manufacturer can produce goods after receiving the demands of customers, lower bounds are given for the quantity of final products that are generated in the MTS phase.

According to (16e), we obtain the following:

$$\frac{c_5 - v_5}{p_2 + u_2 - v_4 - v_5 - m_2} = \Pr\left(D_1 < a_1 + q_4 - q_5, D_2 > a_2 + q_5\right). \tag{21}$$

We replace the second and third probabilities on the right-hand side of (16b) with the terms on the left-hand side of (21) and (16d). The optimality condition (16b) can be rewritten as follows:

$$\left(m_2 + v_4 + v_5 - w_2\right)\Pr\left(D_2 < a_2\right) = m_2 + c_4 + c_5 - r_2. \tag{22}$$

Thereafter, we obtain the following:

$$\Pr\left(D_2 < a_2\right) = \frac{m_2 + c_4 + c_5 - r_2}{m_2 + v_4 + v_5 - w_2} \tag{23}$$

which is recognized as the optimality condition of the classical newsvendor problem for product 2 that is generated in the MTS phrase. The right-hand fraction is adjusted upwards for the cost of assembling the components into the products in the ATO phase.

We now determine the bounds for the total procurement quantity of each component. The optimality condition (16c) can be rewritten as follows:

$$\begin{aligned}\frac{p_1 + u_1 - c_3 - v_4 - m_1}{p_1 + u_1 - v_3 - v_4 - m_1} &= \Pr\left(D_1 < a_1 + q_3\right) \\ &+ \frac{p_2 + u_2 - v_4 - v_5 - m_2}{p_1 + u_1 - v_3 - v_4 - m_1} \\ &\cdot \Pr\left(D_1 > a_1 + q_3, D_2 > a_2 + q_4 - q_3\right)\end{aligned} \tag{24}$$

which can be recognised as a newsvendor solution that is adjusted downwards when the common component is insufficient. This adjustment is weighted by the ratio of opportunity costs of selling rather than salvaging in the ATO phase between products 2 and 1. The second probability on the right-hand side depends on the demand correlation. Therefore, (24) indicates that the procurement quantity of component 3 is nonincreasing in demand correlation. Therefore, the common component becomes a constraint that results from the low amount of additional products when the demands for these two products tend to be high.

By disregarding the nonnegative second term on the right-hand side, we obtain the upper bound for the optimal value of $(a_1 + q_3)$, which can be expressed as follows:

$$\Pr\left[D_1 < \left(a_1 + q_3\right)^u\right] = \frac{p_1 + u_1 - c_3 - v_4 - m_1}{p_1 + u_1 - v_3 - v_4 - m_1}. \tag{25}$$

Similarly, (16c) can be rewritten as follows:

$$\begin{aligned}&\frac{p_1 + u_1 - c_3 - v_4 - m_1}{p_1 + u_1 - v_3 - v_4 - m_1} = \Pr\left(D_1 < a_1 + q_3\right) \\ &\quad + \frac{p_2 + u_2 - v_4 - v_5 - m_1}{p_1 + u_1 - v_3 - v_4 - m_1}\Pr\left(D_1 > a_1 + q_4 - q_5, D_2\right. \\ &\quad \left. > a_2 + q_4 - q_3\right) - \frac{p_2 + u_2 - v_4 - v_5 - m_2}{p_1 + u_1 - v_3 - v_4 - m_1}\Pr\left(a_1\right. \\ &\quad \left. + q_4 - q_5 < D_1 < a_1 + q_3, D_1 + D_2 > a_1 + a_2 + q_4\right) \\ &\quad = \Pr\left(D_1 < a_1 + q_3\right) + \frac{p_2 + u_2 - v_4 - v_5 - m_2}{p_1 + u_1 - v_3 - v_4 - m_1} \\ &\quad \cdot \Pr\left(\min\left(D_1, a_1\right) + \min\left(a_2, D_2\right) > a_1 + a_2 + q_4\right) \\ &\quad - \frac{p_2 + u_2 - v_4 - v_5 - m_2}{p_1 + u_1 - v_3 - v_4 - m_1}\Pr\left(a_1 + q_4 - q_5 < D_1\right. \\ &\quad \left. < a_1 + q_3, D_1 + D_2 > a_1 + a_2 + q_4\right) = \Pr\left(D_1 < a_1\right. \\ &\quad \left. + q_3\right) + \frac{c_4 - v_4}{p_1 + u_1 - v_3 - v_4 - m_1} \\ &\quad - \frac{p_2 + u_2 - v_4 - v_5 - m_2}{p_1 + u_1 - v_3 - v_4 - m_1}\Pr\left(a_1 + q_4 - q_5 < D_1\right. \\ &\quad \left. < a_1 + q_3, D_1 + D_2 > a_1 + a_2 + q_4\right) \le \Pr\left(D_1 < a_1\right. \\ &\quad \left. + q_3\right) + \frac{c_4 - v_4}{p_1 + u_1 - v_3 - v_4 - m_1}.\end{aligned} \tag{26}$$

Therefore, we can obtain the lower bound $(a_1 + q_3)^l$ for the optimal value of $(a_1 + q_3)$, which can be characterised as follows:

$$\Pr\left[D_1 < \left(a_1 + q_3\right)^l\right] = \frac{p_1 + u_1 - c_3 - c_4 - m_1}{p_1 + u_1 - v_3 - v_4 - m_1}. \tag{27}$$

The gap of fractals between the upper and lower bounds can be computed as follows:

$$\frac{c_4 - v_4}{p_1 + u_1 - v_3 - v_4 - m_1}. \tag{28}$$

In many practical situations, the gap is closed in cases where the difference between the unit cost of the common component and the salvaged unit of the common component is small. Thus, tight bounds can be obtained.

Equation (16d) can also be rewritten as follows:

$$\begin{aligned}&\Pr\left(\min\left(D_1 - a_1, q_3\right) + \min\left(D_2 - a_2, q_5\right) < q_4\right) \\ &\quad = \frac{p_2 + u_2 - c_4 - v_5 - m_2}{p_2 + u_2 - v_4 - v_5 - m_2}.\end{aligned} \tag{29}$$

Apparently, the term on the left-hand side of (29) refers to the cumulative distribution function of the demand for the common component in the ATO phase. The ratio on the right- hand side, which is referred to as the critical fractal, balances the cost of being understocked (a lost sale worth $(p_2 + u_2 - c_4 - v_5 - m_2)$) and the total costs of being either overstocked or understocked (where the cost of being overstocked is $c_4 - v_5$). An extra unit of the common component leads to one extra sale when the product demand does not satisfy the usage of final products and when the specific component is available. The extra unit comes at a cost and can be salvaged when the number of final products exceeds the demand.

Given that $\Pr[\min(D_1, a_1) + \min(a_2, D_2) < a_1 + a_2 + q_4] > \Pr(D_1 + D_2 < a_1 + a_2 + q_4)$, the upper bound follows immediately, which can be characterised as follows:

$$\Pr\left[D_1 + D_2 < (a_1 + a_2 + q_4)^u\right] = \frac{p_2 + u_2 - c_4 - v_5 - m_2}{p_2 + u_2 - v_4 - v_5 - m_2}. \tag{30}$$

Intuitively, the left-hand side of (30) indicates that the procurement quantity of the common component decreases along with demand. A higher correlation between procurement quantity and demand decreases the risk-pooling effect.

This bound becomes tighter for a highly negatively correlated demand.

By using the lower bounds of the specific components and relation $q_4 \geq \max(q_3, q_5)$, we obtain the lower bound as follows:

$$q_4{}^l = \max\left[(a_1 + q_3)^l, (a_2 + q_5)^l\right]. \tag{31}$$

We set about bounding the total procurement quantity of component 5. By using (16e), we obtain the following:

$$\begin{aligned}\frac{p_2 + u_2 - v_4 - c_5 - m_1}{p_2 + u_2 - v_4 - v_5 - m_1} &= \Pr(D_2 < a_2 + q_5) \\ &\quad + \Pr(D_1 > a_1 + q_4 - q_5, D_2 > a_2 + q_5).\end{aligned} \tag{32}$$

By excluding the second probability on the right-hand side of (32) that has a nonnegative value, we obtain the lower bound on the optimal value of $a_2 + q_5$ as follows:

$$\Pr\left[D_2 < (a_2 + q_5)^l\right] = \frac{p_2 + u_2 - c_4 - c_5 - m_1}{p_2 + u_2 - v_4 - v_5 - m_1}. \tag{33}$$

Equation (16e) can also be rewritten as follows:

$$\begin{aligned}\frac{p_2 + u_2 - v_4 - c_5 - m_1}{p_2 + u_2 - v_4 - v_5 - m_1} &= \Pr(D_2 < a_2 + q_5) \\ &\quad + \Pr(D_1 > a_1 + q_4 - q_5, D_2 > a_2 + q_5).\end{aligned} \tag{34}$$

The structure of (34) is similar to that of (24) except that the second term on the right-hand side of (34) has a weight that is equal to one because product 2 has a lower priority in terms of common component allocation.

By excluding the nonnegative second term on the right-hand side of (35), we obtain the following:

$$\Pr\left[D_2 < (a_2 + q_5)^u\right] = \frac{p_2 + u_2 - v_4 - c_5 - m_1}{p_2 + u_2 - v_4 - v_5 - m_1} \tag{35}$$

Table 2: Optimal inventory-production decisions in different production system.

Production system	a_1	a_2	q_3	q_4	q_5
MTS and ATO (with common components)	250	167	503	749	560
MTS and ATO (without common components)	250	167	407	840	433
ATO and MTS	581	516	657	1257	600

which characterises the upper bound $(a_2 + q_5)^u$ of the optimal value of $a_2 + q_5$. Given the second probability on the right-hand side of (35), the procurement of component 5 is adjusted downwards when the common component is insufficient (in this case, an increase in component 5 cannot contribute to the increase in profit).

In a hybrid MTS/ATO production system, the components acquired at the beginning of the period are divided into two parts. Some of these components are used for production and the others are saved for use in later production. We have set bounds for the total procurement quantity of these components and the amount of final products that are generated in the MTS phase. Therefore, the bounds for the components that are saved for ATO can be obtained easily.

5. Numerical Experiments

In this section, we report the results of a numerical experiment that is designed to demonstrate the advantage of the hybrid MTS and ATO production system. The base parameters used in our numerical experiments are the following.

Let $D_1 \sim U[0, 1000]$, $D_2 \sim U[0, 1000]$, $p_1 = 40$, $p_2 = 30$, $c_3 = 5$, $c_4 = 7$, $c_5 = 3$, $m_1 = 8$, $m_2 = 6$, $r_1 = 18$, $r_2 = 15$, $u_1 = 3$, and $u_2 = 1$. In order to assess the influence of common components strategy on our model, we made a comparison with hybrid MTS and ATO production system that does not share common components. Meanwhile, we introduced optimal product and components stocking decisions and expected profit of pure MTS and pure ATO production systems into our model to further analyze the differences.

As is shown in Table 2, we can see that the optimal product stocking decisions of MTS mode (a_1^*, a_2^*) have the biggest quantity while the optimal product stocking decisions of hybrid MTS and ATO production system with common components share the same volume with the same system without common components. On the components stocking (q_3^*, q_4^*, q_5^*) aspect, ATO mode has the biggest volume; and in hybrid MTS and ATO production system with common components, common components' stocking volume is less than that of all the exclusive components together ($q_3^* + q_5^* > q_4^*$); lastly, in the hybrid system with no shared components, stocking volume of products and exclusive components together equals that in ATO mode.

As is shown in Table 3, it is obvious that the expected profit of a hybrid system is higher than that of a single ATO or MTS system. The highest profit is 10927 when the manufacturer uses the hybrid MTS and ATO production

Table 3: Expected profit of the manufacturer in different production system.

Production system	Expected profit	Profit improvement (compared to MTS)
MTS and ATO (with common components)	10927	16.3%
MTS and ATO (without common components)	10490	11.6%
ATO	10057	7.03%
MTS	9396	

system with common components, followed by the expected profit of the hybrid MTS and ATO production system with no shared components, which is 10490. Apparently, due to the risk-pooling effect it brought about, commonality is a contributor to the 4.7%-increase of the expected profit. Moreover, although the scale economies effect of pure MTS mode has brought down the unit production cost, the hybrid MTS and ATO production system with common components still has 16.3% more profit than it. This is because, in the pure MTS mode, the manufacturer only stocks products rather than materials or components to satisfy customers' demand and this strategy will take up large amount of floating capital. On the contrary, in pure ATO mode, the manufacturer only stocks components and they are assembled after demand information is required. This strategy can avoid producing unnecessary products but requires more flexibility which would bring higher production cost.

According to the statistics above, we can conclude the following.

(1) The hybrid MTS and ATO production system reduces uncertain demand risk by the method combined assemble-in-advance in the MTS phase with assemble-in-advance and ATO phase; meanwhile, the expected profit of manufacturer is improved in return. This implies the assemble-in-advance strategy is an efficient way to cater for the risk from the uncertainties of both the demand.

(2) Our example shows that commonality can bring about a higher profit than would be attainable without commonality and contribute to the reduction of forecast error. In the context of this paper, after applying commonality strategy, the inventory level of common component will decrease while those exclusive components will increase; owing to the risk-pooling effect of commonality, total inventories q_3+q_5 can be larger than q_4.

6. Summary and Future Research Directions

We have addressed in this paper the optimal product and inventory decision problem that arises from a hybrid MTS/ATO production system with commonality strategy under demand uncertainty. By analysing the balance between MTO at a lower cost and ATO at a higher cost operation, an optimal decision model is presented based on the two-stage production and inventory decision. Consequently, a balanced trade-off between the low unit production cost of MTS and the flexibility of ATO arises. According to the Karush-Kuhn-Tucker condition, the optimality conditions are found in a set of adjusted newsvendor-like solutions in which two products share one component. We have also studied the bounds and properties for the total procurement quantity of the components that are acquired at the beginning of the period and the amount of final products that are manufactured in the MTS phase.

Finally, we conduct numerical experiments to validate the model and the advantages of the hybrid MTS and ATO production system with common components. Research results show that a hybrid MTS and ATO production system can not only effectively respond to emergent orders and market demands using product inventory in MTS phase, but also reduce demand uncertainty risk by smoothing customers' demand using components in the ATO phase. And the risk-pooling effect of commonality strategy effectively reduces inventory cost.

Although our paper recommends some management techniques to assembly manufacturing enterprises, our findings can be extended in the following ways:

(1) finding optimal production-inventory decisions in a multiperiod condition with an uncertain context;

(2) exploring from a whole supply chain perspective the collaboration among material suppliers, manufacturers, wholesalers, and retailers as well as the related inventory control problems;

(3) considering the randomness of supplier output and supply uncertainty; inventory control and optimisation of multisupplier and multiproduct collaborative delivery that is also a promising research direction.

Appendix

Proof of Proposition 1

For any given (D_1, D_2), let

$$
\begin{aligned}
A_1^1 &= \min(D_1, a_1), \\
A_1^2 &= \min(D_2, a_2), \\
A_2^2 &= \left[\min\left(D_2 - a_2, q_5, q_4 - A_2^1\right)\right]^+, \\
A_1^1 + A_2^1 &= \min(D_1, a_1 + q_3), \\
A_1^1 + A_2^1 + A_1^2 + A_2^2 &= \min\Big(D_1 + A_1^1 + A_2^1, q_5 + A_1^1 \\
&\quad + A_2^1 + A_1^2, q_4 + A_1^1 + A_1^2\Big),
\end{aligned}
$$

$$
\begin{aligned}
g(Q) = E\Bigg\{&\Bigg[\sum_{i=1}^{2} p_i s_i - m_i y_i^A - u_i (D_i - s_i)^+ \\
&+ w_i (a_i - D_i)^+ - r_i a_i\Bigg] + v_3 \left(q_3 - y_1^A\right)^+ + v_4 \Big(q_4 \\
&- y_1^A - y_2^A\Big)^+ + v_5 \left(q_5 - y_2^A\right)^+ - c_3 q_3 - c_4 q_4
\end{aligned}
$$

$$\begin{aligned} - c_5 q_5 \Big\} &= (m_1 + v_3 + v_4 - w_1) A_1^1 + (m_2 + v_4 \\ &+ v_5 - w_2) A_1^2 + [(p_1 + u_1 - v_3 - v_4 - m_1) - (p_2 \\ &+ u_2 - v_4 - v_5 - m_2)] \left(A_1^1 + A_2^1\right) + (p_2 + u_2 - v_4 \\ &- v_5 - m_2) \left(A_1^1 + A_2^1 + A_1^2 + A_2^2\right) + \sum_{i=1}^{2} (w_i - r_i) a_i \\ &+ \sum_{j=3}^{5} (v_i - c_i) q_j - \sum_{i=1}^{2} u_i D_i. \end{aligned} \tag{A.1}$$

Note that, A_1^1, $A_1^1 + A_2^1$, $A_1^1 + A_2^1 + A_1^2 + A_2^2$ are all concave in Q and $Z(Q)$ is a linear combination of concave functions

$$\begin{aligned} &\text{Since } (m_1 + v_3 + v_4 - w_1) > 0, \\ &(m_2 + v_4 + v_5 - w_2) > 0, \\ &(p_1 + u_1 - v_3 - v_4 - m_1) \\ &\quad > (p_2 + u_2 - v_4 - v_5 - m_2) > 0, \\ &w_1 - r_1 < 0, \\ &w_2 - r_2 < 0, \\ &v_3 > c_3, \\ &v_4 > c_4, \\ &v_5 > c_5 \end{aligned} \tag{A.2}$$

and therefore, $g(Q)$ is a concave function in Q.

For any Q_1, Q_2, and $\lambda \in [0, 1]$, let

$$\begin{aligned} Q_\lambda &= \lambda Q_1 + (1 - \lambda) Q_2, \\ Z(Q_1) &= g(Q_1), \\ Z(Q_2) &= g(Q_2). \end{aligned} \tag{A.3}$$

Then

$$\begin{aligned} Z(Q_\lambda) \geq g(Q_\lambda) &= g[\lambda Q_1 + (1 - \lambda) Q_2] \\ &\geq \lambda g(Q_1) + (1 - \lambda) g(Q_2) \\ &= \lambda Z Q_1 + (1 - \lambda) Z Q_2. \end{aligned} \tag{A.4}$$

Hence

$$Z(Q_\lambda) \geq \lambda Z Q_1 + (1 - \lambda) Z Q_2. \tag{A.5}$$

$Z(Q)$ is concave in Q. This completes the proof.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The work that is described in this paper was supported by the Talented Person Foundation of Central South Forestry University of Science and Technology, Forestry Engineering Postdoctoral Research Center, and Graduate Education and Degree Innovation Foundation of Central South University (2014). The authors would like to thank the guest editors and the anonymous referees for their helpful comments and constructive suggestions on an earlier version of the paper.

References

[1] K. Fu, V. N. Hsu, and C.-Y. Lee, "Inventory and production decisions for an assemble-to-order system with uncertain demand and limited assembly capacity," *Operations Research*, vol. 54, no. 6, pp. 1137–1150, 2006.

[2] Y. Xiao, J. Chen, and C.-Y. Lee, "Optimal decisions for assemble-to-order systems with uncertain assembly capacity," *International Journal of Production Economics*, vol. 123, no. 1, pp. 155–165, 2010.

[3] J.-S. Song and P. Zipkin, "Supply chain operations: assemble-to-order systems," in *Handbooks in Operations Research and Management Science*, S. C. Graves and A. Kok, Eds., pp. 561–596, Elsevier, 2003.

[4] I. Moon and S. Choi, "Distribution free procedures for make-to-order (MTO), make-in-advance (MIA), and composite policies," *International Journal of Production Economics*, vol. 48, no. 1, pp. 21–28, 1997.

[5] H. Tsubone, Y. Ishikawa, and H. Yamamoto, "Production planning system for a combination of make-to-stock and make-to-order products," *International Journal of Production Research*, vol. 40, no. 18, pp. 4835–4851, 2002.

[6] M. Kalantari, M. Rabbani, and M. Ebadian, "A decision support system for order acceptance/rejection in hybrid MTS/MTO production systems," *Applied Mathematical Modelling*, vol. 35, no. 3, pp. 1363–1377, 2011.

[7] C. A. Soman, D. P. van Donk, and G. Gaalman, "Combined make-to-order and make-to-stock in a food production system," *International Journal of Production Economics*, vol. 90, no. 2, pp. 223–235, 2004.

[8] M. Perona, N. Saccani, and S. Zanoni, "Combining make-to-order and make-to-stock inventory policies: an empirical application to a manufacturing SME," *Production Planning & Control*, vol. 20, no. 7, pp. 559–575, 2009.

[9] A. Eynan and M. J. Rosenblatt, "The impact of component commonality on composite assembly policies," *Naval Research Logistics*, vol. 54, no. 6, pp. 615–622, 2007.

[10] S. Benjaafar and M. Elhafsi, "Production and inventory control of a single product assemble-to-order system with multiple customer classes," *Management Science*, vol. 52, no. 12, pp. 1896–1912, 2006.

[11] Z. Pang, H. Shen, and T. C. E. Cheng, "Inventory rationing in a make-to-stock system with batch production and lost sales," *Production and Operations Management*, vol. 23, no. 7, pp. 1243–1257, 2014.

[12] B. Pal, S. S. Sana, and K. Chaudhuri, "A multi-echelon production-inventory system with supply disruption," *Journal of Manufacturing Systems*, vol. 33, no. 2, pp. 262–276, 2014.

[13] J.-S. Song and Y. Zhao, "The value of component commonality in a dynamic inventory system with lead times," *Manufacturing*

and Service Operations Management, vol. 11, no. 3, pp. 493–508, 2009.

[14] M. S. Hillier, "The costs and benefits of commonality in assemble-to-order systems with a (Q, r) -policy for component replenishment," *European Journal of Operational Research*, vol. 141, no. 3, pp. 570–586, 2002.

[15] S. Ma, W. Wang, and L. Liu, "Commonality and postponement in multistage assembly systems," *European Journal of Operational Research*, vol. 142, no. 3, pp. 523–538, 2002.

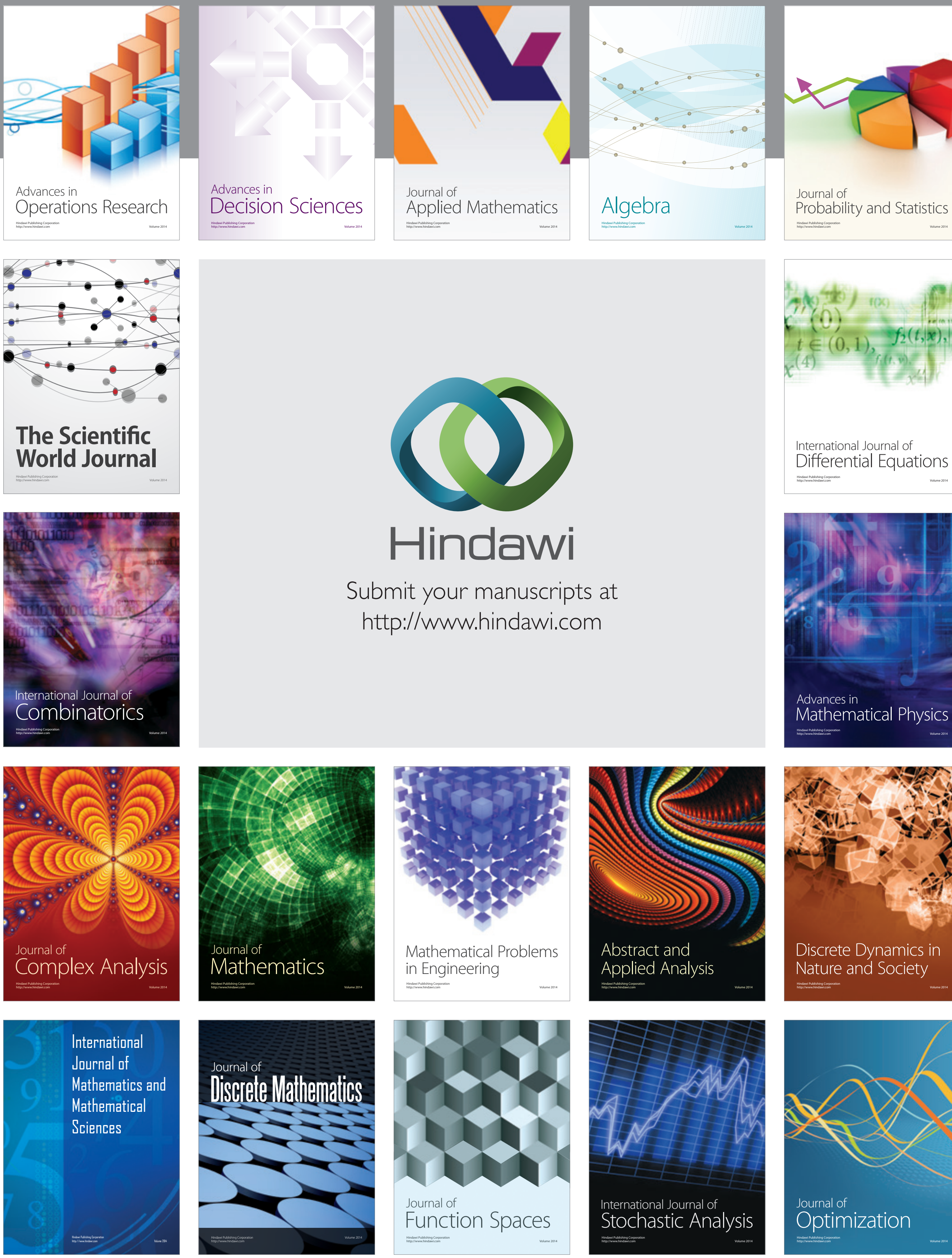
[16] E. Mohebbi and F. Choobineh, "The impact of component commonality in an assemble-to-order environment under supply and demand uncertainty," *Omega*, vol. 33, no. 6, pp. 472–482, 2005.

[17] V. N. Hsu, C. Y. Lee, and K. C. So, "Optimal component stocking policy for assemble-to-order systems with lead-time-dependent component and product pricing," *Management Science*, vol. 52, no. 3, pp. 337–351, 2006.

[18] Y.-B. Xiao, J. Chen, and C.-Y. Lee, "Single-period two-product assemble-to-order systems with a common component and uncertain demand patterns," *Production and Operations Management*, vol. 19, no. 2, pp. 216–232, 2010.

[19] F. Bernstein, G. A. DeCroix, and Y. Wang, "The impact of demand aggregation through delayed component allocation in an assemble-to-order system," *Management Science*, vol. 57, no. 6, pp. 1154–1171, 2011.

[20] Y. Gerchak, M. J. Magazine, and B. A. Gamble, "Component commonality with service level requirements," *Management Science*, vol. 34, no. 6, pp. 753–760, 1988.


Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
Algebra
Journal of Probability and Statistics
The Scientific World Journal
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Differential Equations
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
Journal of Optimization