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Research Article

Existence of Multiple Solutions for a Class of Biharmonic Equations

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By a symmetric Mountain Pass Theorem, a class of biharmonic equations with Navier type boundary value at the resonant and nonresonant case are discussed, and infinitely many solutions of the equations are obtained.

1. Introduction and Main Results

In this paper, we study the following fourth-order elliptic equation:

$$\begin{gathered} \Delta^2 u + c\Delta u = \mu h(x)|u|^{p-2}u + f(x,u), \quad x \in \Omega, \\ u = \Delta u = 0, \quad x \in \partial\Omega, \end{gathered} \tag{1}$$

where Δ^2 is the biharmonic operator, c is a constant, $\Omega \subset R^N$ is a bounded smooth domain, $1 < p < 2$, $\mu \geq 0$ is a parameter, $h \in L^\infty(\Omega)$, $h(x) \geq 0$, $h(x) \not\equiv 0$, $f(x,s)$ is a continuous function on $\overline{\Omega} \times R$.

This fourth-order semilinear elliptic problem can be considered as an analogue of a class of second-order problems which have been studied by many authors. A main tool of seeking solutions of the problem is the Mountain Pass Theorem (see [1–3]). In [4], Pei studied the following problem:

$$\begin{gathered} \Delta^2 u + c\Delta u = f(x,u), \quad x \in \Omega, \\ u = \Delta u = 0, \quad x \in \partial\Omega, \end{gathered} \tag{2}$$

and obtained at least three nontrivial solutions by using the minimax method and Morse theory.

Problem (1) has been studied extensively in recent years; we refer the reader to [5–11] and the references therein. In [12], the author showed that the problem (1) admits at least three (or four or five) nontrivial solutions by using the minimax method and Mountain Pass Theory.

However, to the best of author's knowledge, there have been very few results dealing with (1) using a symmetric Mountain Pass Theorem. This paper will make some contribution in the research field. In this paper, we study the problem (1) by a symmetric Mountain Pass Theorem at the resonant and nonresonant case and obtain infinitely many solutions of the equation.

Consider eigenvalue problem

$$\begin{gathered} -\Delta u = \lambda u, \quad x \in \Omega, \\ u = 0, \quad x \in \partial\Omega. \end{gathered} \tag{3}$$

Let us denote that $\lambda_k (k \in N)$ are the eigenvalues and $\varphi_k (k \in N)$ are the corresponding eigenfunctions of the eigenvalue problem (3). It is well known that $0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \cdots \leq \lambda_k \to +\infty$, and the first eigenfunction $\varphi_1 > 0$, $x \in \Omega$.

It is easy to see that $\lambda_k(\lambda_k - c)$, $k = 1, 2, \ldots$, are eigenvalues of the problem

$$\begin{gathered} \Delta^2 u + c\Delta u = \Lambda u, \quad x \in \Omega, \\ u = \Delta u = 0, \quad x \in \partial\Omega, \end{gathered} \tag{4}$$

and $\varphi_k (k = 1, 2, \ldots)$ are still the corresponding eigenfunctions.

Let $H = H^2(\Omega) \cap H_0^1(\Omega)$ be the Hilbert space equipped with the inner product

$$\langle u, v \rangle_H = \int_\Omega (\Delta u \Delta v + \nabla u \nabla v)\, dx, \tag{5}$$

and the deduced norm

$$\|u\|_H = \left[\int_\Omega \left(|\Delta u|^2 + |\nabla u|^2\right) \mathrm{d}x\right]^{1/2}. \tag{6}$$

Suppose that $c < \lambda_1$. Let us define a norm of the space H as follows:

$$\|u\| = \left[\int_\Omega \left(|\Delta u|^2 - c|\nabla u|^2\right) \mathrm{d}x\right]^{1/2}. \tag{7}$$

It is easy to verify that the norm $\|\cdot\|$ is equivalent to the norm $\|\cdot\|_H$ on H, and for all $u \in H$, the following Poincaré inequality holds:

$$\|u\|^2 \geq \lambda_1 (\lambda_1 - c) |u|_2^2, \tag{8}$$

where $|u|_2^2 = \int_\Omega |u|^2 dx$.

Throughout this paper, the weak solutions of (1) are the critical points of the associated functional

$$\begin{aligned}\Phi_\mu(u) &= \frac{1}{2}\left(\int_\Omega \left(|\Delta u|^2 - c|\nabla u|^2\right) \mathrm{d}x\right) \\ &\quad - \frac{\mu}{p}\int_\Omega h(x)|u|^p \mathrm{d}x - \int_\Omega F(x,u)\,\mathrm{d}x, \quad \forall u \in H,\end{aligned} \tag{9}$$

where $F(x,u) = \int_0^u f(x,t)\,dt$.

Obviously $\Phi_\mu \in C^1(H,R)$, and for all $u, \varphi \in H$,

$$\begin{aligned}\left\langle \Phi_\mu'(u), \varphi\right\rangle &= \int_\Omega (\Delta u \Delta \varphi - c\nabla u \nabla \varphi\ \mathrm{d}x) \\ &\quad - \mu\int_\Omega h(x)|u|^{p-2}u\varphi\,\mathrm{d}x - \int_\Omega f(x,u)\varphi\,\mathrm{d}x.\end{aligned} \tag{10}$$

In order to establish solutions for problem (1), we make the following assumptions:

(F_0) $h \in L^\infty(\Omega)$, $h(x) \geq 0$;

(F_1) $f(x,0) = 0$, $f(x,-t) = -f(x,t)$, for all $x \in \Omega$, $t \in R$;

(F_2) $\lim_{|t|\to 0} f(x,t)/t = \alpha$, $\lim_{|t|\to\infty} f(x,t)/t = \beta$ uniformly for a.e. $x \in \Omega$, where $0 \leq \alpha < \lambda_k(\lambda_k - c) < \beta$, or $\beta = \lambda_k(\lambda_k - c)$;

(F_3) $\lim_{|t|\to\infty}(f(x,t)t - 2F(x,t)) = -\infty$ uniformly in $x \in \Omega$.

Definition 1 (see [13]). Let $\Phi \in C^1(X,R)$, we say that Φ satisfies the Cerami condition at the level $c \in R$, if any sequence $\{u_n\} \subset X$ with

$$\Phi(u_n) \longrightarrow c, \qquad (1 + \|u_n\|)\Phi'(u_n) \longrightarrow 0, \quad \text{as} \quad n \longrightarrow \infty \tag{11}$$

possesses a convergent subsequence; Φ satisfies the (C) condition if Φ satisfies $(C)_c$ for all $c \in R$.

Lemma 2. *Let $X_k = \operatorname{span}\{\varphi_1, \varphi_2, \ldots, \varphi_k\}$; $H = X_k \oplus X_k^\perp$; then*

$$\begin{aligned}&\|u\|^2 \leq \lambda_k(\lambda_k - c)|u|_2^2, \quad \forall u \in X_k; \\ &\|u\|^2 \geq \lambda_{k+1}(\lambda_{k+1} - c)|u|_2^2, \quad \forall u \in X_k^\perp,\ k \geq 2.\end{aligned} \tag{12}$$

Proof. It is similar to the proof of Lemma 2.5 in [13]. □

Theorem 3 (see [14] a symmetric Mountain Pass Theorem). *Suppose that $\Phi \in C^1(E,R)$ is even, $\Phi(\theta) = 0$, and*

(i) *there exist ρ, $\alpha > 0$, and a finite dimensional linear subspace Z such that*

$$\Phi|_{Z^\perp \cap \partial B_\rho(\theta)} \geq \alpha, \tag{13}$$

(ii) *there exist a sequence of linear subspace $\overline{Z}_m$, $\dim(\overline{Z}_m) = m$, and $R_m > 0$ such that*

$$\Phi(x) \leq 0, \quad \forall x \in \overline{Z}_m \setminus B_{R_m}, \quad m = 1, 2, \ldots. \tag{14}$$

If Φ satisfies (PS) condition, then Φ possesses infinitely many distinct critical points corresponding to positive critical values.

Remark 4. If Φ satisfies the (C) condition, Theorem 3 still holds.

The main results of this paper are as follows.

Theorem 5. *Assume that (F_0)–(F_2) hold, and $c < \lambda_1$, $\lambda_k(\lambda_k - c) < \beta$, β is not an eigenvalue of (4); then there exist $\mu^* > 0$ such that for $\mu \in (0, \mu^*)$, (1) has infinitely many solutions.*

Theorem 6. *Assume that $(F_0)-(F_3)$ hold, and $c < \lambda_1$, $\beta = \lambda_k(\lambda_k - c)$; then there exist $\mu^* > 0$ such that for $\mu \in (0, \mu^*)$, (1) has infinitely many solutions.*

2. Proofs of Theorems

Proof of Theorem 5. (i) Assume that $\{u_n\} \subset H$ is a (C) sequence, that is,

$$(1 + \|u_n\|)\Phi_\mu'(u_n) \longrightarrow 0, \qquad \Phi_\mu(u_n) \longrightarrow c. \tag{15}$$

We claim that $\{u_n\}$ is bounded. Assume as a contradiction that $|u_n|_2 \to \infty$. Setting $v_n = u_n/|u_n|_2$, then $|v_n|_2 = 1$. Without loss of generality, we assume

$$\begin{aligned}&v_n \rightharpoonup v \text{ in } H, \qquad v_n \longrightarrow v \text{ in } L^2(\Omega), \\ &v_n \longrightarrow v \text{ a.e.} \quad x \in \Omega.\end{aligned} \tag{16}$$

From (15) we know that

$$\left|\int_\Omega (\Delta v_n \Delta\varphi - c\nabla v_n \nabla\varphi)\,\mathrm{d}x - \mu\int_\Omega h(x)\frac{|u_n|^{p-2}u_n}{|u_n|_2}\varphi\,\mathrm{d}x - \int_\Omega \frac{f(x,u_n)}{|u_n|_2}\varphi\,\mathrm{d}x\right|$$

$$= \frac{1}{|u_n|_2} \left| \left\langle \Phi'_\mu (u_n), \varphi \right\rangle \right| \le \frac{1}{|u_n|_2} \left\| \Phi'_\mu (u_n) \right\| \|\varphi\| \longrightarrow 0, \quad \forall \varphi \in H. \tag{17}$$

Next we consider the two possible cases: (a)$v \neq 0$, (b)$v = 0$. In case (a), from (F_2) we derive

$$|u_n| = |v_n| |u_n|_2 \longrightarrow \infty, \quad \lim_{n\to\infty} \frac{f(x,u_n)}{u_n} = \beta. \tag{18}$$

For $v_n \to v$ in $L^2(\Omega)$, we have

$$\lim_{n\to\infty} \frac{f(x,u_n)}{|u_n|_2} = \lim_{n\to\infty} \frac{f(x,u_n)}{u_n} v_n = \beta v, \quad \text{a.e.} \quad x \in \Omega. \tag{19}$$

In case (b), we have

$$\frac{f(x,u_n)}{|u_n|_2} \le c|v_n| \longrightarrow 0. \tag{20}$$

Then

$$\lim_{n\to\infty} \int_\Omega \frac{f(x,u_n)\varphi}{|u_n|_2} \mathrm{d}x = \lim_{n\to\infty} \int_\Omega \frac{f(x,u_n) v_n}{u_n} \varphi \, \mathrm{d}x = \int_\Omega \beta v \varphi \, \mathrm{d}x, \quad \forall \varphi \in H. \tag{21}$$

By (17) and $\lim_{n\to\infty} \int_\Omega h(x)(|u_n|^{p-2}u_n/|u_n|_2)\varphi dx = 0$, we have

$$\int_\Omega (\Delta v \Delta \varphi - c \nabla v \nabla \varphi) \, \mathrm{d}x = \int_\Omega \beta v \varphi \mathrm{d}x, \quad \forall \varphi \in H. \tag{22}$$

We can easily see that $v \not\equiv 0$. In fact, if $v \equiv 0$, then $|v|_2 = 0$, which contradicts $\lim_{n\to\infty}|v_n|_2 = |v|_2 = 1$. Hence, β is an eigenvalue of the problem (4); this contradicts our assumption. Then $\{u_n\}$ is bounded; there exist a subsequence of $\{u_n\}$(we can also denote by $\{u_n\}$) and $u \in H$, such that $u_n \rightharpoonup u$ in H. By Lemma 2 and (15), we have

$$\begin{aligned} \|u_n - u_m\| &= \mu \int_\Omega h(x) |u_n - u_m|^p \mathrm{d}x \\ &\quad + \int_\Omega f(x, u_n - u_m)(u_n - u_m) \, \mathrm{d}x \\ &\quad + o(1) \|u_n - u_m\| \\ &\le \mu |h|_\infty \int_\Omega |u_n - u_m|^p \mathrm{d}x \\ &\quad + \int_\Omega f(x, u_n - u_m)(u_n - u_m) \, \mathrm{d}x \\ &\quad + o(1) \|u_n - u_m\|, \end{aligned} \tag{23}$$

where $o(1) \to 0$, and

$$\left| \int_\Omega f(x, u_n - u_m)(u_n - u_m) \, \mathrm{d}x \right| \le |f(x, u_n - u_m)|_2 |u_n - u_m|_2, \tag{24}$$

as $n, m \to \infty$. Hence $u_n \to u$ in H. We verify $\Phi_\mu(u)$ satisfies (C) condition.

(ii) There exists some $\rho, \gamma > 0$ such that $\Phi_\mu|_{X_k^\perp \cap \partial B_\rho(\theta)} \ge \gamma > 0$, where $B_\rho(\theta) = \{u \in H : \|u\| \le \rho\}$.

By (F_1) and (F_2), taking $\sigma \in (2, 2^*)$, for any given $\varepsilon > 0$, there exists $C_1 > 0$ such that

$$F(x,u) \le \frac{1}{2}(\alpha + \varepsilon)|u|^2 + C_1 |u|^\sigma, \quad \forall x \in \Omega, \tag{25}$$

where

$$2^* = \begin{cases} \dfrac{2N}{N-2}, & N > 2, \\ +\infty, & N \le 2. \end{cases} \tag{26}$$

Taking $\varepsilon > 0$ such that $\alpha + \varepsilon < \lambda_k(\lambda_k - c)$, combining Lemma 2, Poincaré inequality, and Sobolev embedding, we have

$$\begin{aligned} \Phi_\mu(u) &\ge \frac{1}{2}\left(\int_\Omega \left(|\Delta u|^2 - c|\nabla u|^2 \right) \mathrm{d}x \right) \\ &\quad - \frac{\mu |h|_\infty}{p} \int_\Omega |u|^p \mathrm{d}x - \int_\Omega F(x,u) \, \mathrm{d}x \\ &\ge \frac{1}{2}\|u\|^2 - \frac{\alpha+\varepsilon}{2}|u|_2^2 - \frac{\mu|h|_\infty}{p}|u|_p^p - C_1|u|_\sigma^\sigma \\ &\ge \frac{1}{2}\left(1 - \frac{\alpha+\varepsilon}{\lambda_{k+1}(\lambda_{k+1}-c)}\right)\|u\|^2 - \mu C_2 \|u\|^p - C_3\|u\|^\sigma \\ &= \left(\frac{1}{2}\left(1 - \frac{\alpha+\varepsilon}{\lambda_{k+1}(\lambda_{k+1}-c)}\right) \right. \\ &\quad \left. - \mu C_2 \|u\|^{p-2} - C_3 \|u\|^{\sigma-2} \right) \|u\|^2, \end{aligned} \tag{27}$$

where C_2, C_3 are constant.

Let $g(t) = \mu C_2 t^{p-2} + C_3 t^{\sigma-2}$; we claim that there exists t_0 such that

$$g(t_0) < \frac{1}{2}\left(1 - \frac{\alpha+\varepsilon}{\lambda_{k+1}(\lambda_{k+1}-c)}\right). \tag{28}$$

It is easy to see that $g(t)$ has a minimum at $t_0 = ((\mu C_2(2-p))/(C_3(\sigma-2)))^{1/(\sigma-p)}$; substituting t_0 in $g(t)$, we have

$$g(t_0) = \mu^{(\sigma-2)/(\sigma-p)}\left[\left(C_2\left(\frac{C_2(2-p)}{C_3(\sigma-2)}\right)\right)^{(p-2)/(\sigma-p)} + \left(C_3\left(\frac{C_2(2-p)}{C_3(\sigma-2)}\right)\right)^{(\sigma-2)/(\sigma-p)}\right] < \frac{1}{2}\left(1-\frac{\alpha+\varepsilon}{\lambda_{k+1}(\lambda_{k+1}-c)}\right), \tag{29}$$

where $0 < \mu < \mu^* = [1/2(1-((\alpha+\varepsilon)/(\lambda_{k+1}(\lambda_{k+1}-c))))/((C_2(C_2(2-p)/C_3(\sigma-2)))^{(p-2)/(\sigma-p)} + (C_3(C_2(2-p)/C_3(\sigma-2)))^{(\sigma-2)/(\sigma-p)})]^{(2-\sigma)/(\sigma-p)}$.

We take $\|u\| = \rho = t_0 > 0$, then there exists $\gamma > 0$ such that $\Phi_\mu|_{X_k^\perp\cap\partial B_\rho(\theta)} \geq \gamma > 0$, where $B_\rho(\theta) = \{u \in H : \| u \| \leq \rho\}$.

(iii) There exists $R_k > \rho$, such that

$$\Phi_\mu(u) \leq 0, \quad \forall u \in X_k \setminus B_{R_k}(\theta), \quad k = 1, 2, \ldots. \tag{30}$$

For $\beta > \lambda_k(\lambda_k - c)$, (F_2) implies that for any $\varepsilon > 0$, there exists $C_4 > 0$ such that

$$F(x,u) \geq \frac{1}{2}(\beta-\varepsilon)|u|^2 - C_4. \tag{31}$$

Taking $\varepsilon > 0$ such that $\beta - \varepsilon > \lambda_k(\lambda_k - c)$. By Lemma 2 and $(\mu/p)\int_\Omega h(x)|u|^p \mathrm{d}x \geq 0$, we have

$$\begin{aligned}\Phi_\mu(u) &= \frac{1}{2}\left(\int_\Omega\left(|\Delta u|^2 - c|\nabla u|^2\right)\mathrm{d}x\right) - \frac{\mu}{p}\int_\Omega h(x)|u|^p\,\mathrm{d}x - \int_\Omega F(x,u)\,\mathrm{d}x \\ &\leq \frac{1}{2}\|u\|^2 - \frac{\beta-\varepsilon}{2}|u|_2^2 + C_4|\Omega| \\ &\leq \frac{1}{2}\left(1-\frac{\beta-\varepsilon}{\lambda_k(\lambda_k-c)}\right)\|u\|^2 + C_4|\Omega| \longrightarrow -\infty, \quad \forall u \in X_k, \text{ as } \|u\| \longrightarrow +\infty.\end{aligned} \tag{32}$$

Hence there exists $R_k > \rho$ such that

$$\Phi_\mu(u) \leq 0, \quad \forall u \in X_k \setminus B_{R_k}(\theta), \; k = 1, 2, \ldots. \tag{33}$$

Summing up the above proofs, Φ_μ satisfies all the conditions of Theorem 3 and Remark 4. Then the problem (1) has infinitely many solutions. □

Proof of Theorem 6. Similar to the proof of Theorem 5(i), we have

$$\int_\Omega(\Delta v\Delta\varphi - c\nabla v\nabla\varphi)\,\mathrm{d}x = \int_\Omega \lambda_k v\varphi\,\mathrm{d}x, \quad \forall \varphi \in H. \tag{34}$$

We can easily see that $v \not\equiv 0$. In fact, if $v \equiv 0$, then $|v|_2 = 0$, which contradicts $\lim_{n\to\infty}|v_n|_2 = |v|_2 = 1$. Then v is a corresponding eigenfunction of the eigenvalue λ_k; hence $|u_n| \to \infty$, a.e. $x \in \Omega$.

By (F_3), we have

$$\lim_{n\to\infty}(f(x,u_n)u_n - 2F(x,u_n)) = -\infty \text{ uniformly in } x \in \Omega. \tag{35}$$

It follows from Fatous Lemma that

$$\lim_{n\to\infty}\int_\Omega(f(x,u_n)u_n - 2F(x,u_n))\,\mathrm{d}x = -\infty. \tag{36}$$

On the other hand, (15) implies that

$$\begin{aligned}2c &\longleftarrow 2\Phi_\mu(u_n) - \left\langle\Phi'_\mu(u_n), u_n\right\rangle \\ &= \int_\Omega\left(|\Delta u_n|^2 - c|\nabla u_n|^2\right)\mathrm{d}x - \frac{2\mu}{p}\int_\Omega h(x)|u_n|^p\,\mathrm{d}x - \int_\Omega 2F(x,u_n)\,\mathrm{d}x \\ &\quad - \int_\Omega\left(|\Delta u_n|^2 - c|\nabla u_n|^2\right)\mathrm{d}x + \mu\int_\Omega h(x)|u_n|^p\,dx + \int_\Omega f(x,u_n)u_n\mathrm{d}x \\ &= \int_\Omega(f(x,u_n)u_n - 2F(x,u_n))\,\mathrm{d}x - \left(\frac{2}{p}-1\right)\mu\int_\Omega h(x)|u_n|^p\,\mathrm{d}x,\end{aligned} \tag{37}$$

where $\lim_{n\to\infty}(2/p-1)\mu\int_\Omega h(x)|u_n|^p\,dx = \infty$, which contradicts (36); hence $\{u_n\}$ is bounded. Similar to the proof of Theorem 5(i), we have $\{u_n\} \to u$ in H. Hence we verify $\Phi_\mu(u)$ satisfies (C) condition.

Similar to [15], let $H(x,t) = F(x,t) - (1/2)\lambda_k(\lambda_k - c)t^2$, and $f(x,t) = \lambda_k(\lambda_k - c)t + h(x,t)$; then

$$\lim_{|t|\to\infty}\frac{2H(x,t)}{t^2} = 0, \qquad \lim_{|t|\to\infty}(h(x,t)t - 2H(x,t)) = -\infty. \tag{38}$$

It follows that for every $N > 0$, there exists $R_N > 0$ such that

$$h(x,t)t - 2H(x,t) \leq -N, \quad \forall t \in R, \quad |t| \geq R_N, \quad \text{a.e. } x \in \Omega. \tag{39}$$

For $t > 0$, we have

$$\frac{d}{dt}\left[\frac{H(x,t)}{t^2}\right] = \frac{h(x,t)t - 2H(x,t)}{t^3}; \tag{40}$$

over the interval $[t,s] \subset [T,+\infty)$, we have

$$\frac{H(x,s)}{s^2} - \frac{H(x,t)}{t^2} \leq \frac{N}{2}\left(\frac{1}{s^2} - \frac{1}{t^2}\right). \tag{41}$$

Letting $s \to +\infty$, we see that $H(x,t) \geq (N/2)$, $t \in R$, $t \geq R_N$, a.e. $x \in \Omega$. In a similar way, we have $H(x,t) \geq (N/2)$, $t \in R$, $t \leq -R_N$, a.e. $x \in \Omega$. Hence

$$\lim_{|t|\to\infty} H(x,t) = +\infty. \quad \text{a.e.} \quad x \in \Omega. \tag{42}$$

By Lemma 2 and $h(x) \geq 0$, we have

$$\begin{aligned}\Phi_\mu(u) &= \frac{1}{2}\left(\int_\Omega \left(|\Delta u|^2 - c|\nabla u|^2\right) dx\right) \\ &\quad - \frac{1}{2}\lambda_k(\lambda_k - c)\int_\Omega u^2 \, dx \\ &\quad - \frac{\mu}{p}\int_\Omega h(x)|u|^p \, dx - \int_\Omega H(x,u)\, dx \\ &\leq -\int_\Omega H(x,u)\, dx \longrightarrow -\infty, \quad \forall u \in X_k, \\ &\qquad \text{as } \|u\| \longrightarrow +\infty.\end{aligned} \tag{43}$$

Hence there exists $R_k > \rho$ such that

$$\Phi_\mu(u) \leq 0, \quad \forall u \in X_k \setminus B_{R_k}(\theta), \; k = 1, 2, \ldots. \tag{44}$$

Summing up the above proofs and Theorem 5 (ii), Φ_μ satisfies all the conditions of Theorem 3 and Remark 4, then the problem (1) has infinitely many solutions. □

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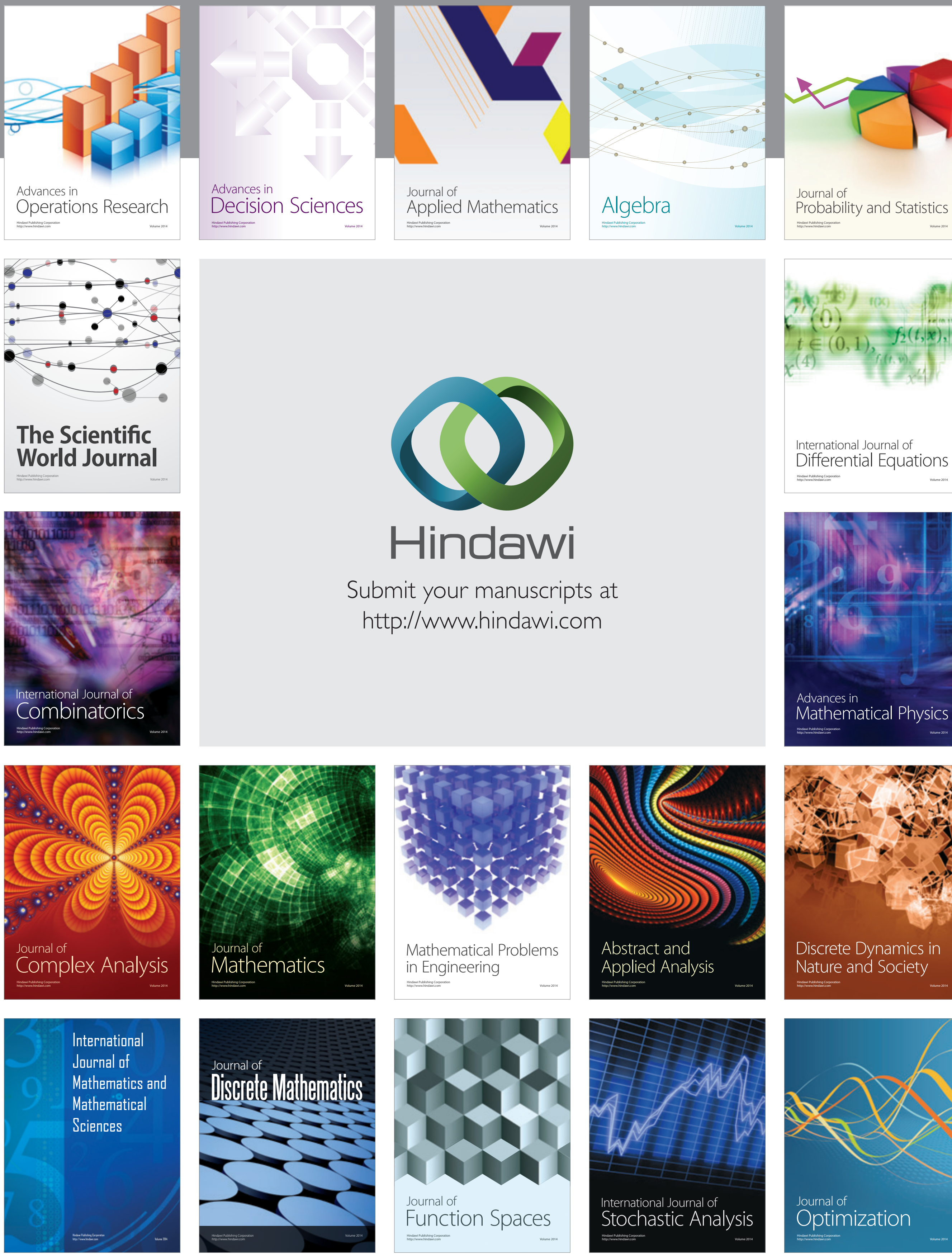
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