

From Information Geometry to Newtonian Dynamics*

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Abstract

Newtonian dynamics is derived from prior information codified into an appropriate statistical model. The basic assumption is that there is an irreducible uncertainty in the location of particles so that the state of a particle is defined by a probability distribution. The corresponding configuration space is a statistical manifold the geometry of which is defined by the information metric. The trajectory follows from a principle of inference, the method of Maximum Entropy. No additional “physical” postulates such as an equation of motion, or an action principle, nor the concepts of momentum and of phase space, not even the notion of time, need to be postulated. The resulting entropic dynamics reproduces the Newtonian dynamics of any number of particles interacting among themselves and with external fields. Both the mass of the particles and their interactions are explained as a consequence of the underlying statistical manifold.

1 Introduction

It is widely assumed that geometry is useful because it describes properties of the real world. Indeed, Euclidean geometry may very well have been the first successful physics theory, the first example of a “law of nature”. Later developments such as Riemannian geometry and the theory of fiber bundles have only strengthened this conception: geometry works because it lies at the very core of physics. Thus, it may be surprising, at least at first sight, to find that the same methods of geometry have also turned out to be useful in statistical inference, a separate field that makes no claims to authority on natural phenomena. It could just be a coincidence but perhaps it is not.

Perhaps the laws of physics are deeply geometrical because they are practical rules to process information about the world and geometry is the uniquely natural tool to do just that. This notion, that *the laws of physics are not laws of*

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nature but rules of inference, seems outrageous but deserves serious attention. The evidence supporting it is already considerable. Indeed, most of the formal structure of statistical mechanics [1] and of quantum theory [2] can already be derived from principles of inference (consistency, probabilities, entropy, etc.).

The objective of this paper is to use well established principles of inference to derive Newtonian dynamics from relevant prior information codified into a statistical model. The challenge, of course, is to accomplish this task without assuming what we want to derive. One must not assume equations of motion or principles of least action, and in particular, one must not assume the concept of momentum and the associated phase space, and not even the notion of an absolute Newtonian time.

The first step is to construct a suitable statistical model of the space of states of a system of particles. A most remarkable fact is that the statistical configuration space is automatically endowed with a geometry and that this “information” geometry turns out to be unique [3][4].

Next we tackle the dynamics: Given the initial and the final states, what trajectory is the system expected to follow? In the usual approach one postulates an equation of motion or an action principle that presumably reflects a “law of nature.” For us the dynamics follows from a principle of inference, the method of Maximum Entropy, and we show that with a suitable choice of the statistical manifold the resulting “entropic dynamics” [5][6] reproduces Newtonian dynamics.

The entropic dynamics approach allows us to see familiar notions such as time, mass and interactions from an unfamiliarly fresh perspective. For example, there is no reference to an external time but there is an internal “intrinsic” time that is a measure of the change of the system itself. Thus, the Newtonian universe turns out to be its own clock, and the familiar Newtonian time is not particularly fundamental but merely a convenient definition designed to make motion look as simple as possible. Both the mass of the particles and their interactions are explained in terms of an irreducible uncertainty of their positions; they are features of the underlying statistical manifold.

2 Configuration space as a statistical manifold

Let us start with a single particle moving in space: the configuration space is a three dimensional manifold with some unknown metric tensor $g_{ij}(x)$. Our main assumption is that there is a certain fuzziness to space; there is an irreducible uncertainty in the location of the particle. Thus, when we say the particle is at the point x what we mean is that its “true” position y is somewhere in the vicinity of x . This leads us to associate a probability distribution $p(y|x)$ to each point x and the configuration space is thus transformed into a statistical manifold: a point x is no longer a structureless dot but a probability distribution.

Remarkably there is a *unique* measure of the extent to which the distribution at x can be distinguished from the neighboring distribution at $x + dx$. It is the information metric of Fisher and Rao [3]. Thus, physical space, when viewed as

a statistical manifold, inherits a metric structure from the distributions $p(y|x)$. We will assume that the originally unspecified metric $g_{ij}(x)$ is precisely the information metric induced by the distributions $p(y|x)$.

In [6] we proposed that a Gaussian model,¹

$$p(y|x) = \frac{\gamma^{1/2}(x)}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} \gamma_{ij}(x) (y^i - x^i)(y^j - x^j) \right], \quad (1)$$

where $\gamma = \det \gamma_{ij}$, incorporates the physically relevant information which consists of an estimate of the particle position,

$$\langle y^i \rangle = \int dy p(y|x) y^i = x^i, \quad (2)$$

and of its uncertainty given by the covariance matrix,

$$\langle (y^i - x^i)(y^j - x^j) \rangle = \int dy p(y|x) (y^i - x^i)(y^j - x^j) = \tilde{\gamma}^{ij}(x), \quad (3)$$

where $\tilde{\gamma}^{ij}$ is the inverse of γ_{ij} , $\tilde{\gamma}^{ik} \gamma_{kj} = \delta_j^i$.

Unfortunately the expected values in eqs.(2) and (3) are not covariant under coordinate transformations. Indeed, the transformation $y^i = f^i(y)$ does not lead to $x^i = f^i(x)$ because in general $\langle f(y) \rangle \neq f(\langle y \rangle)$ except when uncertainties are small. Our Gaussian model can at best be an approximation valid when $p(y|x)$ is sharply localized in a very small region within which curvature effects are negligible. Fortunately this is all we need for our present purpose.

[As an interesting aside we note that it is possible to devise fully covariant models. Here is an example: Let $\gamma_{ij}(x)$ be a positive definite tensor field and let us use it as if it were a metric tensor, $d\ell^2 = \gamma_{ij} dx^i dx^j$. Let $\ell(x, y)$ be the γ -length along the γ -geodesic from the point x to the point y . The proposed distribution is

$$p(y|x) = \frac{1}{\zeta} \gamma^{1/2}(y) \exp -\frac{\ell^2(x, y)}{2\sigma^2(x)}, \quad (4)$$

which is a manifestly covariant object: the normalization constant ζ , the γ -length $\ell(x, y)$, the scalar field $\sigma(x)$, and $dy \gamma^{1/2}(y)$ are all invariants. From this model we can compute a second metric, the information metric g_{ij} , which need not in general coincide with γ_{ij} . In the limit of small uncertainties (after absorbing σ into γ_{ij}) one recovers eq.(1).]

3 The Information Metric

The information distance between $p(y|\theta)$ and $p(y|\theta + d\theta)$ where the θ^a are parameters is calculated from (see e.g., [3])

$$d\ell^2 = G_{ab} d\theta^a d\theta^b \quad \text{with} \quad G_{ab} = \int dy p(y|\theta) \frac{\partial \log p(y|\theta)}{\partial \theta^a} \frac{\partial \log p(y|\theta)}{\partial \theta^b}. \quad (5)$$

¹We adopt the standard summation convention: repeated indices are summed over.

Consider the 9-dimensional space of Gaussians

$$p(y|x, \gamma) = \frac{\gamma^{1/2}}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} \gamma_{ij} (y^i - x^i)(y^j - x^j) \right]. \quad (6)$$

Here the parameters θ^a include the three x^i plus six independent elements of the symmetric matrix γ_{ij} . Eq.(5) gives the information distance between $p(y|x, \gamma)$ and $p(y|x + dx, \gamma + d\gamma)$ as

$$d\ell^2 = G_{ij} dx^i dx^j + G_k^{ij} d\gamma_{ij} dx^k + G^{ij kl} d\gamma_{ij} d\gamma_{kl}, \quad (7)$$

where

$$G_{ij} = \gamma_{ij}, \quad G_k^{ij} = 0, \quad \text{and} \quad G^{ij kl} = \frac{1}{4} (\tilde{\gamma}^{ik} \tilde{\gamma}^{jl} + \tilde{\gamma}^{il} \tilde{\gamma}^{jk}). \quad (8)$$

($\tilde{\gamma}^{ik}$ is the inverse of γ_{kj} .) Therefore,

$$d\ell^2 = \gamma_{ij} dx^i dx^j + \frac{1}{2} \tilde{\gamma}^{ik} \tilde{\gamma}^{jl} d\gamma_{ij} d\gamma_{kl}. \quad (9)$$

This is the metric of the full 9-dimensional manifold, but it is not what we need.

What we want is the metric of the embedded 3-dimensional submanifold where $\gamma_{ij} = \gamma_{ij}(x)$ is some function of x . To find the induced metric we cannot just substitute $d\gamma_{ij} = \partial_k \gamma_{ij} dx^k$ into eq.(9) because under a change of coordinates dx^k transforms as a tensor but the ordinary derivative $\partial_k \gamma_{ij}$ does not. In a model of physical space the i indices in x^i cannot be treated independently from the ij indices that appear in γ_{ij} because any transformation that changes the x^i also changes the γ_{ij} . Accordingly, we require that $d\gamma_{ij} = \nabla_k \gamma_{ij} dx^k$ where ∇_k is the covariant derivative and the corresponding induced information metric is

$$g_{ij} = \gamma_{ij} + \frac{1}{2} \tilde{\gamma}^{ac} \tilde{\gamma}^{bd} \nabla_i \gamma_{ab} \nabla_j \gamma_{cd}. \quad (10)$$

Normally one is given a manifold of probability distributions and the problem is to find the corresponding information metric. In order to do physics we are also concerned with the inverse problem: we want to design statistical manifolds with the appropriate geometries. We want to find the covariance field tensor $\gamma_{ij}(x)$ that leads to a given metric tensor $g_{ij}(x)$. Thus, we regard eq.(10) as a set of differential equations for $\gamma_{ij}(x)$. Since $\nabla_k g_{ij} = 0$,² a straightforward substitution shows that the solution is

$$\gamma_{ij}(x) = g_{ij}(x). \quad (11)$$

In words: *information distance is measured in units of the local uncertainty.* This beautifully simple but non-trivial result is valid in the low uncertainty regime where eq.(1) holds. The uniqueness of the solution (11), and whether it also holds in high curvature regions, such as near singularities, remains to be ascertained.

²The choice of the Levi-Civita connection is justified in the next section.

4 Entropic Dynamics for a single particle

The key to the question “Given an initial and a final state, what trajectory is the system expected to follow?” lies in the implicit assumption that there exists a continuous trajectory. A large change is the result of a succession of very many small changes and *therefore* we only need to determine what a short segment of the trajectory looks like. The idea behind entropic dynamics is that as the system moves from a point x to a neighboring point $x + \Delta x$ it must pass through a halfway point [5].

The basic dynamical question can now be rephrased as follows: The system is initially described by the probability distribution $p(y|x)$ and we are given the information that it has moved to one of the neighboring states in the family $p(y|x')$ where the x' lie on the plane halfway between the initial x and the final $x + \Delta x$. Which $p(y|x')$ do we select? The answer is given by the method of maximum (relative) entropy, ME. The selected distribution is that which maximizes the entropy of $p(y|x')$ *relative* to the prior $p(y|x)$ subject to the constraint that x' is equidistant from x and $x + \Delta x$. The result is that the selected x' minimizes the distance to x and therefore the three points x , x' and $x + \Delta x$ lie on a straight line.

Since any three neighboring points along the trajectory must line up, the trajectory predicted by entropic dynamics is the geodesic that minimizes the length

$$J = \int_{\lambda_i}^{\lambda_f} d\lambda [g_{ij} \dot{x}^i \dot{x}^j]^{1/2} \quad \text{with} \quad \dot{x}^i = \frac{dx^i}{d\lambda}, \quad (12)$$

where λ is any parameter that labels points along the curve, $x^i = x^i(\lambda)$.

Incidentally, note that in entropic dynamics there is one family of curves that is singled out as special: these are the minimal-length geodesics. From the purpose of building useful physics models no additional structure is needed and thus none will be introduced. It is therefore natural to use this same family of curves to *define* the notion of parallelism: the minimal-length geodesics are defined to be the straightest curves. This definition leads to the Levi-Civita connection which is equivalent to the condition $\nabla_k g_{ij} = 0$ assumed in the previous section. (See e.g. [7])

The simplest statistical model is a three-dimensional manifold of spherically symmetric Gaussians with constant variance σ_0^2 . The corresponding information metric is

$$g_{ij}^{(0)}(x) = \gamma_{ij}^{(0)}(x) = \frac{1}{\sigma_0^2} \delta_{ij}, \quad (13)$$

which we recognize as the familiar metric of flat Euclidean space. It is reassuring that already in such a simple model entropic dynamics reproduces the familiar straight line trajectories that are commonly associated with Galilean inertial motion. But this is too simple; non-trivial dynamics requires some curvature.

We are thus led to consider a slightly more complicated model of spherically symmetric Gaussians where the variance is a non-uniform scalar field $\sigma^2(x)$. It

is convenient to write the corresponding information metric as the Euclidean metric eq.(13) modulated by a (positive) conformal factor $\Phi(x)$,

$$g_{ij}(x) = \gamma_{ij}(x) = \frac{\Phi(x)}{\sigma_0^2} \delta_{ij} , \quad (14)$$

with $\sigma^2(x) = \sigma_0^2/\Phi(x)$.³

It is convenient to rewrite the length eq.(12) with the metric (14) in the form

$$J = 2^{1/2} \int_{\lambda_i}^{\lambda_f} d\lambda L(x, \dot{x}) , \quad (15)$$

with a ‘‘Lagrangian’’ function

$$L(x, \dot{x}) = [\Phi(x)T_\lambda(\dot{x})]^{1/2} \quad \text{with} \quad T_\lambda(\dot{x}) = \frac{1}{2\sigma_0^2} \delta_{ij} \dot{x}^i \dot{x}^j . \quad (16)$$

The geodesics follow from the Lagrange equations,

$$\frac{d}{d\lambda} \frac{\partial L}{\partial \dot{x}^i} = \frac{\partial L}{\partial x^i} , \quad (17)$$

or

$$\frac{1}{\sigma_0^2} \left(\frac{\Phi}{T_\lambda} \right)^{1/2} \frac{d}{d\lambda} \left[\left(\frac{\Phi}{T_\lambda} \right)^{1/2} \frac{dx^i}{d\lambda} \right] = \frac{\partial \Phi}{\partial x^i} . \quad (18)$$

These rather formidable equations can be simplified considerably once we notice that the parameter λ is quite arbitrary. Let us replace the original λ with a new parameter t given by

$$dt = \left(\frac{T_\lambda}{\Phi} \right)^{1/2} d\lambda \quad \text{or} \quad \frac{d}{dt} = \left(\frac{\Phi}{T_\lambda} \right)^{1/2} \frac{d}{d\lambda} . \quad (19)$$

In terms of the new t the equation of motion simplifies to

$$\frac{1}{\sigma_0^2} \frac{d^2 x^i}{dt^2} = \frac{\partial \Phi}{\partial x^i} . \quad (20)$$

From eq.(19) the new t is such that

$$\Phi = T_\lambda \left(\frac{d\lambda}{dt} \right)^2 = T_t \quad \text{where} \quad T_t = \frac{1}{2\sigma_0^2} \delta_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt} . \quad (21)$$

Eqs.(20) and (21) are equivalent to Newtonian dynamics. To make it explicit we introduce a ‘‘mass’’ m and a ‘‘potential’’ $\phi(x)$ through a mere change of notation,

$$\frac{1}{\sigma_0^2} = m \quad \text{and} \quad \Phi(x) = -\phi(x) + E \quad (22)$$

³The effect of $\Phi(x)$ is a local dilation. Since each side of a small triangle at x is dilated by the same factor $\Phi(x)$ its angles remain unchanged. Such angle-preserving transformations are called conformal.

where the constant E reflects the freedom to add a constant to the potential. The result is Newton's equation,

$$m \frac{d^2 x^i}{dt^2} = - \frac{\partial \phi}{\partial x^i} , \quad (23)$$

and energy conservation,

$$\frac{1}{2} m \delta_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt} + \phi(x) = E , \quad (24)$$

Thus, the constant E is interpreted as energy.

We have just derived $F = ma$ purely from principles of inference applied to the relevant information codified into a statistical model! From eq.(12) onwards our inference approach is formally identical to the Jacobi action principle of classical mechanics [8] but we did not need to know this. Indeed, by a wild stretch of our historical imagination it is perhaps conceivable that had Newton, Lagrange, and Jacobi known less physics and much more inference they might have invented their subject along these lines. Had history actually followed this unlikely course we might not have used the notions of mass m or potential $\phi(x)$ and instead we would have referred to the particle's "intrinsic" position uncertainty σ_0 , and how it is modulated throughout space by the field $\Phi(x)$.

The derivation above serves to illustrate the main idea but suffers from two important limitations. First, it applies to a single particle with a fixed constant energy and this means that we deal with an isolated system. Second, while it is true that we have identified a convenient and very suggestive parameter t , how do we know that it actually represents "true" time? Is t the universal Newtonian time or just a parameter that applies only to one particular isolated particle? The original formulation in terms of the "Jacobi" action, eq.(15), is completely timeless; how and where did time sneak in?

The solution to both these problems emerges as we apply the formalism to the motion of the only system known to be completely isolated: the whole universe. Then the fact that the energy is a fixed constant does not represent a restriction. And further, since the preferred time parameter would be associated to the whole universe, it would not be at all inappropriate to call it the *universal* time.

5 The whole universe: many particles

To simplify our notation we will consider a universe that consists of $N = 2$ particles. The generalization to arbitrary N is trivial. For the 2-particle system the position $x = (x_1, x_2)$ is denoted by 6 coordinates x^A with $A = 1, 2, \dots, 6$. Let $x^A = (x^{i_1}, x^{i_2})$ with $i_1 = 1, 2, 3$ for particle 1 and $i_2 = 4, 5, 6$ for particle 2. A point in the $N = 2$ configuration space is a Gaussian distribution,

$$p(y|x) = \frac{\gamma^{1/2}(x)}{(2\pi)^{3/2}} \exp \left[-\frac{1}{2} \gamma_{AB}(x) (y^A - x^A)(y^B - x^B) \right] . \quad (25)$$

The simplest model for two (possibly non-identical) particles assigns uniform variances σ_1^2 and σ_2^2 to each particle. The corresponding metric, analogous to eq.(13), is

$$g_{AB}^{(0)} = \gamma_{AB}^{(0)} = m_{AB} , \quad (26)$$

where m_{AB} is a constant 6×6 diagonal matrix,

$$m_{AB} = \begin{bmatrix} \delta_{i_1 j_1} / \sigma_1^2 & 0 \\ 0 & \delta_{i_2 j_2} / \sigma_2^2 \end{bmatrix} , \quad (27)$$

where each entry represents a 3×3 matrix. The metric m_{AB} describes a flat space; the trajectories are familiar “straight” lines and the particles move independently of each other; they do not interact. As before, non-trivial dynamics requires the introduction of curvature and the simplest way to do this is through an overall conformal field $\Phi(x)$ with $x = (x_1, x_2)$. Thus we propose

$$g_{AB}(x) = \gamma_{AB}(x) = \Phi(x) m_{AB} . \quad (28)$$

The equation of motion for the $N = 2$ universe is the geodesic that minimizes

$$J = 2^{1/2} \int_{\lambda_i}^{\lambda_f} d\lambda L(x_1, x_2, \dot{x}_1, \dot{x}_2) , \quad (29)$$

where

$$L(x, \dot{x}) = [\Phi(x) T_\lambda(\dot{x})]^{1/2} \quad \text{and} \quad T_\lambda(\dot{x}) = \frac{1}{2} m_{AB} \dot{x}^A \dot{x}^B . \quad (30)$$

The Lagrange equations yield,

$$m_{AB} \left(\frac{\Phi}{T_\lambda} \right)^{1/2} \frac{d}{d\lambda} \left[\left(\frac{\Phi}{T_\lambda} \right)^{1/2} \frac{dx^B}{d\lambda} \right] = \frac{\partial \Phi}{\partial x^A} , \quad (31)$$

which suggests introducing a new parameter t defined by

$$dt = \left(\frac{T_\lambda}{\Phi} \right)^{1/2} d\lambda \quad \text{or} \quad \frac{d}{dt} = \left(\frac{\Phi}{T_\lambda} \right)^{1/2} \frac{d}{d\lambda} . \quad (32)$$

In terms of the new parameter the equations of motion are

$$m_{AB} \frac{d^2 x^A}{dt^2} = \frac{\partial \Phi}{\partial x^A} , \quad (33)$$

which, since m_{AB} is a diagonal matrix, is

$$\frac{1}{\sigma_n^2} \frac{d^2 x^{i_n}}{dt^2} = \frac{\partial}{\partial x^{i_n}} \Phi(x_1, x_2) , \quad (34)$$

for each of the particles, $n = 1, 2$. Note that the motion of particle 1 depends on the location of particle 2: *these are interacting particles!*

The new time parameter t , eq.(32), is such that

$$\Phi = T_\lambda \left(\frac{d\lambda}{dt} \right)^2 = T_t \quad \text{where} \quad T_t = \frac{1}{2} m_{AB} \frac{dx^A}{dt} \frac{dx^B}{dt} . \quad (35)$$

As before, the equivalence to Newtonian dynamics is made explicit by a change of notation,

$$\frac{1}{\sigma_n^2} = m_n \quad \text{and} \quad \Phi(x) = -\phi(x) + E . \quad (36)$$

The result is

$$m_n \frac{d^2 x^{i_n}}{dt^2} = -\frac{\partial}{\partial x^{i_n}} \phi(x_1, x_2) \quad \text{and} \quad \frac{1}{2} m_{AB} \frac{dx^A}{dt} \frac{dx^B}{dt} + \phi(x_1, x_2) = E . \quad (37)$$

The constant E is the total energy of the universe and there are no restrictions on the energy of individual subsystems.

For the conformal factor $\Phi(x_1, x_2)$ we can choose anything we want. For example,

$$\Phi(x_1, x_2) = -v_1(x_1) - v_2(x_2) - u(x_1, x_2) + E , \quad (38)$$

so the particles can interact with external potentials v_1 and v_2 and also with each other through $u(x_1, x_2)$.

The definition of time t required taking into account all the particles in the universe. This is in accord with the ephemeris time defined by astronomers. We started with a completely timeless theory, eq.(29), and in fact, no *external* time has been introduced. What we have is a convenient t parameter associated to the change of the total system, which in this case is the whole universe. The universe is its own clock; it measures universal time. Incidentally, note that the reparametrization that allowed us to introduce a Newtonian time was possible only because the same conformal factor $\Phi(x)$ applies equally to all particles.

Entropic dynamics offers a new perspective on the concepts of mass and interactions. To see this note that since γ_{AB} is diagonal the distribution (25) turns out to be a product,

$$p(y|x) = p(y_1|x_1, x_2)p(y_2|x_1, x_2) . \quad (39)$$

Note that although the model represents interacting particles the distribution is a product: the uncertain variables y_1 and y_2 are statistically independent. The coupling arises through conditioning on $x = (x_1, x_2)$.

Let us focus our attention on particle 1; similar remarks also apply to particle 2. The distribution $p(y_1|x_1, x_2)$ is a spherically symmetric Gaussian,

$$p(y_1|x_1, x_2) \propto \exp \left[-\frac{1}{2\sigma_1^2(x_1, x_2)} \delta_{ij} (y^i - x^i)(y^j - x^j) \right] . \quad (40)$$

The uncertainty in the position of particle 1 is given by

$$\sigma_1(x_1, x_2) = [\Phi(x_1, x_2)m_1]^{-1/2} . \quad (41)$$

The mass m_1 is interpreted in terms of a uniform background contribution to the uncertainty. Mass is a manifestation of an uncertainty in location; higher mass reflects a lower uncertainty. On the other hand, interactions arise from the non-uniformity of $\sigma_1(x_1, x_2)$ that depends on the location of other particles through the modulating field $\Phi(x_1, x_2)$. It is worthwhile to note that even though this is a non-relativistic model there already appears a “unification” between mass and (potential) energy: they are different aspects of the same thing, the position uncertainty.

6 Final remarks

We emphasize that the model we have proposed does not take into account all the dynamical information that we know is relevant – relativistic and quantum effects have not been included. Our model is very restricted. For example, our model invokes two apparently unrelated metrics. There is the metric δ_{ij} of flat 3-dimensional Euclidean space that appears in the kinetic energies and there is the information metric g_{ij} that accounts for mass and interactions and applies to the curved configuration space. This is a reflection of the fact that a system of N particles is described as a point in a $3N$ -dimensional configuration space. A better model would have N points living within the same evolving 3-dimensional space.

Furthermore, we have not provided any rationale for how to choose the modulating field $\Phi(x)$. Just as Newton deliberately refrained from explaining the origin of his inverse square forces – *hypothesis non fingo* – so have we refrained from offering any physical hypothesis about the underlying fuzziness of space. It is reasonable to expect that a derivation of general relativity as an example of entropic dynamics would yield important insights on this matter. Preliminary steps in this direction appeared in [6].

What we have done is to show, by exhibiting an explicit example, that the tools of inference – probability, information geometry and entropy – are sufficiently rich that one can construct entropic dynamics models that reproduce recognizable laws of physics. Perhaps all laws of physics can be derived in this way.

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