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Periodic boundary value problems for nonlinear first-order impulsive dynamic equations on time scales

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Abstract

By using the classical fixed point theorem for operators on cone, in this article, some results of one and two positive solutions to a class of nonlinear first-order periodic boundary value problems of impulsive dynamic equations on time scales are obtained. Two examples are given to illustrate the main results in this article.
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Keywords: time scale, periodic boundary value problem, positive solution, fixed point, impulsive dynamic equation

1 Introduction

Let $\mathbf{T}$ be a time scale, i.e., $\mathbf{T}$ is a nonempty closed subset of R. Let 0, T be points in $\mathbf{T}$, an interval $(0, T)_{\mathbf{T}}$ denoting time scales interval, that is, $(0, T)_{\mathbf{T}}: = (0, T) \cap \mathbf{T}$. Other types of intervals are defined similarly.

The theory of impulsive differential equations is emerging as an important area of investigation, since it is a lot richer than the corresponding theory of differential equations without impulse effects. Moreover, such equations may exhibit several real world phenomena in physics, biology, engineering, etc. (see [1-3]). At the same time, the boundary value problems for impulsive differential equations and impulsive difference equations have received much attention [4-18]. On the other hand, recently, the theory of dynamic equations on time scales has become a new important branch (see, for example, [19-21]). Naturally, some authors have focused their attention on the boundary value problems of impulsive dynamic equations on time scales [22-36]. However, to the best of our knowledge, few papers concerning PBVPs of impulsive dynamic equations on time scales with semi-position condition.

In this article, we are concerned with the existence of positive solutions for the following PBVPs of impulsive dynamic equations on time scales with semi-position condition

$$\begin{cases} x^{\Delta}(t) + f(t, x(\sigma(t))) = 0, \quad t \in J := [0, T]_{\mathbf{T}}, \quad t \neq t_k, \quad k = 1, 2, \ldots, m, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), \quad k = 1, 2, \ldots, m, \\ x(0) = x(\sigma(T)), \end{cases} \tag{1.1}$$

where $\mathbf{T}$ is an arbitrary time scale, $T > 0$ is fixed, $0, T \in \mathbf{T}$, $f \in C(J \times [0, \infty), (-\infty, \infty))$, $I_k \in C([0, \infty), [0, \infty))$, $t_k \in (0, T)_{\mathbf{T}}$, $0 < t_1 < \ldots < t_m < T$, and for each $k = 1, 2,\ldots, m$, $x(t_k^+) = \lim_{h\to 0^+} x(t_k + h)$ and $x(t_k^-) = \lim_{h\to 0^-} x(t_k + h)$ represent the right and left limits of $x(t)$ at $t = t_k$. We always assume the following hypothesis holds (semi-position condition):

(H) There exists a positive number M such that

$$Mx - f(t, x) \geq 0 \text{ for } x \in [0, \infty), \quad t \in [0, T]_{\mathbf{T}}.$$

By using a fixed point theorem for operators on cone [37], some existence criteria of positive solution to the problem (1.1) are established. We note that for the case $\mathbf{T} = R$ and $I_k(x) \equiv 0$, $k = 1, 2,\ldots, m$, the problem (1.1) reduces to the problem studied by [38] and for the case $I_k(x) \equiv 0$, $k = 1, 2,\ldots, m$, the problem (1.1) reduces to the problem (in the one-dimension case) studied by [39].

In the remainder of this section, we state the following fixed point theorem [37].

Theorem 1.1. Let X be a Banach space and $K \subset X$ be a cone in X. Assume Ω_1, Ω_2 are bounded open subsets of X with $0 \in \Omega_1 \subset \bar{\Omega}_1 \subset \Omega_2$ and $\Phi: K \cap (\bar{\Omega}_2 \backslash \Omega_1) \to K$ is a completely continuous operator. If

(i) There exists $u_0 \in K\backslash\{0\}$ such that $u - \Phi u \neq \lambda u_0$, $u \in K \cap \partial \Omega_2$, $\lambda \geq 0$; $\Phi u \neq \tau u$, $u \in K \cap \partial\Omega_1$, $\tau \geq 1$, or

(ii) There exists $u_0 \in K\backslash\{0\}$ such that $u - \Phi u \neq \lambda u_0$, $u \in K \cap \partial\Omega_1$, $\lambda \geq 0$; $\Phi u \neq \tau u$, $u \in K \cap \partial\Omega_2$, $\tau \geq 1$.

Then Φ has at least one fixed point in $K \cap (\bar{\Omega}_2 \backslash \Omega_1)$.

2 Preliminaries

Throughout the rest of this article, we always assume that the points of impulse t_k are right-dense for each $k = 1, 2,\ldots,m$.

We define

$$\begin{aligned} PC = \{x \in [0, \sigma(T)]_{\mathbf{T}} \to R : x_k \in C(J_k, R),\ k = 0, 1, 2, \ldots, m \text{ and there exist} \\ x(t_k^+) \text{ and } x(t_k^-) \text{ with } x(t_k^-) = x(t_k),\ k = 1, 2, \ldots, m\}, \end{aligned}$$

where x_k is the restriction of x to $J_k = (t_k, t_{k+1}]_{\mathbf{T}} \subset (0, \sigma(T)]_{\mathbf{T}}$, $k = 1, 2,\ldots, m$ and $J_0 = [0, t_1]_{\mathbf{T}}$, $t_{m+1} = \sigma(T)$.

Let

$$X = \{x : x \in PC, \quad x(0) = x(\sigma(T))\}$$

with the norm $\|x\| = \sup_{t\in[0,\sigma(T)]_{\mathbf{T}}} |x(t)|$, then X is a Banach space.

Lemma 2.1. Suppose $M > 0$ and $h: [0, T]_{\mathbf{T}} \to R$ is rd-continuous, then x is a solution of

$$x(t) = \int_0^{\sigma(T)} G(t, s)h(s)\Delta s + \sum_{k=1}^{m} G(t, t_k)I_k(x(t_k)), \quad t \in [0, \sigma(T)]_{\mathbf{T}},$$

$$\text{where } G(t, s) = \begin{cases} \dfrac{e_M(s, t)e_M(\sigma(T), 0)}{e_M(\sigma(T), 0) - 1}, & 0 \leq s \leq t \leq \sigma(T), \\ \dfrac{e_M(s, t)}{e_M(\sigma(T), 0) - 1}, & 0 \leq t < s \leq \sigma(T), \end{cases}$$

if and only if x is a solution of the boundary value problem

$$\begin{cases} x^{\Delta}(t) + Mx(\sigma(t)) = h(t), & t \in J := [0,T]_{\mathbf{T}}, \quad t \neq t_k, \quad k = 1,2,\ldots,m, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), & k = 1,2,\ldots,m, \\ x(0) = x(\sigma(T)). \end{cases}$$

Proof. Since the proof similar to that of [34, Lemma 3.1], we omit it here.

Lemma 2.2. Let $G(t, s)$ be defined as in Lemma 2.1, then

$$\frac{1}{e_M(\sigma(T),0)-1} \le G(t,s) \le \frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1} \quad \text{for all } t,s \in [0,\sigma(T)]_{\mathbf{T}}.$$

Proof. It is obviously, so we omit it here.

Remark 2.1. Let $G(t, s)$ be defined as in Lemma 2.1, then $\int_0^{\sigma(T)} G(t,s)\Delta s = \frac{1}{M}$.

For $u \in X$, we consider the following problem:

$$\begin{cases} x^{\Delta}(t) + Mx(\sigma(t)) = Mu(\sigma(t)) - f(t,u(\sigma(t))), & t \in [0,T]_{\mathbf{T}}, \quad t \neq t_k, \quad k = 1,2,\ldots,m, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)), & k = 1,2,\ldots,m, \\ x(0) = x(\sigma(T)). \end{cases} \tag{2.1}$$

It follows from Lemma 2.1 that the problem (2.1) has a unique solution:

$$x(t) = \int_0^{\sigma(T)} G(t,s)h_u(s)\Delta s + \sum_{k=1}^{m} G(t,t_k)I_k(x(t_k)), \quad t \in [0,\sigma(T)]_{\mathbf{T}},$$

where $h_u(s) = Mu(\sigma(s)) - f(s, u(\sigma(s)))$, $s \in [0, T]_{\mathbf{T}}$.

We define an operator $\Phi: X \to X$ by

$$\Phi(u)(t) = \int_0^{\sigma(T)} G(t,s)h_u(s)\Delta s + \sum_{k=1}^{m} G(t,t_k)I_k(u(t_k)), \quad t \in [0,\sigma(T)]_{\mathbf{T}}.$$

It is obvious that fixed points of Φ are solutions of the problem (1.1).

Lemma 2.3. $\Phi: X \to X$ is completely continuous.

Proof. The proof is divided into three steps.

Step 1: To show that $\Phi: X \to X$ is continuous.

Let $\{u_n\}_{n=1}^{\infty}$ be a sequence such that $u_n \to u$ $(n \to \infty)$ in X. Since $f(t, u)$ and $I_k(u)$ are continuous in x, we have

$$\left|h_{un}(t) - h_u(t)\right| = \left|M(u_n - u) - (f(t,u_n) - f(t,u))\right| \to 0 (n \to \infty),$$
$$\left|I_k(u_n(t_k)) - I_k(u(t_k))\right| \to 0 (n \to \infty).$$

So

$$\begin{aligned} &\left|\Phi(u_n)(t) - \Phi(u)(t)\right| \\ &= \left|\int_0^{\sigma(T)} G(t,s)[h_{u_n}(s) - h_u(s)]\Delta s + \sum_{k=1}^{m} G(t,t_k)[I_k(u_n(t_k)) - I_k(u(t_k))]\right| \\ &\le \frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1}\left[\int_0^{\sigma(T)} \left|h_{u_n}(s) - h_u(s)\right| \Delta s + \sum_{k=1}^{m} \left|I_k(u_n(t_k)) - I_k(u(t_k))\right|\right] \to 0 (n \to \infty), \end{aligned}$$

which leads to $||\Phi u_n - \Phi u|| \to 0$ $(n \to \infty)$. That is, $\Phi: X \to X$ is continuous.

Step 2: To show that Φ maps bounded sets into bounded sets in X.

Let $B \subset X$ be a bounded set, that is, $\exists\ r > 0$ such that $\forall\ u \in B$ we have $||u|| \le r$. Then, for any $u \in B$, in virtue of the continuities of $f(t, u)$ and $I_k(u)$, there exist $c > 0$, $c_k > 0$ such that

$$\left|f(t,u)\right| \le c, \quad \left|I_k(u)\right| \le c_k, \quad k = 1, 2, \ldots, m.$$

We get

$$\begin{aligned}\left|\Phi(u)(t)\right| &= \left|\int_0^{\sigma(T)} G(t,s)h_u(s)\Delta s + \sum_{k=1}^m G(t,t_k)I_k(u(t_k))\right| \\ &\le \int_0^{\sigma(T)} G(t,s)\left|h_u(s)\right|\Delta s + \sum_{k=1}^m G(t,t_k)\left|I_k(u(t_k))\right| \\ &\le \frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1}\left[\sigma(T)(Mr+c) + \sum_{k=1}^m c_k\right].\end{aligned}$$

Then we can conclude that Φu is bounded uniformly, and so $\Phi(B)$ is a bounded set.

Step 3: To show that Φ maps bounded sets into equicontinuous sets of X.

Let $t_1, t_2 \in (t_k, t_{k+1}]_{\mathbf{T}} \cap [0, \sigma(T)]_{\mathbf{T}}$, $u \in B$, then

$$\begin{aligned}&\left|\Phi(u)(t_1) - \Phi(u)(t_2)\right| \\ &\le \int_0^{\sigma(T)} \left|G(t_1,s) - G(t_2,s)\right|\left|h_u(s)\right|\Delta s + \sum_{k=1}^m \left|G(t_1,t_k) - G(t_2,t_k)\right|\left|I_k(u(t_k))\right|.\end{aligned}$$

The right-hand side tends to uniformly zero as $|t_1 - t_2| \to 0$.

Consequently, Steps 1-3 together with the Arzela-Ascoli Theorem shows that $\Phi: X \to X$ is completely continuous.

Let

$$K = \{u \in X : u(t) \ge \delta \|u\|, \quad t \in [0, \sigma(T)]_{\mathbf{T}}\},$$

where $\delta = \dfrac{1}{e_M(\sigma(T),0)} \in (0,1)$. It is not difficult to verify that K is a cone in X.

From condition (H) and Lemma 2.2, it is easy to obtain following result:

Lemma 2.4. Φ maps K into K.

3 Main results

For convenience, we denote

$$\begin{aligned}f^0 &= \limsup_{u\to 0^+} \max_{t\in[0,T]_{\mathbf{T}}} \frac{f(t,u)}{u}, \quad f^\infty = \limsup_{u\to\infty} \max_{t\in[0,T]_{\mathbf{T}}} \frac{f(t,u)}{u}, \\ f_0 &= \liminf_{u\to 0^+} \min_{t\in[0,T]_{\mathbf{T}}} \frac{f(t,u)}{u}, \quad f_\infty = \liminf_{u\to\infty} \min_{t\in[0,T]_{\mathbf{T}}} \frac{f(t,u)}{u}.\end{aligned}$$

and

$$I_0 = \lim_{u\to 0^+} \frac{I_k(u)}{u}, \quad I_\infty = \lim_{u\to\infty} \frac{I_k(u)}{u}.$$

Now we state our main results.

Theorem 3.1. Suppose that

(H_1) $f_0 > 0, f^\infty < 0, I_0 = 0$ for any k; or

(H_2) $f_\infty > 0, f^0 < 0, I_\infty = 0$ for any k.

Then the problem (1.1) has at least one positive solutions.

Proof. Firstly, we assume (H_1) holds. Then there exist $\varepsilon > 0$ and $\beta > \alpha > 0$ such that

$$f(t,u) \geq \varepsilon u, \quad t \in [0,T]_{\mathbf{T}}, \quad u \in (0,\alpha], \tag{3.1}$$

$$I_k(u) \leq \frac{[e_m(\sigma(T),0)-1]\varepsilon}{2Mme_M(\sigma(T),0)} u, u \in (0,\alpha], \quad \text{for any } k, \tag{3.2}$$

and

$$f(t,u) \leq -\varepsilon u, \quad t \in [0,T]_{\mathbf{T}}, \quad u \in [\beta,\infty). \tag{3.3}$$

Let $\Omega_1 = \{u \in X: ||u|| < r_1\}$, where $r_1 = \alpha$. Then $u \in K \cap \partial\Omega_1$, $0 < \delta\alpha = \delta\, ||u|| \leq u(t) \leq \alpha$, in view of (3.1) and (3.2) we have

$$\begin{aligned}
\Phi(u)(t) &= \int_0^{\sigma(T)} G(t,s)h_u(s)\Delta s + \sum_{k=1}^{m} G(t,t_k)I_k(u(t_k)) \\
&\leq \int_0^{\sigma(T)} G(t,s)(M-\varepsilon)u(\sigma(s))\Delta s + \sum_{k=1}^{m} G(t,t_k)\frac{[e_M(\sigma(T),0)-1]\varepsilon}{2Mme_M(\sigma(T),0)}u(t_k) \\
&\leq \frac{(M-\varepsilon)}{M}\|u\| + \frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1}\sum_{k=1}^{m}\frac{[e_M(\sigma(T),0)-1]\varepsilon}{2Mme_M(\sigma(T),0)}\|u\| \\
&= \frac{\left(M-\frac{\varepsilon}{2}\right)}{M}\|u\| \\
&< \|u\|, t \in [0,\sigma(T)]_{\mathbf{T}},
\end{aligned}$$

which yields $||\Phi(u)|| < ||u||$.

Therefore

$$\Phi u \neq \tau u, \quad u \in K \cap \partial\Omega_1, \quad \tau \geq 1. \tag{3.4}$$

On the other hand, let $\Omega_2 = \{u \in X: ||u|| < r_2\}$, where $r_2 = \frac{\beta}{\delta}$.

Choose $u_0 = 1$, then $u_0 \in K\backslash\{0\}$. We assert that

$$u - \Phi u \neq \lambda u_0, \quad u \in K \cap \partial\Omega_2, \quad \lambda \geq 0. \tag{3.5}$$

Suppose on the contrary that there exist $\bar{u} \in K \cap \partial\Omega_2$ and $\bar{\lambda} \geq 0$ such that

$$\bar{u} - \Phi\bar{u} = \bar{\lambda}u_0.$$

Let $\varsigma = \min_{t\in[0,\sigma(T)]_{\mathbf{T}}}\bar{u}(t)$, then $\varsigma \geq \delta \|\bar{u}\| = \delta r_2 = \beta$, we have from (3.3) that

$$\begin{aligned}\bar{u}(t) &= \Phi(\bar{u})(t) + \bar{\lambda} \\ &= \int_0^{\sigma(T)} G(t,s)h_{\bar{u}}(s)\Delta s + \sum_{k=1}^{m} G(t,t_k)I_k(\bar{u}(t_k)) + \bar{\lambda} \\ &\geq \int_0^{\sigma(T)} G(t,s)h_{\bar{u}}(s)\Delta s + \bar{\lambda} \\ &\geq \frac{(M+\varepsilon)}{M}\varsigma + \bar{\lambda}, \quad t \in [0,\sigma(T)]_{\mathbf{T}}.\end{aligned}$$

Therefore,

$$\varsigma = \min_{t\in[0,\sigma(T)]_{\mathbf{T}}} \bar{u}(t) \geq \frac{(M+\varepsilon)}{M}\varsigma + \bar{\lambda} > \varsigma,$$

which is a contradiction.

It follows from (3.4), (3.5) and Theorem 1.1 that Φ has a fixed point $u^* \in K \cap (\bar{\Omega}_2\backslash\Omega_1)$, and u^* is a desired positive solution of the problem (1.1).

Next, suppose that (H_2) holds. Then we can choose $\varepsilon' > 0$ and $\beta' > \alpha' > 0$ such that

$$f(t,u) \geq \varepsilon' u, \quad t \in [0,T]_{\mathbf{T}}, \quad u \in [\beta', \infty), \tag{3.6}$$

$$I_k(u) \leq \frac{[e_M(\sigma(T),0)-1]\varepsilon'}{2Mme_M(\sigma(T),0)}u, \quad u \in [\beta',\infty) \text{ for any } k, \tag{3.7}$$

and

$$f(t,u) \leq -\varepsilon' u, \quad t \in [0,T]_{\mathbf{T}}, \quad u \in (0,\alpha']. \tag{3.8}$$

Let $\Omega_3 = \{u \in X: ||u|| < r_3\}$, where $r_3 = \alpha'$. Then for any $u \in K \cap \partial\Omega_3$, $0 < \delta ||u|| \leq u(t) \leq ||u|| = \alpha'$.

It is similar to the proof of (3.5), we have

$$u - \Phi u \neq \lambda u_0, \quad u \in K \cap \partial\Omega_3, \quad \lambda \geq 0. \tag{3.9}$$

Let $\Omega_4 = \{u \in X: ||u|| < r_4\}$, where $r_4 = \frac{\beta'}{\delta}$. Then for any $u \in K \cap \partial\Omega_4$, $u(t) \geq \delta ||u|| = \delta r_4 = \beta'$, by (3.6) and (3.7), it is easy to obtain

$$\Phi u \neq \tau u, \quad u \in K \cap \partial\Omega_4, \quad \tau \geq 1. \tag{3.10}$$

It follows from (3.9), (3.10) and Theorem 1.1 that Φ has a fixed point $u^* \in K \cap (\bar{\Omega}_4\backslash\Omega_3)$, and u^* is a desired positive solution of the problem (1.1).

Theorem 3.2. Suppose that

(H_3) $f^0 < 0, f^\infty < 0$;

(H_4) there exists $\rho > 0$ such that

$$\min\{f(t,u) - u | t \in [0,T]_{\mathbf{T}}, \quad \delta\rho \leq u \leq \rho\} > 0; \tag{3.11}$$

$$I_k(u) \leq \frac{[e_M(\sigma(T),0)-1]}{Mme_M(\sigma(T),0)}u, \quad \delta\rho \leq u \leq \rho, \quad \text{for any } k. \tag{3.12}$$

Then the problem (1.1) has at least two positive solutions.

Proof. By (H_3), from the proof of Theorem 3.1, we should know that there exist β'' $>\rho>\alpha''>0$ such that

$$u-\Phi u\neq\lambda u_0,\quad u\in K\cap\partial\Omega_5,\quad \lambda\geq 0, \tag{3.13}$$

$$u-\Phi u\neq\lambda u_0,\quad u\in K\cap\partial\Omega_6,\quad \lambda\geq 0, \tag{3.14}$$

where $\Omega_5=\{u\in X: ||u||<r_5\}$, $\Omega_6=\{u\in X: ||u||<r_6\}$, $r_5=\alpha''$, $r_6=\dfrac{\beta''}{\delta}$.

By (3.11) of (H_4), we can choose $\varepsilon>0$ such that

$$f(t,u)\geq(1+\varepsilon)u,\quad t\in[0,T]_{\mathbf{T}},\quad \delta\rho\leq u\leq\rho. \tag{3.15}$$

Let $\Omega_7=\{u\in X: ||u||<\rho\}$, for any $u\in K\cap\partial\Omega_7$, $\delta\rho=\delta\,||u||\leq u(t)\leq||u||=\rho$, from (3.12) and (3.15), it is similar to the proof of (3.4), we have

$$\Phi u\neq\tau u,\quad u\in K\cap\partial\Omega_7,\quad \tau\geq 1. \tag{3.16}$$

By Theorem 1.1, we conclude that Φ has two fixed points $u^{**}\in K\cap(\bar{\Omega}_6\backslash\Omega_7)$ and $u^{***}\in K\cap(\bar{\Omega}_7\backslash\Omega_5)$, and u^{**} and u^{***} are two positive solution of the problem (1.1).

Similar to Theorem 3.2, we have:

Theorem 3.3. Suppose that

(H_4) $f_0>0$, $f_\infty>0$, $I_0=0$, $I_\infty=0$;

(H_5) there exists $\rho>0$ such that

$$\max\{f(t,u)|t\in[0,T]_{\mathbf{T}},\quad \delta\rho\leq u\leq\rho\}<0.$$

Then the problem (1.1) has at least two positive solutions.

4 Examples

Example 4.1. Let $\mathbf{T}=[0,1]\cup[2,3]$. We consider the following problem on $\mathbf{T}$

$$\begin{cases} x^{\Delta}(t)+f(t,x(\sigma(t)))=0,\quad t\in[0,3]_{\mathbf{T}},\quad t\neq\dfrac{1}{2},\\ x\left(\dfrac{1^+}{2}\right)-x\left(\dfrac{1^-}{2}\right)=I\left(x\left(\dfrac{1}{2}\right)\right),\\ x(0)=x(3),\end{cases} \tag{4.1}$$

where $T=3$, $f(t,x)=x-(t+1)x^2$, and $I(x)=x^2$

Let $M=1$, then, it is easy to see that

$$Mx-f(t,x)=(t+1)x^2\geq 0 \text{ for } x\in[0,\infty),\quad t\in[0,3]_{\mathbf{T}},$$

and

$$f_0\geq 1,\quad f^{\infty}=-\infty,\quad \text{and } I_0=0.$$

Therefore, by Theorem 3.1, it follows that the problem (4.1) has at least one positive solution.

Example 4.2. Let $\mathbf{T}$ = [0, 1] ∪ [2,3]. We consider the following problem on $\mathbf{T}$

$$\begin{cases} x^{\Delta}(t) + f(t, x(\sigma(t))) = 0, \quad t \in [0,3]_{\mathbf{T}}, \quad t \neq \frac{1}{2}, \\ x\left(\frac{1}{2}^{+}\right) - x\left(\frac{1}{2}^{-}\right) = I\left(x\left(\frac{1}{2}\right)\right), \\ x(0) = x(3), \end{cases} \tag{4.2}$$

where $T = 3, f(t,x) = 4e^{1-4e^2}x - (t+1)x^2e^{-x}$, and $I(x) = x^2e^{-x}$.

Choose $M = 1$, $\rho = 4e^2$, then $\delta = \frac{1}{2e^2}$, it is easy to see that

$$Mx - f(t,x) = x(1 - 4e^{1-4e^2}) + (t+1)x^2e^{-x} \geq 0 \text{ for } x \in [0,\infty), \quad t \in [0,3]_{\mathbf{T}},$$
$$f_0 \geq 4e^{1-4e^2} > 0, \quad f_\infty \geq 4e^{1-4e^2} > 0, \quad I_0 = 0 \quad, I_\infty = 0,$$

and

$$\max(f(t,u)\,|t \in [0,T]_{\mathbf{T}}, \delta\rho \leq u \leq \rho\} = \max\{f(t,u)\,|\,t \in [0,3]_{\mathbf{T}}, 2 \leq u \leq 4e^2\} = 16e^{3-4e^2}(1-e) < 0.$$

Therefore, together with Theorem 3.3, it follows that the problem (4.2) has at least two positive solutions.

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Competing interests
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