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Behavior of solutions of a third-order dynamic equation on time scales

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Abstract

In this paper, we will establish some sufficient conditions which guarantee that every solution of the third-order nonlinear dynamic equation

$$(r_1(t)(r_2(t)x^{\Delta}(t))^{\Delta})^{\Delta} + P(t,x(t),x^{\Delta}(t)) + F(t,x(t)) = 0$$

oscillates or converges to zero on an arbitrary time scale $\mathbb{T}$.

1 Introduction

The theory of time scales, which has recently received a lot of attention, was introduced by Stefan Hilger in his Ph.D. Thesis [1] in order to unify continuous and discrete analysis. A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the reals, and the cases when this time scale is equal to the reals or to the integers represent the classical theories of differential and of difference equations. A book on the subject of time scales by Bohner and Peterson [2] summarizes and organizes much of the time scale calculus. Many other interesting time scales exist and they give rise to plenty of applications, among them the study of population dynamic models (see [3]). In the last few years, there has been much research activity concerning the oscillation and nonoscillation of solutions of some dynamic equations on time scales, and we refer the reader to the papers [3–21].

Following this trend, in this paper, we will consider the third-order nonlinear dynamic equation

$$\bigl(r_1(t)\bigl(r_2(t)x^{\Delta}(t)\bigr)^{\Delta}\bigr)^{\Delta} + P\bigl(t,x(t),x^{\Delta}(t)\bigr) + F\bigl(t,x(t)\bigr) = 0, \quad t \geq t_0, \tag{1}$$

where $r_1(t)$, $r_2(t)$ are positive, real-valued, rd-continuous functions defined on the time scale interval $[a,b]$ (throughout $a,b \in \mathbb{T}$ with $a < b$). We assume throughout that:

(i) $F : \mathbb{T} \times \mathbb{R} \to \mathbb{R}$ is a function such that $uF(t,u) > 0$, $\frac{F(t,u)}{u} \geq f(t)$ for $u \neq 0$,
(ii) $P : \mathbb{T} \times \mathbb{R}^2 \to \mathbb{R}$ is a function such that $\frac{P(t,u,v)}{u} \geq q(t)$, $uP(t,u,v) > 0$ for $u \neq 0$,
(iii) $q, f : \mathbb{T} \to \mathbb{R}$ are rd-continuous functions such that $q(t) + f(t) > 0$ for all $t \in \mathbb{T}$.

Since we are interested in oscillatory behavior, we suppose that the time scale under consideration is not bounded above, *i.e.*, it is a time scale interval of the form $[a,\infty)$. By a solution of (1), we mean a nontrivial real-valued function satisfying equation (1) for $t \geq a$. A solution $x(t)$ of (1) is said to be oscillatory if it is neither eventually positive nor eventually negative; otherwise, it is called nonoscillatory. Equation (1) is said to be oscillatory if

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all its solutions are oscillatory. Our attention is restricted to those solutions of (1) which exist on some half-line $[t_x, \infty)$ and satisfy $\sup\{|x(t)| : t > t_0\} > 0$ for any $t_0 \geq t_x$.

2 Some lemmas

In this section, we state and prove some lemmas which we will need in the proofs of our main results.

Lemma 2.1 *Suppose that x is an eventually positive solution of* (1) *and*

$$\int_{t_0}^{\infty} \frac{1}{r_1(t)} \Delta t = \infty, \qquad \int_{t_0}^{\infty} \frac{1}{r_2(t)} \Delta t = \infty. \tag{2}$$

Then there is a $t_1 \in [t_0, \infty)$ such that either

(i) $x(t) > 0$, $x^{\Delta}(t) > 0$, $(r_2(t)x^{\Delta}(t))^{\Delta} > 0$, $t \in [t_1, \infty)$, *or*

(ii) $x(t) > 0$, $x^{\Delta}(t) < 0$, $(r_2(t)x^{\Delta}(t))^{\Delta} > 0$, $t \in [t_1, \infty)$.

Proof Let x be an eventually positive solution of (1). Then there exists $t_1 \in [t_o, \infty)$ such that $x(t) > 0$ for $t \in [t_1, \infty)$. From (1) we have

$$\begin{aligned}\bigl(r_1(t)\bigl(r_2(t)x^{\Delta}(t)\bigr)^{\Delta}\bigr)^{\Delta} &= -P\bigl(t, x(t), x^{\Delta}(t)\bigr) - F\bigl(t, x(t)\bigr) \\ &\leq -x(t)q(t) - x(t)f(t) \\ &< 0\end{aligned}$$

for $t \in [t_1, \infty)$. Hence, $r_1(t)(r_2(t)x^{\Delta}(t))^{\Delta}$ is strictly decreasing on $[t_1, \infty)$. We claim that $r_1(t)(r_2(t)x^{\Delta}(t))^{\Delta} > 0$ on $[t_1, \infty)$. Assume not, then there is a $t_2 \in [t_1, \infty)$ such that $r_1(t)(r_2(t)x^{\Delta}(t))^{\Delta} < 0$, $t \in [t_2, \infty)$. Then we can choose a negative constant c and $t_3 \in [t_2, \infty)$ such that $r_1(t)(r_2(t)x^{\Delta}(t))^{\Delta} \leq c < 0$ for $t \in [t_3, \infty)$. Dividing by $r_1(t)$ and integrating from t_3 to t, we obtain

$$r_2(t)x^{\Delta}(t) \leq r_2(t_3)x^{\Delta}(t_3) + c \int_{t_3}^{t} \frac{\Delta s}{r_1(s)}.$$

Letting $t \to \infty$, then $r_2(t)x^{\Delta}(t) \to -\infty$ by (2). Thus, there is a $t_4 \in [t_3, \infty)$ such that for $t \in [t_4, \infty)$, $r_2(t)x^{\Delta}(t) \leq r_2(t_4)x^{\Delta}(t_4) < 0$. Dividing by $r_2(t)$ and integrating from t_4 to t, we obtain

$$x(t) - x(t_4) \leq r_2(t_4)x^{\Delta}(t_4) \int_{t_4}^{t} \frac{\Delta s}{r_2(s)},$$

which implies that $x(t) \to -\infty$ as $t \to \infty$ by (2), a contradiction with the fact that $x(t) > 0$. Hence, we have

$$\bigl(r_2(t)x^{\Delta}(t)\bigr)^{\Delta} > 0, \quad t \in [t_1, \infty).$$

This implies that $r_2(t)x^{\Delta}(t)$ is strictly increasing on $[t_1, \infty)$. It follows from this that either $r_2(t)x^{\Delta}(t) < 0$ on $[t_1, \infty)$ or $r_2(t)x^{\Delta}(t)$ is eventually positive and the proof is complete. □

Lemma 2.2 *Assume that*

$$\int_{t_0}^{\infty} \varphi(t)\Delta t = \infty, \tag{3}$$

where $\varphi(t) = q(t) + f(t)$ *and* $x(t)$ *is a solution of* (1) *that satisfies Case* (ii) *in Lemma* 2.1. *Then* $\lim_{t\to\infty} x(t) = 0$.

Proof Let x be a solution of (1) satisfying Case (ii) in Lemma 2.1, that is,

$$x(t) > 0, \qquad x^{\Delta}(t) < 0, \qquad \bigl(r_2(t)x^{\Delta}(t)\bigr)^{\Delta} > 0, \quad t \in [t_1, \infty).$$

Then $\lim_{t\to\infty} x(t) = b \geq 0$. Assume $b > 0$ and we now show that this leads to a contradiction. From (1) and $x \geq b$,

$$\begin{aligned}
\bigl(r_1(t)\bigl(r_2(t)x^{\Delta}(t)\bigr)^{\Delta}\bigr)^{\Delta} &= -P\bigl(t, x(t), x^{\Delta}(t)\bigr) - F\bigl(t, x(t)\bigr) \\
&\leq -x(t)q(t) - x(t)f(t) \\
&= -x(t)\bigl(q(t) + f(t)\bigr) \\
&\leq -b\varphi(t),
\end{aligned}$$

where $\varphi(t) = q(t) + f(t)$, for $t \in [t_1, \infty)$. Now let

$$u(t) = r_1(t)\bigl(r_2(t)x^{\Delta}(t)\bigr)^{\Delta}, \quad t \in [t_1, \infty),$$

then we have

$$u^{\Delta}(t) \leq -b\varphi(t), \quad t \in [t_1, \infty).$$

Integrating the last inequality from t_1 to t, we have

$$u(t) \leq u(t_1) - b\int_{t_1}^{t} \varphi(s)\Delta s.$$

Using (3) it is possible to choose a $t_2 \in [t_1, \infty)$, sufficiently large, such that $u(t) < 0$ for all $t \in [t_2, \infty)$, which is a contradiction, and this completes the proof. □

Lemma 2.3 *Assume that* $x(t)$ *is a solution of* (1) *satisfying Case* (i) *of Lemma* 2.1. *Then there exists* $t_1 \in [t_0, \infty)$ *such that*

$$x^{\Delta}(t) \geq \frac{\delta(t, t_1)r_1(t)}{r_2(t)}\bigl(r_2(t)x^{\Delta}(t)\bigr)^{\Delta} \quad \textit{for } t \geq t_1, \tag{4}$$

where $\delta(t, t_1) = \int_{t_1}^{t} \frac{\Delta s}{r_1(s)}$.

Proof From Case (i) of Lemma 2.1, we have $x(t)$ as a solution of (1) satisfying

$$x(t) > 0, \qquad x^{\Delta}(t) > 0, \qquad \bigl(r_2(t)x^{\Delta}(t)\bigr)^{\Delta} > 0$$

for $t \geq t_1$. Using $x(t)$ is a solution of (1), we get $(r_1(t)(r_2(t)x^\Delta(t))^\Delta)^\Delta < 0$ and hence $r_1(t)(r_2(t)x^\Delta(t))^\Delta$ is decreasing on $[t_1, \infty)$. Hence,

$$\begin{aligned} r_2(t)x^\Delta(t) &= r_2(t_1)x^\Delta(t_1) + \int_{t_1}^t \frac{r_1(s)(r_2(s)x^\Delta(s))^\Delta}{r_1(s)} \Delta s \\ &\geq r_1(t)\delta(t,t_1)\bigl(r_2(t)x^\Delta(t)\bigr)^\Delta, \quad t \geq t_1, \end{aligned}$$

and this leads to (4) and the proof is complete. □

3 Main results

In this section, we establish some sufficient conditions which guarantee that every solution $x(t)$ of (1) oscillates on $[t_0, \infty)$ or converges as $t \to \infty$.

Theorem 3.1 *Assume that* (2) *holds. Furthermore, assume that there exists a positive function $z(t)$ such that $z^\Delta(t)$ is rd-continuous on $[t_0, \infty)$ and if*

$$\limsup_{t\to\infty} \int_{t_1}^t \left(z(s)\varphi(s) - \frac{(z^\Delta(s))^2}{4Q(s,t_1)} \right) \Delta s = \infty, \tag{5}$$

for all sufficiently large t_1, where $Q(t,t_1) = \frac{z(t)\delta(t,t_1)}{r_2(t)}$, $\varphi(t) = q(t) + f(t)$, then every solution $x(t)$ of (1) *is oscillatory or $\lim_{t\to\infty} x(t)$ exists* (*finite*).

Proof Let $x(t)$ be a nonoscillatory solution of (1). We only consider the case when $x(t)$ is eventually positive, since the case when $x(t)$ is eventually negative is similar. By Lemma 2.1 either Case (i) or Case (ii) in Lemma 2.1 holds. Assume $x(t)$ satisfies Case (i) in Lemma 2.1. Define the Riccati-type function $w(t)$ by

$$w(t) = z(t)\frac{r_1(t)(r_2(t)x^\Delta(t))^\Delta}{x(t)}, \quad t \geq t_1. \tag{6}$$

By the product rule,

$$\begin{aligned} w^\Delta(t) &= r_1\bigl(\sigma(t)\bigr)\bigl(r_2x^\Delta(t)\bigr)^\Delta\bigl(\sigma(t)\bigr)\left[\frac{z(t)}{x(t)}\right]^\Delta + \frac{z(t)}{x(t)}\bigl(r_1(t)\bigl(r_2(t)x^\Delta(t)\bigr)^\Delta\bigr)^\Delta \\ &= r_1\bigl(\sigma(t)\bigr)\bigl(r_2x^\Delta(t)\bigr)^\Delta\bigl(\sigma(t)\bigr)\left[\frac{z(t)}{x(t)}\right]^\Delta + \frac{z(t)}{x(t)}\bigl(-P\bigl(t,x(t),x^\Delta(t)\bigr) - F\bigl(t,x(t)\bigr)\bigr). \end{aligned}$$

Using (1) we have

$$w^\Delta(t) \leq -z(t)q(t) - z(t)f(t) + z^\Delta(t)\frac{w^\sigma(t)}{z^\sigma(t)} - z(t)\frac{x^\Delta(t)}{x(t)x^\sigma(t)}\bigl(r_1(t)\bigl(r_2(t)x^\Delta(t)\bigr)^\Delta\bigr)^\sigma.$$

Using (4) and $x^\Delta(t) > 0$, we obtain

$$\begin{aligned} w^\Delta(t) &\leq -z(t)q(t) - z(t)f(t) + z^\Delta(t)\frac{w^\sigma(t)}{z^\sigma(t)} \\ &\quad - z(t)\frac{r_1(t)(r_2(t)x^\Delta(t))^\Delta(r_1(t)(r_2(t)x^\Delta(t))^\Delta)^\sigma\delta(t,t_1)}{r_2(t)(x^\sigma(t))^2}. \end{aligned}$$

Since $r_1(t)(r_2(t)x^{\Delta}(t))^{\Delta}$ is decreasing, we have

$$\begin{aligned} w^{\Delta}(t) &\le -z(t)\varphi(t) + z^{\Delta}(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)} - z(t)\frac{\delta(t,t_1)}{r_2(t)}\frac{(w^{\sigma}(t))^2}{(z^{\sigma}(t))^2} \\ &= -z(t)\varphi(t) + z^{\Delta}(t)\frac{w^{\sigma}(t)}{z^{\sigma}(t)} - Q(t,t_1)\frac{(w^{\sigma}(t))^2}{(z^{\sigma}(t))^2}, \end{aligned} \tag{7}$$

where $\varphi(t) = q(t) + f(t)$ and $Q(t,t_1) = \frac{z(t)\delta(t,t_1)}{r_2(t)}$. From (7) we have

$$\begin{aligned} w^{\Delta}(t) &\le -z(t)\varphi(t) + \frac{(z^{\Delta}(t))^2}{4Q(t,t_1)} - \left[\sqrt{Q(t,t_1)}\frac{w^{\sigma}(t)}{z^{\sigma}(t)} - \frac{1}{2}\sqrt{\frac{1}{Q(t,t_1)}}z^{\Delta}(t)\right]^2 \\ &\le -\left[z(t)\varphi(t) - \frac{(z^{\Delta}(t))^2}{4Q(t,t_1)}\right]. \end{aligned} \tag{8}$$

Integrating (8) from t_1 to t, we obtain

$$-w(t_1) \le w(t) - w(t_1) \le -\int_{t_1}^{t}\left[z(s)\varphi(s) - \frac{(z^{\Delta}(s))^2}{4Q(s,t_1)}\right]\Delta s, \tag{9}$$

which yields

$$\int_{t_1}^{t}\left[z(s)\varphi(s) - \frac{(z^{\Delta}(s))^2}{4Q(s,t_1)}\right]\Delta s \le w(t_1) \tag{10}$$

for all large t. This is contrary to (5) and so Case (i) is not possible. If Case (ii) in Lemma 2.1 holds, then clearly $\lim_{t\to\infty} x(t)$ exists (finite). □

On the basis of Lemma 2.2 and Theorem 3.1, we have the following results.

Corollary 3.2 *If* (2) *and* (3) *hold, then every solution of* (1) *is either oscillatory or* $\lim_{t\to\infty} x(t) = 0$.

Corollary 3.3 *Assume that* (2) *holds. If*

$$\limsup_{t\to\infty}\int_{t_1}^{t}\left(s\varphi(s) - \frac{r_2(s)}{4s\delta(s,t_1)}\right)\Delta s = \infty,$$

then every solution of (1) *is either oscillatory or* $\lim_{t\to\infty} x(t)$ *exists.*

Example 3.4 We consider the third-order dynamic equation

$$x^{\Delta^3}(t) + \frac{a(1 + (x^{\Delta}(t))^2)}{t}x(t) + \frac{b}{t}x(t) = 0, \quad a > 0, b > 0, t \ge 1, \tag{11}$$

where $r_1(t) = r_2(t) = 1$, $P(t,x(t),x^{\Delta}(t)) = a(1 + (x^{\Delta}(t))^2)x(t)/t$, $F(t,x(t)) = bx(t)/t$. Let $f(t) = b/t$ and $q(t) = a/t$. It is not difficult to verify that all conditions of Corollary 3.3 are satisfied. Hence, every solution of equation (11) is oscillatory or satisfies $\lim_{t\to\infty} x(t) = 0$.

Now we establish a Kamenev-type (see [18]) oscillation criterion for (1).

Theorem 3.5 *Assume that* (2) *holds. Furthermore, assume that there is a positive function* $z(t)$ *such that* $z^{\Delta}(t)$ *is rd-continuous on* $[t_0,\infty)$, *and for all sufficient large* t_1,

$$\limsup_{t\to\infty}\frac{1}{t^m}\int_{t_1}^{t}(t-s)^m\left[z(s)\varphi(s)-\frac{(z^{\Delta}(s))^2}{4Q(s,t_1)}\right]\Delta s=\infty, \tag{12}$$

where $m\geq 1$, $Q(t,t_1)$ *and* $\varphi(t)$ *are as in Theorem* 3.1. *Then every solution of* (1) *is either oscillatory or* $\lim_{t\to\infty}x(t)$ *exists.*

Proof Proceeding as in Theorem 3.1, we assume that (1) has a nonoscillatory solution, say $x(t)>0$ for all $t\geq t_1$, where t_1 is chosen so large that Lemma 2.1 and Lemma 2.3 hold. By Lemma 2.1 there are two possible cases. First, if Case (i) holds, then by defining again $w(t)$ by (6) as in Theorem 3.1, we have $w(t)>0$ and (8), that is,

$$\left[z(t)\varphi(t)-\frac{(z^{\Delta}(t))^2}{4Q(t,t_1)}\right]\leq -w^{\Delta}(t).$$

Therefore,

$$\int_{t_1}^{t}(t-s)^m\left[z(s)\varphi(s)-\frac{(z^{\Delta}(s))^2}{4Q(s,t_1)}\right]\Delta s\leq -\int_{t_1}^{t}(t-s)^m w^{\Delta}(s)\Delta s. \tag{13}$$

Integrating by parts of the right-hand side leads to

$$\begin{aligned}\int_{t_1}^{t}(t-s)^m w^{\Delta}(s)\Delta s&=(t-s)^m w(s)\big|_{t_1}^{t}-\int_{t_1}^{t}h(t,s)w^{\sigma}(s)\Delta s\\&=-(t-t_1)^m w(t_1)-\int_{t_1}^{t}h(t,s)w^{\sigma}(s)\Delta s,\end{aligned} \tag{14}$$

where $h(t,s):=((t-s)^m)^{\Delta_s}$. Note that

$$h(t,s)=\begin{cases}-m(t-s)^{m-1}, & \mu(s)=0;\\ \frac{(t-\sigma(s))^m-(t-s)^m}{\mu(s)}, & \mu(s)>0,\end{cases}$$

and $m\geq 1$, $h(t-s)\leq 0$ for $t\geq\sigma(s)$. It follows from (14) that

$$\int_{t_1}^{t}(t-s)^m w^{\Delta}(s)\Delta s\geq -(t-t_1)^m w(t_1).$$

Then from (13) we have

$$\int_{t_1}^{t}(t-s)^m\left[z(s)\varphi(s)-\frac{(z^{\Delta}(s))^2}{4Q(s,t_1)}\right]\Delta s\leq (t-t_1)^m w(t_1).$$

Then

$$\frac{1}{t^m}\int_{t_1}^{t}(t-s)^m\left[z(s)\varphi(s)-\frac{(z^{\Delta}(s))^2}{4Q(s,t_1)}\right]\Delta s\leq\left(\frac{t-t_1}{t}\right)^m w(t_1).$$

Hence,

$$\limsup_{t\to\infty} \frac{1}{t^m} \int_{t_1}^{t} (t-s)^m \left[z(s)\varphi(s) - \frac{(z^\Delta(s))^2}{4Q(s,t_1)} \right] \Delta s \le w(t_1),$$

which is a contradiction of (12). If Case (ii) holds, then, as before, $\lim_{t\to\infty} x(t)$ exists and the proof is complete. □

Competing interests
The author declares that they have no competing interests.

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