BOUNDARY VALUE PROBLEMS

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to Vivien, Karen, and Natalie

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Preface

This book is designed for use in a one-year course in boundary value problems for both beginning graduate students in mathematics and advanced graduate students in other disciplines in engineering and the physical sciences. It deals mainly with the boundary value problems of linear partial differential euqations of second order, in particular, problems relating to the three fundamental equations of mathematical physics, namely, the wave equation, the heat equation, and Laplace's equation.

There are two objectives in our approach. The first goal is to obtain a formal solution of a given boundary value problem either by the method of separation of variables for all applicable linear equations or by the method of d'Alembert's solution in the wave equation. This formal solution is, in general, represented either by a series or an integral. In order to obtain such a representation, the theory of Sturm-Liouville problems, Fourier series, and Fourier transforms have been developed. The second goal is to verify that the formal solution is actually a solution of the boundary value problem. The formal solution must satisfy the problem with the required smooth properties, and the resulting solution must be unique and depend continuously on the initial and boundary data. The verification of solutions depends on the concept of uniform convergence of the series or the improper integral. In turn, the uniqueness and stability of the solution depend on the method of the energy integral for the wave equation and the maximum principles for the heat equation and Laplace's equation.

The first six chapters of this book cover the boundary value problems of linear partial differential equations in two independent variables with development of the relevant theory. In Chapter 7, we treat the boundary value problems in three independent variables. In some boundary value problems involving cylindrical and spherical domains, the method of separation of variables leads us to two singular differential equations, namely, Bessel's equation and Legendre's equation. Hence, some important properties of Bessel functions and Legendre polynomials are treated in that chapter. Some of the treatment in this book follows that of Cannon [2], Indritz [8], Tolstov [11], and Weinberger [13].

As a prerequisite for a course based on this book, the student should have completed a course in advanced calculus and an elementary course on ordinary differential equations. The problems at the end of each chapter are chosen to fit the level of the course and, in some cases, supplement the contents of the chapter.

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