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Nonlinear statistics of quantum transport in chaotic cavities

D. V. Savin,[1] H.-J. Sommers,[2] and W. Wieczorek[2]
[1]*Department of Mathematical Sciences, Brunel University, Uxbridge UB8 3PH, United Kingdom*
[2]*Fachbereich Physik, Universität Duisburg-Essen, 47048 Duisburg, Germany*
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In the framework of the random matrix approach, we apply the theory of Selberg's integral to problems of quantum transport in chaotic cavities. All the moments of transmission eigenvalues are calculated analytically up to the fourth order. As a result, we derive exact explicit expressions for the skewness and kurtosis of the conductance and transmitted charge as well as for the variance of the shot-noise power in chaotic cavities. The obtained results are generally valid at arbitrary numbers of propagating channels in the two attached leads. In the particular limit of large (and equal) channel numbers, the shot-noise variance attends the universal value $1/64\beta$ that determines a universal Gaussian statistics of shot-noise fluctuations in this case.

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Quantum transport of noninteracting electrons in mesoscopic systems can be conventionally described in the framework of scattering theory.[1,2] The conductance and shot noise are the brightest and, perhaps, most frequently considered examples. Being expressed in terms of transmission eigenvalues T_i of a conductor, the dimensionless conductance g and the zero-frequency shot-noise power p of the two-terminal setup at zero temperature are, respectively, given by

$$g=\sum_i^n T_i, \quad p=\sum_i^n T_i(1-T_i). \tag{1}$$

Here, $n\equiv\min(N_1,N_2)$, where $N_{1,2}$ is the number of scattering channels in each of the two attached leads.

Generally, the positive quantities $T_i\leqslant 1$ are mutually correlated random numbers whose fluctuations depend on the conductor's nature. In the case of chaotic cavities considered below, the joint distribution of T_i is believed to be provided by the random matrix theory (RMT) and reads as follows:[1]

$$\mathcal{P}(\{T_i\})=\mathcal{N}_\beta^{-1}\prod_{i=1}^{n} T_i^{\alpha-1}\prod_{j<k}|T_j-T_k|^\beta, \tag{2}$$

with $\alpha=\frac{\beta}{2}(|N_1-N_2|+1)$ and normalization constant $\mathcal{N}_\beta$. The symmetry index β (=1, 2, or 4) distinguishes between the three standard RMT classes (orthogonal, unitary, or symplectic ensembles, respectively) which are realized depending on the presence or the absence of time-reversal and spin-flip symmetry in the system.[1,3]

The exact RMT results for the average and variance of the conductance are well known for quite a long time[1,4,5] and that for the average shot-noise power has become available only recently[6] (see also Refs. 7–9). As concerns higher order cumulants of these quantities, their exact RMT expressions valid at arbitrary $N_{1,2}$ have not been reported in the literature so far, a progress in this direction being announced very recently.[10]

In this paper, we present a thorough analysis of this question by further developing and applying the theory of Selberg's integral to the problem. As was recently recognized,[6] such an approach is a powerful nonperturbative method suited particularly well for the studies of moments and counting statistics (see also Refs. 10 and 11). It represents a useful alternative to orthogonal polynomial[3] or diagrammatic[12] approaches, especially in the situation when the finite dimensionality of relevant random matrices becomes important.

Selberg's integral appears naturally in the problem first as an integral determining the normalization constant,

$$\mathcal{N}_\beta=\prod_{j=1}^{n-1}\frac{\Gamma\left[1+\frac{\beta}{2}(1+j)\right]\Gamma\left(\alpha+\frac{\beta}{2}j\right)\Gamma\left(1+\frac{\beta}{2}j\right)}{\Gamma\left(1+\frac{\beta}{2}\right)\Gamma\left[1+\alpha+\frac{\beta}{2}(n+j-1)\right]}. \tag{3}$$

This expression assures that Eq. (2) is a probability density, being generally valid for discrete n and continuous α and β.[13] To study the cumulants of g and p, one needs to know the moments $\langle T_1^{n_1}\cdots T_k^{n_k}\rangle$, with $\langle\cdots\rangle$ standing for the integration over the joint probability density (2) and $n_i\geqslant 0$. Here, we calculate all the moments with $\Sigma_i n_i\leqslant 4$ by deriving a set of new algebraic relations for them and reducing the moments to the forms of Selberg's integral. Presenting the relevant technical details at the end of the paper, we now discuss applications of the obtained results to various linear and nonlinear statistics on the transmission eigenvalues.

Transmitted charge cumulants. The statistics of charge q (in units of e) transmitted through the cavity over the observation time is usually described by means of the current or charge cumulants $\langle\langle q^m\rangle\rangle$. Following Levitov and co-workers,[14,15] it is convenient to use a general formula for the cumulant generating function expressed in terms of the transmission eigenvalues as follows: $\Sigma_m\frac{\lambda^m}{m!}\langle\langle q^m\rangle\rangle=\langle\Sigma_i\ln[1+T_i(e^\lambda-1)]\rangle$. One finds that the first cumulant, $\langle\langle q\rangle\rangle=n\langle T_1\rangle$, gives the conductance, the RMT average of which is known[4] to be

$$\langle\langle q\rangle\rangle\equiv\langle g\rangle=\frac{N_1N_2}{N-1+\frac{2}{\beta}}, \quad N\equiv N_1+N_2, \tag{4}$$

while $\langle\langle q^2\rangle\rangle=n(\langle T_1\rangle-\langle T_1^2\rangle)$ yields shot noise as follows:[6]

$$\langle\langle q^2\rangle\rangle \equiv \langle p\rangle = \langle g\rangle \frac{\left(N_1-1+\frac{2}{\beta}\right)\left(N_2-1+\frac{2}{\beta}\right)}{\left(N-2+\frac{2}{\beta}\right)\left(N-1+\frac{4}{\beta}\right)}. \quad (5)$$

An equivalent to RMT derivation of these and related results within a semiclassical approach may be found in Ref. 16.

The charge cumulants are an example of linear statistics on the T_i, which is fully determined by the transmission eigenvalue density, $\rho(T)=\langle\Sigma_i\delta(T-T_i)\rangle$. However, the latter is analytically known only in some limiting cases of a few[4,7] or many[17] open channels, restricting the use of $\rho(T)$ for the calculation of $\langle\langle q^m\rangle\rangle$ and full counting statistics[18,19] to these cases.

In the general situation of arbitrary $N_{1,2}$, one can alternatively consider the joint probability distribution (2) and exploit its simple algebraic structure (i.e., that of the Selberg integral kernel) to derive exact relations for its moments.[6] For example, Eqs. (15) and (16) presented below yield straightforwardly and in a uniform way exact results, Eqs. (4), (5), and (9), for $\langle g\rangle$, $\langle p\rangle$, and $\mathrm{var}(g)$, respectively. This approach was recently extended further to find the third cumulant $\langle\langle q^3\rangle\rangle=\langle(q-\langle q\rangle)^3\rangle$ exactly.[11] For completeness and later use, we write down this result [following from Eq. (18) below] as follows:

$$\frac{\langle\langle q^3\rangle\rangle}{\mathrm{var}(q)} = \frac{\left(1-\frac{2}{\beta}\right)^2-(N_1-N_2)^2}{\left(N-3+\frac{2}{\beta}\right)\left(N-1+\frac{6}{\beta}\right)}. \quad (6)$$

As to the fourth cumulant of the transmitted charge, its explicit expression can be found from Eqs. (15)–(19) according to $\langle\langle q^4\rangle\rangle=n[\langle T_1\rangle-7\langle T_1^2\rangle+12\langle T_1^3\rangle-6\langle T_1^4\rangle]$, the result being too lengthy to be reported here. In the case of the single-mode leads, $N_1=N_2=1$, one gets $\langle\langle q^4\rangle\rangle=-\frac{2}{105},-\frac{1}{30},-\frac{1}{30}$ at the values of $\beta=1,2,4$, respectively. In the opposite semiclassical limit of many channels, $N_{1,2}\gg 1$, we represent the outcome of our calculation as the following $\frac{1}{N}$ expansion:

$$\frac{\langle\langle q^4\rangle\rangle}{\mathrm{var}(q)} = \frac{N_1^4-8N_1N_2^3+12N_1^2N_2^2-8N_1^3N_2+N_2^4}{N^4} + \frac{6(\beta-2)(N_1-N_2)^2(2N_1^2-7N_1N_2+2N_2^2)}{\beta N^5} + \mathcal{O}(1/N^2). \quad (7)$$

The leading order term agrees with the result obtained by a different method in Ref. 18. The next order term gives a weak localization correction which vanishes at $\beta=2$ or $N_1=N_2$. In the latter case of symmetric cavities, one further finds

$$\langle\langle q^4\rangle\rangle = \frac{n}{64}\left(1-\frac{\beta^2-6\beta+4}{2\beta^2n^2}\right)+\mathcal{O}\left(\frac{1}{n^2}\right). \quad (8)$$

Conductance cumulants. As is clear from the discussion, the presented method is equally applied to nonlinear statistics determined by different transmission eigenvalues as well. The simplest example of such a quantity is the variance of the conductance, the exact RMT result of which reads[1,6]

$$\frac{\mathrm{var}(g)}{\langle g\rangle} = \frac{2\left(N_1-1+\frac{2}{\beta}\right)\left(N_2-1+\frac{2}{\beta}\right)}{\beta\left(N-2+\frac{2}{\beta}\right)\left(N-1+\frac{2}{\beta}\right)\left(N-1+\frac{4}{\beta}\right)}. \quad (9)$$

We note that $\mathrm{var}(g)=2\langle g\rangle\langle p\rangle/\beta N_1N_2$ makes a relation of Eq. (9) to the linear statics (4) and (5) considered above.

The third cumulant of the conductance, the so-called skewness, can also be found from Eqs. (15)–(18) and be represented after some algebra in the following compact form:

$$\frac{\langle\langle g^3\rangle\rangle}{\mathrm{var}(g)} = \frac{4\left[\left(1-\frac{2}{\beta}\right)^2-(N_1-N_2)^2\right]}{\beta\left(N-3+\frac{2}{\beta}\right)\left(N-1+\frac{2}{\beta}\right)\left(N-1+\frac{6}{\beta}\right)}. \quad (10)$$

One finds immediately that $\langle\langle g^3\rangle\rangle=8[\langle g\rangle/\beta N_1N_2]^2\langle\langle q^3\rangle\rangle$. It is also worth noting that the skewness vanishes for symmetric cavities ($N_1=N_2$) at $\beta=2$. This holds generally for any odd cumulant of g (or q), as the corresponding distribution becomes symmetric around $\frac{n}{2}$ (or $\frac{1}{2}$) in this case.[20]

By our method, we have also calculated the fourth cumulant $\langle\langle g^4\rangle\rangle$ which is related to the conductance kurtosis. The fourth moment of the conductance is found to be determined by moments of T_i as $\langle g^4\rangle=n[\langle T_1^4\rangle+(n-1)(3\langle T_1^2T_2^2\rangle+4\langle T_1T_2^3\rangle)+6(n-1)(n-2)\langle T_1T_2T_3^2\rangle+(n-1)(n-2)(n-3)\langle T_1T_2T_3T_4\rangle]$, the corresponding cumulant being given by the standard formula. Since the resulting explicit expression appears to be too cumbersome, we restrict our consideration to the limiting cases of the single-mode and many-mode leads. In the former case, we get $\langle\langle g^4\rangle\rangle=-\frac{32}{4725},-\frac{1}{120},-\frac{1}{540}$ at $\beta=1,2,4$, respectively, whereas in the latter case of $N_{1,2}\gg 1$, we arrive at the following expression:

$$\frac{\langle\langle g^4\rangle\rangle}{\mathrm{var}(g)} = \frac{24}{\beta^2N^6}\left[(N_1-N_2)^2(N_1^2+N_2^2-4N_1N_2)+\frac{\beta-2}{\beta N}(12(N_1^4+N_2^4)-64N_1N_2(N_1^2+N_2^2)+105N_1^2N_2^2)\right]+\mathcal{O}(1/N^4). \quad (11)$$

One can readily see that higher cumulants contribute in the next order of $\frac{1}{N}$, thus the conductance distribution gets more Gaussian-like as N grows.[21] This tendency becomes even stronger for symmetric cavities at $\beta=2$, as then $\langle\langle g^3\rangle\rangle=0$ identically and $\langle\langle g^4\rangle\rangle$ vanishes in the leading and next-to-leading orders. In this case of $N_{1,2}=n\gg 1$, one gets

$$\langle\langle g^4\rangle\rangle = \frac{3}{128\beta^3n^3}\left(1-\frac{2}{\beta}+\frac{(\beta+2)^2}{2\beta^2n}\right)+\mathcal{O}\left(\frac{1}{n^5}\right). \quad (12)$$

Shot-noise variance. Now, we discuss statistics of the shot-noise power. Its average value is given by Eq. (5), whereas its second cumulant, the variance, is determined by $\mathrm{var}(p)=n[\langle T_1^2\rangle-2\langle T_1^3\rangle+\langle T_1^4\rangle]+n(n-1)[\langle T_1T_2\rangle-2\langle T_1T_2^2\rangle+\langle T_1^2T_2^2\rangle]-n^2[\langle T_1\rangle-\langle T_1^2\rangle]^2$. However, as in all other cases of

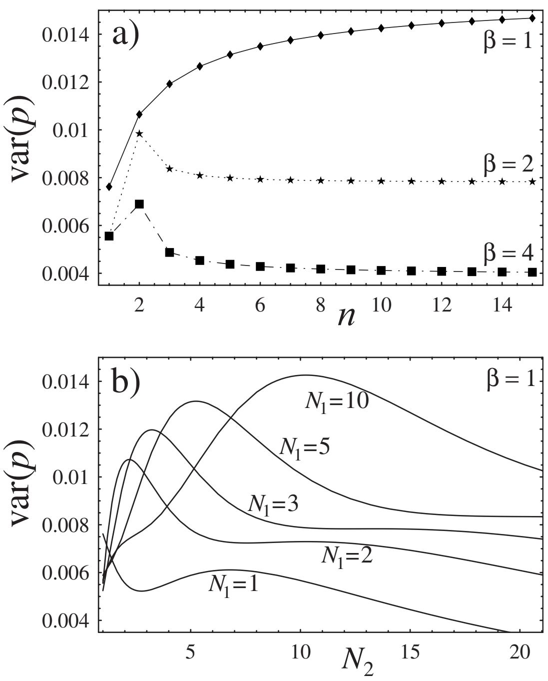

FIG. 1. The variance of the shot-noise power in chaotic cavities as a function of the channel numbers in the leads. (a) The case of symmetric cavities ($N_1=N_2=n$) for three RMT ensembles, when var(p) saturates at the universal value $(64\beta)^{-1}$ at large n. (b) The case of asymmetric cavities with preserved time-reversal symmetry ($\beta=1$) for fixed number N_1 of channels in the one lead and varied one N_2 in the other lead. The shot-noise variance shows a well-developed maximum at $N_2\approx N_1$ and then diminishes down to zero according to $\mathrm{var}(p)\approx 2N_1(N_1-1+\frac{2}{\beta})/\beta N_2^2$ as N_2 grows.

the fourth order cumulants considered above, the explicit result for var(p) cannot be casted in a compact form, so we present the limiting cases again. In the single-mode case, we get $\mathrm{var}(p)=\frac{4}{525},\frac{1}{180},\frac{1}{180}$ corresponding at $\beta=1,2,4$. In the many channel case, we obtain the following expansion:

$$\frac{\mathrm{var}(p)}{\langle p\rangle}=\frac{2}{\beta N^5}\left[N_1^4+N_2^4-4N_1N_2(N_1-N_2)^2+\frac{\beta-2}{\beta N}(9(N_1^4+N_2^4)-42N_1N_2(N_1^2+N_2^2)+70N_1^2N_1^2)\right]+\mathcal{O}(1/N^3). \tag{13}$$

As usual, the first weak localization correction vanishes for unitary symmetry, $\beta=2$. The next order correction can also be found and reads as follows:

$$\mathrm{var}(p)\approx\frac{1}{64\beta}\left(1+\frac{\beta-2}{\beta n}+\frac{4+\beta(\beta-2)}{2\beta^2n^2}\right), \tag{14}$$

where we have omitted the terms $\sim n^{-3}$ and set $N_1=N_2$ for simplicity. The general dependence of the shot-noise variance on the channel numbers in the leads is illustrated on Fig. 1.

Covariance of g and p. It is also instructive to consider statistical correlations between the conductance and the shot-noise power, which can be characterized by their covariance, $\mathrm{cov}(g,p)=\langle gp\rangle-\langle g\rangle\langle p\rangle$. This quantity involves moments of T_i up to the third order. Surprisingly, the resulting exact formula for $\mathrm{cov}(g,p)/\mathrm{var}(g)$ turns out to be given precisely by the right-hand side of Eq. (6). This expression vanishes in symmetric cavities at $\beta=2$ that can be easily seen again as a consequence of the symmetry of the distributions. In this case, therefore, g and p become uncorrelated on the level of their averages and it would be interesting to understand to which extent this holds for their higher moments in general.

Moments of $\{T_i\}$ *and Selberg's integral*. Finally, we discuss the derivation of general moments $\langle T_1^{n_1}\cdots T_k^{n_k}\rangle$ at arbitrary positive α and β. Moments with all $n_i=1$ as well as $\langle T_1^2\rangle$ can be found from recursion relations already given in Mehta's book,[3] which read as follows:

$$\Pi_m\equiv\langle T_1T_2\cdots T_m\rangle=\prod_{j=1}^{m}\frac{\alpha+\frac{\beta}{2}(n-j)}{\alpha+1+\frac{\beta}{2}(2n-j-1)}, \tag{15}$$

$$\langle T_1^2\rangle=\frac{[\alpha+1+\beta(n-1)]\Pi_1-\frac{\beta}{2}(n-1)\Pi_2}{\alpha+2+\beta(n-1)}. \tag{16}$$

To calculate moments containing higher powers of T_m, one may note that

$$\langle T_1T_2\cdots T_m^k\rangle=\left\langle T_1T_2\cdots\frac{\partial}{\partial T_m}\frac{T_m^{k+1}}{k+1}\right\rangle$$

and employ a partial integration here (for $\alpha>0$). This yields a contribution $\langle T_1T_2\cdots\rangle'_{n-1}$ at the upper boundary $T_m=1$, where notation $\langle\cdots\rangle'_{n-1}$ has been introduced for an averaging over a joint density of eigenvalues $T_1,\ldots,T_{n-1}$, which contains in addition a factor $\Pi_{i=1}^{n-1}|1-T_i|^{\beta}$. This case is also contained in the general form of Selberg's integral. In particular, the corresponding analog of Eq. (15) is found to be

$$\Pi'_m\equiv\langle T_1\cdots T_m\rangle'_{n-1}=\prod_{j=1}^{m}\frac{\alpha+\frac{\beta}{2}(n-j-1)}{\alpha+1+\frac{\beta}{2}(2n-j-1)}. \tag{17}$$

In this way, we are able to calculate all the moments up to the fourth order. For moments of the third order, we have

$$\langle T_1^3\rangle=\frac{\alpha+\frac{\beta}{2}(n-1)(1-2\langle T_1T_2^2\rangle)}{\alpha+3+\beta(n-1)}, \tag{18a}$$

$$\langle T_1T_2^2\rangle=\frac{\left[\alpha+\frac{\beta}{2}(n-1)\right]\Pi'_1-\frac{\beta}{2}(n-2)\Pi_3}{\alpha+2+\beta(n-1)}. \tag{18b}$$

The fourth order moments are given as follows:

$$\langle T_1^4\rangle=\frac{\alpha+\frac{\beta}{2}(n-1)(1-2\langle T_1T_2^3\rangle-\langle T_1^2T_2^2\rangle)}{\alpha+4+\beta(n-1)}, \tag{19a}$$

$$\langle T_1T_2^3\rangle = \frac{\left[\alpha+\frac{\beta}{2}(n-1)\right]\Pi_1' - \frac{\beta}{2}\langle T_1^2T_2^2\rangle}{\alpha+3+\beta(n-1)} - \frac{\beta(n-2)\langle T_1T_2T_3^2\rangle}{\alpha+3+\beta(n-1)}, \quad (19b)$$

$$\langle T_1^2T_2^2\rangle = \frac{\left[\alpha+\frac{\beta}{2}(n-1)\right]\langle T_1^2\rangle_{n-1}'}{\alpha+2+\beta\left(n-\frac{3}{2}\right)} - \frac{\beta}{2}\frac{(n-2)\langle T_1T_2T_3^2\rangle}{\alpha+2+\beta\left(n-\frac{3}{2}\right)}, \quad (19c)$$

$$\langle T_1T_2T_3^2\rangle = \frac{\left[\alpha+\frac{\beta}{2}(n-1)\right]\Pi_2' - \frac{\beta}{2}(n-3)\Pi_4}{\alpha+2+\beta(n-1)}, \quad (19d)$$

and

$$\langle T_1^2\rangle_{n-1}' = \frac{[\alpha+1+\beta(n-2)]\Pi_1' - \frac{\beta}{2}(n-2)\Pi_2'}{\alpha+2+\beta(n-1)}. \quad (20)$$

At last, expressions (15) and (17) taken at $m=1,\ldots,4$ make the above algebraic system of equations be closed.

In conclusion, we have applied essentially the theory of Selberg's integral to problems of quantum transport in chaotic cavities. The cumulants up to the fourth order for current and conductance fluctuations and up to the second order for shot noise have been calculated exactly at arbitrary channel numbers and symmetry parameter β. We note that the proposed method can also be used for determining the corresponding distributions in closed form suitable for an analytic work in the case of a few open channels as well as for numerical implementations.[10,22] Our analysis of shot-noise statistics suggests that in close analogy with universal conductance fluctuations,[21] we may characterize universal fluctuations of shot noise by their cumulants as follows:

$$\langle\langle p^m\rangle\rangle \propto \langle p\rangle^{2-m}. \quad (21)$$

In the limit of large (and equal) channel numbers, Eq. (21) yields the Gaussian distribution which is peaked at $\langle p\rangle \approx \frac{n}{4}$ and has a width given by the universal value of var(p) $\approx \frac{1}{64\beta}$. It would be highly interesting to understand how our findings, which are relevant to the zero-dimensional (RMT) case, can be extended to higher dimensions where some analytical results are already known.[23,24]

It would also be desirable to compare our results with the relevant experimental data. For example, measurements of up to the fifth cumulants of the transmitted charge have been recently done in a weakly coupled quantum dot.[25] However, performing similar experiments for a lower impedance device, such as a perfectly open chaotic cavity with several modes in the leads, requires a much higher detector resolution that represents a current experimental challenge. On the other hand, the conductance distribution in chaotic cavities[26] has been directly measured in open quantum dots[27] and also tested recently by means of experiments with microwaves.[28] A comparison with our results in this case could be, however, not so straightforward, as it involves taking into account effects of dephasing and absorption[26,29] as well.

As another potential application of our results, we mention quantized transport in graphene p-n junctions, which has been very recently studied both experimentally[30] and theoretically.[31] The exact RMT results for higher cumulants of the conductance and noise may be useful for understanding possible mechanisms of edge mode mixing in the bipolar regime leading to chaotic transport there.[31] Further work in this and in the other directions mentioned above is needed.

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