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*Research Article*

# $H_\infty$ Consensus for Discrete-Time Multiagent Systems

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An $H_\infty$ consensus problem of multiagent systems is studied by introducing disturbances into the systems. Based on $H_\infty$ control theory and consensus theory, a condition is derived to guarantee the systems both reach consensus and have a certain $H_\infty$ property. Finally, an example is worked out to demonstrate the effectiveness of the theoretical results.

## 1. Introduction

The research booms for consensus problems of multiagent systems following DeGroot's literature [1]. Consensus problem of multiagent systems has attracted considerable attention in recent years. Among various studies of linear-consensus algorithms, a noticeable phenomenon is the fact that algebraic graph theory and linear matrix inequalities (LMIs) play an important role in dealing with consensus problems.

There has been a tremendous amount of interest in consensus problems of multiagent systems. In 2003, Jadbabaie et al. explained the phenomenon mentioned in [2] by theoretical techniques like undirected graph theory, algebraic theory, and the special properties of stochastic matrices in [3]. The distributed complete consensus problems for continuous-time have been intensively investigated [4–15], since the theoretical framework of consensus problems for first-order multiagent networks was proposed and solved by Olfati-Saber and Murray in [16]. They presented the conditions on consensus in terms of graphs for three cases. Furthermore, Ren and Beard presented looser conditions to guarantee the complete consensus based on the graph theory in [17] than the results in [16]. The consensus problem for discrete-time multiagent systems has attracted numerous researchers from mathematics, physics, biology, sociology, control science, and computer science. Xiao and Wang investigated the consensus problems for discrete-time multiagent system with time-varying delays; it is worth mentioning that the augmentation system was first proposed to deal with the consensus problem in [18]. In [19], the cluster consensus problem for first-order discrete-time multiagent system based on the stochastic matrix theory was solved.

In recent years, many researchers analysed the external disturbances effects on stability and the convergence performance of networks, which is called $H_\infty$ consensus. In [20], the $H_\infty$ consensus problems were investigated for both discrete-time and continuous-time multiagent systems. In [21], the sufficient conditions to guarantee all agents achieve $H_\infty$ consensus were obtained by algebraic graph theory firstly. After that, Guo et al. extended general $H_\infty$ consensus problem to cluster synchronization in [22] and obtained some sufficient conditions to realize cluster synchronization of the Lurie dynamical networks both without time delay and with time delay. In [23], Liu and Chen addressed $H_\infty$ consensus control for second-order multiagent systems; a sufficient condition is derived to guarantee $H_\infty$ consensus for the systems by Lyapunov-Krasovkii functional.

Inspired by the above analysis, for discrete-time multiagent systems, we discussed a distributed $H_\infty$ consensus problem of which the objective is to design appropriate protocols to reach consensus while satisfying the described $H_\infty$ performance for multiagent system with external disturbances. The paper is organized as follows. Section 2 presents the mathematical model as well as relevant graph theory. Some analysis results on $H_\infty$ are derived in Section 3. A simulation result is presented in Section 4. The conclusion is given in Section 5.

*Notations.* Throughout this paper, the following notations are used: $I$ denotes the identity matrix with an appropriate dimension; $\|\cdot\|_2$ denotes the space of square integrable vector functions over $[0,\infty)$; $*$ represents an ellipsis for the term introduced by symmetry.

## 2. Preliminaries

*2.1. Graph Theory.* Denote by $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ an undirected graph with $n$ nodes, where $\mathcal{V}=\{1,2,\ldots,n\}$ and $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$ indicate the set of vertices and edges, respectively. Here, $(i,j)\in\mathcal{E}$ if agent $j$ can communicate with agent $i$. In this paper, we assume that there is no self-loop in the graph; that is, $(i,i)\overline{\in}\mathcal{E}$. Let $\mathcal{N}_i=\{j\in\mathcal{V}\mid(j,i)\in\mathcal{E}\}$ be the neighborhood set of vertex $i$. For any edge of the undirected graph $\mathcal{G}$, $(i,j)\in\mathcal{E}$ if and only if edge $(j,i)\in\mathcal{E}$. The term of path refers to a sequence of distinct vertices. A path $\mathcal{P}$ between two vertices $v_0$ and $v_k$ is the sequence $\{v_0,\ldots,v_k\}$, where $(v_{i-1},v_i)\in\mathcal{E}$ for $i=1,2,\ldots,k$. Graph $\mathcal{G}$ is called connected if there exists a path between any two vertices in $\mathcal{G}$. Let $\mathcal{A}=[a_{ij}]\in R^{n\times n}$ be the adjacency matrix of $\mathcal{G}$, where $a_{ij}\geq 0$ if and only if $(j,i)\in\mathcal{E}$; otherwise $a_{ij}=0$. Define $\mathcal{L}=\mathcal{D}-\mathcal{A}$ as the Laplacian matrix of the undirected graph $\mathcal{G}$, where $\mathcal{D}=[d_{ii}]\in R^{n\times n}$ is diagonal with $d_{ii}=\sum_{j=1}^{n}a_{ij}$. The degree of node $i$ is defined as $d_i=\sum_{j=1}^{n}a_{ij}$.

*2.2. Mathematical Models.* We suppose that a multiagent system under consideration consists of $n$ agents with discrete-time dynamics, and the dynamics of $i$th agent is given by

$$\begin{aligned} x_i(k+1) &= x_i(k)+u_i(k)+b_iw_i(k),\\ z_i(k) &= c_ix_i(k)+d_iw_i(k), \\ & \qquad k=0,1,\ldots, \end{aligned} \tag{1}$$

where $x_i(k)$ is the state, $u_i(k)$ is the control input to be designed, and $w_i(k)$ and $z_i(k)$ are the external disturbance and the output, respectively. Here, we assume parameters $b_i$, $c_i$, and $d_i$ are all known constants.

Here, for (1), we design the following consensus protocol:

$$u_i(k)=\sum_{j=1}^{n}a_{ij}\left(x_j(k)-x_i(k)\right), \tag{2}$$

where $a_{ij}\geq 0$ is the entry of weighted matrix $\mathcal{A}$.

To end this section we give two definitions.

*Definition 1.* Given the dynamic system (1), one says that protocol (2) asymptotically solves an $H_\infty$ consensus problem if the states of agents satisfy $\lim_{k\to\infty}\|x_i(k)-x_j(k)\|=0$, $\forall i,j\in\mathcal{N}$.

*Definition 2.* Let $x_i(0)$ be the initial state of agent $v_i$. Then protocol (2) is said to solve an average consensus problem asymptotically if the states of agents satisfy $\lim_{t\to\infty}\|x_i(k)-\sum_{j=1}^{n}x_j(0)/n\|=0$, $\forall i\in\mathcal{N}$.

## 3. Analysis Results

In this section, we discuss an $H_\infty$ consensus problem of the proposed protocol for the discrete-time multiagent system. We first transfer system (1) into one in the form of matrix for completing theory analysis conveniently.

Let

$$\tilde{x}_i(k)=x_i(k)-\frac{\sum_{j=1}^{n}x_j(0)}{n}, \tag{3}$$

and it is easy to know that $\tilde{x}_i(k)\to 0$ when $k\to+\infty$.

Based on (1) and (3), one gets

$$\tilde{x}_i(k+1)=\tilde{x}_i(k)+\tilde{u}_i(k)+B\tilde{w}_i(k). \tag{4}$$

Define $\tilde{x}(k)=[\tilde{x}_1(k),\ldots,\tilde{x}_n(k)]^T$, $z(k)=[z_1(k),\ldots,z_n(k)]^T$, and $w(k)=[w_1(k),\ldots,w_n(k)]^T$, and then (1) can be rewritten into a vector form as

$$\begin{aligned} \tilde{x}(k+1) &= \widetilde{A}\tilde{x}(k)+\widetilde{B}w(k),\\ z(k) &= \widetilde{C}\tilde{x}(k)+\widetilde{D}w(k), \end{aligned} \tag{5}$$

where

$$A=\left(a_{ij}\right)=\begin{cases} a_{ij}, & i\neq j;\\ 1-\displaystyle\sum_{j=1,j\neq i}^{n}a_{ij}, & i=j. \end{cases} \tag{6}$$

And $\widetilde{B}=\operatorname{diag}\{b_1,b_2,\ldots,b_n\}$, $\widetilde{C}=\operatorname{diag}\{c_1,c_2,\ldots,c_n\}$, and $\widetilde{D}=\operatorname{diag}\{d_1,d_2,\ldots,d_n\}$.

Let

$$H=\begin{bmatrix} 1 & -1 & 0 & \cdots & 0 & 0 & 0\\ 0 & 1 & -1 & \cdots & 0 & 0 & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & 1 & -1 & 0\\ 0 & 0 & 0 & \cdots & 0 & 1 & -1 \end{bmatrix}_{(n-1)\times n}. \tag{7}$$

It is obvious that $H$ is a full row rank matrix. We introduce a linear transformation $\overline{x}(k)=H\tilde{x}(k)$ for (5) and note that $\tilde{x}(k)=H^T(HH^T)^{-1}\overline{x}(k)$. Thus, (5) is transformed to

$$\begin{aligned} \overline{x}(k+1) &= H\tilde{x}(k)=H\widetilde{A}\tilde{x}(k)+H\tilde{f}(k)\\ &= H\widetilde{A}H^T\left(HH^T\right)^{-1}\overline{x}+H\tilde{f}(k). \end{aligned} \tag{8}$$

Furthermore, (4) can be written as

$$\begin{aligned} \overline{x}(k+1) &= A\overline{x}(k)+Bw(k),\\ z(k) &= C\overline{x}(k)+Dw(k), \end{aligned} \tag{9}$$

where $A=H\widetilde{A}H^T(HH^T)^{-1}$, $B=H\widetilde{B}$, $C=\widetilde{C}H^T(HH^T)^{-1}$, and $D=\widetilde{D}$.

*Definition 3.* System (9) is said to realize consensus with $H_\infty$ performance index $\gamma>0$, if

(1) system (9) with zero disturbance can reach consensus;

(2) system (9) satisfies $\sum_{k=0}^{\infty} \|z(k)\| \leq \gamma \sum_{k=0}^{\infty} \|w(k)\|$, where $\|\cdot\|$ is the 2-norm of a vector in $R^n$ and $\gamma$ is a constant.

**Lemma 4** (Schur Complement). *$S = \begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{22} \end{bmatrix} \in R^{n\times n}$ is a symmetric matrix, where $S_{11} \in R^{r\times r}$ with $r < n$; then the following three conditions are equivalent:*

(a) $S < 0$;

(b) $S_{11} < 0$, $S_{22} - S_{21}S_{11}^{-1}S_{12} < 0$;

(c) $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{21} < 0$.

Based on the above analysis, we can derive the following main result.

**Theorem 5.** *For given constants $\gamma > 0$, system (1) can achieve $H_\infty$ consensus if there exists a symmetric positive definite matrix $X$, such that*

$$\begin{bmatrix} -X & AX & B & 0 \\ * & -X & 0 & C^T \\ * & * & -\gamma^2 I & D^T \\ * & * & * & -I \end{bmatrix} < 0. \quad (10)$$

*Proof.* We first consider the system without disturbances as follows:

$$\overline{x}(k+1) = A\overline{x}(k), \quad (11)$$

and we prove that system (9) without disturbances is stable.

Define a Lyapunov function as

$$V(x(k), k) = x(k)^T Px(k), \quad (12)$$

where $P$ is a symmetric positive definite matrix.

The difference of Lyapunov function $V(x(k), k)$ yields

$$\begin{aligned} \nabla V(x(k), k) &= V(x(k+1), k+1) - V(x(k), k) \\ &= x(k)^T \left(A^T PA - P\right) x(k). \end{aligned} \quad (13)$$

The difference is negative if and only if

$$A^T PA - P < 0, \quad (14)$$

which is equivalent by Schur Complement to

$$\begin{bmatrix} -P^{-1} & AP^{-1} \\ P^{-1}A & -P^{-1} \end{bmatrix} < 0. \quad (15)$$

Under (10), it is easy to know that the inequality above holds by denoting $X = P^{-1}$.

Hence, system (11) is stable; that is, system (9) without disturbances is stable.

In order to prove that the system has the $H_\infty$ consensus property, that is, satisfying the performance $\sum_{k=0}^{\infty} \|z(k)\|_2 \leq \gamma \sum_{k=0}^{\infty} \|w(k)\|_2$, we define

$$J = \sum_{k=0}^{\infty} \left[z^T(k) z(k) - \gamma^2 w^T(k) w(k)\right]. \quad (16)$$

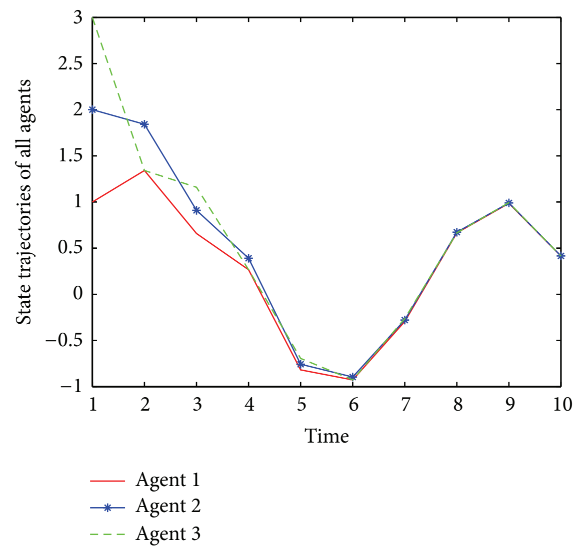


Figure 1: State trajectories of all agents.

Since the system is stable and $\lim_{k\to+\infty} V(x(k), k) = 0$, for $\forall w(k) \in L_2[0, +\infty)$, we have

$$\begin{aligned} J \leq & \sum_{k=0}^{\infty} \left[z^T(k) z(k) - \gamma^2 w^T(k) w(k)\right] \\ & + [V(x(1), 1) - V(x(0), 0)] \\ & + [V(x(2), 2) - V(x(1), 1)] + \cdots \\ & + [V(x(k+1), k+1) - V(x(k), k)] + \cdots. \end{aligned} \quad (17)$$

Since $\nabla V(x(k), k) < 0$, we have

$$\begin{aligned} J & \\ & \leq \sum_{k=0}^{\infty} \left[z^T(k) z(k) - \gamma^2 w^T(k) w(k) + \nabla V(x(k), k)\right]. \end{aligned} \quad (18)$$

Furthermore, letting $\xi(k) = [x^T(k), w^T(k)]^T$, we can obtain

$$J \leq \sum_{k=0}^{\infty} \xi^T(k) S\xi(k), \quad (19)$$

where $S = \begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}$ and $S_{11} = A^T PA - P + C^T C$, $S_{12} = C^T D + A^T PB$, and $S_{22} = -\gamma^2 I + D^T D + B^T PB$. It is easy to know that $J \leq 0$ if $S < 0$. That indicates $\sum_{k=0}^{\infty} \|z(k)\|_2 \leq \gamma \sum_{k=0}^{\infty} \|w(k)\|_2$.

By Schur Complement Lemma, we know that $S < 0$ if and only if

$$\begin{bmatrix} -P^{-1} & A & B & 0 \\ * & -P & 0 & C^T \\ * & * & -\gamma^2 I & D^T \\ * & * & * & -I \end{bmatrix} < 0. \quad (20)$$

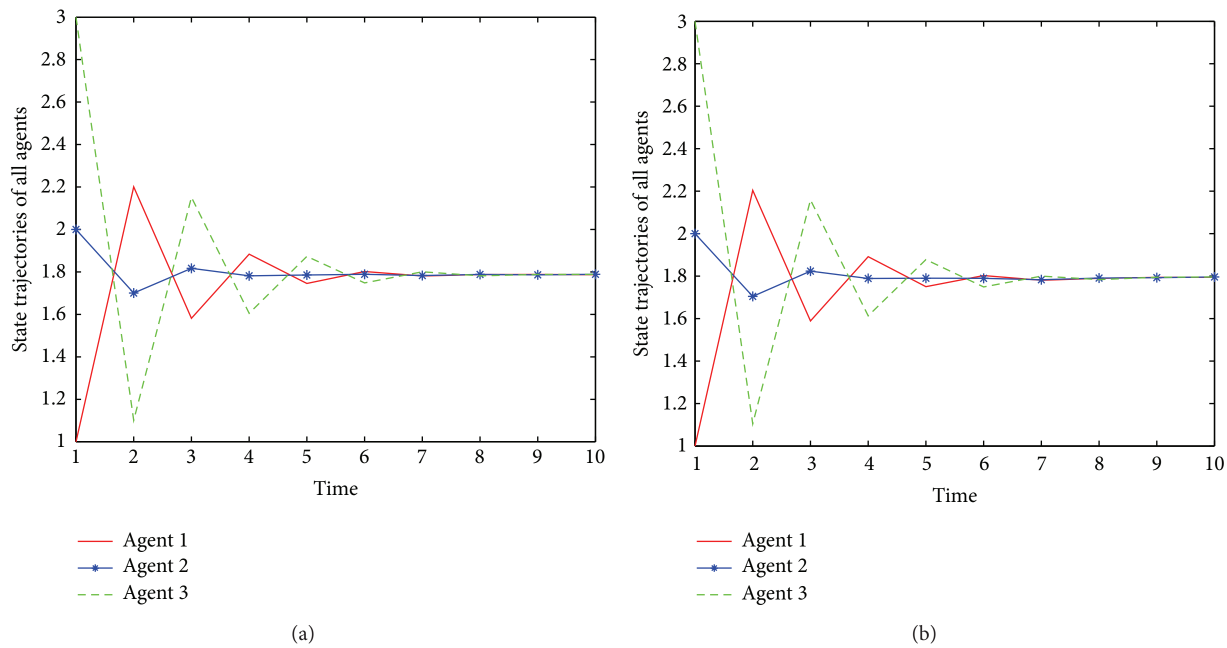


FIGURE 2: (a) State trajectories of all agents when $\gamma = 0.001$. (b) State trajectories of all agents when $\gamma = 0.005$.

Furthermore, by using Schur Complement Lemma again, we can derive equivalently

$$\begin{bmatrix} -P^{-1} & AP^{-1} & B & 0 \\ * & -P^{-1} & 0 & C^T \\ * & * & -\gamma^2 I & D^T \\ * & * & * & -I \end{bmatrix} < 0 \quad (21)$$

which exactly is (10) by letting $P^{-1} = X$. According to Definition 3, we obtain that system (1) can achieve $H_\infty$ consensus. The proof is completed. □

## 4. Simulation

In this section, we provide some simulations of two protocols for system (1) under the effect of different positive parameter $\gamma$ with 3 agents. The state trajectories of all agents are described in all the figures. In particular, the solid curve and the dotted curve describe the state trajectories of agent 1 and agent 3, respectively; the remaining curve describes the state trajectory of agent 2. The horizontal axis and vertical axis describe time and state trajectories of all agents, respectively. In order to verify the effect of positive parameter $\gamma$, we present some simulations for Example 7 under different $\gamma$.

*Example 6.* Consider a multiagent system with 3 agents and the weighted adjacency matrix is

$$A = \begin{bmatrix} 0 & -0.5 & 0.5 \\ -0.5 & 0 & 0.5 \\ -0.5 & 0.5 & 0 \end{bmatrix}. \quad (22)$$

We suppose the disturbance is $w(k) = [\sin(k)\sin(k), \sin(k)]^T$; the positive parameter $\gamma = 0.001$ which is defined in Definition 3. Let the initial values be $x_1(0) = 1$, $x_2(0) = 2$, and $x_3(0) = 3$. The state trajectories of all agents are described in Figure 1. By simple matrix computation, we know that the model satisfies the condition in Theorem 5. It is easy to see that the system reaches consensus (three curves reach coincident) at about time 6 in Figure 1.

*Example 7.* Consider a multiagent system with 3 agents and the weighted adjacency matrix is

$$A = \begin{bmatrix} 0 & 0.8 & 0.2 \\ 0.65 & 0 & 0.35 \\ 0.9 & 0.1 & 0 \end{bmatrix}. \quad (23)$$

We suppose the disturbance is $w(k) = [\sin(k), \sin(k), \sin(k)]^T$; we provide some simulations with different $\gamma$. Let the initial state values be $x(0) = [1, 2, 3]^T$.

In Figure 2(a), the $H_\infty$ consensus for system (1) is under topology (2) and $\gamma = 0.001$; in particular, all the agents converge to a constant consensus state at about time 7. In Figure 2(b), the $H_\infty$ consensus for system (1) is under topology (2) and $\gamma = 0.005$; in particular, all the agents converge to a constant consensus state at about time 7. In Figure 3(a), the $H_\infty$ consensus for system (1) is under topology (2) and $\gamma = 0.02$; in particular, all the agents converge to a time-varying consensus state at about time 8. In Figure 3(b), the $H_\infty$ consensus for system (1) is under topology (2) and $\gamma = 0.08$; in particular, all the agents converge to a time-varying consensus state at about time 7. In Figure 4(a), the $H_\infty$ consensus for system (1) is under topology (2) and $\gamma = 0.1$; in particular,

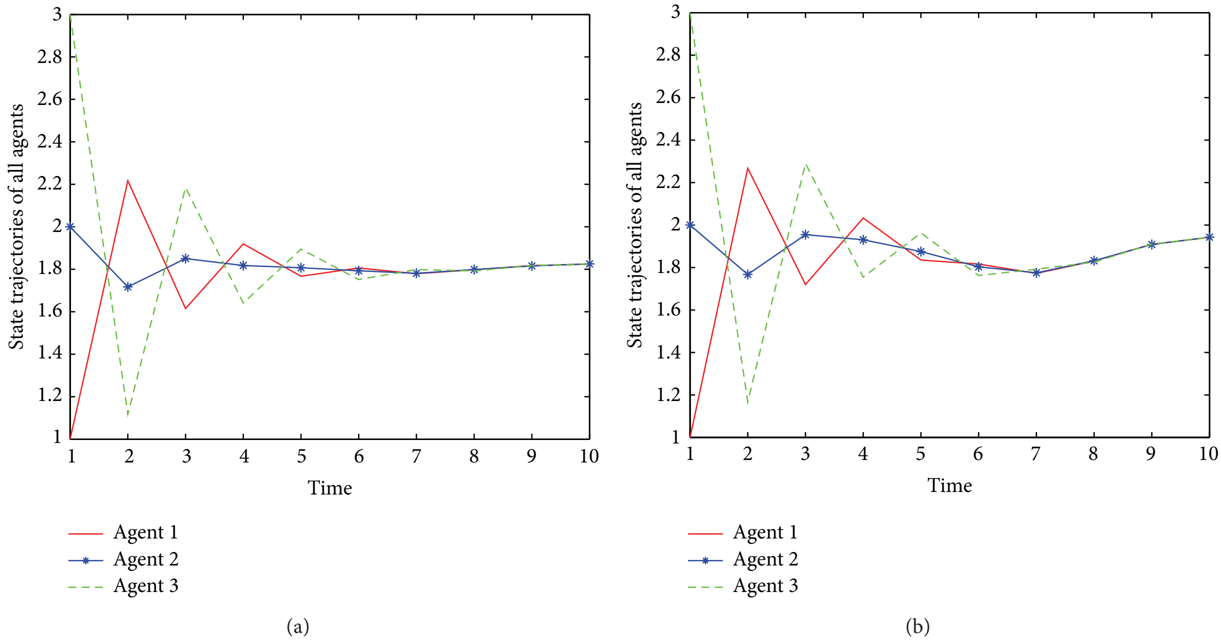


FIGURE 3: (a) State trajectories of all agents when $\gamma = 0.02$. (b) State trajectories of all agents when $\gamma = 0.08$.

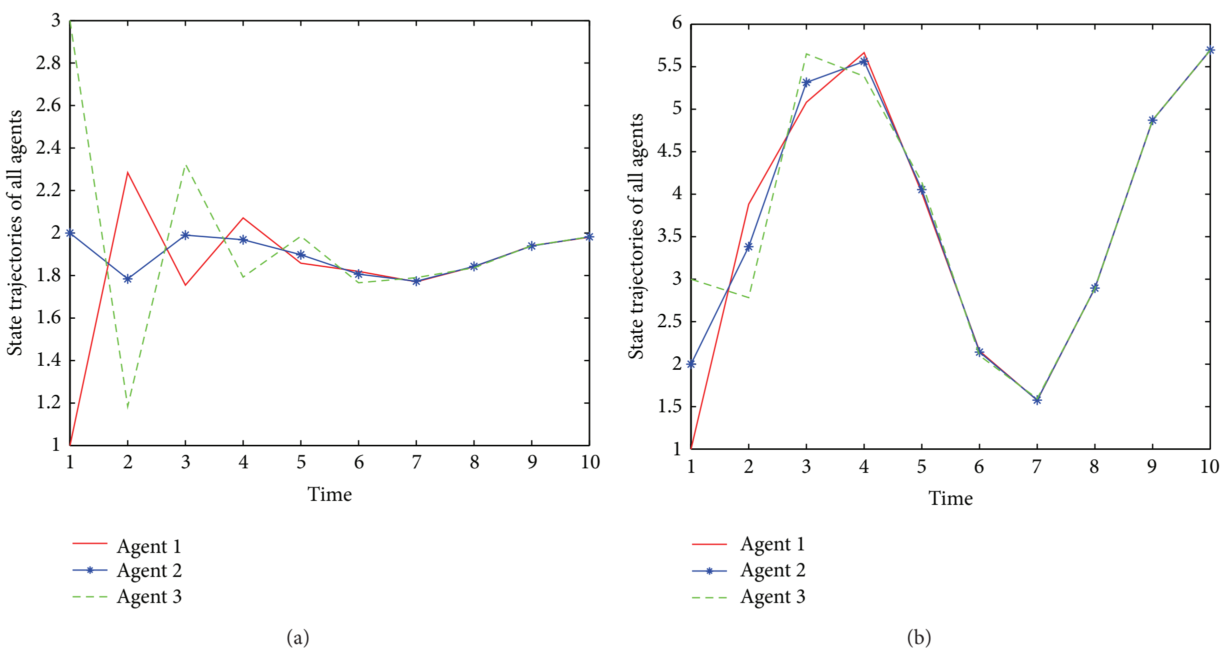


FIGURE 4: (a) State trajectories of all agents when $\gamma = 0.1$. (b) State trajectories of all agents when $\gamma = 2$.

all the agents converge to a time-varying consensus state at about time 7. In Figure 4(b), the $H_\infty$ consensus for system (1) is under topology (2) and $\gamma = 2$; in particular, all the agents converge to a time-varying consensus state at about time 4.5.

## 5. Conclusions

An $H_\infty$ consensus problem for discrete-time multiagent system with external disturbances is investigated. Based on system transformation as well as Lyapunov stability theory, the sufficient condition in form of LMIs is obtained to guarantee $H_\infty$ consensus of discrete-time multiagent with disturbances. A simulation result is finally provided to verify the effectiveness of our theoretical results.

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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