## From Subjective Probabilities to Decision Weights: The Effect of Asymmetric Loss Functions on the Evaluation of Uncertain Outcomes and Events

#### Elke U. Weber

Much of decision aiding uses a divide-and-conquer strategy to help people with risky decisions. Assessing the utility of outcomes and one's degree of belief in their likelihood are assumed to be separable tasks, the results of which can then be combined to determine the preferred alternative. Evidence from different areas of psychology now provides a growing consensus that this assumption is too simplistic. Observed dependencies in the evaluation of uncertain outcomes and the likelihood of the events giving rise to them are frequent and systematic. Dependencies seem to derive from general strategic processes that take into consideration asymmetric costs of over- vs. underestimates of uncertain quantities. This asymmetric-loss-function interpretation provides a psychological explanation for observed judgments and decisions under uncertainty and links them to other judgment tasks. The decision weights estimated when applying dependent-utility models to choices are not simply reflections of perceived subjective probability but a response to several constraints, all of which modify the weight of risky or uncertain outcomes.

Perhaps more than any other social science, psychology maintains an ongoing debate about its status as a coherent field of scholarship (cf. Fowler, 1990; Koch, 1969; Simon, 1992), often expressing genuine concern about the paucity of established and cumulative results and theories. Thus, note the emergence of a consensus on an important behavioral fact from different areas of psychology as well as economics. This article brings together some commonalities in results and in the mechanisms designed to explain them. These should be of theoretical as well as practical interest to anybody interested in human judgments and decisions. With this interpretative review, I attempt to present these often technical results and theories in an integrative and more accessible way. I argue that people's behavior in the judgment and decision situations discussed can be seen as responsive to self- or outwardly imposed constraints in their environment rather than as the result of perceptual or cognitive errors. In particular, I suggest that in situations in which an uncertain quantity needs to be assessed, for example, the probability with which some event will occur or the value of

mon psychological intuition behind the better descriptive fit of these models.

Evaluations of Outcomes and Probability

Are Not Independent

some object, people will be sensitive to the consequences of mis-

judging this quantity and that consequences are often asymmet-

ric for over- as opposed to underassessments. As a result, judg-

ments and choices that incorporate such considerations will of-

ten deviate from normative models, which ignore these

consequences of misjudgments to which people are sensitive. In

this article, I review more general theories in several different

domains that capture these deviations, and I point out the com-

Psychologists, philosophers, mathematicians, and economists have had a long-standing interest in the way people operate under conditions of uncertainty. Given that many events can be predicted only probabilistically, how does one interpret likelihood information? How does one choose between courses of action? Tasks as these require us to estimate the likelihood with which events will occur (be it relative frequency or degree of belief) as well as assess the utility of their outcomes to us. Until recently, formal models of decision making in such situations generally made the assumption that judgments of the utility of an outcome would not be influenced by the probability of the event that determined its occurrence and that perceptions of the likelihood of an outcome would not be influenced by its utility, partly because any dependency between the two evaluations appeared a priori irrational. Just as "a rose is a rose," .5 ought to be .5, and a slight chance ought to be a slight chance, regardless of whether it is a slight chance of catching a cold in February or a slight chance of obtaining a research grant. At a practical level, dependencies between probability and outcome evaluations are descriptively and prescriptively inconvenient

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Correspondence concerning this article should be addressed to Elke U. Weber, Center for Decision Research, Graduate School of Business, University of Chicago, 1101 East 58th Street, Chicago, Illinois 60637.

and contrary to the decomposition philosophy of decision analysis. Even though early experimental evidence suggested the existence of such dependencies (Edwards, 1962b; Halpern & Irwin, 1973; Irwin, 1953; Irwin & Snodgrass, 1966; Marks, 1951), theories of choice before the 1980s, whether descriptive or normative, usually assumed that the utility of risky or uncertain alternatives could be described by a combination of separately assessable outcome and probability or event transformations. Even without relinquishing this simplifying assumption (but at least partially because of it), it has been far from easy to obtain reliable assessments of utility and likelihood (e.g., Farquar, 1984; Hershey, Kunreuther, & Schoemaker, 1982).

Over the last decade, evidence from various areas of psychology and economics has again started to suggest that people's impressions of the subjective values of outcomes and events may not be independent, at least when there is some uncertainty about the value of choice alternatives or about the probability of obtaining different values. This time around, observed dependencies appear sufficiently systematic to be modeled. The dependencies seem to derive from general psychological processes, which provide links to other judgment tasks. In this review, I integrate research from the following areas: (a) interpretations of likelihood conveyed by verbal probability expressions, (b) configurality in social judgments, (c) dependent-utility models of decisions under uncertainty, and (d) developments in the measurement theory of preference.

#### Interpretations of Verbal Probabilities

Probability information often gets communicated verbally. A weatherman forecasts that "rain is likely." A doctor assures her or his patient that "there is a good chance that the prescribed medication will have no side-effects." Wallsten, Budescu, Rapoport, Zwick, and Forsyth (1986) and Mosteller and Youtz (1990) studied people's perceptions of the numerical probability implied by such verbal expressions. Each expression was found to be interpreted as expressing a range of plausible numerical probabilities, but to varying degrees. Thus, people perceive the word likely to imply probabilities anywhere between .5 and .99, with most interpretations around .8 and decreasing frequency of interpretation to both sides.

Verbal expressions have been mapped into numerical estimates for other quantities. For numerical equivalents of expressions of frequency and amount, Bass, Cascio, and O'Connor (1974) found good agreement between judges and no evidence of contextual effects. There is greater variability in people's numerical interpretations of probability expressions, however (Lichtenstein & Newman, 1967; also comments by Clark, Cliff, and Wallsten & Budescu, cited in Mosteller & Youtz, 1990). When they investigated context dependence, Wallsten, Fillenbaum, and Cox (1986) found people's numerical evaluations of probability words dependent on the base rate of the outcome that the word was qualifying. Thus, people interpreted the word likely as conveying a higher numerical probability when it described the probability of rain in London as opposed to rain in Cairo. The numerical equivalent of the perceived probability of an event was a weighted average of the probability implied by

the word *likely* per se and the base rate probability of the event (rain in London vs. rain in Cairo).

The base rate of occurrence may not be the only characteristic of an outcome that influences people's perceptions of its probability of occurrence. Bohner, Bless, Schwarz, and Strack (1988) found that the emotional valence of outcomes had an effect on the initiation of causal reasoning. In analogy, E. U. Weber and Hilton (1990) hypothesized and found that the disutility or negative valence of an outcome also affected people's interpretations of the probability with which the outcome was predicted to occur. Thus, slight chance was interpreted differently when referring to a slight chance of gastric disturbances as opposed to a slight chance of skin cancer, not only because these two outcomes had different a priori base rates, but also because skin cancer was a more severe outcome (with greater negative valence) than gastric disturbances. People's numerical interpretations of probability words depended on the base rate as well as on the severity of the outcome predicted to occur, with more severe outcomes leading to reports of higher numerical probability equivalents for a given probability phrase after controlling for the effects of base rates. Thus, most people gave slight chance of gastric disturbances a higher numerical interpretation than slight chance of skin cancer because of the greater base rate of gastric disturbances, which masked the opposite effect resulting from the severity difference. However, for people who considered their personal base rates of experiencing gastric disturbances and skin cancer to be the same, slight chance of skin cancer got a higher probability interpretation than did slight chance of gastric disturbances because of its greater severity. Cohen and Wallsten (1991) found that greater positive valence of outcomes also increased people's interpretations of associated probability words.

In this experimental task and similar situations, people provide their single best estimate of an uncertain quantity—in this case, a probability level. This leaves room for error in the estimation, both in the direction of overestimating the actual probability value and in the direction of underestimating it. Different consequences for errors that overestimate rather than underestimate the true value, with the difference in consequences depending on the valence of the associated outcomes, are sufficient to give rise to the observed probability-outcome evaluation dependencies. For example, the greater the disutility of an outcome (e.g., skin cancer), the more costly are errors that underestimate the outcome's probability level. Underestimates of the probability implied by *slight chance* could potentially result in death due to insufficient medical monitoring because of inadequate concern. Overestimates of the probability level, on the other hand, carry few costs (some unnecessary monitoring, perhaps) and will give rise to relief when the danger turns out to be smaller than expected. Such asymmetries in the consequences of over- versus underestimates of uncertain quantities are frequently referred to as asymmetric loss functions (e.g., Birnbaum, Coffey, Mellers, & Weiss, 1992).

At a process level, such loss functions may influence probability evaluation through mental simulation processes of the kind postulated in the ambiguity model by Einhorn and Hogarth (1985) and venture theory (Hogarth & Einhorn, 1990). If people evaluate ambiguous verbal-probability expressions by a

judgment heuristic described by Tversky and Kahneman (1974) as "anchoring and adjustment" (i.e., using their best estimate of the expression's per se numerical probability equivalent as an initial anchor and adjusting it upward and downward for the particular situation by mentally simulating possible alternative probability values), then it is plausible that the direction and extent of the simulation (and thus of the adjustment) will be affected by the predicted outcome's utility or disutility. Larger utilities of a positive outcome expected to occur with the uncertain probability may bias people to asymetrically simulate more probability values that exceed the anchor than values smaller than the anchor, out of joyful anticipation, hope, or greed for the positive outcome. Larger disutilities of negative outcomes may lead people to asymmetrically simulate more values greater than the anchor, out of fear of the negative consequences associated with underestimating the probability.

For both positive and negative outcomes, underestimates thus tend to carry greater costs than overestimates (albeit for different reasons) and thus drive mental simulation asymmetrically in the direction of probability values greater than the initial anchor: The more negative the outcome, the greater the cost of inadequate prevention or preparation, due to underestimation of the probability; the more positive the outcome, the greater the cost of forgoing the beneficial effect of overestimating its likelihood of occurrence and thus turning it into a self-fulfilling prophecy. There are, of course, situations in which the loss functions for over- versus underestimates are symmetric or differ in the direction opposite to the one just described. In those situations, our loss function sensitivity interpretation would predict no effects or effects opposite to the ones described above, making the theory empirically testable.

Interpreting the dependencies between the valence of outcomes and the evaluation of the uncertain probability level implicit in vague verbal probability expressions as the result of asymmetric loss functions for over- versus underestimates of the true probability level allows us to connect these results to another area of psychology, namely, social judgment theory. In social judgments, the uncertain quantity that has to be ascertained is not an uncertain probability level, but the value of an uncertain quantity such as the trustworthiness of a politician or the likableness of a new colleague. Even though the uncertain quantities are different, I argue that the same mechanisms are at work in both contexts, namely, asymmetric consequences (loss functions) associated with over- versus underestimates, which influence estimates in ways that minimize such losses.

#### Configurality in Social Judgments

Social judgments, such as deciding for which candidate to vote in an election or how much to pay for a used car, often require people to combine information from a variety of sources. Birnbaum and Stegner (1979) suggested three factors as relevant in the evaluation and use of such information. Two of them, namely, the expertise and the bias of the information source, are related to the credibility of this information. In deciding on a fair price for a used car, for example, one would give greater weight to the value estimate provided by a friend who is a car mechanic than by a friend who is an attorney, because of

the former's greater expertise. Estimates provided by a used-car salesman might be discounted, due to the source's bias. The third factor, and the one of interest to this discussion, relates to the perspective of the decision maker. Given that there is some ambiguity about the "true" fair price of a used car, particular estimates of fair price may be too high or too low. Depending on one's perspective, misestimates have different consequences (i.e., different loss functions). For the buyer of a car, misestimates that exceed the true price are costly, whereas underestimates are in his or her favor. For the seller of a car, the loss function for misestimates is the exact opposite: Underestimates are costly; overestimates are advantageous.

Birnbaum and Stegner (1979) found people to be sensitive to such subtle differences in consequences for the misestimation of price. Provided with several estimates of the value of a used car from sources that varied in expertise and bias, participants were asked to judge the "true" value of the car after being told that they were the agent of either the buyer or the seller. Consistent with their respective loss function for misestimations, those with a buyer's perspective gave greater weight to lower estimates of car prices. Those assigned a seller's perspective gave greater weight to higher estimates.

This effect of perspective on the judgment of "true" car price, TP, given two price estimates by sources x and y could be well described by a two-part model:

$$TP(x, y) = \alpha u(x) + (1 - \alpha)u(y) + \beta |u(x) - u(y)|,$$

$$0 \le \alpha \le 1. \quad (1)$$

The first two components of the model, common to both buyers and sellers, compute a weighted average of the two price estimates u(x) and u(y), with weights  $\alpha$  and  $(1 - \alpha)$  reflecting the respective credibility of the two sources. The third part redistributes some of the weight between the lower and the higher price estimate, consistent with the judge's loss function for overversus underestimates. Buyers' judgments were thus described by a negative configural parameter,  $\beta$ , which takes some weight away from the higher estimate, u(x) and transfers it to the lower estimate, u(y). This can be seen be rewriting Equation 1 for buyers, assuming that u(x) > u(y):

$$TP_{\text{Buyer}}(x, y) = \alpha u(x) + (1 - \alpha)u(y) - \beta[u(x) - u(y)]$$
$$= (\alpha - \beta)u(x) + (1 - \alpha + \beta)u(y). \tag{2}$$

To fit sellers' judgments, a positive configural parameter,  $\beta$ , is necessary, which takes some weight away from the lower estimate u(y) and transfers it to the higher estimate u(x):

$$TP_{Seller}(x, y) = \alpha u(x) + (1 - \alpha)u(y) + \beta [u(x) - u(y)]$$
$$= (\alpha + \beta)u(x) + (1 - \alpha - \beta)u(y). \tag{3}$$

A specific example will illustrate the basic idea of the model. Assume that you have obtained two price estimates for a vintage Volvo, the first, u(x) = \$2,300, from your regular car mechanic and the second, u(y) = \$1,800, from your uncle, who likes to fix his own cars. Because you have greater confidence in the expertise of your car mechanic, you give his estimate a weight of  $\alpha = .7$  and your uncle's estimate the remaining weight of  $1 - \alpha = .3$ . If you intend to buy the car, your concern about the

discrepancy in the two price estimates centers around the possibility that the first one might be too high and that you may end up paying more than the car is worth. On the other hand, if your goal is to sell the Volvo, your concern centers around the possibility that the second estimate is too low and that you may end up getting less than the car is worth. These different concerns might translate into a negative value of  $\beta = -.25$  if you were buying the car and a positive value of  $\beta = .25$  if you were selling the car. Thus, using Equation 2, your best estimate of the true price of the Volvo if you are a buyer would be

$$TP_{Buyer} = .7(\$2,300) + .3(\$1,800) - .25(\$2,300 - \$1,800)$$
$$= \$2,150 - \$125 = \$2,025.$$

Using Equation 3, your best estimate of the true price if you are selling the car would be

$$TP_{Seller} = .7(\$2,300) + .3(\$1,800) + .25(\$2,300 - \$1,800)$$
$$= \$2.150 + \$125 = \$2.275.$$

The  $\beta$  parameter of the model is called *configural* because it reassigns weight to price estimates not on the basis of characteristics of their sources or their absolute magnitude, but solely on the basis of their relative rank in the configuration of other price estimates.

Birnbaum and Stegner's (1979) explanation of perspective effects in social judgment in terms of different loss functions for misestimates is prescient of a similar distinction about perspective made by Lopes (1987) in the context of risky choice. Arguing that many risky decisions consist of finding some point of comfort between hope for good outcomes and fear of bad outcomes, Lopes (1987) presents evidence that people have different comfort levels along the fear-hope continuum, with individuals having a stronger need either for security (driven by fear) or for potential (driven by hope). Individuals with a security perspective are assumed to place greater weight on the low outcomes in the distribution of possible outcomes for a given choice alternative because those outcomes are guaranteed to obtain (i.e., they are guaranteed to get the lowest possible outcome or something better).

If each outcome of a lottery is seen as an estimate of the "true" value of the lottery that will be revealed after the lottery has been played, then an analogy can made between perspective effects in riskless social judgments and perspective effects in risky choice. Just as the "true" price estimated for a car, the value assessment of a lottery may depend on the reliability of the individual estimates (i.e., the stated probabilities of the possible outcomes) as well as on the (possibly asymmetric) consequences of over- versus underassessing the final "true" outcome (e.g., as a function of perspective). In keeping with a loss function explanation, individuals with a security perspective would thus be choosing in accordance with a loss function that is more sensitive to the disappointment of having overiudged the actual outcome of the lottery than to the pleasant surprise of having underjudged it, with the result that more weight is placed on outcomes at the low end of the distribution. Choices of individuals with a potential perspective, on the other hand, are consistent with the assumption that they are more sensitive to the positive consequences of underjudging the final value of the lottery, with the result that greater weight is placed on the high outcomes in the distribution of possible outcomes.

In the next section, I introduce some mathematical formalisms developed by theorists in economics and psychology, which capture these psychological intuitions about judgment and choice under uncertainty. The models presented have in common that they all introduce some dependency into the evaluation of events and outcomes, either as the result of top-down attempts to solve conceptual problems with less complex theories or as the result of bottom-up attempts to fit data. The models differ in the nature of the assumed dependence and in the domain of decision situations for which they were developed. The list is not exhaustive, but it exemplifies the spirit of these models.

## Nonexpected-Utility Models of Choice Under Risk and Uncertainty

In the decision literature, the failure of traditional variants of expected utility (EU) theory to describe a wide range of phenomena observed in people's choices between risky or uncertain alternatives is well documented (see, e.g., Birnbaum, 1992; Lopes, 1990; Schoemaker, 1982). A variety of nonexpectedutility models have tried to accommodate these deviations in behavior from expected-utility theory by replacing the model with one that is no longer a linear function of objective probabilities (for reviews see, e.g., Machina, 1987; M. Weber & Camerer, 1987).

Some of these models have the drawback of predicting violations of the principle of stochastic dominance: They predict that people will not choose an alternative that is as good or better in all outcomes than another alternative. Such counterintuitive predictions occur when the nonlinear transformations of objective probabilities into subjective decision weights are not sufficiently constrained. In the models of Edwards (1954, 1962a) and Kamarkar (1978), or in Kahneman and Tversky's (1979) prospect theory, the utility of an alternative  $X = (p_1, x_1; ...;$  $p_n, x_n$ ), where outcome  $x_1$  occurs with probability  $p_1$  and so on, is defined as  $U(X) = \sum_i \pi[(p_i)u(x_i)]^{1}$  The decision weights  $\pi(p_i)$ that are a function of the objective probabilities are not required to sum to 1. Such models predict that in some situations, people will prefer a choice alternative that is dominated by another alternative (i.e., they will prefer the alternative that is worse or not any better on all possible outcomes). Without the prior editing operation that eliminates dominated alternatives, prospect theory predicts, for example, that people will assign a lower utility to lottery A, which provides a .4 chance at getting \$210, a .4 chance at \$200, and a .2 chance at \$0, than to the dominated lottery B, which provides a .8 chance at \$200 and a .2 chance at \$0, even though lottery A is equally good or better than lottery B on all dimensions. Lottery B was derived from lottery A by reducing the value of A's first outcome from \$210

<sup>&</sup>lt;sup>1</sup> The 1979 version of prospect theory avoids dominance violations by postulating a special editing operation that detects and climinates dominated alternatives.

to \$200, (i.e., to the value of the second outcome). As a result, the probabilities of the two outcomes (.4 chance each at \$210 and \$200) are now considered together (.8 chance at \$200). If the nonlinear weighting of the sum of those two probabilities, that is,  $\pi(p_1 + p_2) = \pi(.8)$ , is greater than the sum of the weights of the probabilities considered separately, that is,  $\pi(p_1) + \pi(p_2)$  $=\pi(.4)+\pi(.4)$ , and if this increase in  $\pi$  outweighs the decrease in outcome utility, then theories like prospect theory will predict violations of stochastic dominance. For our example, prospect theory's value function postulates that  $\pi(.4) = .27$  and that  $\pi(.8) = .65$ . Assuming, for simplicity and without any loss of generality, a linear-utility function: The utility of lottery A would be valued at .27(\$210) + .27(\$200) = \$110.70, which is lower than the utility of dominated lottery B valued at .65(\$200) = \$130, thus predicting that people would choose B over A, a violation of stochastic dominance. Because stochastic dominance seems to hold empirically or is violated only in very special cases (Lopes, 1984; Mellers, Weiss, & Birnbaum, 1992; Tversky & Kahneman, 1986) and because it is useful for economic and other applications by allowing for the comparison of risky decision alternatives in terms of their cumulative distribution functions (Levy, 1992), models that predict violations of the principle are at a disadvantage.

Fortunately, there is a class of nonexpected-utility models for which stochastic dominance violations do not arise. These models create their nonlinearity in decision weights in a more constrained way, by introducing some dependence between the evaluation of probabilities and characteristics of the outcomes. The rank-dependent utility (RDU) models by Quiggin (1982) and Yaari (1987), for example, achieve the nonlinearity in decision weights necessary to account for people's deviations from EU theory by a nonlinear, nondecreasing transformation w that operates not on individual probabilities, but on the cumulative distribution of outcomes, as shown in Equation 4. In their model, the utility of an alternative  $X = (p_1, x_1; \ldots p_n, x_n)$ , with outcomes ordered in increasing order of preference  $[u(x_1) < \ldots < u(x_n)]$ , is defined as

$$RDU(X) = \sum_{i} \pi(p_{i}, X) u(x_{i}), \tag{4}$$

where 
$$\pi(p_i, X) = w(p_i + \dots + p_n) - w(p_{i+1} + \dots + p_n)$$
.

The decision weight  $\pi(p_i, X)$  now is a difference between two expressions that no longer depend only on  $p_i$ , but also depend on the rank of outcome  $x_i$  in relation to other outcomes and thus on the whole distribution of outcomes, X: The first expression is the sum over the probabilities of all outcomes that are at least as great as  $x_i$ ; the second expression is the sum over the probabilities of all outcomes that are greater than  $x_i$ . The dependence on the rank of  $x_i$  comes about because different probability values enter into the two summations, depending on the rank of  $x_i$ . For a linear w function, this does not matter, and the RDU model reduces to the EU model. For nonlinear w functions, however, the decision weight  $\pi(p_i, X)$  given to probability  $p_i$  depends critically on the rank of the associated outcome  $x_i$ . An example will follow shortly to illustrate this dependence of the decision weight on the probability as well as on the rank of the outcome

Just as risk averse and risk seeking have been used as descrip-

tive labels to characterize the shape of nonlinear utility functions in EU theory, some RDU theorists (e.g., Quiggin, 1982) have used the labels *pessimistic* and *optimistic* to characterize the nonlinearity of the probability weighting function w. The pessimistic w function shown in Figure 1 gives greater weight to lower outcomes (i.e., to outcomes with lower ranks); the optimistic w function gives greater weight to larger outcomes (i.e., to outcomes with higher ranks).<sup>4</sup>

This is seen the easiest by way of an example. Consider alternative  $X = (.2, x_1; .2, x_2; .6, x_3)$  where  $u(x_1) < u(x_2) < u(x_3)$ . According to Equation 4, the rank-dependent utility of this alternative is

$$= [w(p_1 + p_2 + p_3) - w(p_2 + p_3)]u(x_1)$$

$$+ [w(p_2 + p_3) - w(p_3)]u(x_2) + w(p_3)u(x_3)$$

$$= [w(1) - w(.8)]u(x_1) + [w(.8) - w(.6)]u(x_2) + w(.6)u(x_3).$$

(5)

When I substitute the appropriate w(p) values read off the w functions in Figure 1, Equation 5 reduces to

$$[1 - .8]u(x_1) + [.8 - .6]u(x_2) + .6u(x_3)$$
  
= .2u(x\_1) + .2u(x\_2) + .6u(x\_3) = EU(X)

for the linear w function,

$$[1 - .62]u(x_1) + [.62 - .36]u(x_2) + .36u(x_3)$$
$$= .38u(x_1) + .26u(x_2) + .36u(x_3)$$

for the pessimistic w function, and

$$[1 - .9]u(x_1) + [.9 - .78]u(x_2) + .78u(x_3)$$
  
= .1u(x\_1) + .12u(x\_2) + .78u(x\_3)

for the optimistic w function.

The pessimistic weighting function takes away a portion of the objective probability weight of the highest outcome,  $x_3$ , (.24 out of .6) and transfers most of it (.18) to the lowest outcome,  $x_1$ , and some of it (.06) to the second lowest outcome,  $x_2$ . The optimistic weighting function takes some weight from the lowest

<sup>&</sup>lt;sup>2</sup> Other related models are summarized in Wakker (1989). The theories by Gilboa (1987) and Schmeidler (1989), for example, provide a generalization of the representation in Equation 4 from risky alternatives with well-specified probability levels to uncertain alternatives where outcomes are the consequences of events with nonspecified, ambiguous, probabilities.

<sup>&</sup>lt;sup>3</sup> This avoids violations of stochastic dominance, because the cumulative distribution function (and hence its transformation) of a dominating alternative is always greater than or equal to that of the dominated alternative.

<sup>&</sup>lt;sup>4</sup> While these labels are appropriate descriptions of the effect of these functions on the relative weighting of outcomes that are gains, they become inaccurate when describing the effect of these functions when they are reflected, as in cumulative prospect theory (Tversky & Kahneman, 1992), to apply to losses. See discussion below.

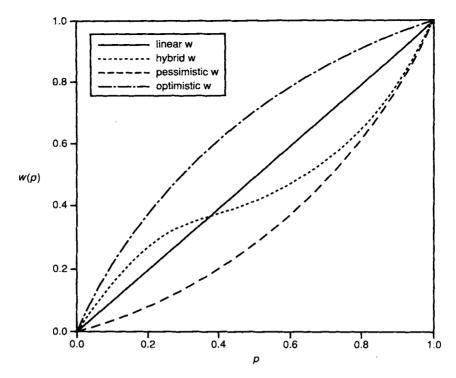


Figure 1. Optimistic, pessimistic, hybrid, and linear probability weighting functions w used in the mapping of cumulative probabilities  $\sigma_i$  into rank-dependent decision weights  $\pi(p_i, X)$ .

and middle outcome (.1 and .08, respectively) and transfers it to the highest outcome. *Pessimism* thus describes choices that indicate that lower ranked outcomes are given greater importance and thus greater weight, almost verbatim the definition of Lopes's (1987) "security" perspective. Equivalently, the transfer of decision weight from lower to higher outcomes by an optimistic w function provides a formalism for her definition of individuals with a "potential" perspective.

Just as in the model of social judgment integration, where the configural weighting parameter  $\beta$  transferred weight from less important to more important pieces of information (Equation 1), the example shows that the w function in rank-dependent utility evaluation transfers weight from less important to more important outcomes, where in both cases importance is defined in terms of the relative ranks of outcomes. The terms configural weighting and rank-dependent weighting thus refer to the same phenomenon.

## Empirical Evidence for Rank Dependence

Throughout the 1980s, Lopes (1984, 1987, 1990) provided solid empirical evidence for the fact that individuals differ in the relative emphasis they put on the security level versus the potential of risky alternatives and that such differences in perspective affect their choices. The original formalization of the security-potential (SP) aspect of Lopes's SP/A theory (A = aspirations) in terms of visual characteristics of Lorenz curves (i.e., cumulative probability and value functions adopted from welfare economics) was perhaps not as tractable and easy to use

as the more recent formalization (Lopes, 1990) in terms of the rank-dependent Quiggin-Yaari function of Equation 4. More important than the formalization, however, is the fact that the theory provides a psychological mechanism and justification for the existence of rank dependence and attempts to explain it as a function of individual and situational variables.

In a parallel to the perspective effects in the riskless social judgment situation described earlier, that is, in the task of deciding on the true price of a used car as either a buyer or a seller, Birnbaum, Coffey, Mellers, and Weiss (1992) found support for effects of perspective in risky choice. Whereas Lopes (1987) assumed that a person's relative emphasis on the high versus the low end of the distribution of outcomes was primarily a stable individual-difference characteristic, Birnbaum et al. used a situation in which differential configural weighting of outcomes was induced by the task. People were asked to judge the value of monetary lotteries either from a buyer's, a seller's, or a neutral perspective. Resulting differences in judgments were best explained by strategic differences in the configural weighting of lower versus higher ranked outcomes as a function of the assigned perspective with its associated loss function for misvaluations. Buyers of a lottery tended to assign more weight to the lower-ranked outcomes when setting a price, in keeping with the interpretation that for them the consequences of overvaluing the lottery (i.e., to lose money by receiving an outcome lower than the purchase price) hurt more than the consequences of undervaluing it (i.e., to forgo money by not purchasing a profitable lottery). Sellers, on the other hand, tended to assign

more weight to the higher ranked outcomes when setting a price, because for them the consequences of undervaluing the lottery (i.e., to lose money by selling a profitable lottery too cheaply) hurt more than the consequences of overvaluing it (i.e., to forgo money by not selling a not-so-profitable lottery).

Another model developed as a descriptive account of empirical data is prospect theory (Kahneman & Tversky, 1979), which has been very influential by virtue of accounting for a large number of observed deviations from EU theory with a small set of additional assumptions. One, derived from psychophysics, assumes that people tend to encode events in a relative rather than an absolute way and thus will encode outcomes as deviations (gains or losses) from a readily available reference point (e.g., the status quo) rather than in terms of final wealth. The utility of outcomes is thus described by a value function that is defined over losses and gains. Another assumption, derived from observing behavior such as people's reluctance to engage in coin tosses for symmetric gains and losses, assumes that people evaluate losses differently from gains, expressed by a value function that is different and steeper for losses than for gains. The third assumption, also empirically derived, assumes that people do not treat certainty (0 or 1) as the endpoints of the probability continuum but give it special status. As a result, they will divide choice alternatives into outcomes that will obtain for sure and those that are merely probable, and they give greater weight to certain outcomes (modeled by a weighting function  $\pi(p)$  with discontinuities at the two endpoints). This last assumption gives rise to rank dependence for two-outcome choice alternatives with outcomes that are both on the same side of the reference point.

The recent update of prospect theory (Tversky & Kahneman, 1992) eliminates two limitations of the original version, namely, its restriction to choice alternatives with at most two nonzero outcomes and its insufficient constraints on the decisionweighting function as discussed above. Cumulative prospect theory has kept its original value function (concave for gains; convex and steeper for losses) but replaced its decision-weighting function with the Quiggin-Yaari rank-dependent transformation of cumulative probabilities (Equation 4). Expanding the original insight that people treat losses and gains differently, the new prospect theory assumes that people divide a choice alternative (X) into a loss portion ( $X^-$ ) and a gain portion ( $X^+$ ), evaluate each portion separately in the rank-dependent fashion of Equation 4, and subsequently combine the two. This allows for the nature of the rank dependence (i.e., the degree of optimism or pessimism) to differ for losses and gains, making the decision weights not just rank but also sign dependent. In notation similar to the models above, the rank- and sign-dependent utility (RSDU) of alternative  $X = (p_{-m}, x_{-m}; \dots p_{-1}, x_{-1}; p_0,$  $x_0; p_1, x_1; \ldots; p_n, x_n$ ), with  $u(x_{-m}) < \ldots < u(x_n)$ , is defined by cumulative prospect theory as

RSDU(X) = RSDU(X<sup>-</sup>) + RSDU(X<sup>+</sup>), (6)  
where RSDU(X<sup>+</sup>) = 
$$\sum_{i} \pi^{+}(p_{i}, X)u(x_{i})$$
 and  
 $\pi^{+}(p_{i}, X) = w^{+}(p_{i} + \dots + p_{n}) - w^{+}(p_{i+1} + \dots + p_{n}),$   
 $0 \le i \le n - 1,$ 

and 
$$\pi^+(p_n, X) = w^+(p_n),$$
 (7)

and where RSDU( $X^-$ ) =  $\Sigma_i \pi^-(p_i, X) u(x_i)$  and

$$\pi^{-}(p_{i}, X) = w^{-}(p_{-m} + \dots + p_{i}) - w^{-}(p_{-m} + \dots + p_{i-1}),$$

$$1 - m \le i \le 0, \text{ and } \pi^{-}(p_{-m}, X) = w^{-}(p_{-m}).$$
 (8)

Tversky and Kahneman (1992) estimated  $w^+$  and  $w^-$  functions from individual- and group-choice data and obtained functions that both were similar to the hybrid w function shown in Figure 1 with a pessimistic upper portion and an optimistic lower portion. Given that the summations over outcomes for the  $\pi^+$  and  $\pi^-$  decision weights shown in Equations 7 and 8 are mirror images of each other (i.e., starting with low-ranked outcomes and going up for positive outcomes and starting with high-ranked outcomes and going down for negative outcomes), similar-shaped w+ and w- functions imply that whatever weighting occurs in the gain domain will be reflected into its mirror image in the loss domain. The pessimistic upper portion of the estimated  $w^+$  and  $w^-$  functions thus reflects overweighting of lower ranked positive outcomes and higher ranked negative outcomes (i.e., an overweighting of outcomes close to the status quo). The optimistic lower portion of the  $w^+$  and  $w^-$  functions in turn reflects overweighting of high-ranked positive outcomes and low-ranked negative outcomes (i.e., an overweighting of extreme outcomes).

The labels optimism and pessimism suggested by Quiggin (1982) thus no longer describe the effect of these functions on the weighting of negative outcomes when the cumulation of outcomes is reflected for gains and losses, as described in Equations 7 and 8. A psychological interpretation of optimism as the belief that good things tend to happen to one would predict that optimistic individuals would put greater weight on high-ranked negative outcomes when confronted with a range of possible negative outcomes (i.e., they would put greater weight on the smaller losses). Conversely, pessimistic individuals would put greater weight on low-ranked negative outcomes (i.e., on the larger losses), in line with their expectations that bad things tend to happen to them. Extremity weighting is a more accurate label to describe the effects of the weighting function labeled optimistic in Figure 1 on both gains and losses, because its effect is a shift of weight toward more extreme outcomes (i.e., to the greater wins and the larger losses). Status quo weighting is, conversely, a more accurate description of the effects of the function labeled pessimistic in Figure 1 on both gains and losses, because its effect is a shift of weight toward outcomes close to the status quo (i.e., to the small wins and the small losses).

Another example will help to illustrate the particular differential weighting of outcomes captured by the  $w^+$  and  $w^-$  functions, which were estimated empirically by cumulative prospect theory, and the reflection of this weighting as one goes from gains to losses. Take two lotteries, one with five positive outcomes that each occur with probability .2 and one with five negative outcomes that also each occur with probability .2:  $X^+ = (.2, \$10; .2, \$20; .2, \$30, .2, \$40; .2, \$50)$  and  $X^- = (.2, -\$50; .2, -\$40; .2, -\$30, .2, -\$20; .2, -\$10)$ . Using Equation 7 to evaluate  $X^+$  and Equation 8 to evaluate  $X^-$  and the hybrid  $W^+$  function of Figure 1 to read off the necessary values for both  $W^+$  and  $W^-$ , we obtain

 $RSDU(X^+)$ 

$$= [w^{+}(1) - w^{+}(.8)]u(\$10) + [w^{+}(.8) - w^{+}(.6)]u(\$20)$$

$$+ [w^{+}(.6) - w^{+}(.4)]u(\$30) + [w^{+}(.4) - w^{+}(.2)]u(\$40)$$

$$+ [w^{+}(.2)]u(\$50)$$

$$= (1 - .65)u(\$10) + (.65 - .47)u(\$20) + (.47 - .37)u(\$30)$$

$$+ (.37 - .27)u(\$40) + (.27)u(\$50)$$

$$= (.35)u(\$10) + (.18)u(\$20) + (.10)u(\$30) + (.10)u(\$40)$$

$$+ (.27)u(\$50),$$

and

#### $RSDU(X^{-})$

$$= [w^{-}(.2)]u(-\$50) + [w^{-}(.4) - w^{-}(.2)]u(-\$40)$$

$$+ [w^{-}(.6) - w^{-}(.4)]u(-\$30) + [w^{-}(.8) - w^{-}(.6)]u(-\$20)$$

$$+ [w^{-}(1) - w^{-}(.8)]u(-\$10)$$

$$= (.27)u(-\$50) + (.10)u(-\$40) + (.10)u(-\$30)$$

$$+ (.18)u(-\$20) + (.35)u(-\$10).$$

The example shows numerically the effects observed by Tversky and Kahneman (1992) and captured by their estimated  $w^{+}$ and  $w^-$  functions: The weighting of positive outcomes is the mirror image of the weighting of negative outcomes, and outcomes close to the zero point and outcomes at the extremes of the distribution of positive and negative outcomes are overweighted in relation to outcomes in the middle of the distribution. If this configural redistribution of weight is the result of sensitivity to the consequences of misjudging the final outcome of the lottery, then the loss functions for misassessments of value must have the following characteristics: (a) The consequences of having underjudged the final lottery value cannot be symmetric to the consequences of having overjudged it, and (b) this asymmetry must differ for lotteries with positive and negative outcomes. For positive-outcome lotteries, the function reflecting the consequences of overvaluing the lottery must have a steeper slope than the function reflecting the consequences of undervaluing it. For negativeoutcome lotteries, the opposite must be true: The slope of the function reflecting the consequences of undervaluing the lottery must be steeper than that of the function reflecting the consequences of overvaluation. It is not implausible that the nature of internal (psychological) and external reactions to over-versus underassessments of value is different for lotteries involving gains than for lotteries involving losses. Disappointment about final outcomes lower than expected may quite well be the dominant emotion in the domain of gains, and relief about outcomes that are less negative than expected may quite well be the dominant emotion in the domain of losses.

# Developments in the Measurement Theory of Preference

The final piece of converging evidence for the feasibility of modeling interdependencies in the evaluation of events and outcomes comes from measurement theory. Economists and other practitioners who use models of people's choice behavior as building blocks in theories of more complex phenomena are often hesitant to replace EU theory with descriptively more accurate models about people's behavior, at least partly because the latter have often lacked precise mathematical formulation or desirable scaling properties.

Addressing concerns about the measurement and scaling properties of generalized utility theories, Luce and Narens (1985) investigated how general the representation of utility could be and still retain the desirable property of interval scalability. Their original answer to that question, called the dualbilinear model (1985) and later relabeled RDU (Luce, 1992), was developed for uncertain choice alternatives (i.e., for events with unspecified and thus ambiguous probabilities) for the restricted case of two-outcome lotteries. The result that was important for this article was that the most general interval-scalable utility model allowed for lawful dependencies in the evaluation of events and outcomes. In particular, it allowed for the evaluation of the events by which two outcomes occurred to depend on the preference ranks of the outcomes. Thus, the rank-dependent utility of alternative X, where outcome x occurs when event A happens and outcome y occurs otherwise, depends on whether x is the preferred or less preferred of the two outcomes. One way of expressing this dependence of the evaluation on the preference ranks shows the formal equivalence of this model to the configural weight model of Birnbaum and Stegner (1979) of Equation 1:

$$RDU(X) = s(A)u(x) + [1 - s(A)]u(y) + r(A)|u(x) - u(y)|.$$
 (9)

Equation 9 can be rewritten separately for the case in which  $u(x) \le u(y)$  and the case in which  $u(x) \ge u(y)$ , to show its formal equivalence to Equations 2 and 3:

$$RDU(X) = [s(A) - r(A)]u(x) + [1 - s(A) + r(A)]u(y),$$
if  $u(x) \le u(y)$ ;
$$[s(A) + r(A)]u(x) + [1 - s(A) - r(A)]u(y),$$
if  $u(x) \ge u(y)$ . (10)

In a comparison between Equation 1 and Equation 9, s(A) corresponds to the nonconfigural weight  $\alpha$  and r(A) corresponds to the configural weight  $\beta$ . The total decision weight given to event A depends partly on the event itself (i.e., s(A)) and partly on the preference ranks of the associated outcome: If  $u(x) \le u(y)$ , r(A) will be transferred from the weight given to u(x) to the weight given to u(y); if  $u(x) \ge u(y)$ , the opposite happens.

The two-outcome rank-dependent model was generalized by Luce (1988, 1991) and Luce and Fishburn (1991) to a multioutcome rank- and sign-dependent linear utility model, resulting in the representation also proposed by Tversky and Kahneman (1992) and shown in Equations 6 to 8. Although the representations of Luce and Fishburn's (1991) RSDU model and Tversky and Kahneman's (1992) cumulative prospect theory are identical for uncertain prospects <sup>5</sup> and have attractive scal-

<sup>&</sup>lt;sup>5</sup> Identical, that is, after replacing the summation of numerical probabilities in Equations 7 and 8 by the union of the events giving rise to the outcomes of the uncertain-choice alternative (with outcomes again ordered in increasing order of preference), because the Luce and Fishburn model was formulated for the more general class of choice alternatives

Table 1 Classification of Selected Nonexpected-Utility Theories by Domain of Application and Assumed Dependence Between the Evaluation of Events and Outcomes

Nature of events	Type of dependence			
	Rank dependent		Rank and sign dependent	
	Two outcomes	Multiple outcomes	Two outcomes	Multiple outcomes
Risky		Quiggin (1982) Yaari (1987) Lopes (1990)	Kahneman & Tversky (1979)	Tversky & Kahneman (1992)
Uncertain	Birnbaum & Stegner (1979) Luce & Narens (1985)	Luce (1988) Gilboa (1987) Schmeidler (1989)	Luce (1991)	Luce & Fishburn (1991) Tversky & Kahneman (1992)

ing properties, 6 they have been axiomatized in different ways (i.e., they derive from a different set of behavioral assumptions; see Luce, 1990; Wakker & Tversky, 1991). Perhaps more important, cumulative prospect theory was developed to describe empirical data, and attempts to validate it have concentrated on fitting the representation to observed choice data (Tversky & Kahneman, 1992; for an exception see Wakker, Erev, & Weber, in press). The Luce and Fishburn (1991) RSDU model, on the other hand, was developed as an axiomatic theory, and attempts to validate it have concentrated on the validity of the underlying behavioral assumptions (Brothers, 1990). The convergence in the representation of the two theories, thus, is a noteworthy result.

## Discussion

## Configural Weighting

To provide an organization that relates the different dependent-utility theories mentioned in this article to each other, I summarized them in Table 1 by the nature of the hypothesized dependence between outcome and event evaluation (rank dependence vs. rank and sign dependence) and by the level of generality of the model (events with specified vs. unspecified probabilities; two-outcome vs. multiple-outcome alternatives). Models that apply to uncertain-choice alternatives (i.e., to alternatives whose outcomes depend on events of unspecified probability) are, of course, more general than those that apply only to risky-choice alternatives (i.e., to alternatives in which the probabilities of outcomes are well specified) and include the latter as special cases. What the models listed in Table 1 have in common is that the weighting of the likelihood information about an outcome is configural, that is, it depends not solely on the event or probability level itself but also on the relative rank of the associated outcome in the configuration of other possible outcomes.

I argued in this article that such configural weighting could

be conceptualized as strategic behavior that responded to asymmetric loss functions for over- versus underassessments of the final value of the gamble and that such asymmetric-loss-function arguments can explain not only choices made on the basis of a dependent-utility evaluation but also judgments about ambiguous verbal probabilities and other ambiguous quantities.

For decisions under ignorance (i.e., decisions in the absence of any information about the probability levels of different outcomes), Hurwicz (1951) and Arrow and Hurwicz (1972) proposed a model that could be interpreted as a special case of a configural weight model. In their model, the worst and the best possible outcome of an alternative is weighted proportionately to the decision maker's position on a pessimism—optimism continuum, leading to a maximin choice (i.e., focusing exclusively on the worst possible outcome) in the most pessimistic case and to a maximax choice (i.e., focusing exclusively on the best possible outcome) in the most optimistic case.

#### Alternative Explanations and Boundary Conditions

Camerer (1989) suggested that violations of EU theory must have relatively simple—perhaps perceptual—causes, because animals exhibit the same violations that people do (Battalio, Kagel, & MacDonald, 1985). The evidence reviewed in this article suggests that such simple and general processes seem to exist. However, at least for the phenomena addressed in this article, these processes need not necessarily be perceptual in origin. Instead, in this article, I argued that configural or rank-dependent weighting could be interpreted as strategic or motivational (i.e., a reasonable response that takes into consideration existing constraints that are ignored by the EU model). Judgments and decisions under risk and uncertainty have consequences that can be described by loss functions that are often asymmetric for over- versus underestimates. Asymmetries in consequences and in the implied loss functions for different

in which the probabilities of the events that determine the outcomes are not necessarily specified (see Table 1).

<sup>&</sup>lt;sup>6</sup> The addition of sign dependence and a utility-neutral reference point results in a ratio-scale representation.

<sup>&</sup>lt;sup>7</sup> EU theory implicitly assumes symmetric loss functions (see Birnbaum et al., 1992).

judgments and decisions can either be external and situational (e.g., being in a buyer's or a seller's position when judging the fair price of an item of uncertain value) or can be more permanent characteristics of the way an individual evaluates his or her performance. Processes that would allow people to incorporate such considerations include asymmetric mental simulation (in the case of providing a numerical equivalence for an event described by a verbal probability expression or other events with ill-specified probability levels) or the configural redistribution of weights from potentially less damaging to potentially more damaging outcomes in the case of risky choice.

However, alternative explanations for some of these phenomena exist. Configural weighting, for example, may well have perceptual rather than motivational roots.8 People may give greater weight to those outcomes that are more salient. Thus, sure outcomes may get greater weight than outcomes that occur only probabilistically, and more extreme outcomes may get greater weight than outcomes in the middle of the distribution, simply because they are more noticeable. A loss-function interpretation is, of course, not opposed to an interpretation in terms of attentional salience. Negative consequences can be very potent in attracting attention. What is at issue is the reason for the distribution of attentional focus (and thus decision weight) over different outcomes. If attention is attracted by certain outcomes simply because of their surface characteristics (e.g., being extreme or occurring for sure), then people's choices or judgments should not be affected when, in analogy to a signal-detection experiment, we manipulate the consequences of making different types of misjudgments (i.e., the payoff function or loss function for over- vs. underjudging the final outcome of a lottery). Instead, choices or judgments should be affected by instructions or information displays that redirect attention without changing the consequences of people's answers. If, on the other hand, attention is directed by potential consequences (i.e., the asymmetric loss functions postulated in this article), then perceptual manipulations of attention should show no effect. As with many alternative explanations in psychology, chances are that both perceptual and motivational factors affect the distribution of attention. However, the framework outlined in this article suggests clear ways of distinguishing between the relative contributions of these different mechanisms.

This article tried to provide a consistent psychological rationale for observed dependencies in the evaluation of events and outcomes in a variety of domains and to show that such dependencies (whatever their origin) are sufficiently systematic to model them in simple, mathematically tractable, ways. However, this does not mean that people's evaluations of likelihood and outcomes will always be dependent. The interpretation offered in this article suggests that dependencies can be expected to occur when two conditions are satisfied: (a) There has to be uncertainty about at least one of two quantities (probability of outcomes or value of the alternative) and (b) the consequences for over- versus underjudging that quantity must be asymmetric. In risky choice, there is, by definition, uncertainty about the final outcome of the choice alternatives. In choice under uncertainty, there is uncertainty about the final outcome of the decision as well as about the probability of the events that give rise to the different possible outcomes. However, in both

choice under risk and under uncertainty, apparent dependencies between event and outcome evaluation will only occur when the second condition is also satisfied: when there is an asymmetry in consequences for over- as opposed to underestimating the uncertain quantity.

I discussed the interpretation of vague verbal probability expressions as an example of uncertainty about the probability of an event. There, dependencies between outcome and probability valuation occur because the value of the outcome influences the extent to which consequences for over-versus underestimates of the probability value are asymmetric. For outcomes of little positive or negative value, neither over- nor underestimation of their probability matters much. As an outcome becomes increasingly more positive or negative, the negative consequences of under- rather than overestimating its probability will start to increase.

I also included some examples of uncertainty about the value of an alternative. When judging the value of a commodity of uncertain value, asymmetries in consequences for over- versus underestimates arise as a function of the judge's role, perspective, or self-image and lead to a redistribution of weight among the different possible values. This configural redistribution of weight occurs in both riskless- and risky-judgment situations. In riskless situations (e.g., deciding on the true price of a vintage Volvo), the uncertainty about the value of the quantity under judgment arises because conflicting information from different sources needs to be integrated (Anderson, 1981). The weights that get redistributed to minimize the operating loss function are the weights assigned to each source to reflect its credibility, for example,  $\alpha$  and  $(1 - \alpha)$  in Equations 2 and 3. Thus, in riskless social judgment situations, asymmetric loss functions for over- versus underestimates will lead to a dependence between the weighting of a particular source and the rank of the information provided by this source in relation to the information provided by other sources, because only a single weight is estimated for each source, that is,  $(\alpha - \beta)$  for u(x) and  $(1 - \alpha +$  $\beta$ ) for u(y) in Equation 2. Estimating both a source-dependent weight  $\alpha$  and a configural-weight  $\beta$  separately helps to explain this dependence.

The negativity bias frequently reported in impression formation (see Skowronski & Carlston, 1989, and Taylor, 1991, for summaries) is an example of such dependence between the weight given to some information and its rank on the list of other items of information. All other things being equal, more negative (i.e., low-ranked) cues tend to receive greater weight than positive cues. Although a variety of explanations have been suggested to explain this bias, none have been entirely satisfactory (Skowronski & Carlston, 1989). An asymmetric-loss-function explanation seems to be worth exploring, because it is consistent with the effect and makes testable predictions. It predicts a negativity bias when the costs of overestimating the uncertain quantity under judgment exceed the costs of underestimates, a positivity bias when the opposite is true, and no bias when the costs of over-versus underestimates are the same.

In risky-choice situations (e.g., deciding on the utility of a

<sup>&</sup>lt;sup>8</sup> The following interpretation was suggested by Danny Kahneman.

gamble), the uncertainty about the value of the gamble arises because different outcomes can occur with either specified or unspecified probabilities. Here, configural redistribution of weight with the objective of minimizing the judge's loss function for over- versus underestimates of the value of the gamble operates on the weights assigned to each outcome to reflect its probability of occurrence, that is, s(A) and [1 - s(A)] in Equation 10. Thus, in risky-choice situations, asymmetric loss functions for over- versus underestimates of the value of an option (e.g., as the result of being held responsible for unfavorable results) will lead to a dependence between the evaluation of event probability and the value of the associated outcome (i.e., its rank in relation to other outcomes), because only a single decision weight is estimated for each outcome, that is, [s(A) - r(A)]for u(x) and [1 - s(A) + r(A)] for u(y) in Equation 10. Estimating both the event-dependent weight s(A) and the configuralweight r(A) separately again helps to explain this dependence.

In risky choice, the probabilities of the events that bring about the different outcomes are well specified. If these probabilities are believable, there is little reason why the subjectiveprobability weights s(A) should deviate much from the specified objective probabilities. Most nonlinearity in the decision weights will thus be due to the configural weight r(A). In choice under uncertainty, the probabilities of the events that determine the different outcomes are not specified and are thus themselves uncertain. In these situations, the asymmetric loss function for over- versus underestimates of the value of the gamble not only could result in a configural reweighting of the outcomes [r(A)]but also could carry over into the evaluation of the subjective probability of the uncertain events [s(A)]. Along the lines of Irwin's (1953) wishful-thinking effect, people may, for example, overestimate the probability of events leading to desirable outcomes (i.e., those with low costs and high benefits) and underestimate the probability of events leading to undesirable outcomes. Recent evidence (Bar-Hillel & Budescu, 1992) suggests, however, that this does not seem to be the case. In this study, people estimated the probabilities of uncertain events in an unbiased fashion when asked to judge those s(A) probabilities directly. Further support for little distortion in the estimation of subjective probabilities comes from Kirkpatrick and Epstein (1992), who found that people may be aware that two different urns have identical subjective probabilities for the same results, yet may prefer to draw from one of the two urns. Although Kirkpatrick and Epstein argued for the existence of two different conceptual systems that generate different estimates of subjective probability, their results are perhaps more parsimoniously explained by (fairly accurate) subjective probability estimates, s(A), and a configural redistribution of weights, r(A), that does not enter direct probability judgments but operates in addition to the s(A) estimates in people's choices between the two urns.

## Evidence Supporting Loss-Function Interpretation

One might ask whether there is any independent evidence supporting the contention that people are responding to selfor outwardly imposed consequences of providing misestimates when they judge uncertain quantities, such as the value of a risky option or of a used car. A loss-function interpretation of the various effects described in this article would presuppose (a) that the people who provide or make use of judgments and decisions under uncertainty care about the accuracy of these estimates and (b) that people are capable of adjusting their judgments or choices in a way that incorporates any asymmetry in the consequences to them if they provide over-versus underestimates of the actual value.

The large literature on overconfidence of probability estimates and calibration (see Yates, 1990, Chapters 3 and 4, for a comprehensive review) suggests that at least researchers care sufficiently about the accuracy of, in this case, probability estimates to monitor them. Further evidence that people are aware and perhaps afraid of the costs of providing misestimates of uncertain quantities comes from a survey by Wallsten, Zwick, Kemp, and Budescu (1993). When surveying people's preferences for probability information in numerical versus verbal form (where the latter is much more vague), they found that a majority of respondents preferred to receive information from others in the precise numerical form but to give out information in the vague verbal form. This strategic set of preferences maximizes the precision of information received and minimizes the possibility of being called on an inaccurate estimate, where the chance of inaccuracy increases with the degree of precision.

This leaves the question of whether people are capable of adjusting their judgments or choices accordingly. Here, the large body of research on psychophysical judgments that prompted the development of signal-detection theory (e.g., Green & Swets, 1966) is relevant. It shows not only that people are very sensitive to loss functions but also that they are quite capable of adjusting their behavior in a way that minimizes such losses. Furthermore, people seem to be able to do so without much apparent effort and without being necessarily aware of it.

#### Modeling and Scaling Issues

From a purely predictive point of view, dependencies between probability and outcome evaluations can be captured either by making the probability weighting dependent on the outcome distribution, as in the models discussed above, or by making the outcome weighting dependent on the probability distribution, as in a model by Becker and Sarin (1987) called *lottery-dependent* utility. However, when considering the explanatory psychological processes outlined in this article, I found the first modeling alternative to be more useful. 9 Nonindependence between

<sup>&</sup>lt;sup>9</sup> There are other processes that make outcome evaluation dependent on their underlying probabilities. People seem to infer the severity of outcomes from the probability with which they occur (E. U. Weber & Hilton, 1990), probably capitalizing on an ecological, negative correlation between the two variables, as already hypothesized by Edwards (1962b). In the area of motivation and task performance (e.g., Atkinson, 1964; Shapira, 1989), the utility of successfully completing a task has been assumed to derive, at least partially, from the task difficulty, that is, from the probability of successful completion. McCord and Neufville (1984) also report evidence of utility estimates being differentially affected by the associated levels of probability. And in the case of state-or event-dependent preferences, the relative utilities of a set of outcome alternatives (e.g., ways of spending the weekend) may depend on the events (rain or sunshine) that define the probability distribution of the choice situation (see Karni, 1992).

evaluations of outcomes and probabilities for both nonambiguous and ambiguous probabilities seems to be best explained as the result of influences of outcome characteristics on the weighting of the probabilities associated with them.

As a consequence, the decision weights associated with the objective probabilities or events that determine the likelihood of outcomes reflect much more than a person's perception of their subjective probability. Equation 10 shows perhaps most clearly that the decision weight of an outcome contains both a subjective-probability component [s(A)] and a configural-weight component [r(A)] that redistribute weight between outcomes as a function of their rank or sign.

This poses some challenges to the conventional methods of decision analysis, in particular the practical issue of how to obtain (a) valid subjective-probability values and (b) nondistorted utility functions if people's answers to elicitation questions are subject to the constraints that give rise to configurality in the evaluation of outcomes and events. It is perhaps ironic that the method designed to minimize idiosyncratic distortions in the elicitation of subjective probability may, in fact, contribute to such distortions if not interpreted with caution. Influenced by the logical positivism of economics, which allows as evidence only overt behavior, decision analysts usually infer a person's perception of the subjective probabilities of an event from his or her choices between specifically constructed alternatives (e.g., using the equivalent-urns method). However, care needs to be taken in interpreting the decision weights estimated from such choices. In subjective-expected-utility theory (e.g., Edwards, 1962a), the s(p) function defined over objective probabilities was meant to be interpreted as subjective belief about likelihood. Prospect theory's (1979)  $\pi(p)$  decision weights were thought to reflect not perceived likelihood as such, but the "impact" of the event determining the likelihood on the desirability of the choice alternative. Going one step further, observed nonlinearities in the decision weights based on rank-dependent formulations such as the cumulative functional of Equation 4 reflect not only subjective beliefs about the impact of the event determining the probability of the outcome but also subjective beliefs about the desirability or cost of the associated outcome as a function of its rank. The small decision weight of .36 given to the .6 chance of obtaining outcome  $x_3$  when applying the pessimistic w function to the example of Equation 5, for example, may result from the underweighting of larger probabilities as postulated by the original prospect theory but may also result from a more linear weighting of likelihood but a redistribution of importance weights toward lower outcomes (i.e., from a loss function for misexpectations that punishes overestimates more than it rewards underestimates.

Experimental designs that factorially combine probability levels, outcome levels, and the rank and sign of outcomes can, of course, distinguish between these two effects. At least for the case of two-outcome choice alternatives, it is easy to decompose decision weights into a probability-related weight and a configural rank-dependent weight (i.e.,  $\alpha$  and  $\beta$  of Equations 2 and 3; s(A) and r(A) of Equation 10), which can be estimated separately. On the other hand, this amount of effort may not always be necessary. Contrary to the assumption that introspective judgments of subjective probability are unreliable and inaccu-

rate, Bar-Hillel and Budescu (1992) found little evidence of systematic distortions in subjective-probability judgments when these were elicited directly (rather than inferred from choices). Erev, Bornstein, and Wallsten (in press) also obtained very accurate subjective-probability estimates about events in several different domains by asking people for direct numerical judgments. Thus, decision analysts' dogmatic refusal to consider introspective judgments of perceived probability as valid evidence may one day seem as unnecessary in its self-imposed limitations as a behaviorist approach to, say, language acquisition.

The second challenge faced by decision analysis is to obtain stable utility functions for the outcomes under consideration. Violations of procedure invariance, that is, demonstrations that the utility of outcomes appears to depend on the procedure used to infer it, complicate this task and thus have received a lot of attention in recent years (see Birnbaum, 1992, for a summary). Configural-weight models of judgment and choice can help to resolve many (even though not all) of these instances of apparent utility function instability. Configural weighting allows for the representation of judgments across a wide variety of tasks without having to assume that people's utility functions for outcomes are labile. Thus, I discussed earlier how Birnbaum et al. (1992) fit judgments of both the buying and selling price of risky options, using the same utility function for monetary outcomes for both types of judgments, with the help of a configurality parameter. Birnbaum and Sutton (1992) did the same for the utility of money under risky or riskless conditions, providing a possible resolution to the longstanding controversy about whether utility functions estimated from risky versus riskless choices should or should not be the same (see Bell & Raiffa, 1988; Dyer & Sarin, 1982; von Winterfeldt & Edwards, 1986). That is, differences in choices between risky as opposed to riskless alternatives can be explained by assuming that people have the same utility for the outcomes in both contexts but that the uncertainty of not knowing which outcome will obtain in the risky context brings additional configural weighting of those outcomes. Finally, E. U. Weber, Anderson, and Birnbaum (1992) were able to model judgments of both the attractiveness and the riskiness of monetary lotteries as well as account for individual differences in such judgments with the same utility function for outcomes in both tasks and for all subjects. Individual differences and differences between the two judgment tasks could be explained solely by differences in configural weighting. Thus, nonexpected-utility models that explicitly incorporate the configural weights given to outcomes have the ability to provide more consistent, thus presumably more accurate, estimates of the utility of outcomes. By separating the utility of the outcome itself from the weight given to the outcome as a function of its relative rank or the nature of the task (i.e., its configural weight that reflects situation- and task-specific considerations), changes in preference as a function of elicitation method can be attributed to changes in configural weighting, while allowing the utility of the outcome to remain invariant.

### Descriptive Versus Prescriptive Implications

For risky-choice situations, loss-function explanations are related to the notion of disappointment proposed by Bell (1985)

and Loomes and Sugden (1986). Disappointment theory holds that people incorporate their anticipations of their reactions to obtaining outcomes that are smaller or greater than what they expected into their initial decisions. The origin of this expectation of what will be won is not very clearly defined in these models but may very plausibly be an individual's best estimate of what outcome will occur. Outcomes that are larger or smaller than this expectation thus can be seen as indications that the uncertain quantity was misjudged, and disappointment theory postulates that people experience asymmetric psychological costs (i.e., have self-imposed asymmetric loss functions) for outcomes that exceed versus fall short of the expected outcome.

In addition to being subject to internal (i.e., psychological) asymmetric costs, people frequently face external asymmetric consequences, for example, they may be held responsible for outcomes that fall short of the expected outcome in business decisions whereas outcomes that exceed the expected outcome are taken for granted and barely noticed. Thus, Hogarth and Kunreuther (1992), for example, provided some reasons why the expected-utility model may be insufficient to capture the constraints under which actuaries operate when pricing insurance policies for risks with ambiguous losses (i.e., losses with uncertain probabilities or uncertain amounts). Real constraints, such as a tax code that punishes insurance companies more for losses that are larger than expected than rewarding them for losses smaller than expected, may make the ambiguity sensitivity on the part of actuaries normatively more appropriate.

In addition to establishing rank-dependent utility as a reasonable descriptive theory of choice under risk or uncertainty, lossfunction interpretations of configural weighting also have prescriptive implications. If configural weighting is a response to internal or external constraints rather than the result of a cognitive illusion or error, then prescriptive concerns reduce to the question of whether these constraints can or should be ignored by the decision maker and to practical considerations as to how to remove these constraints if the behavior is to be modified. To improve their judgments and decisions, people might want to ask themselves what internal constraints they are imposing onto themselves and what external constraints other people in their decision environment (e.g., spouse, peers, or superiors) are imposing on them. Parallel to the procedures of Janis's decisional balance sheet approach for highlighting the inherent conflict in multiattribute decisions in which decision makers generate, for each choice alternative, a list of all utilitarian and psychological consequences to themselves and other affected parties (see Janis & Mann, 1977, Chapter 6), tacit loss functions that may affect people's estimates of uncertain quantities and thus their choices could be brought out into the open by having decision makers generate a list of rewards or punishments, both physical as well as psychological, that they themselves or other affected parties would impose on either an over- or underassessment of the quantity of uncertain value, be it a probability estimate or an assessment of the value of a risky option. Only when all consequences of over- versus underestimates of the uncertain quantity are fully spelled out, can people decide which of these consequences should affect their decision. In the case of selfimposed psychological consequences, awareness of the negative effects of, for example, disproportionate disappointment about outcomes lower than expected (i.e., self-imposed disproportionate costs of overestimates) may be sufficient to reduce those postdecisional emotions if they are perceived to be dysfunctional. In the case of outwardly imposed costs that, on inspection, appear to lead to loss functions with undesirable consequences for the decision maker, a process of negotiation with the parties imposing these costs may lead to jointly more satisfactory compromises.

#### Conclusion

Configural weighting is likely to occur in judgments and decisions under uncertainty, where the uncertainty may be about the likelihood of an event or about the value of some other quantity. The resulting effect may be large or small, depending on a variety of factors: the external constraints of the situation, the decision maker's perspective and outlook, and the nature of the alternatives that are being judged. In line with the theme of the article, there are asymmetric consequences to paying too much versus too little attention to these configural-weighting effects. Paying too much attention seems to carry little cost: As outlined above, these effects are now easily modeled with only a few additional parameters. Paying too little attention, on the other hand, carries large costs, as configural-weighting models have been able to resolve apparent inconsistencies in the elicitation of utility functions, point to the omission of potentially relevant considerations in the expected-utility model, and finally help to provide more accurate and consistent estimates of subjective probabilities and utilities in situations where all parties agree on the appropriateness of the expected-utility framework as the normative model of choice.

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<sup>&</sup>lt;sup>10</sup> Quiggin (1982) calls his rank-dependent utility model a "theory of anticipated utility."

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