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Distance sensing via magnetic resonance coupling

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Travail confidentiel / *vertrauliche Arbeit* ☐ oui / ja [1] ☒ non / nein	Expert / *Experte* (données complètes) **Sousuke Nakamura | Cho Universität Tokio**	

Titre / *Titel*

Positionsbestimmung über resonante elektromagnetische Kopplung

Description et Objectifs / *Beschreibung und Ziele*

Mobile Geräte und Roboter können drahtlos mit elektrischer Energie versorgt werden, indem ein magnetischer Koppler, welcher auf gekoppelten Schwingkreisen beruht, eingesetzt wird. Weiter ist es möglich durch geeignete Messungen und Signalverarbeitung, die relative Position von Sende- und Empfangsantenne zu bestimmen.

In dieser Arbeit geht es darum

— Zu verstehen, wie die Positionserkennung aus der Messung des Refelxionskoeffizienten des Senders theoretisch funktioniert.

— Dieses Messverfahren zu simulieren.

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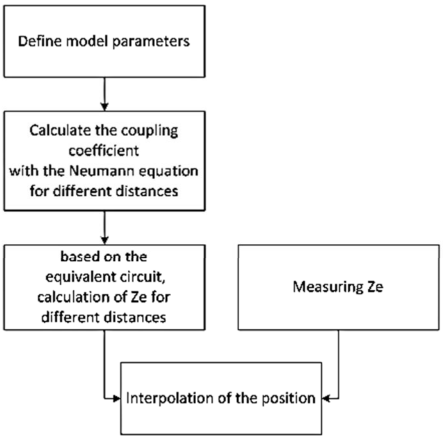
Distance sensing via magnetic resonance coupling

Graduate Orlando Summermatter



Bachelor's Thesis
| 2013 |

Degree course
Systems Engineering

Field of application
Power & Control

Supervising professor
Joseph Moerschell
Expert
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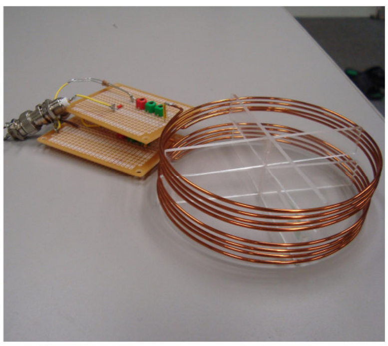
Objectives

The goal is to understand how the principle of distance determination via resonant magnetic coupling based on the researches of Hashimoto laboratory works and to perform measurements.

Methods | Experiences | Results

Mobile devices or robots can be charged wirelessly using the magnetic resonance coupling. Some types of robots work independently. A position location system is necessary for the robot to know where it is. For this reason the possibility to determine the position based on magnetic resonance coupling is investigated. In this way a system handling two tasks, can be developed in one, i.e. charging the battery and locating the position in a building or in a room.

Attention was focused on the distance sensor. The position cannot be calculated directly but it can be determined via a comparison with a table. The reflection coefficient or the equivalent circuit impedance Z_e of the coupled antennas must be measured for this comparison. The necessary calculations for determining the position and the error analysis were made with Matlab. The measurements show that the determination of the distance can be carried out in this manner. However, the range is limited and the area within which the error is small, varies according to the antenna parameters.



Antenna setup used for measurements

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1 TABLE OF CONTENTS

DISTANCE SENSING VIA MAGNETIC RESONANCE COUPLING

2 INTRODUCTION

This work was written as part of a student exchange and was created in collaboration with the Chuo University in Tokyo. I'm worked, in relation to this exchange, in collaboration with the Hashimoto laboratory. The goal is to write a work related to a current topic of this laboratory. This work is based on the work over position sensing based on magnetic resonance coupling from Hashimoto laboratory [1], focusing on the distance sensor.

The idea behind this project is, position sensing and power supply for robots. It is possible to recharge the robot cable less via coils. If it is possible, the position over coils to determine, can be build a system which makes two tasks in one. One possible application is, for example, a factory in which parts via robots from A to B to be transported or robots that act in an intelligent space to facilitate everyday life. The tracking system allows the robot to determine he's position within the factory or the room. If the batteries are discharged, the robot can go to the charging station and recharge himself, with the help of the recharging system, or it can be used the same coils as are used for the position sensing

The first part of the work is to understand how functioning this type of sensor. Therefore, the individual areas were investigated and as possible derive by myself. If possible, have been using Matlab to create functions for the processes to simulate and to understand them better

In the second part of the analysis was focused on the distance sensor and two antennas were produced and measured. The evaluation of the results was performed using Matlab. To improve the knowledge of Matlab was casually reading a book over Matlab [2].

3 THE PRINCIPAL FUNCTIONALITY OF THE PROPOSED SENSOR

The sensor consists of four transmitters and one receiver. The transmitters and the receiver are coils, and they are coiled from copper wire. The transmitters can be individually switched on and off. For the position detection always only one transmitter is active. That's the non-active transmitter not influents the result, is a second switch used to cut off some circuit elements. Thereby changes the resonance frequency of the non-activated transmitters. The basic principle is schematically shown in Figure 1.

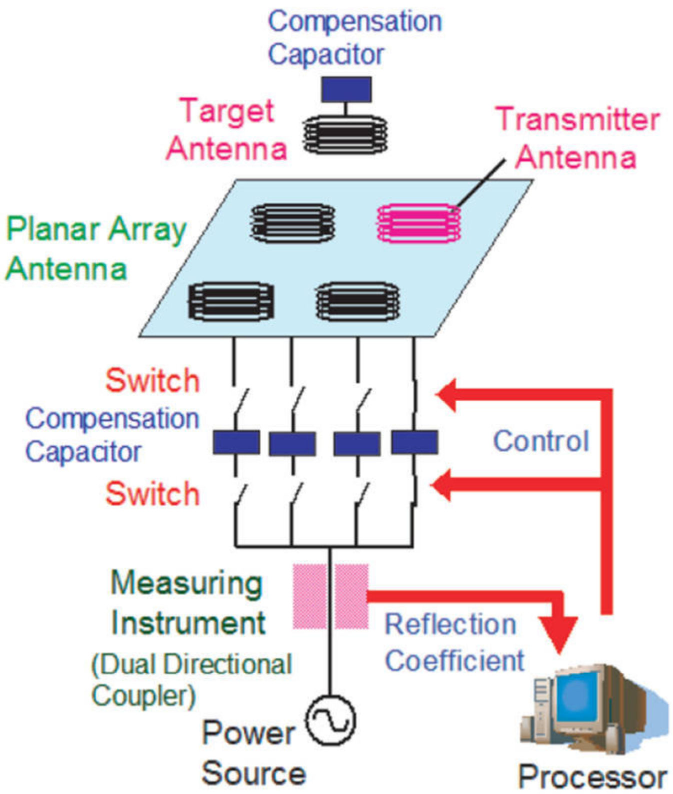

Figure 1 : Schematic illustration of the basic structure [1]

The magnetic resonance coupling with the receiver is incurred if one of the transmitters will be enabled. The coupling coefficient is used to estimate the position. But this coefficient can't be measured. For this reason, the reflection coefficient is measured, and from this the coupling coefficient is determined. With one transmitter antenna, only the distance between the transmitter antenna and the receiver antenna can be estimated. That is why, different transmitter antennas for the position estimation in a three dimensional room are used.

The four transmitter antennas are turned-on one after another, in order to measure the corresponding reflection coefficient. The reflection coefficient vector is made on basis of the measured values.

4 INVESTIGATION, USING THE EQUIVALENT CIRCUIT

In the previous section 3 is assumed that always, only one transmitter antenna is turned-on. For this reason, contains the circuit analysis only the coupling between one transmitter antenna and the receiver antenna. For this case the equivalent circuit shown in Figure 2 can be used. Transmitter and receiver antenna consist of different circuit elements. Each antenna containing, connected in series a resistor $\mathrm{R_i}$, a capacitor C_i and a coil L_i $(\mathrm{i} = 1{,}2)$. Thus, each circuit for the antennas corresponds to a resonant circuit. V_src is the power source and supplies the transmitter antenna with energy. $\mathrm{Z_0}$ is the characteristic impedance of the transmission line between the power source and the transmitter antenna.

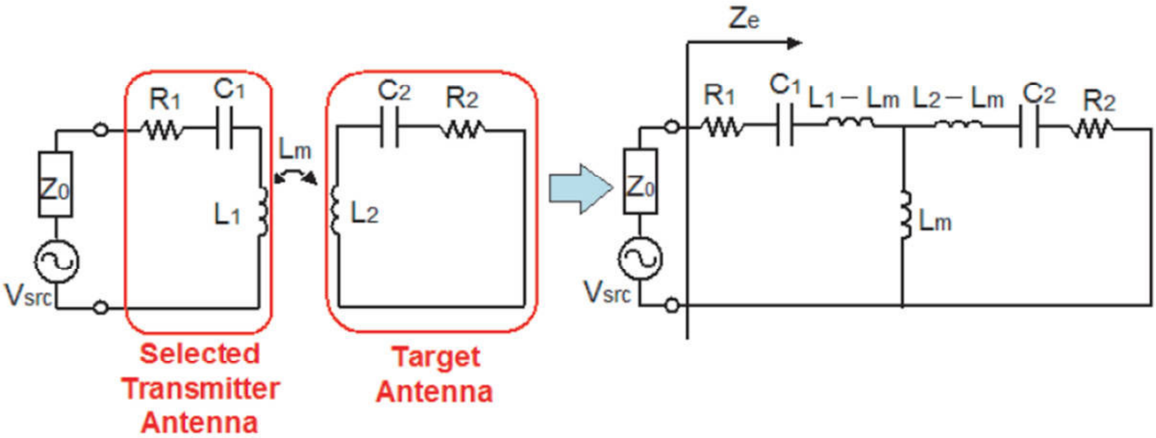

Figure 2 : Equivalent electrical circuit for the magnetic coupling [1]

For reducing the electrical losses, the principle of magnetic resonance coupling is used. The magnetic resonance coupling will be strengthened if both antennas have the same resonance frequency. The resonant frequency for the circuit in Figure 2 is calculated by using the Equation 1 and must be the same for both circuits. The reactance $\mathrm{X_{Ri}}$ must become equal zero to obtain the resonance frequency.

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$$X_{Ri} = \omega_0 L_i - \frac{1}{\omega_0 C_i} = j\omega_0 L_i + \frac{1}{j\omega_0 C_i} = 0 \quad (i = 1{,}2), \qquad \omega_0 = 2\pi f_0$$

Equation 1 : Determining the resonant frequency

f_0 is the resonant frequency ω_0 is the angular frequency from the resonate frequency. If the circuits are coupled, the equivalent circuit diagram from the right side of Figure 2 can be used. The coupling coefficient κ is a dimensionless size and describes the intensity of the magnetic coupling between the antennas. The coupling coefficient is given from Equation 2. L_m is the mutual inductance and changes with the distance between the antennas. The mutual inductance is the only variable size in this equivalent circuit because she's depending from the coupling coefficient. The relation between the coupling coefficient and the mutual inductance, is Described from Equation 2.

$$\kappa = \frac{L_m}{\sqrt{L_1 L_2}}$$

Equation 2 : Coupling coefficient [1]

Z_e is the equivalent impedance of the complete circuit for the transmitting and receiving antenna. Z_e is described as follows.

$$Z_e = R_1 + \frac{1}{j\omega C_1} + j\omega(L_1 - L_m) + \frac{j\omega L_m\left(R_2 + \frac{1}{j\omega C_2} + j\omega(L_2 - L_m)\right)}{j\omega L_m + R_2 + \frac{1}{j\omega C_1} + j\omega(L_1 - L_m)}$$

Concerning to the relationship from the Equation 1, can this equation be simplified.

$$Z_e = R_1 - j\omega_0 L_m + \frac{j\omega_0 L_m(R_2 - j\omega_0 L_m))}{j\omega_0 L_m + R_2 - j\omega_0 L_m)}$$

By forming and simplify the final result is obtained in Equation 3.

$$Z_e = R_1 + \frac{{\omega_0}^2 \, {L_m}^2}{R_2}$$

Equation 3 : Determination of the equivalent resistance Z_e

5 METHOD FOR POSITION ESTIMATION

With this position estimation procedure, is the reflection coefficient used as the measured value. This value changes to in relation with the distance between the antennas. This is the reason why the coupling coefficient can be determined by using the reflection coefficient. The reflection coefficient can be measured by using a bidirectional coupler as shown in Figure 1.

The reflection coefficient Γ is a Physical quantity that describes the relationship between the voltages of an incident to a Reflected wave. If the reflection coefficient is set to the square is obtained a relationship between the incident and reflected power.

The transmitted power from the transmission antenna can be grouped into two areas. One part of the energy receives the receiver antenna, due to the magnetic resonance coupling. The remaining energy flows through the room, and the receiving antenna without the energy is gathering. This part of the energy flows back to the transmitting antenna. The relationship between the received and the reflected power can be described with the coupling coefficient κ. The coupling coefficient indicates the strength of the magnetic coupling. Thus, valid for further analysis are the following relationships.

- The coupling coefficient κ changes in relationship with the mutual position of the antennas
- The reflection coefficient Γ is in relation to the coupling coefficient κ

As described in Section 3 the proposed position sensor has four transmit antennas for determining the position of the receive antenna. For this reason, vectors are introduced to describe the data of the four antennas. The reflection coefficient vector is made by measuring the reflection coefficients for each antenna pairs (each between a transmitting and the receiving antenna). Thus it can be determined the location of the receiving antenna over the reflection coefficient vector. For this, the measured reflection coefficient vector must be converted in to the coupling coefficient vector.

6 CONVERSION OF THE REFLECTION COEFFICIENT IN THE COUPLING COEFFICIENT

The relation between the coupling coefficient and the reflection coefficient can be determined by analysis of the circuit from the section 4 and the description of the Q-factor Analysis. First, the Q-factor is described Equation 4.

$$Q_i = \frac{\omega_0 L_i}{R_i} \; (i = 1{,}2), \qquad Q = \sqrt{Q_1 Q_2}$$

Equation 4 : Determining the Q-factor [1]

The Q-factor is the indicator for the quality of the resonant frequency. The relation between the coupling coefficient and the equivalent circuit impedance Z_e can be described with Equation 3, Equation 2 and Equation 4.

$${L_m}^2 = \kappa^2 (L_1 L_2)$$

By using this relationship to the Equation 3 is obtained the following expression:

$$Z_e = R_1 + \frac{{\omega_0}^2 \, {L_m}^2}{R_2} = R_1 + \frac{{\omega_0}^2 \kappa^2 (L_1 L_2)}{R_2}$$

By inserting the circuit elements, the Equation 4 can also be written as follows:

$$Q = \sqrt{\frac{{\omega_0}^2 L_1 L_2}{R_1 R_2}}$$

If this last Description is compared with the previous equation, the equation for the equivalent circuit impedance Equation 3 is added as follows:

$$Z_e = \kappa^2 Q^2 R_1 + R_1 = (\kappa^2 Q^2 + 1) R_1 = R_1 + \frac{{\omega_0}^2 \, {L_m}^2}{R_2}$$

Equation 5 : supplemented equation for the impedance Z_e

The reflection coefficient is determined from the equivalent circuit impedance Z_e and the impedance of the transmission line to the sensor Z_0, as defined in the Equation 6.

$$\Gamma = \frac{Z_e - Z_0}{Z_e + Z_0}$$

Equation 6 : Determination of the reflection coefficient [1]

The reflection coefficient is translated into the coupling coefficient by using the Equation 5 and the Equation 6. For the calculation are the values for the Q-factor, R_1 and Z_0 as fixed values used. Thus, the reflection coefficient vector $\vec{\Gamma} = (\Gamma_1, \dots, \Gamma_\mathrm{M})$ which describes the measured values of the reflection coefficient between the antenna pairs can be translated into the coupling coefficient vector $\vec{\kappa} = (\kappa_1, \dots, \kappa_M)$, which describes the coupling coefficient between antenna pairs. M corresponds to the number of transmitter antennas.

7 ESTIMATING THE POSITION BASED ON THE COUPLING COEFFICIENT

In the previous section 6, the coupling coefficient is determined by using the measured reflection coefficient. In this section is the position determined, from the previous determination of coupling coefficient and his mathematical relation to the mutual inductance. For this the mutual inductance must be determined mathematically. The mutual inductance can be calculated from the Neumann equation, Equation 7. This equation describes the mutual inductance between two circuits C_1 and C_2 for one winding $N = 1$.

$$L_m = \frac{\mu_0}{4\pi}\int_{C_1}\int_{C_2}\frac{dl_1 dl_2}{r_{12}}$$

Equation 7 : Neumann equation [1]

μ_0 is the permeability of the space, dl_1 and dl_2 are small line elements of the circuits for the transmitter and the receiver antenna. r_{12} is the distance between the line elements dl_1 and dl_2.

7.1 Geometrical relation between the coils

This section will insert the geometric relationship into the Equation 7. For this, is the Scalar product of dl_1 and dl_2 and the relation of r_{12} is analysed. In this case, it is assumed that the transmitter and the receiver antenna are oriented parallel to each other. The geometrical relation is shown in Figure 3. dl_1 and dl_2 can be written as follows:

$$|\,\mathrm{dl}_1| = \frac{d}{2} * \mathrm{d}\theta_1, \qquad |\,\mathrm{dl}_2| = \frac{d}{2} * \mathrm{d}\theta_2$$

The scalar product from dl_1 and dl_2 can be determined as follows.

$$dl_1 dl_2 = |\,\mathrm{dl}_1| * |\,\mathrm{dl}_2| * \cos(\theta_1 - \theta_2) = \frac{d^2}{4} * \cos(\theta_1 - \theta_2) * \mathrm{d}\theta_1\,\mathrm{d}\theta_2$$

Equation 8 : Scalar product of $dl_1 * dl_2$

The distance r_{12} is determined by using the orthogonal projection. For the analysis is the displacements projected on the base of the respective axis. Two of the projected distances are in the same plane as the transmitting antenna. The projection for the height difference is vertical to this plane. The projections are expressed in polar coordinates. The distance r_{12} can be expressed as follows.

$$r_{12}{}^2 = g^2\sin(\varphi)^2 + (gcos(\varphi) - \frac{d}{2}(\cos(\theta_1) - \cos(\theta_2))^2 + (\frac{d}{2}(\sin(\theta_1) - \sin(\theta_2))^2$$

By multiplying out the brackets and grouping the terms we get:

$$r_{12}{}^2 = g^2(\sin(\varphi)^2 + \cos(\varphi)^2) - 2g\cos(\varphi)\frac{d}{2}(\cos(\theta_1) - \cos(\theta_2) + \frac{d^2}{4}(\cos(\theta_1)^2 + \cos(\theta_2)^2 + \sin(\theta_1)^2 + \sin(\theta_2)^2 - 2(\cos(\theta_1)\cos(\theta_2) + \sin(\theta_1)\sin(\theta_2))$$

This expression can be more simplified, by using the following relations:

$$\cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta) = \cos(\alpha - \beta), \qquad und\ \cos(\alpha)^2 + \sin(\alpha)^2 = 1$$

By use of these relations it can be written as follows:

$$r_{12}{}^2 = g^2 + gd\cos(\varphi)(\cos(\theta_1) - \cos(\theta_2) + \frac{d^2}{4}(2 - 2\cos(\theta_1 - \theta_2)$$

The final result is obtained by using the square root of it, and the expression $d/2$ is excluded.

$$r_{12} = \frac{d}{4}\sqrt{8 + 16\left(\frac{g}{d}\right)^2 - 8\cos(\theta_1 - \theta_2) - 16\frac{g}{d}\cos(\varphi)(\cos(\theta_1) - \cos(\theta_2))}$$

Equation 9 : geometrical relation of r_{12}

If the geometrical relation for the small line elements dl_1 and dl_2 Equation 8, and the relation for r_{12} Equation 9 in to the Equation 7 will be inserted, is the result the mutual inductance $\mathrm{L_m}$, how it is described at Equation 10.

$$L_m = N^2 \frac{\mu_0}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{dcos(\theta_1 - \theta_2) d\theta_1 d\theta_2}{\sqrt{8 + 16 \left(\frac{g}{d}\right)^2 - f(\theta_1, \theta_2)}}$$

Equation 10 : Neumann equation under consideration to the geometrical relation [1]

For the Equation 10 is valid:

$$f(\theta_1, \theta_2) = 8cos(\theta_1 - \theta_2) + 16 \frac{g}{d} \cos(\varphi)(\cos\theta_1 - \cos\theta_2)$$

The analytical description of the position is based on the complex description of the mutual inductance not possible. N is the number of turns from the coil, d is the antenna diameter, g is the distance between the antenna centres and φ is the elevation angle. It is assumed that (x1, y1, z1) and (x2, y2, z2) are the central position for the transmitter antenna and the receiver antenna.

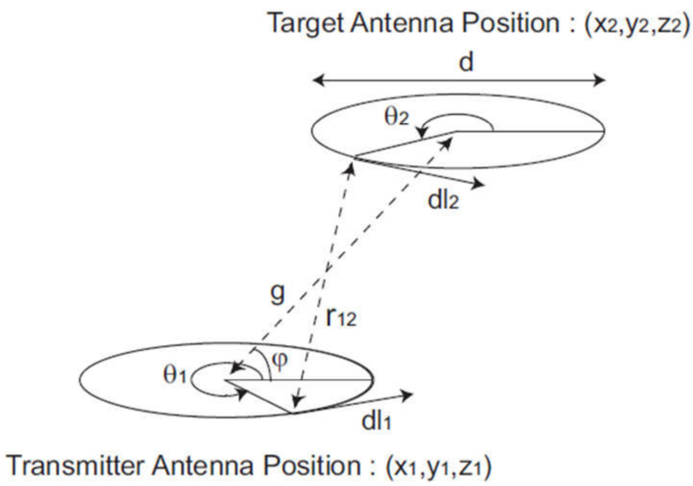

Figure 3 : The geometric relation between the antennas [1]

The distance g between the centre points of the antennas and the elevation angle φ, are described from the Equation 11 and the Equation 12.

$$g = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Equation 11 : Determination of the distance g between the centre points of the coils [1]

$$\varphi = \tan^{-1}\left(\frac{z_2 - z_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}\right)$$

Equation 12 : Determining the elevation angle φ [1]

To receive a relation between the geometrical position and the coupling coefficient is the Equation 2 used at the Equation 10. The result is shown at the Equation 13

$$\kappa = \frac{N^2}{\sqrt{L_1 L_2}} \frac{\mu_0}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{d cos(\theta_1 - \theta_2) d\theta_1 d\theta_2}{\sqrt{8 + 16\left(\frac{g}{d}\right)^2 - f(\theta_1, \theta_2)}}$$

Equation 13 : Geometric description of the coupling coefficient κ [1]

For the Equation 13 is valid:

$$f(\theta_1, \theta_2) = 8 cos(\theta_1 - \theta_2) + 16\frac{g}{d}\cos(\varphi)(\cos\theta_1 - \cos\theta_2)$$

By use of the Equation 11 and Equation 12 into the Equation 13 is obtained a description which gives the coupling coefficient in relation to the coordinates of the coils. Thus, the position can be determined based on the coupling coefficient. This equation can't be analytically solved based on the double integral. On this reason it is not possible to determinate the position analytically.

In addition, the solution set, of possible coordinate points are elements of a circle, which is located in a certain distance to the coil. This relationship is not surprising, since the intensity of a magnetic field at a certain distance around the ring coil is also equal for any point element of a circle around the coil. This relation holds as long the propagation of the field is not affected by external disturbances.

8 BASIC CHARACTERISTICS OF THE DISTANCE SENSOR

The basic characteristics of the distance sensor are analysed by using one pair, corresponding to one transmitter antenna and one receiver antenna. It is assumed that only one transmitter antenna is activated. For simplicity, the orientation of the antennas to each other is neglected, that means that the antennas are oriented parallel to one another.

Further, the antennas are oriented so that they are precisely superposed, corresponding to the case where the elevation angle $\varphi = \frac{\pi}{2}$ [rad] corresponding to Figure 3. Thus, the movement is only in one axis possible.

8.1 Distance estimation method

The distance $\mathrm{g/d}$ is a dimensionless relation between the antennas distance g and the antenna diameter d. This distance is used for the analysis of the sensor characteristic. The distance estimation is made by executing the following two steps.

- Transforming the reflection coefficient into the coupling coefficient.
- Derivation of position corresponding from the coupling coefficient

For the displacement in the z-axis is only one transmitter antenna used for that reason are no vectors used and all values are scalar. For the displacement in the z-axis is used only one transmitter antenna, for that reason are no vectors be used and all values are scalar values. The position is derived from a database comparison. The database is created by using the Equation 14.

$$\kappa = \frac{N^2}{\sqrt{L_1 L_2}} \frac{\mu_0}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{d cos(\theta_1 - \theta_2) d\theta_1 d\theta_2}{\sqrt{8 + 16\left(\frac{g}{d}\right)^2 - 8cos(\theta_1 - \theta_2)}}$$

Equation 14 : Determination of the coupling coefficients of the superposed antennas [1]

For the Equation 14 is valid:

$$\varphi = \frac{\pi}{2}\ [rad]$$

8.2 Relationship between design parameters and model errors

The difference between the determinate values and the real values is called model error. In the second step for the distance estimation, is the position corresponding to the coupling coefficient and the Neumann equation derived. Into the Neumann equation is assumed that the current has in the whole circuit a similar value. Therefore, the Neumann equation cannot be used if the electrical length comes to big. In this case is the result an error, because the current in the circuit is unequally distributed. Below is a closer look at the electrical length.

The electrical length is a physical value and is defined from Equation 15.

$$l = \frac{\pi}{c} fNd$$

Equation 15 : Determination of electrical length [1]

The equation for the electrical length contains the following design parameters: The frequency f, number of turns N of one coil and the antenna diameter d. The natural constant c is the speed of light. The relationship between model error and design parameter is shown from Figure 4 to Figure 6.

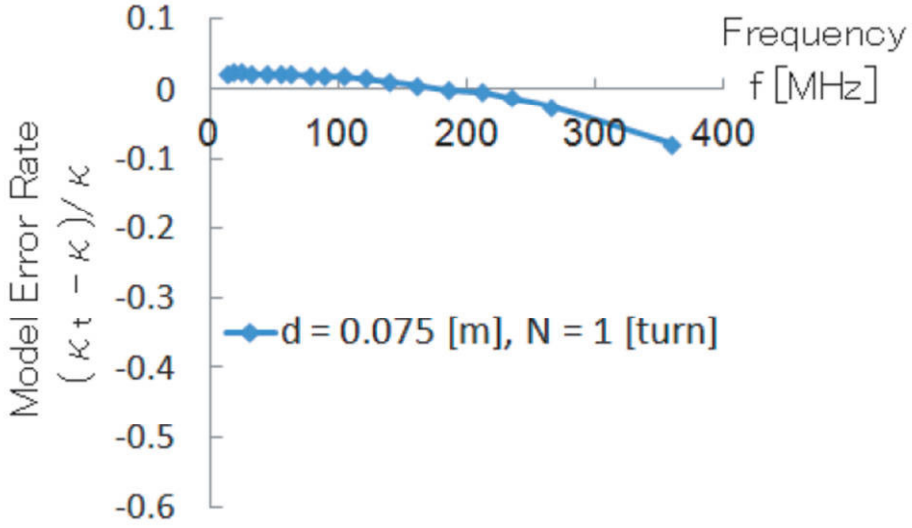

Figure 4 : Model error in relationship to the frequency f [1]

The model error rate is expressed in the base of the coupling coefficient. κ_t is the theoretical value, and it is calculated by using Equation 14. κ is the true value which is determined by magnetic field analysis. This analysis is made by CAD analysis software for electrical components. Thus, the error rate is described as follows:

$$Model\ fehler = \frac{(\boldsymbol{\kappa}_t - \boldsymbol{\kappa})}{\boldsymbol{\kappa}}$$

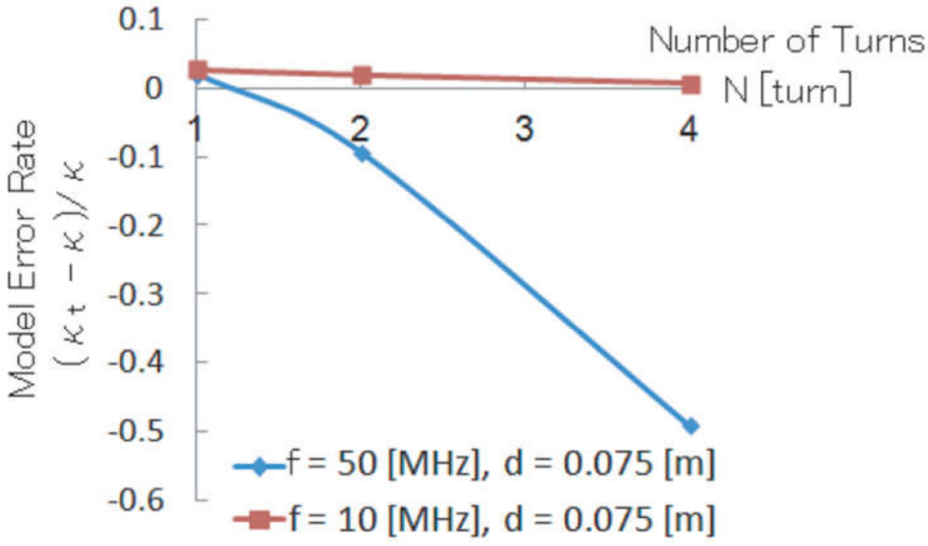

Figure 5 : Model error in relationship to the number of turns N, for two different frequencies f [1]

From Figure 4 to Figure 6 show that's the model error comes smaller if the individual design parameters come smaller. From Equation 15 it's shown that these three parameters form a product for the determination of the electrical length. Thus, it shows that the error increases if the electrical length comes bigger. The error can be limited by limitation of the electrical length.

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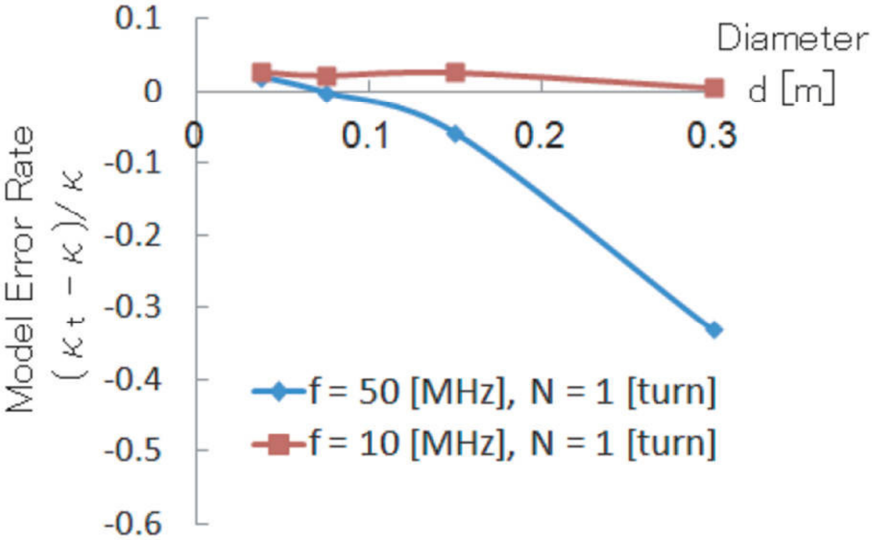

Figure 6 : Model error in relationship to antenna diameter d, for two different frequencies f [1]

8.3 Relationship between effective range and the design parameters

An important relation for a distance sensor is the relation between the design parameters and the effective range. The effective range according to an area in that's the distance error is small. Below, the effective range and the distance error are investigated as a function of design parameters. The distance error is dependent of measurement error and model error. Following is assumed that the model error is eliminated by reason of limiting the electrical length. Thus, the distance error is based on the measurement error. The measurement error is the variation from the measuring instrument.

The variation from the reflection coefficient $\Delta\Gamma$ is being caused from measuring error. The measurement error is given from Equation 16. The measurement error is given from the datasheet of the used measuring instrument [3]. For the measurement, a bi-directional coupler of an Agilent Technologies E-5061A network analyser was used.

$$\Delta\Gamma = \mathrm{a}|\Gamma| + b$$

Equation 16 : Determination of the measurement error for the reflection coefficient [1]

This error characteristic describes the worst case. For this reason, the real values should be below these values. The error characteristic is approximated as a straight with the slope a = 0.0178 and Intercept b = 0.004.

8.3.1 Behaviour of the coupling coefficient measuring errors in relation with the distance

An analytical analysis is not possible because the Equation 14 contains a double integration. In this case are the antennas superposed and the elevation angle is $\varphi = \frac{\pi}{2}\,[\mathrm{rad}]$. Thus, the coupling coefficient can be approximate Equation 17. This approximation permits a part analytical analysis. Equation 14 can with be simplified with Equation 17.

$$\kappa \approx \ e^{\alpha(g/d)} \ \ (\alpha < 0)$$

Equation 17 : approximate determination of the coupling coefficient [1]

With Equation 17 it is possible to describe the coupling coefficient error in relation to the distance. For this, first is determined the absolute error:

$$\Delta\kappa \approx \frac{\partial\kappa}{\partial g} * \Delta g \ \ \approx \left|\frac{\alpha}{d} e^{\alpha\left(\frac{g}{d}\right)}\right| * \Delta g \ \ (\alpha < 0)$$

In the second step is determined the relative error:

$$\frac{\Delta\kappa}{\kappa} \approx \frac{\left|\frac{\alpha}{d} e^{\alpha\left(\frac{g}{d}\right)}\right|}{e^{\alpha\left(\frac{g}{d}\right)}} \Delta g \quad (\alpha < 0)$$

The expression $e^{\alpha(g/d)}$ comes never negative. The coupling coefficient has always a value between 1 and 0. The antenna diameter d is also always positive. Only the damping tor α is a negative value, and stay as absolute value. From this, following the result shown in Equation 18.

$$\frac{\Delta\kappa}{\kappa} \approx |\alpha| \frac{\Delta g}{d} \quad (\alpha < 0)$$

Equation 18 : Approximated relationship between the coupling coefficient errors and the distance error [1]

From Equation 18 is shown that the coupling coefficient error is proportional to the distance error. For this reason, below the distance error is replaced by the coupling coefficient error.

First, the error rate of the coupling coefficient will be determined. The error rate of the coupling coefficient is determined from Equation 5, Equation 6 and Equation 16. By using the Equation 5 and the Equation 6 can the coupling coefficient be defined as written in Equation 19.

$$\kappa = \frac{1}{Q}\sqrt{\frac{1}{\gamma_1}\frac{1+\Gamma}{1-\Gamma} - 1}$$

Equation 19 : Coupling coefficient based on the determination of the reflection coefficient

For the determination of the relative error from the coupling coefficient in relation to the reflection coefficient, must the Equation 19 to the reflection coefficient be derived. The result of the derivation must be divided to Equation 19 and multiplied by the error of the reflection coefficient.

$$\frac{\Delta\kappa}{\kappa} = \frac{1}{\kappa}\frac{\partial\kappa}{\partial\Gamma_{(\kappa)}}\Delta\Gamma = \frac{\left|\frac{1}{Q}\frac{1}{2\sqrt{\frac{1}{\gamma_1}\frac{1+\Gamma}{1-\Gamma} - 1}}\frac{1}{\gamma_1}\frac{(1-\Gamma)+(1+\Gamma)}{(1-\Gamma)^2}\right|}{\frac{1}{Q}\sqrt{\frac{1}{\gamma_1}\frac{1+\Gamma}{1-\Gamma} - 1}} * \Delta\Gamma$$

The amount can be omitted, as the result of potency, and a root is always positive or it must be positive. This allows a simplification.

$$\frac{\Delta\kappa}{\kappa} = \frac{\Delta\Gamma}{\left(\frac{1}{\gamma_1}\frac{1+\Gamma}{1-\Gamma} - 1\right)\gamma_1(1-\Gamma)^2} = \frac{\Delta\Gamma}{\left((1-\gamma_1) + \Gamma(1+\gamma_1)\right)(1-\Gamma)}$$

The final result is shown in Equation 20. κ is the real value, $\hat{\kappa}$ is the calculated value from the reflection coefficient. $\Delta\kappa$ is the absolute value and is defined as the difference between κ and $\hat{\kappa}$.

$$\frac{\Delta\kappa}{\kappa} = \frac{1}{\kappa}\frac{\partial\kappa}{\partial\Gamma_{(\kappa)}}\Delta\Gamma = \frac{\mathrm{a}|\Gamma| + b}{(\gamma_1 + 1)(\Gamma - \frac{\gamma_1 - 1}{\gamma_1 + 1})(1-\Gamma)}$$

Equation 20 : error rate of the coupling coefficient [1]

For the Equation 20 is valid:

$$\frac{\gamma_1 - 1}{\gamma_1 + 1} < \Gamma < 1, \mathrm{a} = 0.0178, \mathrm{b} = 0.004, \ \gamma_1 = \frac{R_1}{Z_0}$$

The error curve from Equation 20 can be calculated and displayed with Matlab. In the first effort is the coupling coefficient calculated from Equation 17. The calculated coupling coefficient will converted into the reflexion coefficient. For this conversion are the Equation 4, Equation 5 and the Equation 6 used. After this, the calculated reflection coefficient is used in Equation 20. In the different plots is always changed one of the following design parameters:

- The frequency f
- The number of turns N
- The antenna diameter d
- The resistance of the transmitter antenna R_1
- The resistance of the transmitter antenna R_2
- The impedance of the transmission line Z_0

This result is not detailed enough because the error in the edge cannot accurate be calculated with the result from Equation 17. One example is shown at Figure 7

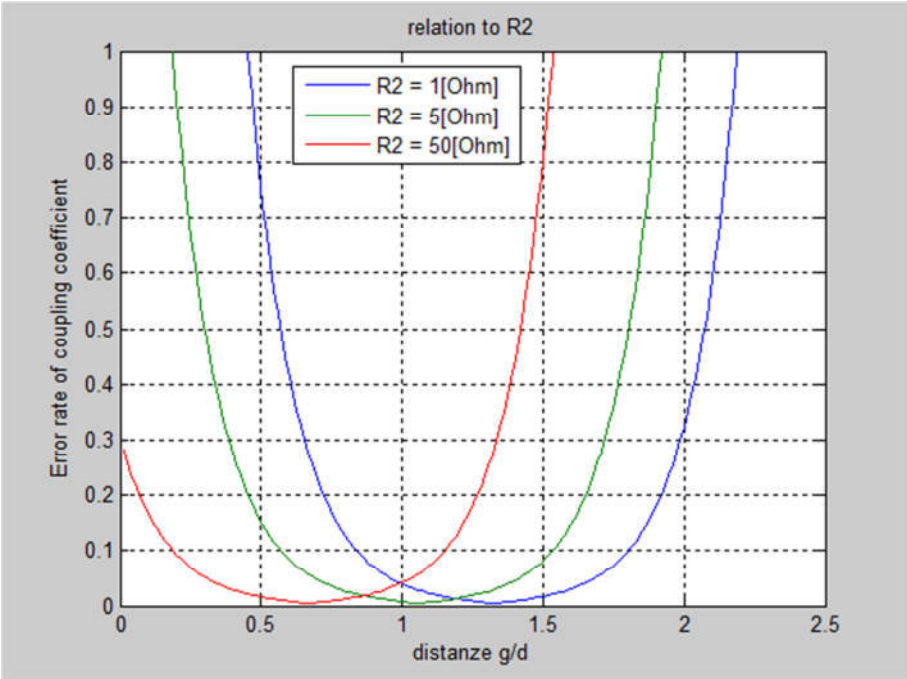

Figure 7 : Error curve based on the coupling coefficient of the Equation 17 for following model parameters: $Z_0 = 50[\Omega]$, $N = 4, f = 10[MHz]$, $d = 0.075[m]$, $R_1 = 1[\Omega], \alpha = -3$

For this analysis the curve has always the same form. Only the position of the curve change. The exact value for the damping factor α is also unknown and it must rough be estimated. The opening of the curve changes as a function of α.

Because the assessment from Equation 17 is not detailed enough, this equation is replaced to the Neumann equation, shown at Equation 14. The rest of the analysis rests same. The calculated coupling coefficient is converted into the reflection coefficient and this is used at Equation 20. The error rate of the coupling coefficient in relation to the distance is shown from Figure 8 to Figure 13. At these plots is always only one design parameter changed.

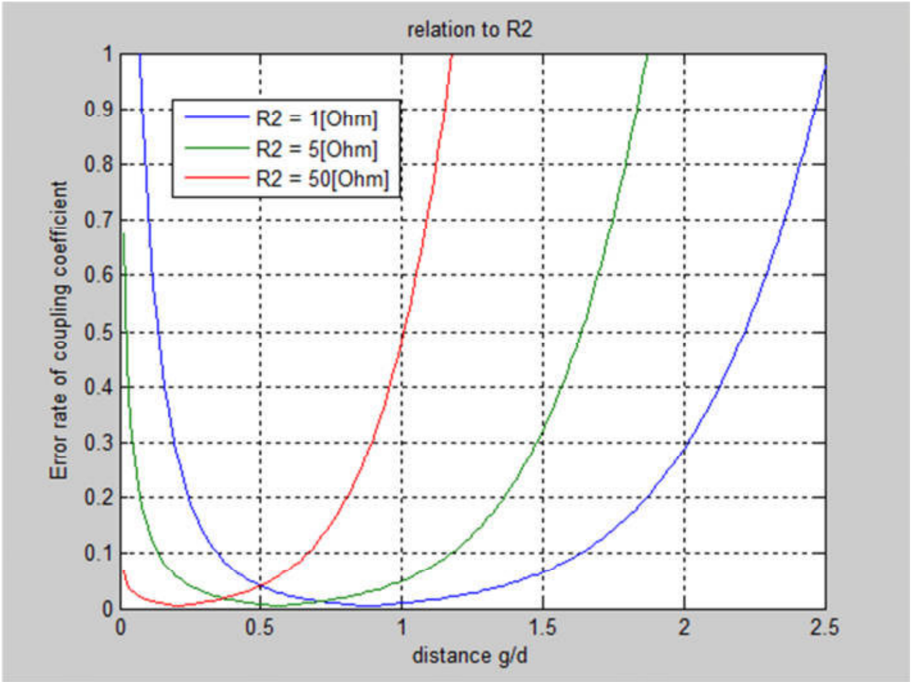

Figure 8 : Error curve for different values of R_2 and the following model parameters: $Z_0 = 50[\Omega],\ N = 4, f = 10[MHz],\ d = 0.075[m],\ R_1 = 1[\Omega]$

The error characteristic is independent of the Q-factor. However, the position of the error curve changes in relation to the Q-factor and thus the area of the effective range.

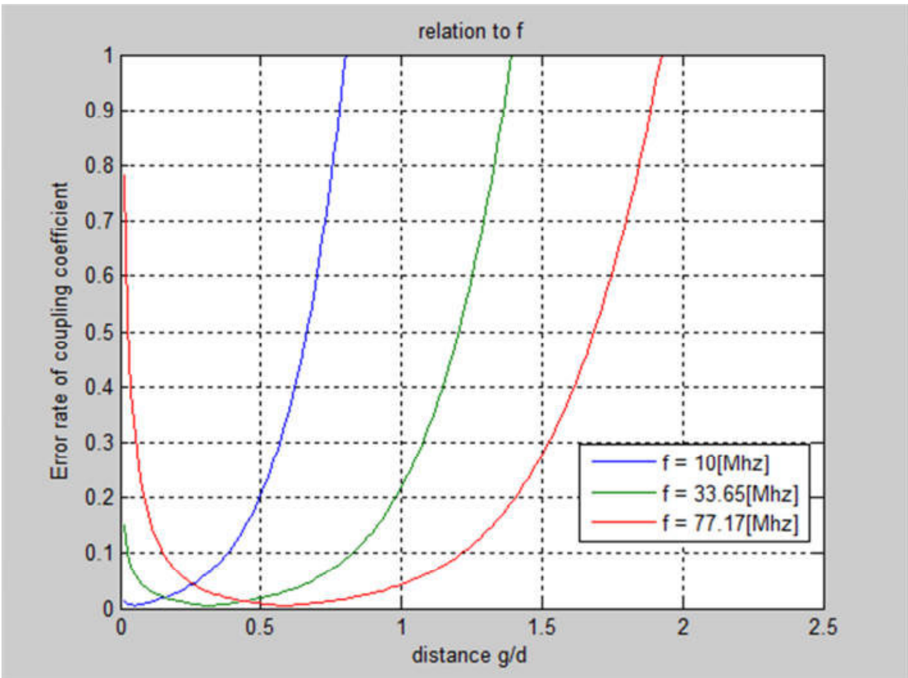

Figure 9 : Error curve for different values of f and the following model parameters: $Z_0 = 50[\Omega],\ N = 1,\ d = 0.075[m],\ R_1 = R_2 = 1[\Omega],$

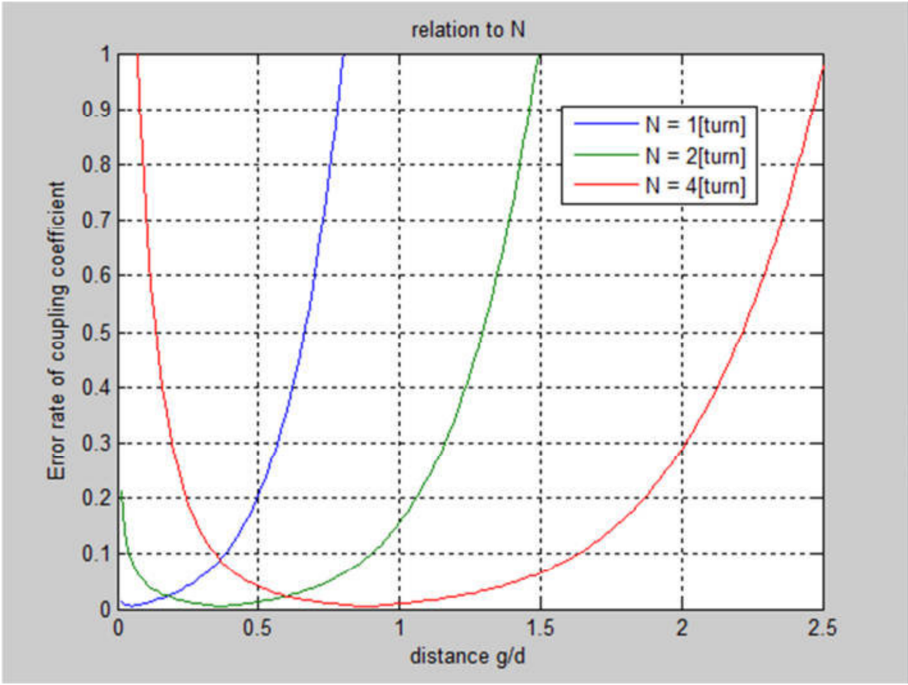

Figure 10 : Error curve for different values of N and the following model parameters: $Z_0 = 50[\Omega],\ f = 10[MHz],\ d = 0.075[m],\ R_1 = R_2 = 1[\Omega]$

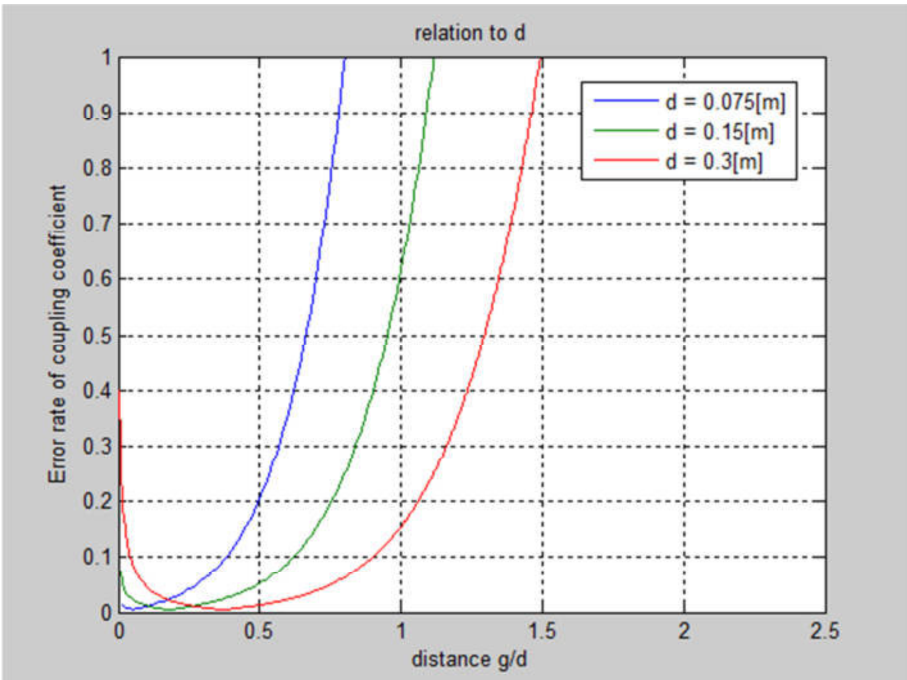

Figure 11 : Error curve for different values of d and the following model parameters: $Z_0 = 50[\Omega],\ N = 1, f = 10[MHz],\ R_1 = R_2 = 1[\Omega]$

The Figure 8 to Figure 13 was made by using the Matlab Funktion *error_Plot* t appendix 4. This function requires 6 under functions appendix 5 to appendix 10.

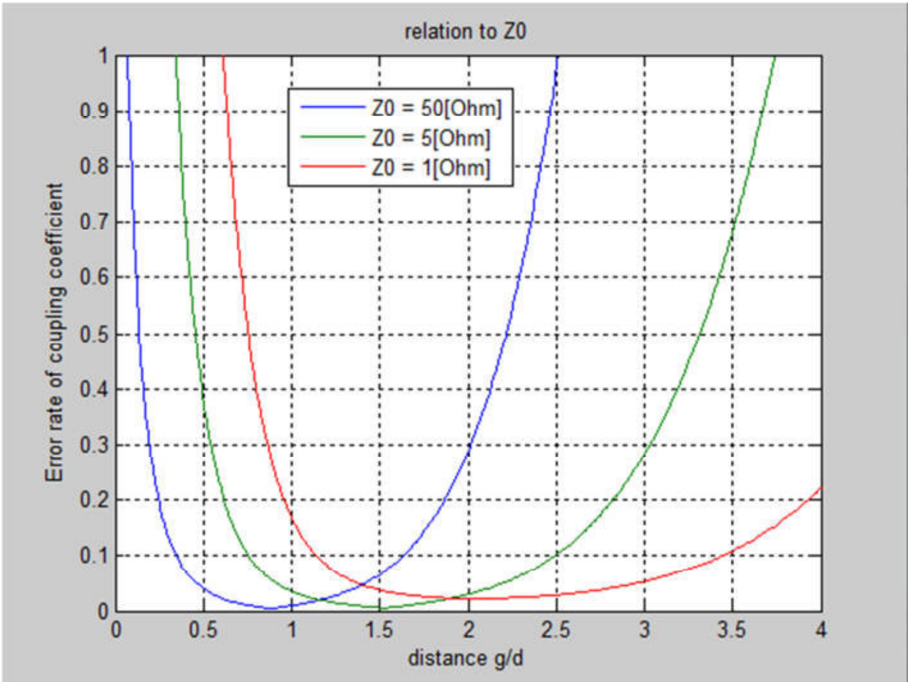

Figure 12 : Error curve for different values of Z_0 and the following model parameters: $f = 10[MHz],\ N = 4,\ \ d = 0.075[m],\ R_1 = \ R_2 = \ 1[\Omega]$

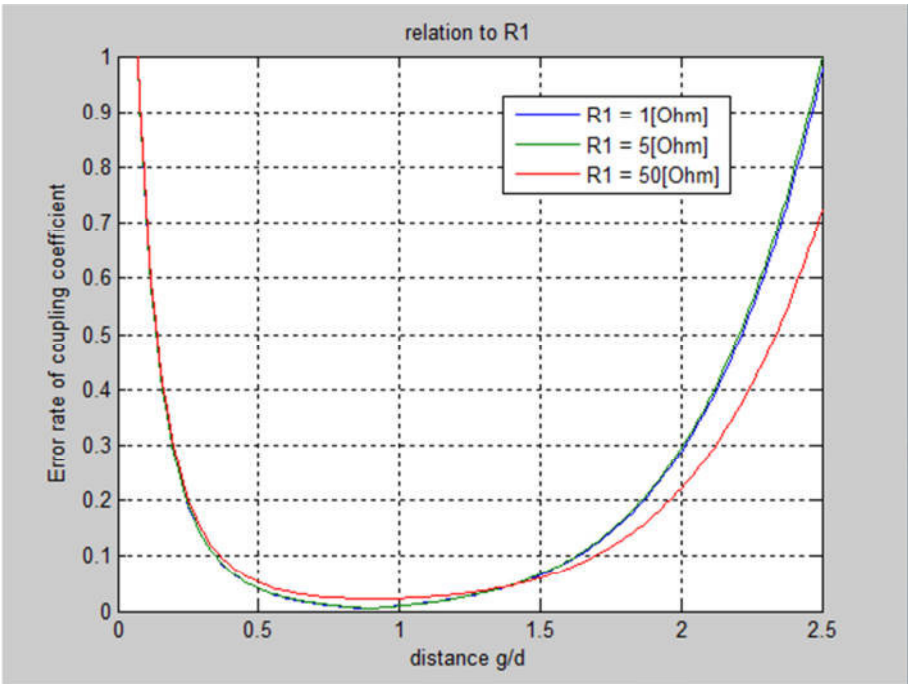

Figure 13 : Error curve for different values of R_1 and the following model parameters: $f = 10[MHz],\ N = 4,\ \ d = 0.075[m],\ R_2 = \ 1[\Omega]$

The error curve is from beginning convex downward up to an effective range. The error curve increase exponentially with increasing distance after overcrossing of this effective range. This behaviour is described in Equation 20. It's shown that the error comes bigger in near field and far field, if the reflection coefficient is in the near of the upper limit 1 and the lower limit $(\gamma\_1 - 1)/(\gamma\_1 + 1)$. This behaviour is more inspected below.

8.3.2 Determination of the minimal error point

The error curve must be exactly analysed, In order to find a suitable operating point. The equation from the error curve has two poles, Equation 20. The curve is only defined, for all values they are between these poles. The behaviour in the peripheral regions of the poles can be determined with the limit consideration

$$lim_{\Gamma\to\frac{\gamma_1-1}{\gamma_1+1}} \frac{\Delta\kappa}{\kappa} = \frac{a\frac{\gamma_1 - 1}{\gamma_1 + 1} + b}{0^+} = \infty$$

Equation 21 : Limit consideration of the lower limit

The reflection coefficient is in Equation 20 so defined that it aspiring from the positive side to the lower limit. As follows is observed the upper limit.

$$lim_{\Gamma\to 1} \frac{\Delta\kappa}{\kappa} = \frac{a + b}{0^+} = \infty$$

Equation 22 : Limit consideration of the upper limit

Thus, it's shown from Equation 21 and Equation 22 that's the error in the peripheral regions aspiring against undefined. This behaviour is also shown from the Figure 8 to Figure 13.

To find out in which region the minimum error point located is, must the Equation 20 be derived to the reflection coefficient. Since the reflection coefficient is in the amount, must the Equation 20 derived for 2 areas. From Figure 14 is shown that the reflection coefficient is a value between 1 and -1. The values for a und b are supposed as fix. For other values of a and b change the area of validity from γ_1.

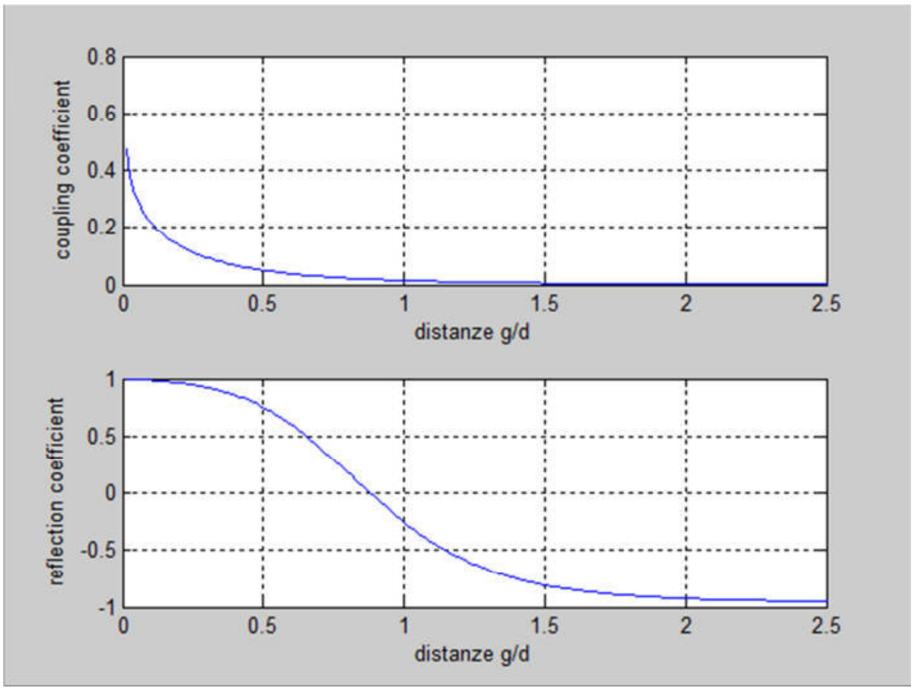

Figure 14 : Change of the reflection coefficient and the coupling coefficient as a function of the distance. This graph was created using the Matlab function *K_T* annex 11

To simplify the derivation, the denominator from Equation 20 will be multiplied out.

$$\frac{\Delta\kappa}{\kappa} = \frac{\mathrm{a}|\Gamma| + b}{-\Gamma^2(\gamma_1 + 1) + 2\Gamma\gamma_1 + 1 - \gamma_1}$$

Equation 23 : out multiplied formula for the error rate of the reflection coefficient

The area of validity is still given by:

$$\frac{\gamma_1 - 1}{\gamma_1 + 1} < \Gamma < 1, \mathrm{a} = 0.0178, \mathrm{b} = 0.004, \ \gamma_1 = \frac{R_1}{Z_0}$$

To obtaining the minimum value of the curve, must the derivation be equal zero. As first, the area which the reflection coefficient is ≥ 0 will be analysed. As result of the derivation is obtained a quadratic equation. This equation has only one defined zero point and this is defined as follows:

$$\Gamma_p = -\frac{b}{a} + \sqrt{\left(\frac{b}{a}\right)^2 + \frac{(\gamma_1 - 1) + 2\frac{b}{a}\gamma_1}{\gamma_1 + 1}}$$

Equation 24 : Minimum error in the positive range of the reflection coefficient

One zero point is not in the defined area, for this reason the only defined zero can be described as shown from Equation 24. γ_1 is the only variable coefficient in this equation. Γ_p can't be negative. Because this, must be locate a valid area for γ_1.

$$\gamma_1 \geq \frac{1}{1 + 2\frac{b}{a}} \approx 0.690$$

At this point the validity range of basic equation may not be forgotten:

$$\frac{\gamma_1 - 1}{\gamma_1 + 1} < \Gamma < 1 = -0{,}183 < \Gamma < 1, \qquad 0 \leq \Gamma_p < 1$$

Thus, all values of the Equation 24 are defined for $\gamma_1 \geq 0.69$.

Following is the area with a negative reflection coefficient inspected. For this, the Equation 23 must be derived for a negative reflection coefficient. Even in this case is the result a quadratic equation. For this equation is only one zero point defined for a negative reflection coefficient. This zero point is defined at the Equation 25.

$$\Gamma_n = \frac{b}{a} - \sqrt{\left(\frac{b}{a}\right)^2 - \frac{-(\gamma_1 - 1) + 2\frac{b}{a}\gamma_1}{\gamma_1 + 1}}$$

Equation 25 : Minimal error in the negative range of the reflection coefficient

γ_1 is the only variable coefficient in this equation. Γ_n can't be positive. For this reason, a range of validity for γ_1 must be defined.

$$\gamma_1 \geq \frac{1}{1 - 2\frac{b}{a}} \approx 1.816$$

Also in this case the validity range from the Equation 23 must be compared with this validity range.

$$\frac{\gamma_1 - 1}{\gamma_1 + 1} < \Gamma < 1 = 0{,}289 < \Gamma < 1 \neq -1 < \Gamma_n < 0$$

It is shown that's no one of the values from the Equation 25 are defined.

Finally, remains the question how the error behaves in the range of $0.690 > \gamma_1 \geq 0$. Since this area is not defined for an analytical consideration. The behaviour must be investigated from a simulation. The behaviour was analysed by using Matlab. The simulations shown that the minimum point of the reflection coefficient is approximate at the zero crossing as long the following values are valid $0.690 > \gamma_1 \geq 0$. Thus, the minimum point of error based on the value γ_1 is defined as follows:

$$\Gamma_{min} = \begin{cases} 0, & 0.690 > \gamma_1 \geq 0 \\ \Gamma_p, & \gamma_1 \geq 0.690 \end{cases}$$

Equation 26 : Determination of the reflection coefficient which the smallest error caused.

The error curve in relation to the reflection coefficient it is calculated with Matlab. For the calculation is the Equation 20 be used. With the same parameters is the minimum error point Γ_p calculated by using the Equation 24. The error rate in this point is obtained by inserting Γ_p into Equation 20. To verify the Equation 24, the obtained point is printed in the same chart as the error curve. The result is shown at the Figure 15. Since the discovered point actually is located at the minimum point of the curve, the Equation 24 can be regarded as correct. The graph of Figure 15 was created by using the Matlab function *Error_XaxisT* appendix 12

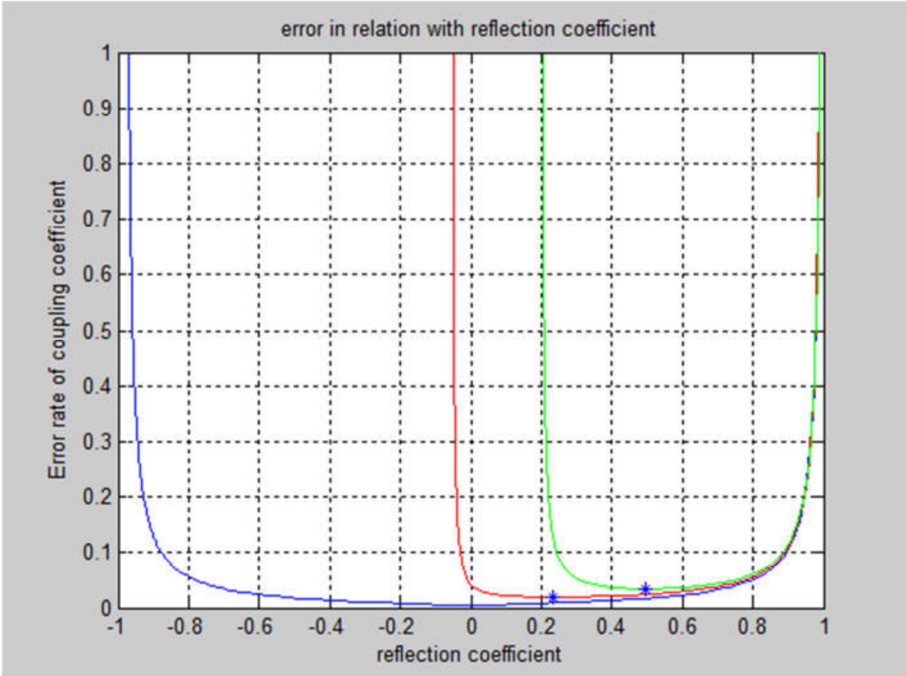

Figure 15 : blue $\gamma_1 = 0.01$, red $\gamma_1 = 0.9$, green $\gamma_1 = 1.5$

From Equation 19, the coupling coefficient can be determined by using the reflection coefficient. This relation can be used at the Equation 26 to determinate, for which value of the coupling coefficient the smallest error exist.

$$\kappa_{min} = \begin{cases} \frac{1}{Q}\sqrt{\frac{1}{\gamma_1} - 1}, & 0.690 > \gamma_1 \geq 0 \\ \frac{1}{Q}\sqrt{\frac{1}{\gamma_1}\frac{1+\Gamma_p}{1-\Gamma_p} - 1}, & \gamma_1 \geq 0.690 \end{cases}$$

Equation 27 : determining the value of the coupling coefficient which have the smallest error

The coupling coefficient comes with higher distance exponentially lower. Based on this, it's shown from the Equation 27 that the distance for the minimum error comes bigger if the Q-factor of the sensor increases. The Q-Factor is described from Equation 4. This equation contains also the resistance of the transmitter antenna R_1. This model Parameter is also contained at γ_1. Because γ_1 influence the error characteristic, it cannot be said that the Q-factor is an independent value. The inductivity of the transmitter and the receiver antenna is given based on their geometrical dimensions. Thus, it rest only the values of f, N, d and R_2 to increase the value of the Q-factor. The values f, N and d are in relation with the model error and this values are limited based on the model error analysis and limitation of the electrical length chapter 8.2. At the Equation 27 the Q-factor can also be written with the model parameters, this is shown in the Equation 28 and Equation 29.

Suppose the transmit antenna resistance R_1 correspond to the copper resistance of the coil. In this case, it can be assumed that $R_1 \approx 1\,[Ohm]$ and the line impedance corresponding $Z_0 \approx 50\,[\Omega]$, then applies $\gamma_1 \approx 0.02 < 0.69$. The value for the minimum error of the coupling coefficient is in this case defined as shown from Equation 28.

$$\kappa_{min} = \frac{1}{Q}\sqrt{\frac{1}{\gamma_1} - 1} = \sqrt{\frac{Z_0 - R_1}{\omega_0 L_1} * \frac{R_2}{\omega_0 L_2}}, \qquad 0.690 > \gamma_1 \geq 0$$

Equation 28 : Minimal error of the coupling coefficient

For values of $\gamma_1 \geq 0.690$ must be considered the Equation 29.

$$\kappa_{min} = \frac{1}{Q}\sqrt{\frac{1}{\gamma_1}\frac{1+\Gamma_p}{1-\Gamma_p} - 1} = \sqrt{\frac{R_2}{{\omega_0}^2 L_2 L_1}\left(Z_0 \frac{1+\Gamma_p}{1-\Gamma_p} - R_1\right)}, \qquad \gamma_1 \geq 0.690$$

Equation 29 : Minimal error of the coupling coefficient

From the Equation 28 und Equation 29 is shown that the effective range decrease if the receiver antenna resistance R_2 increase. This behaviour is confirmed from the Figure 8.

8.3.3 Characteristics of the distance error as a function of the distance

The theoretical consideration of the coupling coefficient error is also valid for the distance error. Because the error rate of the coupling coefficient is proportional to the distance error. This relation is shown at the Equation 18. To simplify the end of the analysis, Z_0 and R_1 are not included in the consideration. There are the following relations.

- The distance error is convex decreasing with distance up to a turning point. From this point the error increases again convex. This means that the error is in the near field as well as in the far field bigger.
- If Q is increased, increases also the distance between the transmitting and the receiving antenna for which is obtained the smallest distance errors.
- The distance error is bigger in the near field and smaller in the far field if the Q-factor is enlarged.

Finally it can be said, that the model parameter must be chooses based on the needed distance. Because the error curve show different areas for the effective range and the minimum error if the design parameters changes.

9 MEASURING OF THE DISTANCE SENSOR

For this measuring were produced two antennas. The measuring contains the equivalent impedance of the two coupled antennas Z_e and the distance between the centre points of the antennas g. The result is a table which contains different values of the impedance Z_e in relation with the distance. The transmitter antenna is supplied over an Agilent Technologies E-5061A Network analyser with a signal frequency of 10 [MHz], the receiver antenna is passive. The Network analyser shows the Smith-chart of the two coupled antennas. The equivalent impedance Z_e can be read from the display.

The transmitting antenna and the receiving antenna are threaded between two sticks. On one of the two sticks is a scale with a pitch of a 0.5 [cm] recorded. The centre of the transmitting antenna is fixed at zero, and the receiving antenna is moved in steps of 0.5 [cm]. The measurement setup is shown in Figure 16

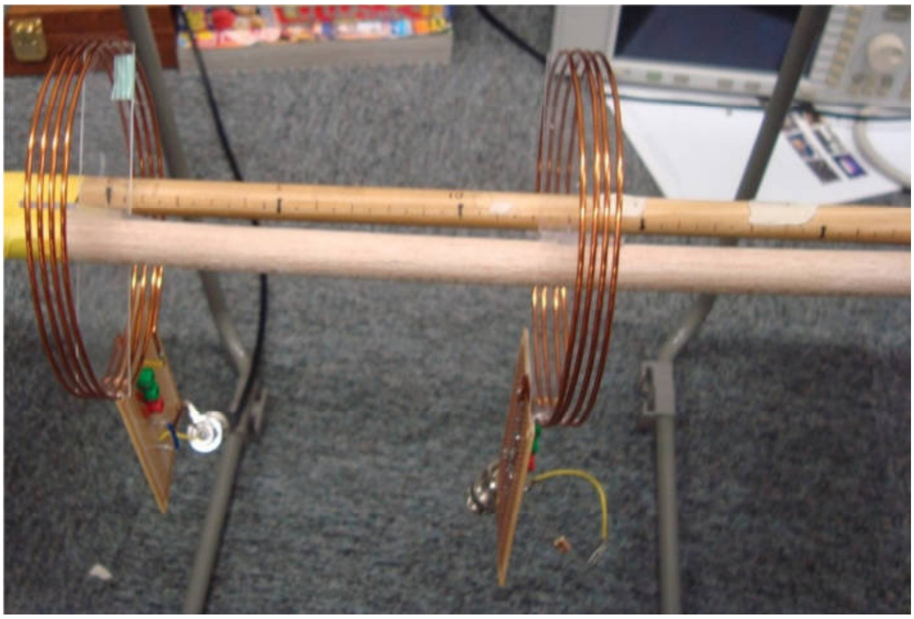

Figure 16 : measurement setup for the distance measuring

To keep the influence of foreign objects as small as possible, the sticks were placed between two chairs so that the coils are in the air. Important for an accurate calculation of the position is the precise as possible determination of the design parameters.

9.1 Measurement of the design parameters

The design parameters of the antennas must be measured. The inductivity can over a LC circuit and a known value of the capacity been determined. In this case is a capacity of 1 [nF] used. From this circuit must be measured the resonance frequency, and over the resonance frequency can be calculated the impedance for each antenna. The resonance frequency can be determined over a Smith-chart or a Bode-diagram. In this case was used the Smith-chart. This Smith-chart is made from an Agilent Technologies E-5061A Network analyser. For both antennas was the resonance frequency, for the capacity from above 2.9 [MHz]. Thus, the resonance frequency can be calculated from Equation 30.

$$L = \frac{1}{C * (2\pi f)^2}$$

Equation 30 : Calculation of the inductance based on the resonance frequency and a known capacitance.

From the calculated inductivity can be determined the needed capacity, for which is obtained the desired resonance frequency. In this case have the capacity a value in the order of 84 [pF]. The resonance frequency is defined from the Equation 31.

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Equation 31 : Determining the resonant frequency

The fine tuning is made with adjustable capacities. Based on this can the desired resonance frequency quite exactly been adjusted. As reference is used a Smith-chart, which is made from an Agilent Technologies E-5061A Network analyser. The marker must be set on the desired resonance frequency. The imaginary part of the antenna impedance must be zero and the real part is corresponding to the antenna resistance R_1 or R_2. The receiver antenna resistance R_2 can be increased via additional resistances. If the receiver antenna resistance R_2 increase, the Q-factor decreases. This is used to make measurements with different Q-factors. For this reason, the receiver antenna resistance is not set as fix. All other model parameters are shown in the Table 1.

Model Parameter		
Designation	Value	Unit
Resonance frequency f	10	MHz
Line impedance Z_0	50	Ω
Number of turns N	4	-
Antenna diameter d	0.1	m
Transmit antenna resistance R_1	1.41	Ω
receiver antenna resistance L_1	3.012	µH
Inductance of the receiving antenna L_2	3.012	µH

Table 1 : Design parameter from measuring setup 1

9.2 Determine the position based on a database interpolation

Since the position cannot be determined analytically another method must be found to determine the position. Therefore, the position determination is based on a database interpolation shown in this section.

First is made a database. This database is made from a function which is programmed with Matlab. The function name is *range* this function is shown into appendix 13. the design parameters from the antennas must be written into the quell code. If the function is executed, they draw a plot which shows the error rate of the coupling coefficient in relation which the distance. This plot based on the chapter 8.3.1. In this chapter is shown that the coupling coefficient error rate is proportional to the distance error. The chart provides information about the area in which the error is not too large to find a suitable measuring range.

The main purpose of this function is to write a table. This table is used for the interpolation of the position. This table looks as shown from Table 2.

Z_e	distance	coupling coefficient	Reflection coefficient	error %	Q-Factor
-	-	-	-	-	
					-

Table 2 : Example of the table for the position estimation

The first line contains only the Q-factor. The Q-factor is only one time be calculated and change not in relation with the distance. From the second line is written the values for the equivalent impedance Z_e, the distance g, the coupling coefficient κ, the reflection coefficient Γ and the error rate of the coupling coefficient in per cent. This function contains also a filter function. With this filter is possible to choose only the values which are under a certain error in per cent, for example only values which have an error under 10%.

The position estimation in self is made from the function *distance_calc* appendix 14. This function needs two tables. The first table contains the measured values for the equivalent impedance Z_e and the distance g. The second table contains the values from the first two columns of the Table 2, the calculated values for the equivalent impedance Z_e and the distance g. From this two tables estimate this function the position based on an interpolation. The formula for the interpolation is shown into the Equation 32

$$i = \frac{Z_{e}\_up - Z_e}{Z_{e}\_up - Z_{e}\_down}, \qquad \hat{g} = i * g\_down + (1 - i)g\_up$$

Equation 32 : Formula for the interpolation

The values for Z_{e}_up and Z_{e}_down are taken from the table for the calculated values, and correspond to the next higher and lower value compared with the measured value of Z_e. g_up is the calculated distance for the Z_{e}_up and g_down the distance for Z_{e}_down. The by this method found distances are compared with the measured distances. The function *distance_calc* gives out a table which contains the measured and the estimated distances. The function plots a graphic which compares the estimated values with the measured values. The plots from Figure 18 and the Figure 21 are made from this function. This function needs also a general offset value.

9.2.1 Results for a receiver antenna resistance without additional resistance

The measured receiver antenna resistance is $R_2 = 1.26\ [Ohm]$. The theoretical effective range is shown into Figure 17. Based on this graphic is choose a measurement range from 3 [cm] to 18 [cm].

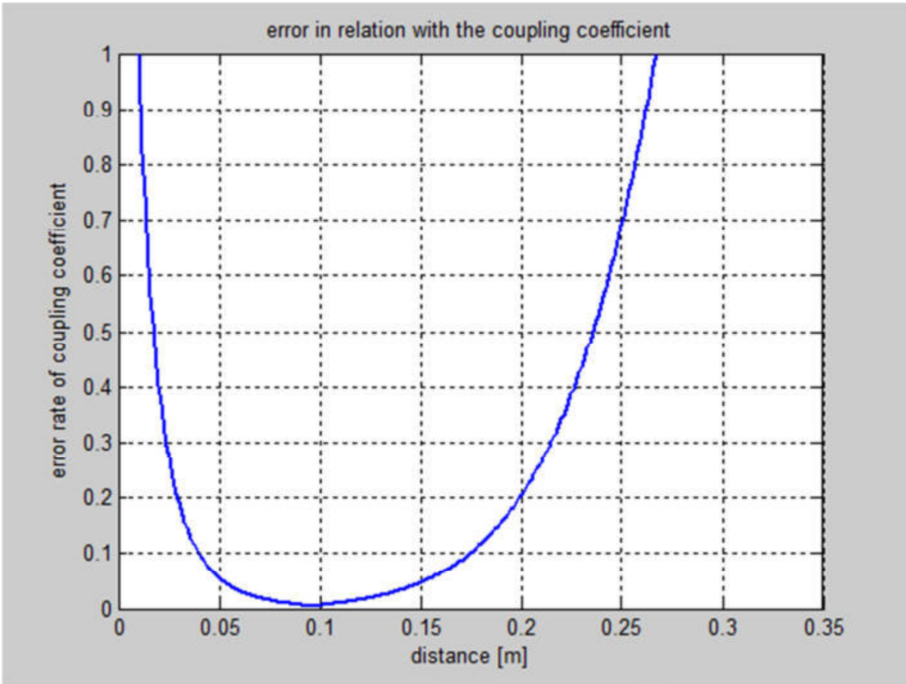

Figure 17 : Error rate in conjunction with the coupling coefficient for $R_2 = 1.26\ [Ohm]$

The calculated value is compared with the ideal value. This comparison is shown at Figure 18. Into the Figure 18 are the calculated values displaced with an offset of 4 [mm]. This offset is for a smaller receiver antenna resistance R_2 bigger. The resistance is with a value of 1.26 [Ohm] relatively small. The curve is displaced already for small changes of R_2 relatively fast. For higher values of R_2 is a measurement error of this value less important.

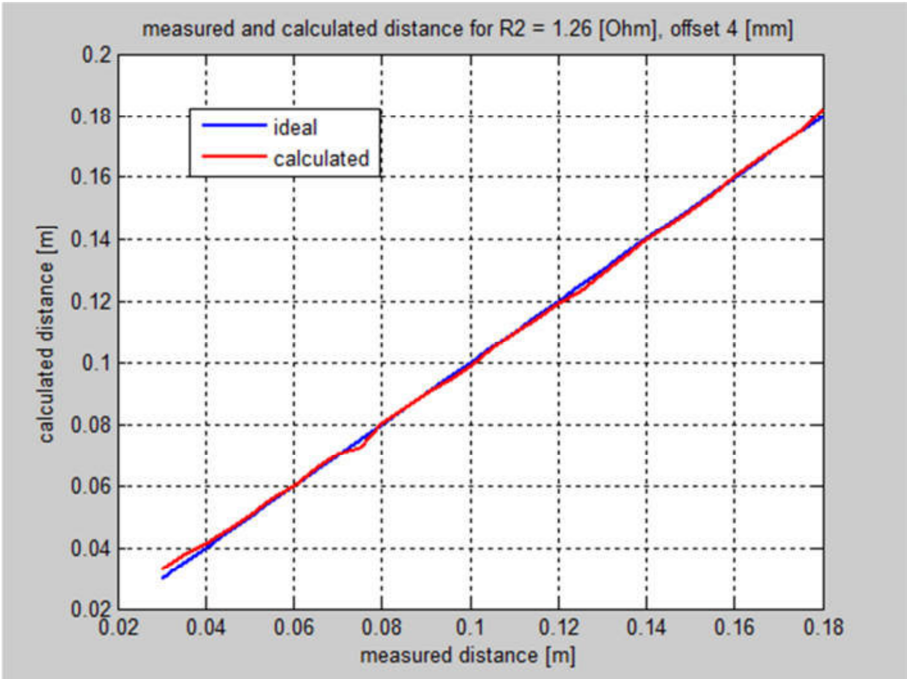

Figure 18 : Comparison of Calculated with Measured distances

The table with the results is into the appendix 1. After using the offset is the error in a range from 4.5 [cm] to 17.5 [cm] except of a few exceptions under 1%. These exceptions are related to reading errors or inaccurate positioning of the antennas. The measurements confirm that, the error at the edge of the range greater is.

On another day is determined the maximal measurable distance. The measured receiver antenna resistance is $R_2 = 1.56\ [Ohm]$. The experimental setup was designed for a greater distance. For this reason is the orientation less exact as for the previous measurement and thus the precision comes less exactly. The measurement steps are also higher for this measurement is a measuring step of 1 [cm] used.

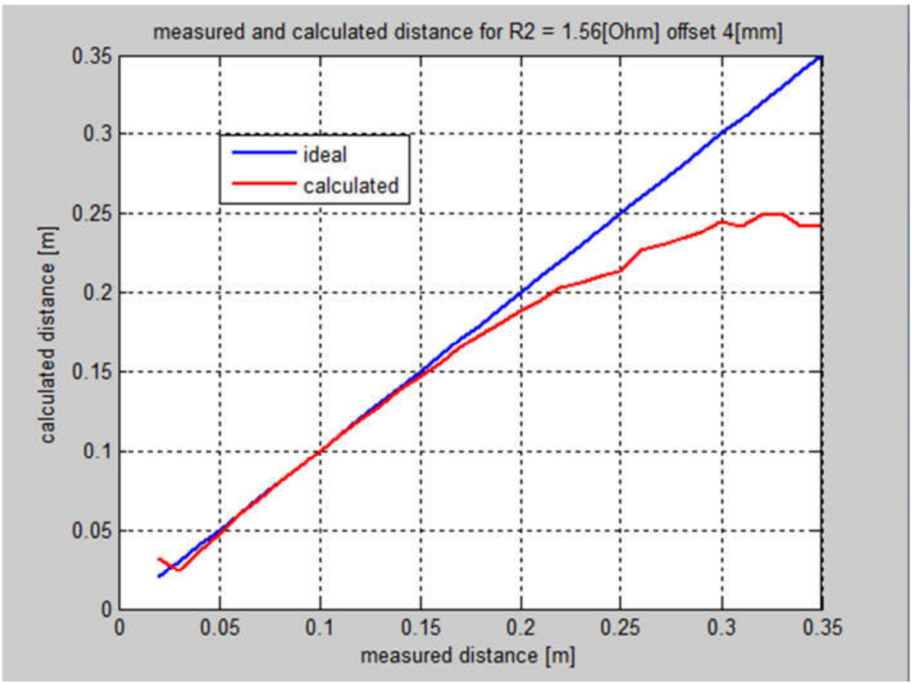

Figure 19 : Determination of the maximum measurable distance

The table with the results is into the appendix 2. The poorer accuracy of the measurement is not disturbing in this case, because it's wanted to determine the maximum measurable distance. From the figure 19 can be seen that the maximum distance is at 30 [cm]. For greater distances, the calculated value remains at 30 [cm].

9.2.2 Results for a receiver antenna resistance of $\mathrm{R}_2 = 46.64\ [Ohm]$

The theoretical effective range is shown into Figure 20. Based on this graphic is choose a measurement range from 2 [cm] to 13 [cm]. The upper section was little increased for a better checking of the error behaviour in the border area.

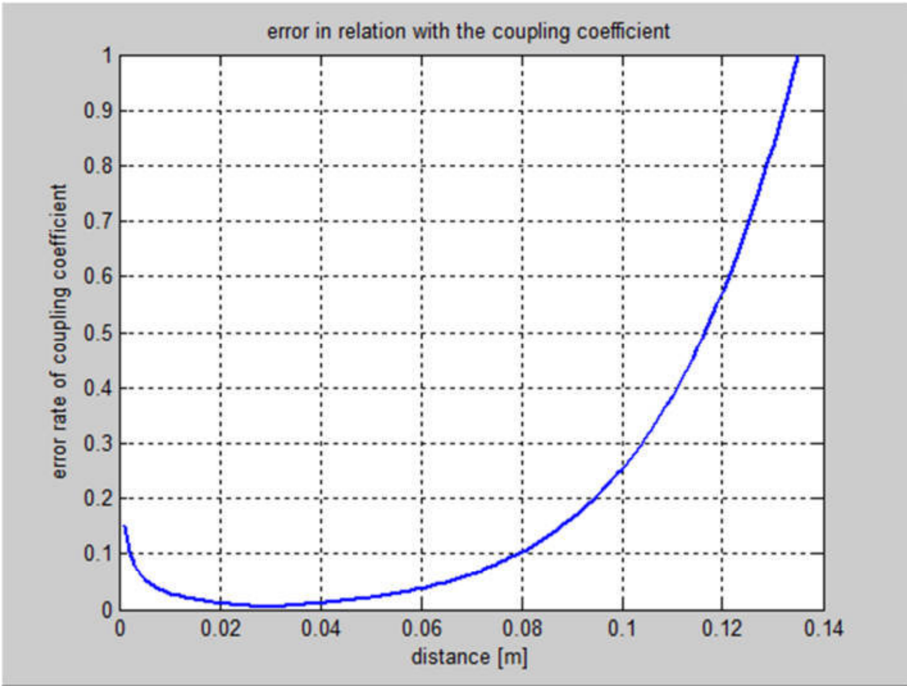

Figure 20 : Error rate in conjunction with the coupling coefficient for $R_2 = 46.64\ [Ohm]$

The calculated value is again compared with the ideal value. This comparison is shown at Figure 21. Into the Figure 21 are the calculated values displaced with an offset of 3 [mm].

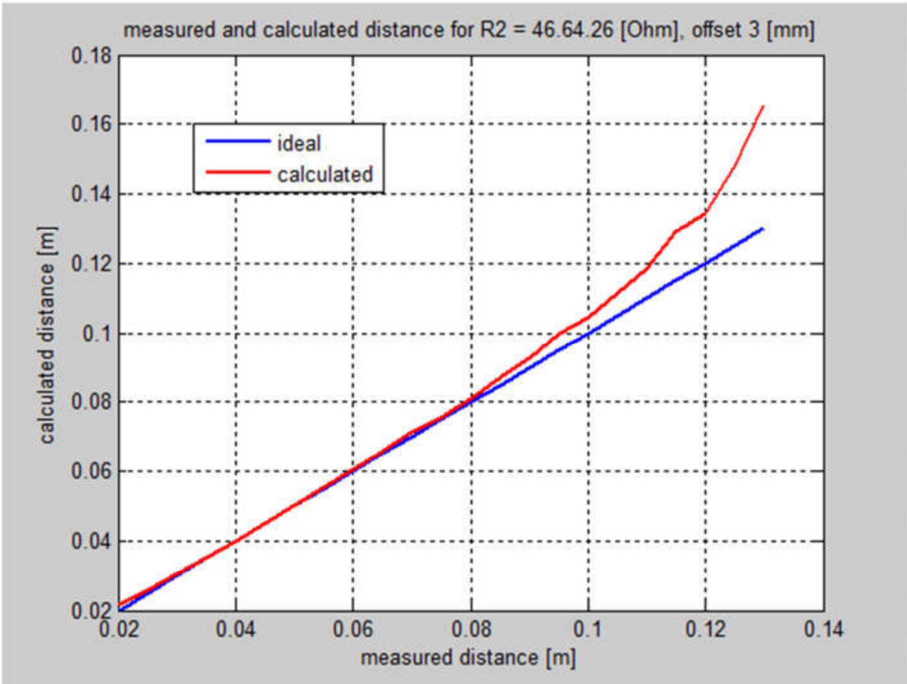

Figure 21 : Comparison of Calculated with Measured distances

The table with the results is in the appendix 3. After using the offset is the error in a range from 3.5 [cm] to 7.5 [cm] except of a two exceptions under 1%. These exceptions are related to reading errors or inaccurate positioning of the antennas. The measurements confirm that, the error at the edge of the range greater is. Especially in the upper region of the measurement is the error larger. The error behaviour in the lower area could not be determined closer since 2 [cm] is the lowest measurable distance due to the geometrical dimensions of the antennas.

10 ALTERNATIVE MEASUREMENT OF $\mathrm{Z_e}$

This section shows a way for measuring the equivalent impedance Z_e without using a network analyser. This is important for building a sensor, which can independent operate. One possibility is, measuring the tension and the current at the entrance of the transmitter antenna. In this case, the measuring point is in front of the transmitter resistance R_1 shown from Figure 2.

10.1 Simulation

For the verification of the possibility to determine the impedance Ze over the tension and the current is made a simulation with Ltspice 4. The circuit as shown from Figure 2 is used. The mutual inductivity L_m is defined from Equation 2. The coupling coefficient is calculated via the function *range* appendix 13. The model parameters are the same as those used in the section 9. The resistance from the receiver antenna R_2 is 46.64 [Ohm]. The simulation is shown into Figure 22.

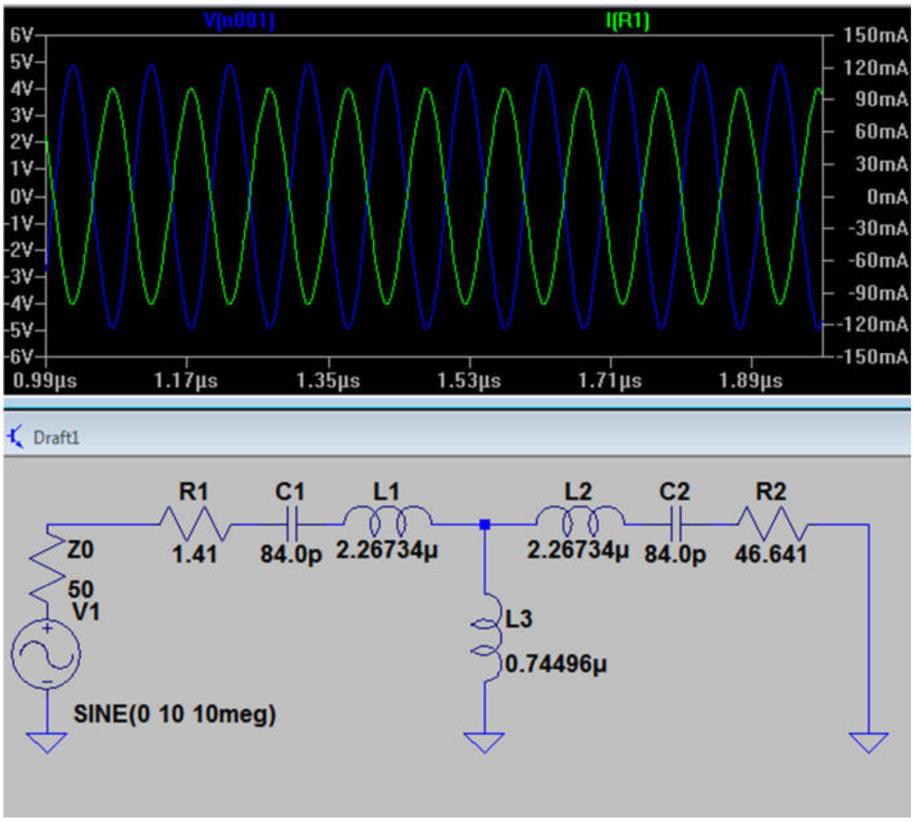

Figure 22 : Simulation to determine Z_e

For the calculation of the coupling coefficient was used a distance of 3 [cm]. The measured RMS value for the tension was 3.43 [V] and the RMS value for the current was 70.18 [mA]. For this values has the amplitude for the impedance $\mathrm{Z_e}$ a value of 48.86 [Ohm]. From the table for the interpolation has $\mathrm{Z_e}$ a value of 48.23 [Ohm].

Thus the simulation shows that it's possible to determine the equivalent impedance $\mathrm{Z_e}$ by using this method.

10.2 Measurement

Unfortunately it was not possible to measure the current because no suitable measuring instrument was available and the time was not sufficient to get a suitable instrument. The voltage was measured with a Tektronix TDS 2004C oscilloscope. For this reason, it cannot be said that this method also works in reality

11 CONCLUSION

The measurements have shown that is possible to build a distance sensor, which based on two antennas. Given the manufacturing tolerances of the antennas and the possible reading errors, the sensor have after adding a general offset a good accuracy.

Since the position is calculated the measurements confirm the accuracy of the formulas. The error behaviour in the peripheral regions could also be confirmed. This proves that the error analysis is also correct.

It has been shown, that's the equivalent impedance of the antenna Z_e can be theoretically determined based on a voltage and current measurement at the entrance of the transmitter antenna. This could not be confirmed in practice, due to time constraints.

To understand how the sensor works has took a lot of time. For this reason, the time was not enough to perform additional measurements, and to measure the position sensor too.

For the calculation of the position it is important to know the resistance of the receiving antenna R_2 as accurately as possible, since the calculation is no longer correct even for small deviations. This consists for some setups of to the copper resistance of the coil. In the case of energy transmission changes the temperature of the coils and with this the value of the receiver antenna resistance R_2.

The effective range of this sensor can be changed. However, it must be said that the precision at short distances came less exactly if the sensor is configured for higher distances

The coupling coefficient decreases exponentially. If it come too small, the measurement is less exactly or impossible. Therefore, it is difficult to produce a sensor for greater distances.

These three characteristics, it is important to get under control to use this sensor successfully. For properties where accuracy does not play a major role is the effective range the total measurable area. For example, it is for a robot in a building not important to know his position of the mm exact.

12 DATE & SIGNATURE

Date: Summermatter Orlando

13 APPENDICES

- Appendix 1: Measuring 1, small receiving antennas resistance R_2
- Appendix 2: Measuring 2, small receiving antennas resistance R_2 determination of the maximum distance
- Appendix 3: Measuring 3, big receiving antennas resistance R_2
- Appendix 4: Code of the Matlab function *error_Plot*
- Appendix 5: Code of the Matlab function *change_d*
- Appendix 6: Code of the Matlab function *change_f*
- Appendix 7: Code of the Matlab function *change_N*
- Appendix 8: Code of the Matlab function *change_R1*
- Appendix 9: Code of the Matlab function *change_R2*
- Appendix 10: Code of the Matlab function *change_Z0*
- Appendix 11: Code of the Matlab function *K_T*
- Appendix 12: Code of the Matlab function *Error_XaxisT*
- Appendix 13: Code of the Matlab function *range*
- Appendix 14: Code of the Matlab function *distance_calc*

14 USED EQUIPMENT

Agilent Technologies E-5061A Netzwerkanalysator

Tektronix TDS 2004C Oszilloskop

15 REFERENCES

[1] N. Sosuke, *Design Evaluation of Position Sensor based on Magnetic Resonance Coupling,* Tokyo, Japan: Hashimoto laboratory, Chuo university, 2011.

[2] O. Beucher, MATLAB und Simulink Eine kursorientierte Einführung, mitp, 2013.

[3] Agilent Technology, *ENA-L RF Network Analyzer E5061A and E062A Data Sheet.*

Measuring 1

Q	R1 [Ω]	R2 [Ω]	L1 & L2 [uH]	Offset [m]
141.98	1.41	1.26	3.012	0.004

Ze measured [Ω]			distance [m]		error	
real part	imaginary part	module	measured	calculet	error [m]	error [%]
1560.200	-1049.900	1880.562	0.0300	0.0328	0.0028	9.500
1198.900	-631.500	1355.047	0.0350	0.0376	0.0026	7.526
998.000	-378.200	1067.258	0.0400	0.0413	0.0013	3.189
801.930	-196.490	825.651	0.0450	0.0453	0.0003	0.750
606.790	-122.060	618.945	0.0500	0.0501	0.0001	0.129
445.710	-87.444	454.207	0.0550	0.0554	0.0004	0.644
348.030	-48.606	351.408	0.0600	0.0599	-0.0001	-0.182
258.400	-38.654	261.275	0.0650	0.0653	0.0003	0.496
201.340	-28.484	203.345	0.0700	0.0701	0.0001	0.096
157.270	90.323	181.362	0.0750	0.0723	-0.0027	-3.614
120.820	-7.746	121.068	0.0800	0.0804	0.0004	0.486
97.940	-3.575	98.005	0.0850	0.0848	-0.0002	-0.241
78.388	-5.372	78.572	0.0900	0.0895	-0.0005	-0.504
63.970	0.630	63.973	0.0950	0.0941	-0.0009	-0.953
52.533	-1.435	52.553	0.1000	0.0986	-0.0014	-1.414
40.516	-1.512	40.544	0.1050	0.1047	-0.0003	-0.268
33.235	-2.683	33.343	0.1100	0.1095	-0.0005	-0.435
27.744	-1.348	27.777	0.1150	0.1142	-0.0008	-0.733
22.886	-0.113	22.886	0.1200	0.1193	-0.0007	-0.620
19.999	-0.726	20.012	0.1250	0.1229	-0.0021	-1.675
16.286	-0.814	16.306	0.1300	0.1287	-0.0013	-1.003
13.549	-1.040	13.589	0.1350	0.1341	-0.0009	-0.674
11.365	-0.644	11.383	0.1400	0.1396	-0.0004	-0.289
9.773	-0.678	9.797	0.1450	0.1445	-0.0005	-0.352
8.505	-0.873	8.549	0.1500	0.1491	-0.0009	-0.571
7.347	-0.409	7.358	0.1550	0.1546	-0.0004	-0.290
6.331	-0.141	6.332	0.1600	0.1603	0.0003	0.191
5.548	-0.349	5.559	0.1650	0.1656	0.0006	0.385
4.993	-0.029	4.993	0.1700	0.1703	0.0003	0.185
4.491	-0.047	4.491	0.1750	0.1752	0.0002	0.135
3.914	-0.199	3.919	0.1800	0.1821	0.0021	1.172

Measuring 2 maximum measurable distance

Q	R1 [Ω]	R2 [Ω]	L1 & L2 [uH]	Offset [m]		
141.98	1.27	1.56	3.012	0.004		
Ze measured [Ω]			distance		error	
real part	imaginary part	module	measured [m]	calculet [m]	error [m]	error [%]
1638.2	-2.6862	1638.2022	0.02	0.0318	0.0118	58.940
2837.2	-14.503	2837.23707	0.03	0.0245	-0.0055	-18.262
1024.3	623.51	1199.1477	0.04	0.0363	-0.0037	-9.310
424.99	371.69	564.597163	0.05	0.0480	-0.0020	-3.905
201.68	204.56	287.262277	0.06	0.0597	-0.0003	-0.522
115.32	126.34	171.057002	0.07	0.0693	-0.0007	-0.992
64.866	72.392	97.2018499	0.08	0.0805	0.0005	0.675
40.375	47.544	62.3744544	0.09	0.0900	0.0000	-0.039
26.816	30.923	40.9307926	0.1	0.0995	-0.0005	-0.495
17.608	19.551	26.3112764	0.11	0.1102	0.0002	0.207
12.423	14.259	18.9116369	0.12	0.1188	-0.0012	-0.975
8.7333	10.318	13.5178272	0.13	0.1282	-0.0018	-1.366
6.3208	7.3539	9.6970283	0.14	0.1383	-0.0017	-1.186
4.8569	5.7071	7.49402882	0.15	0.1469	-0.0031	-2.053
3.8767	4.4734	5.91946876	0.16	0.1555	-0.0045	-2.790
3.102	3.5215	4.69290595	0.17	0.1650	-0.0050	-2.962
2.5511	2.9786	3.92175588	0.18	0.1732	-0.0068	-3.805
2.2388	2.5037	3.35868116	0.19	0.1811	-0.0089	-4.694
1.9372	2.2394	2.96102283	0.2	0.1883	-0.0117	-5.831
1.728	2.0154	2.65477328	0.21	0.1954	-0.0146	-6.947
1.5924	1.7929	2.39796334	0.22	0.2029	-0.0171	-7.777
1.5519	1.6862	2.29165094	0.23	0.2066	-0.0234	-10.179
1.4906	1.6156	2.19819283	0.24	0.2102	-0.0298	-12.408
1.4502	1.5536	2.1252654	0.25	0.2134	-0.0366	-14.655
1.3439	1.3142	1.87967786	0.26	0.2268	-0.0332	-12.780
1.2824	1.3171	1.83828784	0.27	0.2296	-0.0404	-14.945
1.277	1.2449	1.78339704	0.28	0.2339	-0.0461	-16.481
1.2395	1.223	1.74128954	0.29	0.2374	-0.0526	-18.121
1.2109	1.138	1.66172284	0.3	0.2454	-0.0546	-18.206
1.251	1.1476	1.69764153	0.31	0.2416	-0.0684	-22.068
1.2247	1.0761	1.63030098	0.32	0.2490	-0.0710	-22.175
1.1906	1.1046	1.62409037	0.33	0.2498	-0.0802	-24.300
1.2139	1.1753	1.68963999	0.34	0.2424	-0.0976	-28.705
1.2355	1.1501	1.68795446	0.35	0.2426	-0.1074	-30.693

Measuring 3

Q	R1 [Ω]	R2 [Ω]	L1 & L2 [uH]	Offset [m]
23.34	1.41	46,.64	3.012	0.003

Ze measured [Ω]			distance [m]		error	
real part	imaginary part	module	measured	calculet	error [m]	error [%]
112.060	-15.138	113.078	0.020	0.0030	-0.0170	-85.000
80.011	-6.627	80.285	0.025	0.0030	-0.0220	-88.000
56.474	-4.224	56.632	0.030	0.0030	-0.0270	-90.000
41.763	-2.899	41.863	0.035	0.0030	-0.0320	-91.429
30.623	-2.076	30.693	0.040	0.0030	-0.0370	-92.500
22.686	-1.485	22.735	0.045	0.0030	-0.0420	-93.333
17.028	-1.238	17.073	0.050	0.0030	-0.0470	-94.000
13.070	-0.945	13.104	0.055	0.0030	-0.0520	-94.545
9.937	-0.810	9.969	0.060	0.0030	-0.0570	-95.000
7.988	-0.735	8.022	0.065	0.0030	-0.0620	-95.385
6.320	-0.517	6.341	0.070	0.0030	-0.0670	-95.714
5.290	-0.567	5.321	0.075	0.0030	-0.0720	-96.000
4.338	-0.542	4.371	0.080	0.0030	-0.0770	-96.250
3.590	-0.455	3.619	0.085	0.0030	-0.0820	-96.471
3.108	-0.433	3.138	0.090	0.0030	-0.0870	-96.667
2.635	-0.408	2.666	0.095	0.0030	-0.0920	-96.842
2.412	-0.335	2.435	0.100	0.0030	-0.0970	-97.000
2.151	-0.305	2.172	0.105	0.0030	-0.1020	-97.143
1.956	-0.309	1.980	0.110	0.0030	-0.1070	-97.273
1.765	-0.279	1.787	0.115	0.0030	-0.1120	-97.391
1.700	-0.284	1.724	0.120	0.0030	-0.1170	-97.500
1.580	-0.283	1.605	0.125	0.0030	-0.1220	-97.600
1.484	-0.319	1.518	0.130	0.0030	-0.1270	-97.692

```
function [ output_args ] = error_Plot( aprox, single_plot )
%This Function Plot the Error_Rate
%aprox = 1 is for the easy aprosimation
%aprox = default is for the aproximation with the Neumann Formulla
%single_plot = 1 Plot each chart in a new Figur
%single_plot = deafault Plot all charts on the same figure
%
%   The calculation for the error curves is made by under-functions.
%   For each design parameter exist a function, which makes the
%   calculation. for the different design parameters are used the
%   following under-functions:
%
%   frequencie f                      =>      change_f
%   entry impedanz Z0                 =>      change_Z0
%   transmitter resistance R1         =>      change_R1
%   receiver resistance R2            =>      change_R2
%   diameter d                        =>      change_d
%   number of turns N                 =>      change_N

%--------------------------------------------------------------------------
%error calculation for different frequencies
%--------------------------------------------------------------------------

x_f = [];
k_f = [];

%the frequencies can be entered here

f_f = [10e6 33.65e6 77.17e6];

%this loop gives the value one by one to the underfunction for
%the calculation.

for i=1:1:3
 [x_f,k_f(i,:)] = change_f(f_f(i),aprox);

end;

%with this switch can be chosen the plot option. if 1 all curves for one
%parameter are plotted into a new window

switch single_plot
    case 1
         figure
    otherwise
         subplot(3,2,1);
end;

%plot the curves and make the labeling

 plot(x_f,k_f,'LineWidth',1.5);
 ylim([0 1]);
 xlim([0 2.5]);
 title('relation to f');
 xlabel('distance g/d');
```

```
 ylabel('Error rate of coupling coefficient');
 legend('f = 10[Mhz]','f = 33.65[Mhz]','f = 77.17[Mhz]');
 grid

%---------------------------------------------------------------------------
%error calculation for different values of the entry impedanz Z0
%---------------------------------------------------------------------------

 x_Z0 = [];
 k_Z0 = [];

 %the diverent impedances can be entered here

 f_Z0 = [50 5 1];

%this loop gives the value one by one to the underfunction for
%the calculation.

for i=1:1:3
 [x_Z0,k_Z0(i,:)] = change_Z0(f_Z0(i),aprox);

end

%with this switch can be chosen the plot option. if 1 all curves for one
%parameter are plotted into a new window

switch single_plot
    case 1
         figure
    otherwise
         subplot(3,2,2);
end;

%plot the curves and make the labeling

 plot(x_Z0,k_Z0,'LineWidth',1.5);
 ylim([0 1]);
 xlim([0 4]);
 xlabel('distance g/d');
 ylabel('Error rate of coupling coefficient');
 title('relation to Z0');
 legend('Z0 = 50[Ohm]','Z0 = 5[Ohm]','Z0 = 1[Ohm]');
 grid

%---------------------------------------------------------------------------
%error calculation for different values of the transmitter resistance R1
%---------------------------------------------------------------------------

 x_R1 = [];
 k_R1 = [];

%the diverent resistances can be entered here

 f_R1 = [1 5 50];
```

```
%this loop gives the value one by one to the underfunction for
%the calculation.

for i=1:1:3
 [x_R1,k_R1(i,:)] = change_R1(f_R1(i),aprox);

end

%with this switch can be chosen the plot option. if 1 all curves for one
%parameter are plotted into a new window

switch single_plot
    case 1
         figure
    otherwise
         subplot(3,2,3);
end;

%plot the curves and make the labeling

 plot(x_R1,k_R1,'LineWidth',1.5);
 ylim([0 1]);
 xlim([0 2.5]);
 xlabel('distance g/d');
 ylabel('Error rate of coupling coefficient');
 title('relation to R1');
 legend('R1 = 1[Ohm]','R1 = 5[Ohm]','R1 = 50[Ohm]');
 grid

%---------------------------------------------------------------------------
%error calculation for different values of the receiver resistance R2
%---------------------------------------------------------------------------

 x_R2 = [];
 k_R2 = [];

%the diverent resistances can be entered here

 f_R2 = [1 5 50];

%this loop gives the value one by one to the underfunction for
%the calculation.

for i=1:1:3
 [x_R2,k_R2(i,:)] = change_R2(f_R2(i),aprox);

end

%with this switch can be chosen the plot option. if 1 all curves for one
%parameter are plotted into a new window

switch single_plot
    case 1
         figure
    otherwise
```

```
        subplot(3,2,4);
end;

%plot the curves and make the labeling

 plot(x_R2,k_R2,'LineWidth',1.5);
 ylim([0 1]);
 xlim([0 2.5]);
 xlabel('distance g/d');
 ylabel('Error rate of coupling coefficient');
 title('relation to R2');
 legend('R2 = 1[Ohm]','R2 = 5[Ohm]','R2 = 50[Ohm]');
 grid

%-----------------------------------------------------------------------------
%error calculation for different values of the diameter d
%-----------------------------------------------------------------------------

 x_d = [];
 k_d = [];

%the diverent distances can be entered here

 f_d = [0.075 0.15 0.3];

%this loop gives the value one by one to the underfunction for
%the calculation.

for i=1:1:3
 [x_d(i,:),k_d(i,:)] = change_d(f_d(i),aprox);

end

%with this switch can be chosen the plot option. if 1 all curves for one
%parameter are plotted into a new window

switch single_plot
    case 1
        figure
    otherwise
        subplot(3,2,5);
end;

%plot the curves and make the labeling

 plot( x_d(1,:),k_d(1,:),x_d(2,:),k_d(2,:),x_d(3,:),k_d(3,:),...
      'LineWidth',1.5);
 ylim([0 1]);
 xlim([0 2.5]);
 xlabel('distance g/d');
 ylabel('Error rate of coupling coefficient');
 title('relation to d');
 legend('d = 0.075[m]','d = 0.15[m]','d = 0.3[m]');
 grid
```

```
%-----------------------------------------------------------------------------
%error calculation for different values for the number of turns N
%-----------------------------------------------------------------------------

 x_N = [];
 k_N = [];

%the diverent numbers of turns can be entered here

 f_N = [1 2 4];

%this loop gives the value one by one to the underfunction for
%the calculation.

for i=1:1:3
 [x_N,k_N(i,:)] = change_N(f_N(i),aprox);

end

%with this switch can be chosen the plot option. if 1 all curves for one
%parameter are plotted into a new window

switch single_plot
    case 1
         figure
    otherwise
         subplot(3,2,6);
end;

%plot the curves and make the labeling

 plot(x_N,k_N,'LineWidth',1.5);
 ylim([0 1]);
 xlim([0 2.5]);
 xlabel('distance g/d');
 ylabel('Error rate of coupling coefficient');
 title('relation to N');
 legend('N = 1[turn]','N = 2[turn]','N = 4[turn]');
 grid
end
```

```
function [ x, k_error] = change_d( d,aprox )
%change_d calcuate the error rate for the couppling coeffizient in relation
%to the diameter d
%
%this function gives back the error rate of the couppling coeffizient
%and the free dimensional distanze g/d betwen the antennas.

f = 10e6;                   %frequency [Hz]
Z0 = 50;                    %impedanz of the transmission line [Ohm]
N = 1;                      %number of turns []
R2 = 1;                     %resistance of the receiver antenna [Ohm]
R1 = 1;                     %resistance of the transmitter antenna [Ohm]
y1 = R1/Z0;                 %gamma 1 []
u0 = (4e-7)*pi;             %induction constant [V*s/(A*m)]
alpha = -3;                 %damping factor
h1 = (0.015/(4))*N;         %hight of the coil [m]
S_spule=(pi*d^2)/4;         %cross section of the coil [m^2]
a = 0.0178;                 %koefizienten for error calculation
b = 0.004;
g = (0.001:0.001:0.5);      %define the start value, step size and the max
%                            value for the distance g
x = g./d;                   %relation g/d

%calculate the impedance of the coil in relation to the
%geometrical dimensions

L_geo = u0*(S_spule*N^2)/h1;

%with this switch can be chosen the calculation method for the coupling
%coefficient. if 1 the approximate equation is used, otherwise is the
%Neumann equation used

switch aprox
    case 1
    k_aprox = exp(alpha*(g./d));

    otherwise
         for i=1:1:500
         g = i/1000;
         F_neumann = @(x,y) (((N^2)*u0)/(sqrt(L_geo^2)*4*pi)).*...
         ((d.*cos(x-y)./(sqrt(8+16*(g/d)^2-8.*cos(x-y)))));
         k_aprox(i)= integral2(F_neumann,0,2*pi,0,2*pi);

         end;
end;

%calculating the reflection coefficient in relation with the previously
%calculated coupling coefficient

Q = sqrt((4*pi^2*f^2*L_geo^2)/(R1*R2));
Ze = ((Q^2).*(k_aprox.^2).*R1 +R1);
T = (Ze-Z0)./(Ze+Z0);

%calculation of the error rate of the coupling coefficient
```

```
k_error = (a.*abs(T)+b)./((y1+1).*(T-(y1-1)/(y1+1)).*(1-T));

end
```

```
function [ x, k_error] = change_f( f,aprox )
%change_f calcuate the error rate for the couppling coeffizient in relation
%to the frequenz f
%
%this function gives back the error rate of the couppling coeffizient
%and the free dimensional distanze g/d betwen the antennas.


Z0 = 50;                %impedanz of the transmission line [Ohm]
N = 1;                  %number of turns []
d = 0.075;              %antenna diameter [m]
R2 = 1;                 %resistance of the receiver antenna [Ohm]
%L2 =3.012e-6;          %Inductance of the receiver antenna [H]
R1 = 1;                 %resistance of the transmitter antenna [Ohm]
%L1 =3.012e-6;          %Inductance of the transmitter antenna [H]
y1 = R1/Z0;             %gamma 1 []
u0 = (4e-7)*pi;         %induction constant [V*s/(A*m)]
alpha = -3;             %damping factor
h1 = (0.015/(4))*N;     %hight of the coil [m]
S_spule=(pi*d^2)/4;     %cross section of the coil [m^2]
a = 0.0178;             %koefizienten for error calculation
b = 0.004;
g = (0.001:0.001:0.2);  %define the start value, step size and the max
%                        value for the distance g
x = g./d;               %relation g/d

%calculate the impedance of the coil in relation to the
%geometrical dimensions

L_geo = u0*(S_spule*N^2)/h1;

%with this switch can be chosen the calculation method for the coupling
%coefficient. if 1 the approximate equation is used, otherwise is the
%Neumann equation used

switch aprox
    case 1
    k_aprox = exp(alpha*(g./d));

    otherwise
         for i=1:1:200
         g = i/1000;
         F_neumann = @(x,y) (((N^2)*u0)/(sqrt(L_geo^2)*4*pi)).*...
         ((d.*cos(x-y)./(sqrt(8+16*(g/d)^2-8.*cos(x-y)))));
         k_aprox(i)= integral2(F_neumann,0,2*pi,0,2*pi);

         end;
end;

%calculating the reflection coefficient in relation with the previously
%calculated coupling coefficient

Q = sqrt((4*pi^2*f^2*L_geo^2)/(R1*R2));
Ze = ((Q^2).*(k_aprox.^2)+1).*R1;
T = (Ze-Z0)./(Ze+Z0);
```

```
%calculation of the error rate of the coupling coefficient

k_error = (a.*abs(T)+b)./((y1+1).*(T-(y1-1)/(y1+1)).*(1-T));

end
```

```
function [ x, k_error] = change_N( N,aprox )
%change_N calcuate the error rate for the couppling coeffizient in relation
%to the number of turns N
%
%this function gives back the error rate of the couppling coeffizient
%and the free dimensional distanze g/d betwen the antennas.

f = 10e6;                 %frequency [Hz]
Z0 = 50;                  %impedanz of the transmission line [Ohm]
d = 0.075;                %antenna diameter [m]
R2 = 1;                   %resistance of the receiver antenna [Ohm]
%L2 =3.012e-6;            %Inductance of the receiver antenna [H]
R1 = 1;                   %resistance of the transmitter antenna [Ohm]
%L1 =3.012e-6;            %Inductance of the transmitter antenna [H]
y1 = R1/Z0;               %gamma 1 []
u0 = (4e-7)*pi;           %induction constant [V*s/(A*m)]
alpha = -3;               %damping factor
h1 = (0.015/(4))*N;       %hight of the coil [m]
S_spule=(pi*d^2)/4;       %cross section of the coil [m^2]
a = 0.0178;               %koefizienten for error calculation
b = 0.004;
g = (0.001:0.001:0.2);    %define the start value, step size and the max
%                          value for the distance g
x = g./d;                 %relation g/d

%calculate the impedance of the coil in relation to the
%geometrical dimensions

L_geo = u0*(S_spule*N^2)/h1;

%with this switch can be chosen the calculation method for the coupling
%coefficient. if 1 the approximate equation is used, otherwise is the
%Neumann equation used

switch aprox
    case 1
    k_aprox = exp(alpha*(g./d));

    otherwise
        for i=1:1:200
        g = i/1000;
        F_neumann = @(x,y) (((N^2)*u0)/(sqrt(L_geo^2)*4*pi)).*...
        ((d.*cos(x-y)./(sqrt(8+16*(g/d)^2-8.*cos(x-y)))));
        k_aprox(i)= integral2(F_neumann,0,2*pi,0,2*pi);

        end;
end;

%calculating the reflection coefficient in relation with the previously
%calculated coupling coefficient

Q = sqrt((4*pi^2*f^2*L_geo^2)/(R1*R2));
Ze = ((Q^2).*(k_aprox.^2)+1).*R1;
T = (Ze-Z0)./(Ze+Z0);
```

```
%calculation of the error rate of the coupling coefficient

k_error = (a.*abs(T)+b)./((y1+1).*(T-(y1-1)/(y1+1)).*(1-T));

end
```

```
function [ x, k_error] = change_R1( R1,aprox )
%change_R1 calcuate the error rate for the couppling coeffizient in relation
%to the transmitter resistance R1
%
%this function gives back the error rate of the couppling coeffizient
%and the free dimensional distanze g/d betwen the antennas.

f = 10e6;                  %frequency [Hz]
Z0 = 50;                   %impedanz of the transmission line [Ohm]
N = 4;                     %number of turns []
d = 0.075;                 %antenna diameter [m]
R2 = 1;                    %resistance of the receiver antenna [Ohm]
%L2 =3.012e-6;             %Inductance of the receiver antenna [H]
%L1 =3.012e-6;             %Inductance of the transmitter antenna [H]
y1 = R1/Z0;                %gamma 1 []
u0 = (4e-7)*pi;            %induction constant [V*s/(A*m)]
alpha = -3;                %damping factor
h1 = (0.015/(4))*N;        %hight of the coil [m]
S_spule=(pi*d^2)/4;        %cross section of the coil[m^2]
a = 0.0178;                %koefizienten for error calculation
b = 0.004;
g = (0.001:0.001:0.2);     %define the start value, step size and the max
%                           value for the distance g
x = g./d;                  %relation g/d

%calculate the impedance of the coil in relation to the
%geometrical dimensions

L_geo = u0*(S_spule*N^2)/h1;

%with this switch can be chosen the calculation method for the coupling
%coefficient. if 1 the approximate equation is used, otherwise is the
%Neumann equation used

switch aprox
    case 1
    k_aprox = exp(alpha*(g./d));

    otherwise
        for i=1:1:200
        g = i/1000;
        F_neumann = @(x,y) (((N^2)*u0)/(sqrt(L_geo^2)*4*pi)).*...
        ((d.*cos(x-y)./(sqrt(8+16*(g/d)^2-8.*cos(x-y)))));
        k_aprox(i)= integral2(F_neumann,0,2*pi,0,2*pi);

        end;
end;

%calculating the reflection coefficient in relation with the previously
%calculated coupling coefficient

Q = sqrt((4*pi^2*f^2*L_geo^2)/(R1*R2));
Ze = ((Q^2).*(k_aprox.^2)+1).*R1;
T = (Ze-Z0)./(Ze+Z0);
```

```
%calculation of the error rate of the coupling coefficient

k_error = (a.*abs(T)+b)./((y1+1).*(T-(y1-1)/(y1+1)).*(1-T));

end
```

```
function [ x, k_error] = change_R2( R2,aprox )
%change_R2 calcuate the error rate for the couppling coeffizient in relation
%to the receiver resistance R2
%
%this function gives back the error rate of the couppling coeffizient
%and the free dimensional distanze g/d betwen the antennas.


f = 10e6;                   %frequency [Hz]
Z0 = 50;                    %impedanz of the transmission line [Ohm]
N = 4;                      %number of turns []
d = 0.075;                  %antenna diameter [m]
%L2 =3.012e-6;              %Inductance of the receiver antenna [H]
R1 = 1;                     %resistance of the transmitter antenna [Ohm]
%L1 =3.012e-6;              %Inductance of the transmitter antenna [H]
y1 = R1/Z0;                 %gamma 1 []
u0 = (4e-7)*pi;             %induction constant [V*s/(A*m)]
alpha = -3;                 %damping factor
h1 = (0.015/(4))*N;         %hight of the coil [m]
S_spule=(pi*d^2)/4;         %cross section of the coil [m^2]
a = 0.0178;                 %koefizienten for error calculation
b = 0.004;
g = (0.001:0.001:0.2);      %define the start value, step size and the max
%                            value for the distance g
x = g./d;                   %relation g/d

%calculate the impedance of the coil in relation to the
%geometrical dimensions

L_geo = u0*(S_spule*N^2)/h1;

%with this switch can be chosen the calculation method for the coupling
%coefficient. if 1 the approximate equation is used, otherwise is the
%Neumann equation used

switch aprox
    case 1
    k_aprox = exp(alpha*(g./d));

    otherwise
         for i=1:1:200
         g = i/1000;
         F_neumann = @(x,y) (((N^2)*u0)/(sqrt(L_geo^2)*4*pi)).*...
         ((d.*cos(x-y)./(sqrt(8+16*(g/d)^2-8.*cos(x-y)))));
         k_aprox(i)= integral2(F_neumann,0,2*pi,0,2*pi);

         end;
end;

%calculating the reflection coefficient in relation with the previously
%calculated coupling coefficient

Q = sqrt((4*pi^2*f^2*L_geo^2)/(R1*R2));
Ze = ((Q^2).*(k_aprox.^2)+1).*R1;
T = (Ze-Z0)./(Ze+Z0);
```

```
%calculation of the error rate of the coupling coefficient

k_error = (a.*abs(T)+b)./((y1+1).*(T-(y1-1)/(y1+1)).*(1-T));

end
```

```
function [ x, k_error] = change_Z0( Z0,aprox )
%change_Z0 calcuate the error rate for the couppling coeffizient in
%relation to the impedanze Z0.
%
%this function gives back the error rate of the couppling coeffizient
%and the free dimensional distanze g/d betwen the antennas.


f = 10e6;                 %frequency [Hz]
N = 4;                    %number of turns []
d = 0.075;                %antenna diameter [m]
R2 = 1;                   %resistance of the receiver antenna [Ohm]
%L2 =3.012e-6;            %Inductance of the receiver antenna [H]
R1 = 1;                   %resistance of the transmitter antenna [Ohm]
%L1 =3.012e-6;            %Inductance of the transmitter antenna [H]
y1 = R1/Z0;               %gamma 1 []
u0 = (4e-7)*pi;           %induction constant [V*s/(A*m)]
alpha = -3;               %damping factor
h1 = (0.015/(4))*N;       %hight of the coil [m]
S_spule=(pi*d^2)/4;       %cross section of the coil [m^2]
a = 0.0178;               %koefizienten for error calculation
b = 0.004;
g = (0.001:0.001:0.4);    %define the start value, step size and the max
%                          value for the distance g
x = g./d;                 %relation g/d

%calculate the impedance of the coil in relation to the
%geometrical dimensions

L_geo = u0*(S_spule*N^2)/h1;

%with this switch can be chosen the calculation method for the coupling
%coefficient. if 1 the approximate equation is used, otherwise is the
%Neumann equation used

switch aprox
    case 1
    k_aprox = exp(alpha*(g./d));

    otherwise
        for i=1:1:400
        g = i/1000;
        F_neumann = @(x,y) (((N^2)*u0)/(sqrt(L_geo^2)*4*pi)).*...
        ((d.*cos(x-y)./(sqrt(8+16*(g/d)^2-8.*cos(x-y)))));
        k_aprox(i)= integral2(F_neumann,0,2*pi,0,2*pi);

        end;
end;

%calculating the reflection coefficient in relation with the previously
%calculated coupling coefficient

Q = sqrt((4*pi^2*f^2*L_geo^2)/(R1*R2));
Ze = ((Q^2).*(k_aprox.^2)+1).*R1;
T = (Ze-Z0)./(Ze+Z0);
```

```
%calculation of the error rate of the coupling coefficient

k_error = (a.*abs(T)+b)./((y1+1).*(T-(y1-1)/(y1+1)).*(1-T));

end
```

```
function [ ] = K_T( )
%plot into two different windows the coupling coefficient and the
%reflection coefficient in relation with the distance

%   the design parameter must be defined at the quel text of the function

f = 10e6;                   %frequency [Hz]
Z0 = 50;                    %impedanz of the transmission line [Ohm]
N = 4;                      %number of turns []
d = 0.075;                  %antenna diameter [m]
R2 = 1;                     %resistance of the receiver antenna [Ohm]
%L2 =3.012e-6;              %Inductance of the receiver antenna [H]
R1 = 1;                     %resistance of the transmitter antenna [Ohm]
%L1 =3.012e-6;              %Inductance of the transmitter antenna [H]
h1 = (0.015/(4))*N;         %hight of the coil [m]
u0 = (4e-7)*pi;             %induction constant [V*s/(A*m)]
S_spule=(pi*d^2)/4;         %cross section of the coil [m^2]
g = (0.001:0.001:0.2);
x = g./d;

%calculating the inductance of the coil L1 or L2, in relation to the
%geometrical dimensions. theoretical is L1 = L2 in this case

L_geo = u0*(S_spule*N^2)/h1;

%calculating the coupling coefficient with the Neumann Formula

 for i=1:1:200
        g = i/1000;
        F_neumann = @(x,y)  (((N^2)*u0)/(sqrt(L_geo^2)*4*pi)).*...
                    ((d.*cos(x-y)./(sqrt(8+16*(g/d)^2-8.*cos(x-y)))));
        k_aprox(i)= integral2(F_neumann,0,2*pi,0,2*pi);
 end

%calculating the reflection coefficient in relation with the previously
%calculated coupling coefficient

Q = sqrt((4*pi^2*f^2*L_geo^2)/(R1*R2));
Ze = ((Q^2).*(k_aprox.^2)+1).*R1;
T = (Ze-Z0)./(Ze+Z0);

%plot into two different windows the coupling coefficient and the
%reflection coefficient in relation with the distance

subplot(2,1,1)
 plot(x,k_aprox,'LineWidth',1.5);
 %ylim([0 1]);
 xlim([0 2.5]);
 xlabel('distanze g/d');
 ylabel('coupling coefficient');
 grid

 subplot(2,1,2)
 plot(x,T,'LineWidth',1.5);
 %ylim([0 1]);
```

```
 xlim([0 2.5]);
 xlabel('distanze g/d');
 ylabel('reflection coefficient');
 grid

end
```

```
function [T,k_error, Tmin,k_min] = error_XaxisT( y1 )

%This function describes the error rate of coupling coefficient in
%relation to the reflection coefficient for diverent parameters of y1.
%
%The function plot the graph and shows the minimum point of the error rate
%for different values of y1,
%
%this function need as input a value of y1 that's bigger than y1 > 0.690
%that the minimum point is ploted. the minimum point will not plotet for
%values of y1 < 0.690.


f = 10e6;                   %frequency [Hz]
Z0 = 50;                    %impedanz of the transmission line [Ohm]
N = 4;                      %number of turns []
d = 0.075;                  %antenna diameter [m]
R2 = 1;                     %resistance of the receiver antenna [Ohm]
R1 = y1*Z0;                 %resistance of the transmitter antenna [Ohm]
h1 = (0.015/(4))*N;         %hight of the coil [m]
u0 = (4e-7)*pi;             %induction constant [V*s/(A*m)]
S_spule=(pi*d^2)/4;         %cross section of the coil [m^2]
a = 0.0178;                 %koefizienten for error calculation
b = 0.004;

%calculating the inductance of the coil L1 or L2, in relation to the
%geometrical dimensions. theoretical is L1 = L2 in this case

L_geo = u0*(S_spule*N^2)/h1;

    for i=1:1:1000
        g = i/1000;
        F_neumann = @(x,y) (((N^2)*u0)/(sqrt(L_geo^2)*4*pi)).*...
                    ((d.*cos(x-y)./(sqrt(8+16*(g/d)^2-8.*cos(x-y)))));
        k_aprox(i)= integral2(F_neumann,0,2*pi,0,2*pi);

    end;

%calculating the reflection coefficient in relation with the previously
%calculated coupling coefficient

Q = sqrt((4*pi^2*f^2*L_geo^2)/(R1*R2));
Ze = ((Q^2).*(k_aprox.^2)+1).*R1;
T = (Ze-Z0)./(Ze+Z0);

%calculation of the error rate of the coupling coefficient

k_error = (a.*abs(T)+b)./((y1+1).*(T-(y1-1)/(y1+1)).*(1-T));

%calculation of the minimum error point from the error rate curve of the
%coupling coefficient.

Tmin = -b/a+sqrt((b/a)^2+(y1-1+2*b*y1/a)/(y1+1));
k_min = (a*abs(Tmin)+b)/((y1+1)*(Tmin-(y1-1)/(y1+1))*(1-Tmin));
```

```
%if y 1 is bigger than 0.69 can the minimum point be calculated and it
%will by printet in the plot. if y1 smaller than 0.69 the minimum point
%can not be printet.

 if y1 > 0.68;
    plot(T,k_error, Tmin,k_min,'r*');
    sR1 = num2str(R1);
    sTmin = num2str(Tmin);
    sk_min = num2str(k_min);
    legend(['R1 = ' sR1 '[Ohm]'] ,...
            ['Tmin = ' sTmin char(10) 'kmin = ' sk_min]);
 else
    plot(T,k_error);
    sR1 = num2str(R1);
    legend(['R1 = ' sR1 '[Ohm]']);
 end

 ylim([0 1]);
 xlim([-1 1]);
 xlabel('reflection coefficient');
 ylabel('Error rate of coupling coefficient');
 title('error in relation with reflection coefficient');
 grid

end
```

```
function [tabelf ] = range( )
%Plot the error characteristic from coupling factor in relation
%to the distance and give back a table with parameters for the database
%matching
%
%   The basic parameter must be changed into the original text of
%   this function. the function return a filtered table from the imported
%   values for the database matching.
%   over the filter can be chosen which maximum error in percent the
%   values in the tabell have. the table has the following structure
%
%   | Ze | distance | couppling coef. | reflection coef. | error % | Q |
%   |----|----------|-----------------|------------------|---------|---|
% 1 | -  |    -     |        -        |        -         |    -    |   |
%   |----|----------|-----------------|------------------|---------|---|
% 2 |    |          |                 |                  |         | - |
%
%   the first line contains only the Q factor. from line 2 starts the
%   other variables, and Q is not writing anymore.


f = 10e6;                %frequency [Hz]
Z0 = 50;                 %impedanz of the transmission line [Ohm]
N = 4;                   %number of turns []
d = 0.1;                 %antenna diameter [m]
%R2 = 1.261;             %resistance of the receiver antenna [Ohm]
R2 = 1.56;
L2 =3.012e-6;            %inductance of the receiver antenna [H]
%R1 = 1.41;              %resistance of the transmitter antenna [Ohm]
R1 = 1.27;
L1 =3.012e-6;            %inductance of the transmitter antenna [H]
y1 = R1/Z0;              %gamma 1 []
u0 = (4e-7)*pi;          %induction constant [V*s/(A*m)]
a = 0.0178;              %koefizienten for error calculation
b = 0.004;
max_distance = 400;      %the coupling coefficient is calculated up to
                         %this distance [mm]
error_max = 200;         %mximum error rate [%] (used for the filter)

%calculating the coupling coefficient with the Neumann Formula for the
%different distance steps

    for i=1:1:max_distance
         g = i/1000;
         x_g(i) = g;
         F_neumann = @(x,y) (((N^2)*u0)/(sqrt(L1*L2)*4*pi)).*...
                     ((d.*cos(x-y)./(sqrt(8+16*(g/d).^2-8.*cos(x-y)))));
         k_aprox(i)= integral2(F_neumann,0,2*pi,0,2*pi);

    end;

%calculating the reflection coefficient in relation with the previously
%calculated coupling coefficient

Q = sqrt((4*pi^2*f^2*L1*L2)/(R1*R2));
```

```
Ze = ((Q^2).*(k_aprox.^2)+1).*R1;
T = (Ze-Z0)./(Ze+Z0);

%calculation of the error rate of the coupling coefficient

k_error = (a.*abs(T)+b)./((y1+1).*(T-(y1-1)/(y1+1)).*(1-T));

%creates a table of important values
tabel(1,6) = Q;
    for i=1:1:400
         tabel(1+i,1)= Ze(i);
         tabel(1+i,2)= x_g(i);
         tabel(1+i,3)= k_aprox(i);
         tabel(1+i,4)= T(i);
         tabel(1+i,5)= k_error(i)*100;
    end

%with the 'if' condition can be chosen the max error in percent

tabelf(1,6)=Q;
    for q=1:1:400
        if error_max > tabel(1+q,5);
            tabelf = [tabelf ; tabel(1+q,:)];

        end
    end

%plot the error rate of the coupling factor in relation with the distance

plot(x_g,k_error,'LineWidth',1.5);
ylim([0 10]);
%xlim([0 1]);
xlabel('distance [m]');
ylabel('error rate of coupling coefficient');
title('error in relation with the coupling coefficient');
grid

end
```

```
function [ out_tabel ] = distance_Calc( measured,calculated,offset )
%This function needs two tables and a scalar value as input. The function
%returns a table which contains the measured and the interpolated value
%for the position.
%
%This function plots also a chart which compare the measured with the
%interpolated values.
%
%   measured conains the table with the measured values. calculated
%   contains the table with the calculated values. the values for the
%   calculated Ze must start with the high values. Ze is in the correct
%   order when the function "range" is used for the calculation. the
%   offset is the scalar value
%
%   the tables have to look as follows:
%
%         measured                calculated              out_tabel
%
%   |  Ze  |  distance   |    |  Ze  |  distance   |   |  Ze_mes  |  Ze_calc |
%   |------|-------------|    |------|-------------|   |----------|----------|
%   |      |             |    |  up  |             |   |          |          |
%   |      |             |    |  to  |             |   |          |          |
%   |      |             |    | low  |             |   |          |          |


Ze_up = 0;                  %the next higher calculated value, compared to
                            %the measured value
Ze_down = 0;                %the next lower calculated value, compared to
                            %the measured value
g_up = 0;                   %the corresponding distance for Ze_up
g_down = 0;                 %the corresponding distance for Ze_down

%determination of the table lengths

[lengt1]=size(measured);
[lengt2]=size(calculated);

%do the interpolation for each measured resistance value

for i=1:1:lengt1

%determination of Ze_up, Ze_down, g_up and g_down

    for q=1:1:(lengt2-1)
        if calculated(q,1)>= measured(i,1)
            Ze_up = calculated(q,1);
            g_up = calculated(q,2);
            Ze_down = calculated(1+q,1);
            g_down = calculated(1+q,2);
        end
    end

%makes the interpolation and write the results in the table which
%it gives back
```

```
    a = (Ze_up-measured(i,1))/(Ze_up-Ze_down);
    out_tabel(i,2)= a*g_down +(1-a)*g_up+offset;
    out_tabel(i,1)= measured(i,2);

end

%plot the interpolated distance and the ideal distance in the same chart
%and make the labeling

plotx = out_tabel(:,1);
ploty1 = out_tabel(:,1);
ploty2 = out_tabel(:,2);
plot(plotx,ploty1,plotx,ploty2,'r','LineWidth',1.5);
grid on
xlabel('measured distance [m]');
ylabel('calculated distance [m]');
off = num2str(offset);
title(['measured and calculated distance for R2 = 1.56[Ohm] offset ' ...
        off '[mm]']);
legend('ideal','calculated');
```