Supplementary Materials for

Proton structure from the measurement of $2S-2P$ transition frequencies of muonic hydrogen

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Data acquisition and analysis

Data acquisition

On average, about 580 muons μ^- enter the hydrogen target per second. About 330 μ^-/s of these are detected in both muon entrance detectors. This starts the data acquisition system and triggers the laser system provided the laser is ready. After each laser shot, we impose a dead-time of 2 ms on the laser system to ensure stable laser operation, in particular of the Raman cell. This dead-time results in 200 muon-induced laser triggers per second, termed “laser-on” events. For the remaining 130 μ^-/s the data acquisition system is started, too, but no laser trigger is produced. Hence they are named “laser-off” events.

For each event, a 12 μs long “event gate” is opened during which x-rays from muonic hydrogen are recorded in 20 large-area avalanche photo diodes (LAAPDs). In addition, two sets of plastic scintillators record signals from electrons created in muon decay. The μ-decay electrons can also produce signals in the LAAPDs.

Data analysis

In the data analysis, we require that a muonic x-ray is followed by a μ-decay electron within a certain time window, termed “delayed electron time window”, or “dele-window”. This “dele-”requirement reduces the background in the x-ray spectra by about an order of magnitude.

The muon decay electrons are forced on helical orbits by the 5 Tesla magnetic field in the target area. Hence there is a possibility that such a decay electron traverses more than one detector (LAAPD or scintillator). It turned out in the refined analysis presented here, that such multi-hit electrons create a particularly "clean" data set. In contrast, spurious detector signals can create the signature of a single-hit electron, which results in a data set with higher background. In a similar manner, the amount of energy deposited by the electron in an LAAPD or plastic scintillator has an influence on the amount of spurious electron-like signals: The larger the detector signal, the cleaner the electron data set. Finally, the background of the electron-like signals depends on the time difference between the x-ray and the electron. Therefore we optimized the "dele-window" individually for each of the various electron-like signal types, using the "laser-off" data set.

Improvements in the data analysis

One of the main advances in the present analysis, compared to the one in [6], is the introduction of different "classes" of delayed electrons, along the lines described above. We divided the total data set into two classes "I" and "II". Class "I" contains the x-rays which were followed by a μ-decay electron that met the criteria for a "clean" electron identification, i.e. a signal in more than one detector, or at a time within the "dele-window" where the signal-to-background (S/B) ratio is better than a certain threshold. Hence, class "I" has a good S/B, at the expense of a lower total number of events. Class "II" events are the majority of the events, with a significantly smaller S/B.

Other advances in the present analysis include an optimized LAAPD signal analysis, more careful energy and time calibration of the detectors, and refined energy cuts for the muonic K_α x-rays.

Laser resonances

The two measured μp resonances are shown in Fig. 3. For each resonance we plot class "I" and "II" data sets separately. The control data set "laser-off" is shown in Fig. 3, too. The insets show the corresponding time spectra in the region of the laser-induced events. The time of an event is determined by the time of the K_α x-ray after the muon entrance signal.

For the resonance, the number of events in the laser time window is divided by the number of "prompt" events (see Fig. 1A), and plotted as a function of laser frequency. The laser time window is the time at which the laser light was illuminating the muonic hydrogen atoms. It is indicated by the vertical lines in the insets in Fig. 3. Normalizing the number of laser-induced events by the number of "prompt" events helps to correct for different measurement times per laser frequency. Typically, we recorded between 6 and 13 hours of data per laser frequency.

On the peak of the resonance we observed about 6 events per hour, whereas the background in the resonance plots corresponds to approximately 1 event per hour.

Laser-off data is plotted as a function of laser frequency, too. This is done only to demonstrate the flatness of the background. This procedure seems justified because the laser-off events were taken interleaved with the laser-on events at the corresponding laser frequency.

Resonance fit

The two resonances obtained for each transition are fitted simultaneously using six free parameters: both class “I” and “II” resonances share a common position and width, but the signal and flat background amplitudes are independent for the two spectra. A line shape model based on optical Bloch equations for the $2S$–$2P$–$1S$ system is used to fit the resonances accounting for saturation and broadening effects. Laser pulse energy variations can lead to systematic line shifts if the pulse energy is systematically higher on one side of the resonance. To avoid this systematic shift we recorded the laser pulse energy for each laser shot and included this information in the line shape model, resulting in corrections of the resonance positions of typically 50 MHz.

Muonic hydrogen theory and two-photon exchange effects

We have recently reviewed the available theory of the $n = 2$ energy levels in muonic hydrogen [13]. In general, the theory is well established and several authors have double-checked all relevant terms. The reader is referred to Ref. [13] and references therein for all details. Here we would only like to elaborate briefly on the choice of the two-photon exchange (TPE) contribution in Eq. 7, because the TPE effects have recently been under discussion [24,25,26,43].

In the framework of quantum field theory the two-photon exchange contribution ΔE_{TPE} [19,24,25,26] is the sum of an elastic part $\Delta E_{\mathrm{el}} = 0.0295(16)$ meV [24,25], a non-pole term $\Delta E_{\mathrm{np}} = -0.0048$ meV [24], a subtraction term $\Delta E_{\mathrm{sub}} = -0.0042(10)$ meV [25], and an inelastic (proton polarizability) contribution of $\Delta E_{\mathrm{pol}} = 0.0127(5)$ meV [24]. The sum $\Delta E_{\mathrm{el}} + \Delta E_{\mathrm{np}}$ is in agreement with the corresponding approximate expression [16,17,18] $6/7 \times 0.0091 R^3_{(2)}$ meV, where the $6/7$ accounts for recoil and

$$R^3_{(2)} = \int \mathrm{d}^3r \int \mathrm{d}^3r' \; r^3 \, \rho_E(\boldsymbol{r} - \boldsymbol{r'})\rho_E(\boldsymbol{r'})$$

is the third Zemach moment which has been calculated from the measured proton form factor: $R^3_{(2)} = 2.71(13)$ fm^3 [38] and $R^3_{(2)} = 2.85(8)$ fm^3 [5]. Note that Eq. 7 relies on the full TPE calculation including an “on-shell” part which is only approximately described by the $R^3_{(2)}$ term [24]. More details are given in [13].

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