Determination of the nuclear level densities and radiative strength function for 43 nuclei in the mass interval 28≤A≤200

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Abstract. The determination of nuclear level densities and radiative strength functions is one of the most important tasks in low-energy nuclear physics. Accurate experimental values of these parameters are critical for the study of the fundamental properties of nuclear structure. The step-like structure in the dependence of the level densities ρ on the excitation energy of nuclei E_{ex} is observed in the two-step gamma cascade measurements for nuclei in the $28 \leq A \leq 200$ mass region. This characteristic structure can be explained only if a co-existence of quasi-particles and phonons, as well as their interaction in a nucleus, are taken into account in the process of gamma-decay. Here we present a new improvement to the Dubna practical model for the determination of nuclear level densities and radiative strength functions. The new practical model guarantees a good description of the available intensities of the two step gamma cascades, comparable to the experimental data accuracy.

1 Introduction

The development of theoretical models of nuclear structures requires a set of experimental information of the excited levels density, ρ, (with given quantum numbers) and of the values of the partial width (radiative strength function), Γ, of all possible decay channels. Correct interpretation of the dynamics of the nuclear transitions, in a broad variety from the simple low-lying levels (e.g., quasi-particle or phonon structure) to the very complex compound-states is possible by the theoretical calculations if those experimental data are available. One of the most suitable techniques for determination of required nuclear mater parameters (ρ and Γ) is the two-step gamma cascades methods based on measurement of gamma coincidences following neutron capture [1].

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Based on the experimental data collected by two-step gamma cascades experiment a model for description the gamma-decay of neutron resonance was developed at JINR, Dubna [2, 3]. In this model the level density ρ of quasi-particles in any nucleus is defined using the known model of n-quasi-particle levels. Here we presented the improved version of this model taking into account shell inhomogeneities of the single-particle level spectra and their influence on the functions: $\rho=\varphi(E_{ex})$ and $\Gamma=\psi(E_1)$, where E_{ex} is the excitation energy and E_1 is primary transition energy. The experimental results of two step gamma cascades intensity for 43 nuclei in the $28 \leq A \leq 200$ mass region were fitting by this model. This provide us possibility to extract parameters of nuclear structure such as breaking thresholds of the second and the third Cooper pairs, ratio of the collective level density to the total one or level parity.

2 Dubna two-step gamma cascades method

The two-step gamma-cascades method for obtaining information about the nuclear structure parameters following the thermal neutron captures was developed at FLNP, JINR, DUBNA [2, 3]. From amount of gamma-gamma coincidences the method allows to choose registration events of full energy of two-gamma transition cascade with a sufficiently low background. And the experimental intensity distributions of cascades to the final levels of compound-nucleus with excited energy below ~500–800 keV are obtained from these coincidences. Using the nuclear spectroscopy procedures allows decomposing the initial spectrum on primary and secondary transmission components of cascades with an acceptable uncertainty [2, 3].

The basic idea of this method comes from specific dependence of the two-step gamma- cascade intensity on the partial radiative width Γ and the density of excited levels:

$$I_{\gamma\gamma} = \sum_{\lambda,f} \sum_{i} \frac{\Gamma_{\lambda i}}{\Gamma_{\lambda}} \frac{\Gamma_{if}}{\Gamma_{i}} = \sum_{\lambda,f} \frac{\Gamma_{\lambda i}}{\langle \Gamma_{\lambda i} \rangle m_{\lambda i}} n_{\lambda i} \frac{\Gamma_{if}}{\left\langle \Gamma_{if} \right\rangle m_{if}} \tag{1}$$

where $\Gamma_{\lambda i}$ and Γ_{if} are the partial radiative widths corresponding to the primary and to the secondary transitions; $n_{\lambda i} = \rho \Delta E_i$ is the number of the excited intermediate levels in a certain interval of the excitation energy ΔE_i; $\langle \Gamma_{\lambda i} \rangle$ and $\langle \Gamma_{if} \rangle$ are the average values of the corresponding intervals of the nucleus excitation energy widths; $m_{\lambda i}$ and m_{if} are the number of levels in the same intervals. When this method was developed for the first time it was based on an interactive calculation. Using iterative process with "randomly" chosen functions ρ and Γ, it is possible to obtain the most probable values of level density and radiative width (or radiative strength function).

3 Model of the gamma-decay of neutron resonance

Here we present improved version of the model for the gamma-decay of neutron resonance [2] which can explain the experimental data based on combination of phenomenological and theoretical representations.

The level density, described by an expression for density ρ_l of Fermi levels, was taken from the model of density Ω_n of n-quasi-particle states [4]:

$$\rho_l = \frac{(2J+1)\exp\left(-(J+1/2)^2/2\sigma^2\right)}{2\sqrt{2\pi}\sigma^3} \cdot \Omega_n(E_{ex}), \Omega_n(E_{ex}) = \frac{g^n(E_{ex}-U_l)^{n-1}}{((n/2)!)^2(n-1)!} \tag{2}$$

Here J is the spin quantum number, g = $6a/\pi^2$ is the density of the single-particle states near Fermi-surface, σ is the cut-off factor (a and σ values were taken from the back-shifted Fermi-gas model

[5]), and U_l, is the energy of the l-th Cooper pair breaking threshold. The effect of the collective enhancement was also included in this model by the coefficient C_{col} of the collective enhancement of the vibrational level density (or both vibrational and rotational ones for deformed nuclei). For a given excitation energy, E_{ex}, the phenomenological coefficient is determined by a theoretical description that can be found in Ref. [3]:

$$C_{coll} = A_l \exp(\sqrt{(E_{ex} - U_l)/E_\nu} - (E_{ex} - U_l)/E_\mu) + \beta \tag{3}$$

where A_l are parameters of density for the vibrational levels above the breaking point for each l-th Cooper pair, E_μ and E_ν determine the change in the nuclear entropy and the change of the quasi-particles excitation energies, respectively. Coefficients A_l for different pairs are fitted independently, as it was done in Ref. [2]. Coefficient β is used for a description of the rotation level density.

Radiative strength functions for $E1$- and $M1$-transitions are determined in this model by Ref. [6]:

$$\begin{aligned} k(E1, E_\gamma) + k(M1, E_\gamma) = w \frac{1}{3\pi^2\hbar^2c^2A^{2/3}} \frac{\sigma_G\Gamma_G^2(E_\gamma^2 + \kappa 4\pi^2T^2)}{(E_\gamma^2 - E_G^2)^2 + E_\gamma^2\Gamma_\gamma^2} + \\ + P\delta^- \exp(\alpha_p(E_\gamma - E_p)) + P\delta^+ \exp(\beta_p(E_p - E_\gamma)) \end{aligned} \tag{4}$$

with fitting normalization parameter w and coefficient κ; thermodynamic temperature T; the location of the center of the giant dipole resonance E_G, with width Γ_G and cross section σ_G in the maximum for each nucleus. For description of experimental data of Ref. [3] it is necessary to add one or several narrow peaks to the strength function is based on the data of Ref. [3]. The second summand of Eq. (5) corresponds to the left slope of the peak (energies below the maximum), and the third summand is the right slope (energies above the maximum). Position E_p in the energy scale, amplitudes $P\delta^+$ and $P\delta^-$ and slope parameters α_p and β_p are fitted for each peak independently. At $E_1 \approx B_n$ the fitted ratios Γ_{M1}/Γ_{E1} of $E1$- and $M1$-strength functions are normalized to known experimental values, and their sum Γ_λ is normalized to the total radiation width of the resonance.

The influence of the shell correction δE on the density of the quasi-particle levels were tested in this work. It was done by using the $a(A)$ value, which depends on the excitation energy, included linearly in the parameter of the single-particle density g (see Eq. (2)). For a nucleus with mass A and excitation energy E_{ex}, $a(A)$ is expressed, as [3]:

$$a(A) = \tilde{a}(1 + ((1 - exp(\gamma E_{ex}))\delta E/E_{ex})) \tag{5}$$

where asymptotic value is $\tilde{a} = 0.114 \cdot A + 0.162 \cdot A^{2/3}$ and $\gamma = 0.054$. The δE values slightly varied relative to their evaluations [3] in order to keep an average spacing between neutron resonances (see [2]).

In our model the set of common parameters for fitting (see Eqs. (2, 3)) were:

1) the break up thresholds energies U_l up to l=4,

2) the E_μ and E_ν parameters, which are common for all Cooper pairs

3) the mutually independent parameters A_l of the density of vibrational levels above the break up threshold U_l

4) the coefficients w, κ and β

5) the ratio r of negative parity and the total level density.

Those parameters were used for the description of the intensity $I_{\gamma\gamma}$ (E_1) for 43 nuclei, in the framework of the proposed model.

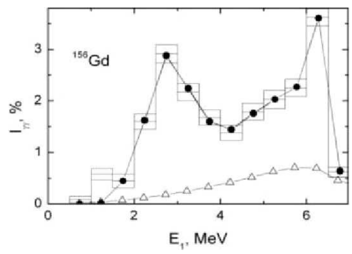

Figure 1. Histogram - experimental cascade intensity and its uncertainties for ^{156}Gd as function of primary cascade quanta E_1. Points - the best fit of the presented practical model; triangles - a calculation of $I_{\gamma\gamma}$ using models of Ref. [5, 6]. Recorded threshold for cascade gammas is $E_\gamma = 520$ keV.

a) **b)**

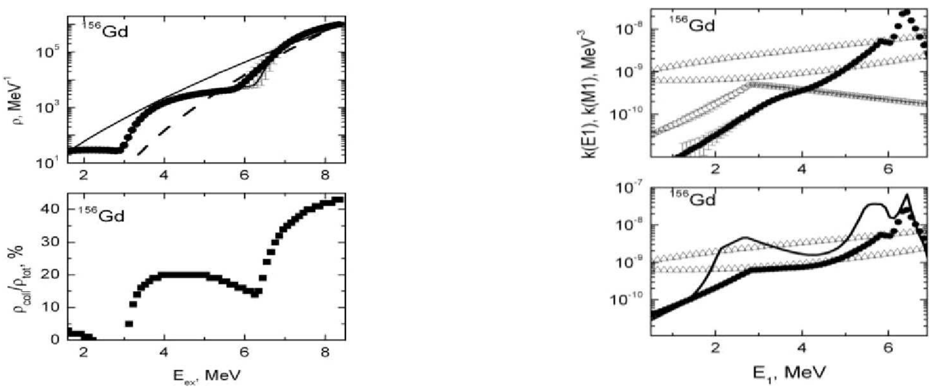

Figure 2. **a)** Level density of ^{156}Gd. *Top:* points are the best fit of level density (uncertainties – scatter of fits for different sets of initial parameters); dashed and solid lines are the level density calculated using the model of Ref. [5], with taking into account the shell correction δE (6) and without δE, correspondingly. *Bottom:* fitted ratio of density of collective levels to the total level density. **b)** Strength function for ^{156}Gd. *Top:* solid points are the best fit of the strength function of $E1$-transitions; open points are the best fit of the strength function of $M1$-transitions. *Bottom:* solid points are a sum of $E1$- and $M1$- strength functions; dash line is the sum of strength functions multiplied by $\rho_{\mathrm{mod}}/\rho_{\mathrm{exp}}$ ratio (Ref. [7]). Calculations using the model of Ref. [6] (lower triangles) and using the model of Ref. [8] (upper triangles) were fulfilled with $k(M1)$= const.

4 Results and discussion

A solution of the system of Eq. (1) is performed by the Monte-Carlo method. The nonlinearity of the strongly correlated equations of the system (1) produces an uncertainty of extracting the ρ and Γ parameters from $I_{\gamma\gamma}$ intensities.

Experimental data on $I_{\gamma\gamma}$ (E_1) are usually obtained with a small total uncertainty and averaged over 500 keV energy intervals. The results for ^{156}Gd are shown, in more detail, in Figs. 1–2. The best fits to $I_{\gamma\gamma}$ (E_1), as well as the fitted level densities and strength functions, are compared to corresponding values calculated using the statistical model. The results and corresponding calculations of level density and radiative strength function for the rest of the investigated nuclei will not be shown in

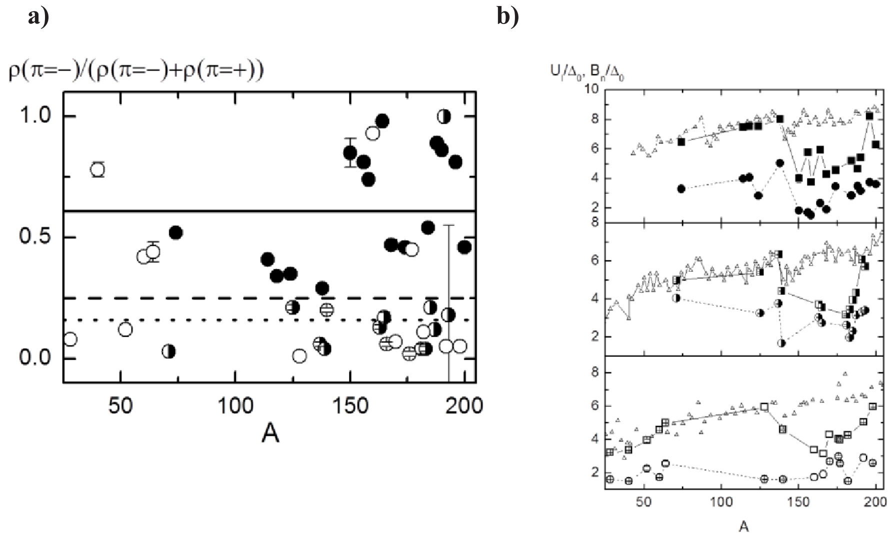

Figure 3. a) A-dependence of the ratios U_l/Δ_0, for the second (points) and the third (squares) Cooper pairs. Full points – even-even, half-open points are even-odd and open points are odd-odd compound nuclei. Triangles – the mass dependence of B_n/Δ_0 ratio. **B)** Mass dependence of the ratio of the level density with negative parity to the total level density at the upper energy border of the E_d and their averages for even-even nuclei (solid lines), even-odd (dashed lines) and odd-odd nuclei (dotted lines). Full points – even-even, half-open points – even-odd and open points – odd-odd compound nuclei.

this publication. However, we are presented here obtained results for some of parameters of nuclear structure.

One important parameter is the breaking thresholds for Cooper pairs. In the present analysis was confirmed the previous results about the connection between the shape of the investigated nucleus and the breaking thresholds. That was established for the first time in our prior analysis [3]. As the breaking thresholds differ for nuclei with various nucleon parities and depend on the average pairing energy (Δ_0) of the last nucleon, the mass dependencies for the ratios of the break up thresholds of the second and the third Cooper pairs to Δ_0, as well as the mass dependence of the binding energy to Δ_0, are presented in Fig. 3. As it can be seen in Fig. 3, there is a noticeable difference in U_2/Δ_0 and U_3/Δ_0 ratios for spherical and deformed nuclei in contrast to B_n/Δ_0.

In this work it was also obtained information about levels parity. For determination of the part $r = \rho(\pi-)/(\rho(\pi-) + \rho(\pi+))$ of levels $\rho(\pi-)$ with negative parity, a linear extrapolation for r value was applied in the $E_d \le E_{ex} \le B_n$ energy interval. At that, in the B_n point we use generally accepted assumption, that $\rho(\pi-) = 0.5(\rho(\pi-) + \rho(\pi+))$, and $\rho(\pi-)$ value in this energy point was fixed, and at the E_d energy the $\rho(\pi-)$ value varied.

The calculated ratios of density of the levels with negative parity to the total level density are shown in Fig. 3. The averages of these ratios are 0.61(22), 0.25(28) and 0.16(16) for even-even, even-odd and odd-odd nuclei, respectively (and for odd-even ^{177}Lu it is 0.65(1)). Hence, the behavior of the gamma-decay process is different for nuclei of various nucleon parities.

5 Conclusion

In this work we presented new variant of model for gamma decay of neutron resonance, taking into account shell inhomogeneities of the single-particle level spectra. We used this model for fitting the experimental intensity of two-step gamma cascades and to obtain information about parameters of nuclear structure.

The data on Cooper pair break-up energies, obtained with a high accuracy, are sufficient to conclude that the dynamics of interaction between superfluid and normal phases of a nucleus depends on its' shape. Our model allows for a separate determination of the density of vibrational levels between the breaking thresholds of the Cooper pairs.

Unfortunately, an existence of the sources of uncertainties of the sought ρ and Γ functions is a fundamental problem, and it is inevitable for any nuclear model used for experimental data analysis and for predictions of the spectra and cross sections. There are also fluctuations of the intensities of gamma-transitions in different nuclei, which has a contribution to the systematical error. Nevertheless, the practical model showed one possibility to describe the data of the two-step experiments with the accuracy that exceeds the statistical one.

For future development of reliable model of cascade gamma decay new experimental data are necessary. Because of that, ^{108}Ag, ^{110}Ag, ^{104}Rh and ^{56}Mn nuclei will be investigated by two step gamma cascade method.

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