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Behavior patterns of online users and the effect on information filtering

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Understanding the structure and evolution of web-based user-item bipartite networks is an important task since they play a fundamental role in online information filtering. In this paper, we focus on investigating the patterns of online users' behavior and the effect on recommendation process. Empirical analysis on the e-commercial systems show that users' taste preferences are heterogeneous in general but their interests for niche items are highly clustered. Additionally, recommendation processes are investigated on both the real networks and the reshuffled networks in which real users' behavior patterns can be gradually destroyed. We find that the performance of personalized recommendation methods is strongly related to the real network structure. Detailed study on each item shows that most hot items are accurately recommended and their recommendation accuracy is robust to the reshuffling process. However, the accuracy for niche items is relatively low and drops significantly after removing users' behavior patterns. Our work is also meaningful in practical sense since it reveals an effective direction to improve the accuracy and the robustness of the existing recommender systems.

1. Introduction

Complex networks have been studied intensively for more than a decade. The rapid development of network science has greatly helped us to understand and model real systems [1]. So far, many systems have been described by networks, such as the transportation system where nodes are airports and links are airlines [2–5], the neural system where nodes are neurons and links are synapses [6], the social system where nodes are people and links are the social interactions [7,8], the power grid where nodes are power plants and links are power cables [9]. Some other systems coupled by two different elements are modeled by the bipartite networks. For example, the e-commercial systems consisting of online users and items [10], the scientific collaboration system consisting of authors and papers [11], family name inheritance system consisting of babies and names [12] are naturally described by such networks.

The e-commercial systems have brought giant benefit to our daily lives. Nowadays, we can simply order books, movies, clothes etc. from the online retailer even at home. However, like a coin has both sides, internet also brings us overabundant information so that we always have too many candidate products to compare. In order to solve the problem, many recommendation algorithms such as collaborative filtering [13,14], content-based analysis [15], spectral analysis [16] and iterative self-consistent refinement [17] were developed to filter irrelevant information. Some physical dynamics on the bipartite networks, including mass diffusion [18] and heat conduction process [19], have also been applied to design recommendation algorithms. The hybrid algorithm combining mass diffusion and heat conduction is shown to obtain significant improvement in both recommendation accuracy and item diversity [20]. Very recently, the performance of diffusion-based recommendation methods has been further enhanced by the preferential diffusion process [21] and modified heat conduction [22].

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Table 1
Properties of the used datasets.

Network	Users	Items	Links	Sparsity
Movielens	943	1682	82,520	$5.20 \cdot 10^{-2}$
Netflix	3000	3000	197,248	$2.19 \cdot 10^{-2}$
Delicious	10,000	232,657	1,233,997	$5.30 \cdot 10^{-4}$
Amazon	99,622	645,056	2,036,091	$3.17 \cdot 10^{-5}$

Previous works have found that people's behavior is far different from random and obeys certain predictable rules [23,24]. From the network point of view, the users' online behavior will emerge some typical statistical patterns on the network structure [25,26]. Besides, the network structure properties have been shown to affect the recommendation process [27–29]. Therefore, users' behavior patterns will inevitably influence the recommendation result. Actually, users' behavior patterns are commonly believed to be beneficial for recommendation since related algorithms generally filter relevant information by cooperating the history of similar users. If users randomly choose items, the recommender systems will not have valuable information to refer to. Hence, how much the recommendation performance relies on these behavior patterns of users can be an interesting problem. It is not only helpful for understanding the effect of network structure on recommendation process, but also meaningful in improving the robustness of the existing recommender systems.

In this paper, we focus on understanding online users' statistical behavior pattern and the related effect on recommendation. We will compare the real bipartite networks with the randomized counterpart networks (i. e. the reshuffled networks) in which real users' behavior pattern can be gradually removed. Actually, some specific properties of the real networks have been discovered by the comparison to the reshuffled networks such as the loop distribution [30,31], rich club [32,33], community structure [34], assortative [35] and motifs [36]. Here, we find that users' taste preferences are well separated in general, which means that users tend to seek for different items from each other. Users' interests for niche items (i.e. items with small degree) are found to be highly clustered, which indicates that the selectors of niche items enjoy a high similarity. Furthermore, we investigate the recommendation processes on both the real networks and the reshuffled networks. The results show that the performance of popularity-based recommendation methods do not rely on the real network structure while the performance of personalized recommendation methods is strongly related to it. The recommendation accuracy on each item is studied in detail. We find that most personalized recommendation methods can accurately catch users' taste on *hot items* (i.e. items with large degree) and the reshuffling process does not influence the recommendation accuracy for hot items. However, the accuracy for niche items is relatively low and drops significantly after removing users' behavior patterns. Actually, a robust recommender system should enjoy a stable recommendation accuracy even the user-item networks are deliberately attacked with some random links, our finding suggests that preserving the recommendation accuracy for niche items is significant for enhancing the robustness of the recommender system. Since we reveal an effective direction to improve the accuracy and the robustness of the existing recommender systems, this work is meaningful from the practical point of view.

2. Statistical behavior patterns of online users

In this paper, the datasets that we will use are the subsets of data obtained from four online systems: Movielens (http://www.grouplens.com/), Netflix (http://www.netflixprize.com/), Delicious (http://www.delicious.com/) and Amazon (http://www.amazon.com/). These data are random samplings of the whole records of user activities in these websites, the descriptions of data are given in Table 1.

To investigate users' behavior pattern, we will compare the real bipartite networks with the reshuffled networks. In each step of the reshuffling process, we first randomly pick two links from the real network, for example, one is from user i to item α and the other is from user j to item β (throughout this paper we use Greek and Latin letters, respectively, for item- and user-related indices). Then we rewire these two links by i to β and j to α. Hence, the degree of the users and items would not be changed by this reshuffling process while the links in this reshuffled networks are randomized. Denoting T as the reshuffling times and L as the total links in the networks, we fix $T/L = 3$ in the following analysis.

After the reshuffling process, users' degree and items' degree are preserved while the correlation between users and items are destroyed. To begin our comparison, we focus on the average degree of users' selected items. Suppose a user i selects m items with degree k_α ($\alpha = 1, 2, \ldots, m$), we calculate the average degree of the items that he/she selected as $d_i = \frac{\sum_{\alpha=1}^{m} k_\alpha}{m}$. Actually, the distribution of d reflects the heterogeneity of users' preference. When all the users prefer the same items, users' d will be quite close to each other. Consequently, the distribution of d will be narrow. On the contrary, the distribution of d will be quite flat if all the users seek for different items. We then compare the distribution $P(d)$ in real networks with $P(d)$ in the reshuffled networks. As shown in Fig. 1, $P(d)$ in real networks indeed are much boarder than that in the reshuffled networks, which means the real users exhibit heterogeneous preference in choosing items.

Second, for each user we study the inter-similarity among all his/her selected items. Suppose a user i selects m items and the similarity between items α and β is denoted as $s_{\alpha\beta}$, the inter-similarity among all these m items can be obtained by $\widetilde{S}_i = \frac{2\sum_{\alpha=2}^{m}\sum_{\beta=1}^{\alpha} s_{\alpha\beta}}{m(m-1)}$. The similarity $s_{\alpha\beta}$ of two items is calculated by the common neighbor index which is simply the

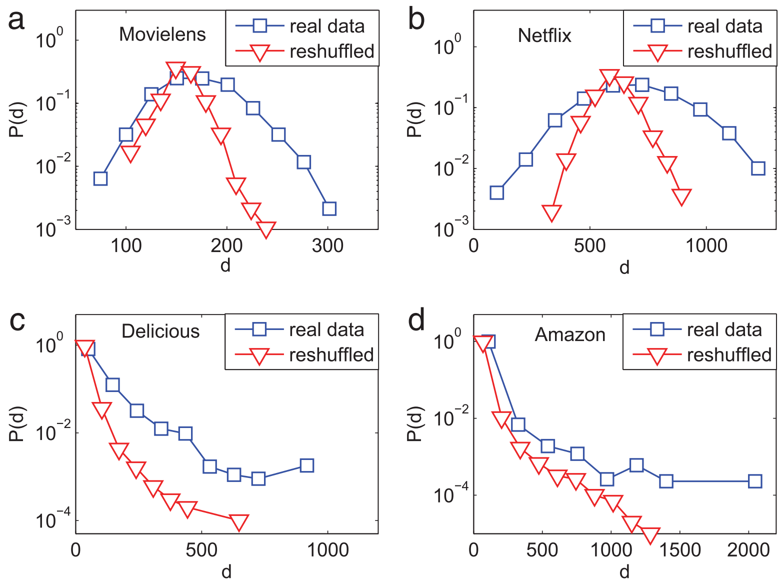

Fig. 1. (Color online) The distribution $P(d)$ in real systems and reshuffled networks where d is the average degree of selected items for each user.

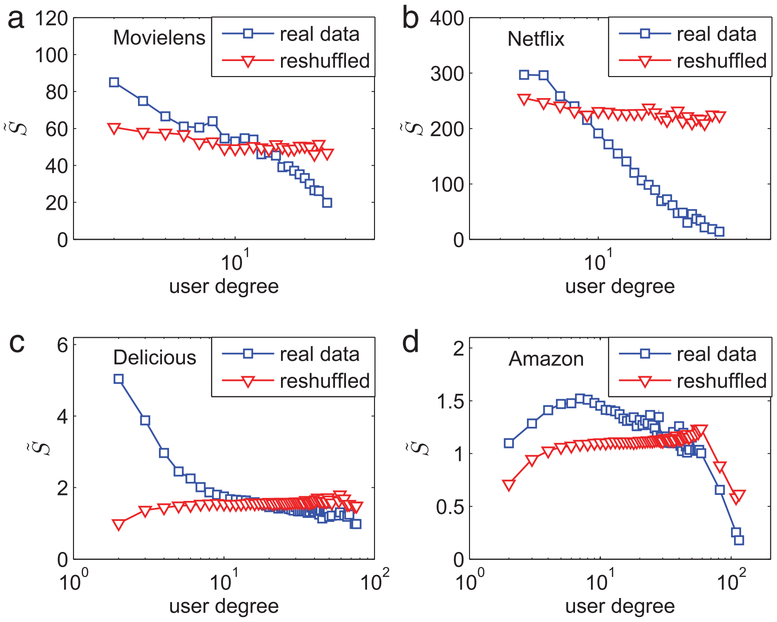

Fig. 2. (Color online) The inter-similarity $\widetilde{S}$ among all the selected items for each user vs user's degree. For a given x, its corresponding $\widetilde{S}$ is obtained by averaging over all the items whose degrees are in the range of $[a(x^2 - x), a(x^2 + x)]$, where a is chosen as $\frac{1}{2}\log 5$ for a better illustration.

overlap number of their neighbors [37,38]. In fact, $\widetilde{S}$ estimates the taste diversity for each single user. Specifically, when a user always selects the same kind of items, the value of $\widetilde{S}$ for him/her will be high. On the other hand, if the interest of a user changes from time to time, his/her $\widetilde{S}$ will be very low. As shown in Fig. 2, compared to the reshuffled networks, the low-degree users in real systems show a higher $\widetilde{S}$ while the high-degree users are with lower $\widetilde{S}$. Actually, as a low-degree user, he/she does not have much experience in exploring new items. He/she is more likely to conservatively choose items in the group which he/she is already familiar with (normally, popular items [25]). Consequently, his/her selected items are very similar. On the contrary, a high-degree user in real systems inclines to search and try different kinds of items (normally, unpopular items [25]). Therefore, his/her selected items will be with low $\widetilde{S}$.

Similarly, for each item we investigate the inter-similarity among all the users who selected it. Assume an item α is chosen by n users and the similarity between user i and j is denoted as s_{ij}, the inter-similarity among all these users is $\widetilde{S}_\alpha = \frac{2\sum_{i=2}^{n}\sum_{j=1}^{i} s_{ij}}{n(n-1)}$ where s_{ij} is again calculated by common neighbor index. Actually, $\widetilde{S}$ here reflects whether a specific

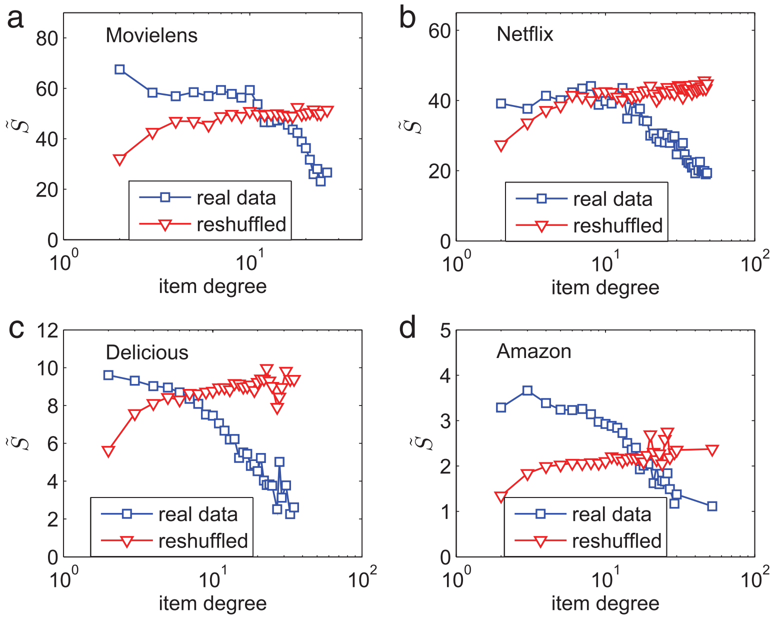

Fig. 3. (Color online) The inter-similarity $\widetilde{S}$ among all the selecting users for each item vs item's degree. The $\widetilde{S}$ is averaged by the same process in Fig. 2.

item is selected by the same group of users. In Fig. 3, we study $\widetilde{S}_\alpha$ in the real networks and the reshuffled networks. For hot items, their selectors in real networks have lower $\widetilde{S}$ than those in the reshuffled networks since hot items in real system are selected by many low-degree users [25]. However, the selectors of niche items in the real networks enjoy higher inter-similarity than those in the reshuffled networks since unpopular items in real networks are normally selected by high-degree users [25]. As we know, the personalized recommendation systems generally filter relevant information by cooperating the history of similar users. The clustering of users' interests for niche items is very meaningful since it makes the limited historical information of these niche items valuable for the recommendation systems to refer to. In following sections, we will detailedly investigate how these users' online behavior patterns affect the recommendation process.

3. Recommendation algorithms and the related features

In order to reveal the effect of users' online behavior patterns on information filtering, we investigate the recommendation process on both the real networks and the reshuffled networks in which real users' behavior patterns are destroyed. We consider four conventional recommendation algorithms including mass diffusion (MD), heat conduction (HC), collaborative filtering (CF), and popularity-based (PR) methods. We will show how the recommendation performance is influenced when we gradually remove users' real behavior patterns.

Let us consider an online system with N users and M items. It can be represented by a bipartite network with adjacency matrix A, where the element $a_{i\alpha} = 1$ if user i has collected item α, and $a_{i\alpha} = 0$, otherwise. As shown in Fig. 4(a), for a target user i, the MD algorithm starts by assigning one unit of resources to items collected by i, and redistributes the resource through the user-item network. We denote the vector $\mathbf{f}^i$ as the initial resources on items, where the α-th component f_α^i is the resource possessed by object α. Recommendations for the user i are obtained by setting the elements in $\mathbf{f}^i$ to be $f_\alpha^i = a_{i\alpha}$, in accordance with the objects the user has already collected. The redistribution is represented by $\widetilde{\mathbf{f}} = W\mathbf{f}$, where

$$W_{\alpha\beta} = \frac{1}{k_\beta} \sum_{l=1}^{N} \frac{a_{l\alpha} a_{l\beta}}{k_l}, \tag{1}$$

is the diffusion matrix, with $k_\beta = \sum_{i=1}^{N} a_{i\beta}$ and $k_l = \sum_{\gamma}^{M} a_{l\gamma}$ denoting the degree of item β and user l, respectively [18]. Physically, the diffusion is equivalent to a three-step random walk starting with k_i units of resources on the target user i. The recommendation score of an item is taken to be the amount of resources on it after the diffusion.

The HC algorithm works similar to the MD algorithm as shown in Fig. 4(b), the only difference is that the diffusion matrix is calculated as

$$W_{\alpha\beta} = \frac{1}{k_\alpha} \sum_{l=1}^{N} \frac{a_{l\alpha} a_{l\beta}}{k_l}. \tag{2}$$

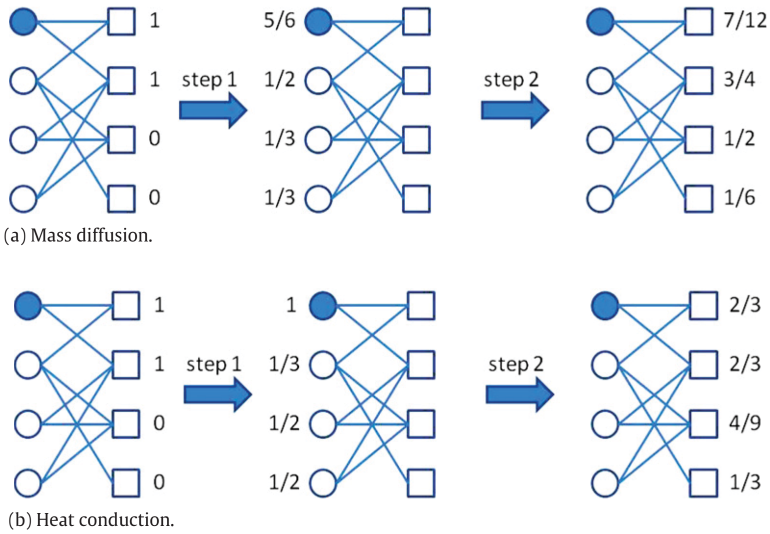

Fig. 4. (Color online) (a) Mass diffusion and (b) heat conduction algorithms (Eqs. (1) and (2)) at work on the bipartite user-item network. Users are shown as circles; items are squares. The target user is indicated by the shaded circle.

Physically, the temperature of a user is considered to be the average temperature of its nearest neighbors (i.e. its connected items) and the temperature of an item is also calculated as the average temperature of its nearest neighbors (i.e. its connected users). The higher the final temperature of an item is, the higher its recommendation score will be [19].

The CF algorithms provide recommendations based on user or item similarities. Here, we consider the item-based CF which has been successfully applied to many online applications such as Amazon (one of the largest online product retailers) [39]. In the item-based CF method, the recommendation score of an item is evaluated based on its similarity with the collected items of the target user. The final recommendation score for each item can be written as

$$\widetilde{f}_\alpha^i = \sum_{\beta=1}^{M} s_{\alpha\beta} a_{i\beta} \tag{3}$$

where $s_{\alpha\beta}$ is the similarity between item α and β [40]. Based on the bipartite networks, many methods can be used to quantify the similarity of two items [38]. Here we apply the Common Neighbors index in the calculation.

The PR algorithms are very simple and commonly used in many websites. In this method, the recommendation score for each item is proportional to its popularity (i.e. the degree of the item).

In these methods, the final recommendation scores for items that user i have already collected are set to 0. The recommendation list for user i is generated by ranking all his/her uncollected items in descending order of their final recommendation scores. Actually, the differences of these recommendation methods have been studied in detail in Ref. [41]. In order to further understand these methods, we calculate the normalized total recommendation score for each item as $F_\alpha = \sum_{i=1}^{N} \frac{\widetilde{f}_\alpha^i}{\max_\beta(\widetilde{f}_\beta^i)}$. The result is shown in Fig. 5. In statistical sense, the MD, CF and PR methods assign high recommendation score to the high degree items. In the HC method, the items with low degree are generally with high recommendation score. Therefore, the MD, CF and PR methods tend to recommend the popular items while the HC method inclines to recommend unpopular items.

4. The effect of users' behavior patterns on information filtering

We then apply all these methods to the real networks and their reshuffled networks to see how users' real behavior patterns affect the recommendation. Similar to the previous work [20], to test the recommendation result we randomly remove 10% of the links (the probe set denoted as E^P). We then apply the algorithms to the remainder (the training set denoted as E^T) to produce a recommendation list for each user.

In order to measure the accuracy of the recommendation result, we make use of the ranking score index [18]. For a target user, the recommender system will return a ranking list of all his uncollected items to him according to the recommendation scores. For each hidden user-item relation (i.e., the link in probe set), we measure the rank of the item in the recommendation list of this user. For example, if there are 1000 uncollected items for user i, and item α is at 10th place, we say the position of this item is 10/1000, denoted by $RS_{i\alpha} = 0.01$. A successful recommendation result is expected to highly recommend the items in the probe set, and thus leading to small ranking score. Averaging over all the hidden user-item relations, we obtain

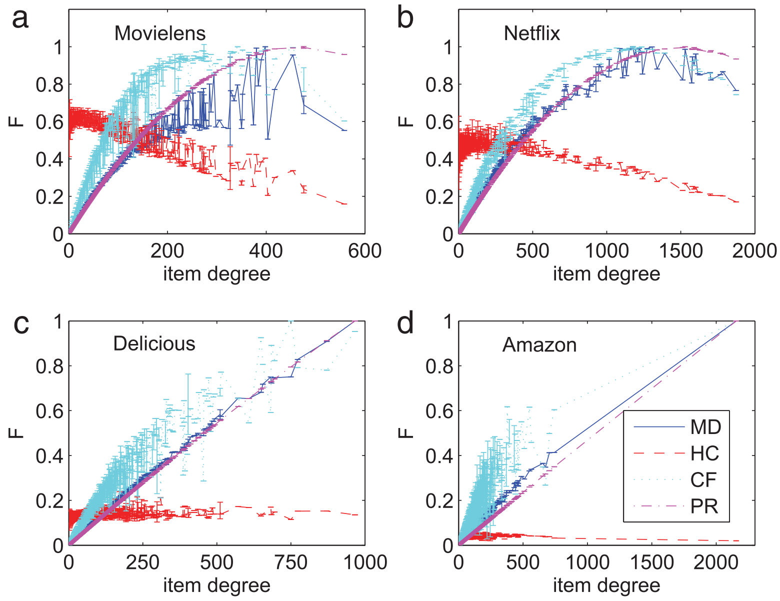

Fig. 5. (Color online) The normalized total recommendation score F vs item degree in different recommendation systems.

the mean value of ranking score to evaluate the recommendation accuracy, namely

$$\langle RS\rangle = \frac{1}{|E^P|}\sum_{i\alpha\in E^P} RS_{i\alpha}, \tag{4}$$

where $i\alpha$ denotes the probe link connecting user i and item α. Clearly, the smaller the ranking score, the higher the algorithm's accuracy, and vice versa.

In Fig. 6, we report how the average ranking scores of different recommendation methods are influenced when we gradually remove real users' behavior patterns. The results show that the ranking score of PR is hardly affected by the reshuffling process. It is reasonable because the PR method does not rely on the detail bipartite network structure and gives the recommendation score for each item simply according to its popularity. On the contrary, the personalized recommendation methods such as the MD, HC and CF methods, which return different recommendation list for each user based on his/her historical information, are influenced. Obviously, the ranking score of the HC method increases most significantly when we reshuffle the networks. In fact, the HC method is considered as an effective method to enhance recommendation diversity by mainly predicting users' preference for niche items [20]. Therefore, the result implies that without the real correlation between users and items, only the information of degree is insufficient for the recommendation systems accurately providing a diverse recommendation. More specifically, as we discussed in the previous section, users' interests for niche items are highly clustered in real systems. Hence, the recommendation systems can predict target user's potential niche items by cooperating the information from his/her similar users. However, in the reshuffled networks users' interests for niche items only slightly overlap, so there is little information from the similar users for the recommendation engines to refer to. It finally leads to the serious increment in the ranking score of the HC method.

As recommendation algorithms which tend to recommend popular items, MD and CF methods are not so sensitive to the reshuffling process as the HC method. In the dense networks like Movielens and Netflix, the ranking scores of MD and CF stay almost unchanged. However, in the sparse networks like Delicious and Amazon, the ranking scores of MD and CF methods show an observable increment. In order to see the effect of the reshuffling process on the MD and CF methods in detail, we study the ranking score for each item, namely

$$\langle RS_\alpha\rangle = \frac{1}{|E^P_\alpha|}\sum_{i\alpha\in E^P_\alpha} RS_{i\alpha}, \tag{5}$$

where E^P_α denotes all the links in the probe set that connect to item α. Then we can see the relation between items' degree and their ranking score, the result is reported in Fig. 7. In real networks, the hot items enjoy low ranking score ($\langle RS\rangle \approx 0$) while the niche items are with high ranking score (it can be even higher than the random recommendation whose $\langle RS\rangle = 0.5$). It suggests that almost all the hot items are accurately recommended while niche items' accuracy is quite low and has plenty of room for improvement. Therefore, in order to design a more effective personalized recommendation method than current ones, it is crucial to solve the cold start problem, i.e. to improve the recommendation for niche items [20,21,42]. Another interesting finding is that only the ranking scores for unpopular items are affected by the reshuffling process while

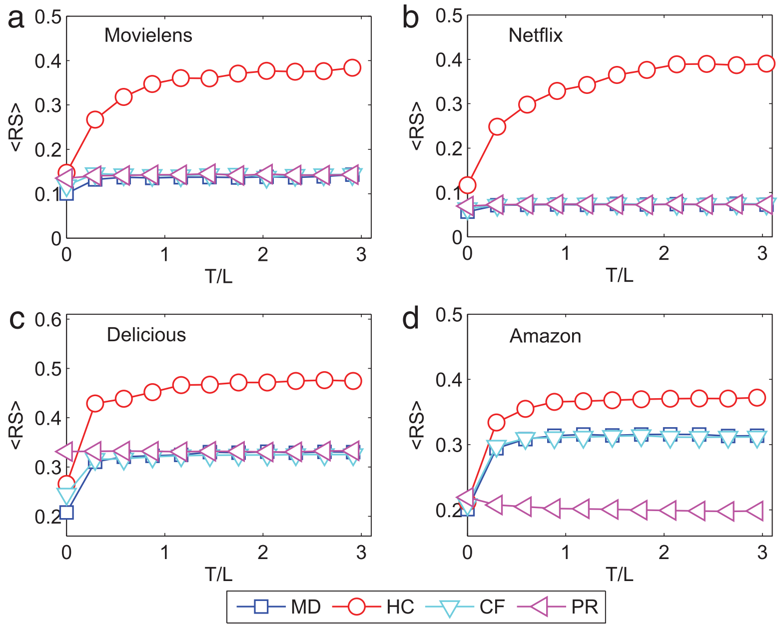

Fig. 6. (Color online) The ranking score $\langle RS \rangle$ of different recommendation methods when reshuffling the real networks. T is the reshuffling steps and L is the total links in the networks.

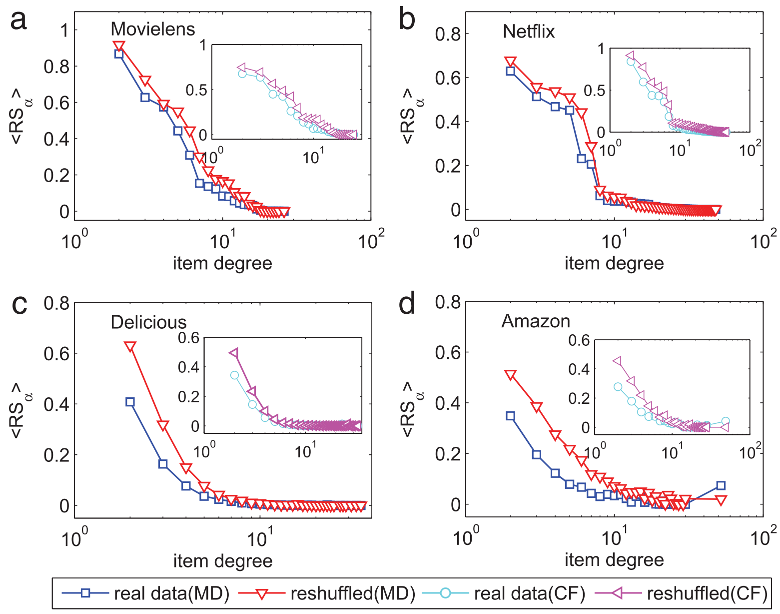

Fig. 7. (Color online) Dependence of ranking score $\langle RS_\alpha \rangle$ on the item degree. The $\langle RS_\alpha \rangle$ is averaged by the same process in Fig. 2. The main figures are the results of the MD method while the inserts are the results of the CF method. In the reshuffled networks, $T/L = 3$.

the ranking score for popular items stays almost the same. Since lots of items are with low degree in the sparse networks such as Delicious and Amazon, the average ranking scores for MD and CF in Fig. 6(c) and (d) increase with the reshuffling process. In the Movielens and Netflix networks where the links are relatively dense, fewer items are with low degree in these networks. Accordingly, the average ranking scores for MD and CF in Fig. 6(a) and (b) do not increase much. From the practical point of view, a robust recommender system should enjoy a stable recommendation accuracy even the user-item networks are deliberately attacked with some random links. Therefore, if we want to enhance the robustness of the recommender system, the most effective way is to preserve the recommendation accuracy for niche items since they are sensitive to the random links.

The previous study reveals that hybrid of the MD and HC methods can result in significant improvement in both recommendation accuracy and item diversity [20]. Actually, this hybrid method is implementable because the HC method can effectively catch the users' taste for niche items. As the recommendation accuracy for the HC method in the reshuffled networks is close to the random recommendation ($\langle RS \rangle = 0.5$), the hybrid method is impossible to be carried out in the systems where users randomly choose their items. It indicates that users' behavior patterns in real systems are essential for solving the diversity-accuracy dilemma of recommender systems.

5. Conclusion

The development in network science has greatly improved the function as well as our understanding to many real systems. In recommendation which is considered as a promising way to solve the problem of information overabundance, researchers have designed the network-based recommendation methods to filter irrelevant information. For example, with the help of some typical physics dynamics on the bipartite networks, the mass diffusion and heat conduction algorithms have been proposed to improve the recommendation accuracy and diversity, respectively.

In this paper, we investigate the users' online behavior patterns and related effect on information filtering. we compare the real bipartite networks with the reshuffled networks in which users' behavior patterns are gradually removed. The results reveal the heterogeneity of users' taste preference and highly clustering of users' interests on niche items. In addition, we find that the performance of personalized recommendation methods such as MD, HC and CF is strongly related to the real network structure. We then detailedly study the recommendation accuracy of the personalized methods on each item. The result indicates that hot items enjoy high recommendation accuracy and their accuracy is quite robust to the reshuffling process. On the contrary, niche items cannot be accurately recommended without real users' behavior properties.

Our work is also meaningful in practical aspect. Our result suggests that the niche items are more valuable information than hot items. Those users who co-collect the same niche item are more likely share common interests. Based on this understanding, a wisely designed similarity, such as Resource Allocation index [43,44], may improve the recommendation performance of the conventional collaborative filtering algorithm. Moreover, in order to improve the accuracy of current recommendation engines, our results suggest that it is crucial to improve the recommendation accuracy for niche items. For enhancing the robustness of the recommender engines, the most effective way is to preserve the recommendation accuracy for niche items when the e-commercial systems are added with some noisy information. In this sense, our work may shed some light for developing a new recommender system with both higher accuracy and better reliability.

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