Supplementary Information to the paper: "Geometric Phase Contribution to Quantum Nonequilibrium Many-body Dynamics"

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EFFECTIVE HAMILTONIAN IN THE ROTATING FRAME

Let us consider a more general setup, which extends the example studied in the main text. Suppose that the system is described by some interacting Hamiltonian $\mathcal{H}_0$, which is rotationally invariant with respect to some vector coupling $\boldsymbol{\lambda}$ [1]. This coupling can represent, for example, an external magnetic or electric field, an anisotropic interaction constant, a nematic order parameter, etc. It can also break an internal U(1) symmetry of $\mathcal{H}_0$ like the mixing symmetry between different spin components. At a nonzero value of $\boldsymbol{\lambda}$ the rotational symmetry is thus broken. Now let us consider a dynamical process where this coupling uniformly rotates at a fixed magnitude. Then the time dependent Hamiltonian reads

$$\mathcal{H}(t) = U^{-1}(t)\mathcal{H}_0(\boldsymbol{\lambda}_0)U(t), \tag{1}$$

where $U(t)$ is the unitary operator corresponding to this rotation. In the rotating frame $|\tilde{\psi}(t)\rangle = U^{-1}(t)|\psi(t)\rangle$ the effective Hamiltonian in the Schrödinger equation picks up an additional "centrifugal" term:

$$\begin{aligned} i\hbar\partial_t|\tilde{\psi}\rangle &= \mathcal{H}_{\text{eff}}|\tilde{\psi}\rangle, \\ \mathcal{H}_{\text{eff}} &= \mathcal{H}(t) - i\hbar\omega U^{-1}\partial_\phi U, \end{aligned} \tag{2}$$

where ϕ is the rotational angle and $\omega = \dot{\phi}$ is the frequency. The centrifugal term in the Hamiltonian has a number of interesting properties. (i) It is proportional to the frequency ω and thus it can be used to continuously modify the effective Hamiltonian. (ii) At a constant frequency the effective Hamiltonian is time independent. (iii) The diagonal components of the centrifugal term are given by the connection one-form of the corresponding energy levels. Let us point out that the time independence of the centrifugal term, which follows from its rotational invariance, and its locality imply that the rotations in the generic parameter space do not lead to a continuous heating even in ergodic nonintegrable systems. This situation is opposite to, e.g., Floquet Hamiltonians. Which are usually nonlocal and lead to constant energy absorption in generic interacting systems. Thus we expect that the qualitative results of the main text are valid even if we add arbitrary interactions to the Hamiltonian, provided they preserve the rotational symmetry but break its integrability.

DERIVATION OF EQ. (6)

Here we show how Eq. (6) from the main text can be obtained for a generic two-level system using only the minimal assumption that the time dependence enters as a U(1) rotation with constant frequency. For such a system the Schrödinger equation can be written in the instantaneous basis $\{|\text{gs}\rangle, |\text{es}\rangle\}$ as

$$\partial_t\tilde{a}_{\text{gs}} = -\langle\text{gs}|\partial_t|\text{es}\rangle \exp\left[iE_{\text{gs,es}}(t) - i\Gamma_{\text{gs,es}}(t)\right]\tilde{a}_{\text{es}}, \tag{3}$$

$$\partial_t\tilde{a}_{\text{es}} = -\langle\text{es}|\partial_t|\text{gs}\rangle \exp\left[iE_{\text{es,gs}}(t) - i\Gamma_{\text{es,gs}}(t)\right]\tilde{a}_{\text{gs}}, \tag{4}$$

where $E_{nm}(t)$ and $\Gamma_{nm}(t)$ are the dynamical and geometric phases defined in the main text. Notice that we have applied the following gauge transformation $a_n = \tilde{a}_n \exp\left[\int_{t_i}^{t} \mathrm{d}\tau\left(-i\epsilon_n(\tau) + iA_\tau(|n\rangle)\right)\right]$ to the coefficients introduced as: $|\psi(t)\rangle = a_{\text{gs}}(t)|\text{gs}(t)\rangle + a_{\text{es}}(t)|\text{es}(t)\rangle$. In agreement with the discussion in the previous section all matrix elements appearing in the Schrödinger equation (3), (4) are time independent if $\phi(t) = \omega t$. It is convenient to change variables from the time t to the angle ϕ in (3), (4):

$$\partial_\phi\tilde{a}_{\text{gs}} = -\langle\text{gs}|\partial_\phi|\text{es}\rangle \exp\left[i\frac{E_{\text{gs,es}}(\phi)}{\omega} - i\Gamma_{\text{gs,es}}(\phi)\right]\tilde{a}_{\text{es}}, \tag{5}$$

$$\partial_\phi\tilde{a}_{\text{es}} = -\langle\text{es}|\partial_\phi|\text{gs}\rangle \exp\left[i\frac{E_{\text{es,gs}}(\phi)}{\omega} - i\Gamma_{\text{es,gs}}(\phi)\right]\tilde{a}_{\text{gs}}. \tag{6}$$

Then we differentiate Eq. (6) one more time with respect to ϕ and eliminating $\tilde{a}_{\text{gs}}$ and $\partial_\phi\tilde{a}_{\text{gs}}$ gives a second order differential equation with constant coefficients:

$$\partial_\phi^2\tilde{a}_{\text{es}} - i\left(\frac{\Delta\epsilon}{\omega} - \Delta A_\phi\right)\partial_\phi\tilde{a}_{\text{es}} + \mathsf{g}_{\phi\phi}(|\text{gs}\rangle)\tilde{a}_{\text{es}} = 0, \tag{7}$$

from which it is trivial to derive Eq.(6) in the main text:

$$p_{\text{ex}} = |a_{\text{es}}(\phi_f)|^2 = \mathsf{g}_{\phi\phi}(|\text{gs}\rangle)\frac{\sin^2\left[\frac{1}{2}\Omega(\omega)\phi_f\right]}{\left[\frac{1}{2}\Omega(\omega)\right]^2}, \tag{8}$$

where $\Omega(\omega) := \sqrt{\left[\frac{\Delta\epsilon}{\omega} - \Delta A_\phi\right]^2 + 4\mathsf{g}_{\phi\phi}(|\text{gs}\rangle)}$.

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[1] In general the Hamiltonian $\mathcal{H}_0$ can be invariant with respect to an arbitrary continuous group, not necessarily rotations.