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Essays in Asset Pricing

by

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Declaration of Authorship

I, Andrea Vedolin, hereby declare that I have written this thesis without any inadmissible assistance and that I have not submitted this thesis or parts of it to another university.

Signed:

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“Le done le ciapa senpre la forma del sogno che 'le contien.”

Abstract

My dissertation aims at understanding the impact of uncertainty and disagreement on asset prices. It contains three main chapters. Chapter One gives a general introduction into the topic of partial information and heterogeneous beliefs.

Chapter Two explains the link between credit spreads and the heterogeneous formation of expectations in an economy where agents with different perception of economic uncertainty disagree about future cash flows of a defaultable firm. The intertemporal risk-sharing of disagreeing investors gives rise to three testable implications: First, larger belief heterogeneity increases credit spreads and their volatility. Second, it implies a higher frequency of capital structure arbitrage violations. Third, it reduces expected equity returns of low levered firms, but the link can be reversed for high levered firms. We use a data-set of firm-level differences in beliefs, credit spreads, and stock returns to empirically test these predictions. The economic and statistical significance of the intertemporal risk-sharing channel of disagreement is substantial and robust to the inclusion of control variables such as Fama and French, liquidity, and implied volatility factors.

Chapter Three studies the link between market-wide uncertainty, difference of opinions and co-movement of stock returns. We show that this link plays an important role in explaining the dynamics of equilibrium volatility and correlation risk premia, the differential cross-sectional pricing of index and individual options, and the risk-return profile of several option trading strategies. We use firm-specific data on analyst forecasts and test the model predictions. We obtain the following novel results: (a) The difference of index and individual volatility risk premia is linked to a counter-cyclical common disagreement component about future earnings; (b) This common component helps to explain the differential pricing of index and individual volatility smiles in the cross-section, as well as the time-series of correlation risk premia extracted from option prices; (c) The time series of returns on straddle and dispersion option portfolios reflects a significant time-varying risk premium, which compensates investors for bearing common disagreement risk; (d) Common disagreement is priced in the cross-section of option strategy returns.

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Acknowledgments are so much fun and perhaps the only thing read, but they always get written at the last minute, so they become *truly* heartfelt statements.

The Sisyphean task of finding happiness in what I was doing or trying to come to an end with a paper while pursuing my personal dreams required a single-mindedness that does not leave much room, emotionally speaking, for the demands of any relationship. Often I was a stranger in my own land, constantly searching for my comfort zone. The longer the work took me, the more likely I became a bitter, jaded harpy with a precarious sense of self. I am grateful to more people than I could possibly list here for their help, support, and bearing me over the past years.

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I thank Andrea Buraschi for his guidance and help, foremost for teaching me the essence of how to write a finance paper. And forgiving many of my random email rants. Enrico Di Giorgi and Pietro Veronesi for being in my thesis committee and thoughtful comments.

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I dedicate this thesis to my parents, Catherine and Guido, and my brothers Oliver and Nicolas who taught me that the essentials of happiness are something to love, something to do, and something to hope for. I always find something to do and hope springs eternal. Your ever present love and care is what I always rely on.

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To My Parents & Brothers

Chapter 1

Introduction

When in December 1912 John Pierpont Morgan was asked before a congressional investigating committee how he decided whether to make an investment or not, he replied “The first thing is character.” The lawyer skeptically questioned whether not money or property would be more important, and Morgan countered “A man I do not trust could not get money from me on all the bonds in Christendom.” The gist of this anecdote is simple: Credit business is based on trust and confidence.¹ In an economy absent of confidence, banks hoard money and market liquidity evaporates. Does not this sound all too familiar?

While trying to come to an end with this thesis, we find ourselves in the midst of the severest recession since the 1930ies, and some might even argue that the D-word is just around the corner. Shocking as the disaster on equity markets might be, the freezing up of the flow of credit is far more damaging to the overall health of the economy. Central banks and Treasury departments all around the world have been desperately trying to fight that gridlock and scarcely one day goes by without any new dramatic intervention. The injected liquidity is going through revolving doors and now that interest rates are almost zero, central banks in Europe, Japan, and the U.S. have turned to quantitative

¹In fact, the root of the word credit comes from credo, Latin, for I believe.

easing. Historically, central banks have been the lender of last resort to banks, now, they are becoming the last resort to everyone.²

The current situation from the viewpoint of a financial economist is briefly summarized by the chief economist at Goldman Sachs: “No sane banker with good contacts and clients would do this [hoard money]. It would be a huge arbitrage profit if they wanted to lend, but they don’t.” The reason for this “insanity” is economic uncertainty which started as a local problem with the increase of subprime mortgage defaults and has now affected the real economy. Unquestionably, the concept of economic uncertainty and its measurement is elusive and abstract. In this thesis, I seek at explaining more thoroughly how uncertainty can arise and its potential impact on asset prices.

To give solid ground to my hypotheses, I plot in Figure 1.1 the uncertainty proxies for a cross-section of different industries to illustrate the huge surge of uncertainty in the current crisis. I proxy economic uncertainty by the dispersion of analysts’ forecasts about future earnings of firms.³ There are two notable points: First, an increase of uncertainty is not particular to the current crisis. For example, both around the economic crisis of 2001 and 2008, the Russian default in 1998, and the terrorists’ attacks in September 2001 uncertainty increases and decreases short after. Second, higher uncertainty is observed across all sectors, no matter how different they might be, so one could argue that there is some systemic component in uncertainty which is common across all industries.

Interestingly, the surge in uncertainty emasculates rather quickly. If these effects are transitory, why should we care⁴ one might ask correctly? The reason is that uncertainty has real effects. To see that uncertainty shocks have an impact on the real economy, I

²The definition of the central bank acting as the lender of last resort goes back to the 18th century (see Thornton, 1802 and Bagehot, 1873). The idea was that banks finance opaque assets with a long maturity and short-lived liabilities – a combination that is vulnerable to sudden loss of confidence. To avoid preventable disasters when confidence evaporates, the classical view is that the central bank should lend to illiquid but solvent banks, at a penalty rate, and against collateral deemed to be good under normal times.

³Economic uncertainty can be measured in many different ways. For instance, Bansal and Yaron (2004) use a GARCH process to proxy consumption volatility, Bloom (2009) uses realized stock markets volatility and Alexopoulos and Cohen (2009) extract an index of economic uncertainty on the number of articles that appear in the New York Times which use the terms uncertain and/or uncertainty and economic and/or economy. The drawback of these measures is their inherently backward-looking nature. GARCH or realized volatility and number of newspaper articles can only be calculated from an ex post perspective using historical data. Information extracted from option prices provides fertile ground to extract future expectations of economic agents. Indeed, the VIX, an index constructed from implied volatilities of a continuum of options on the S&P 500, is often used as a proxy of fear or economic uncertainty. However, in the models I study, implied volatility is an endogenous quantity driven by uncertainty/belief disagreement.

⁴Or worse, write a thesis on it.

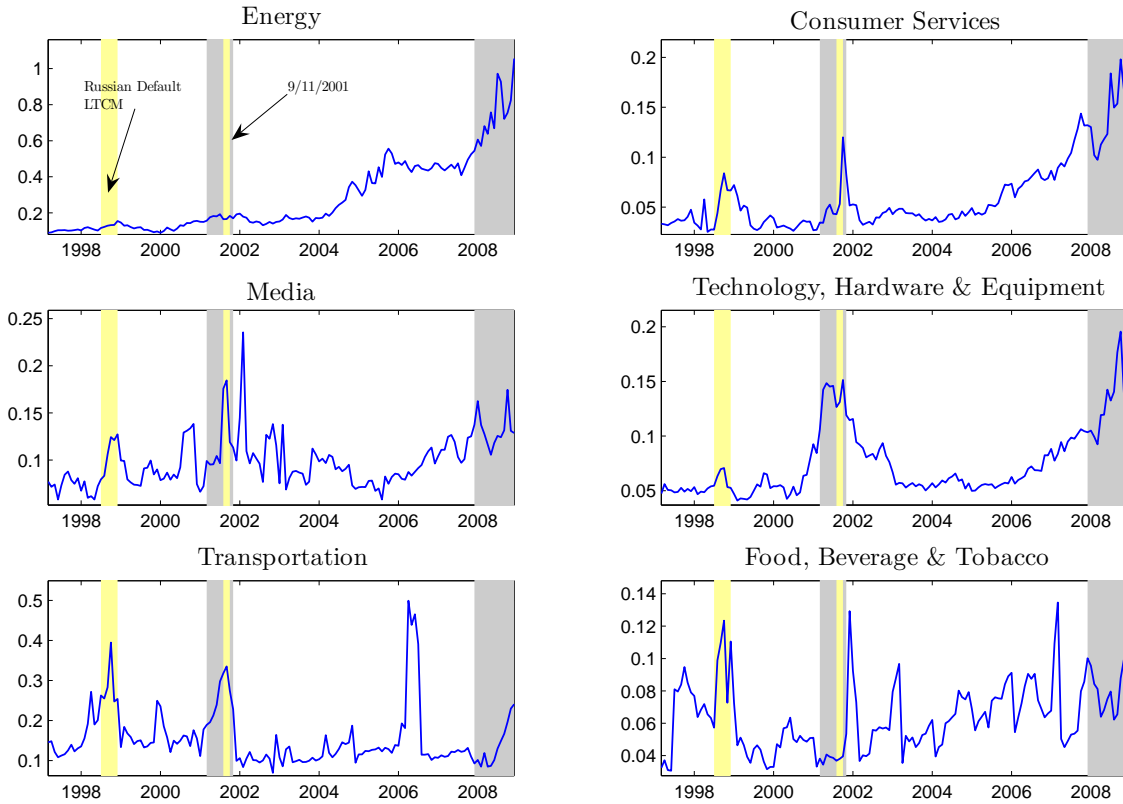


FIGURE 1.1: **Cross-Section of Uncertainty**

I plot sector-wide uncertainty for six different sectors. Industries are defined according to the two-digit GICS code. The yellow shaded areas represent financial crises, the gray shaded areas economic crisis as defined according to the NBER. Data is monthly and runs from March 1997 to December 2008.

estimate a VAR on monthly data using a set of macro-economic variables together with an industry-wide uncertainty proxy.⁵

Figure 1.2 plots the impulse response functions for industrial production (left panel) and employment (right panel) to a one standard deviation shock. We note that both industrial production and employment drop around 0.2% within 10 months and then rebound after almost 20 months for industrial production and 25 months for employment.

⁵The variables in the estimation order are $\log(\text{industrial production})$, $\log(\text{employment})$, hours, $\log(\text{consumer price index})$, $\log(\text{average hourly earnings})$, money stock (M2), Federal Funds Rate, industry wide uncertainty, and $\log(\text{S\&P500})$. This ordering is based on the assumptions that shocks instantaneously influence the stock market, then prices (wages, the CPI, and interest rates) and finally quantities (hours, employment and output). Including the stock returns as the first variable in the VAR ensures the impact of stock returns is already controlled for when looking at the impact of uncertainty shocks. All variables are Hodrick-Prescott (HP) detrended.

What is striking is that the effect of uncertainty is by order of magnitude larger than for instance other standard policy instruments like the federal funds rate or the money stock.

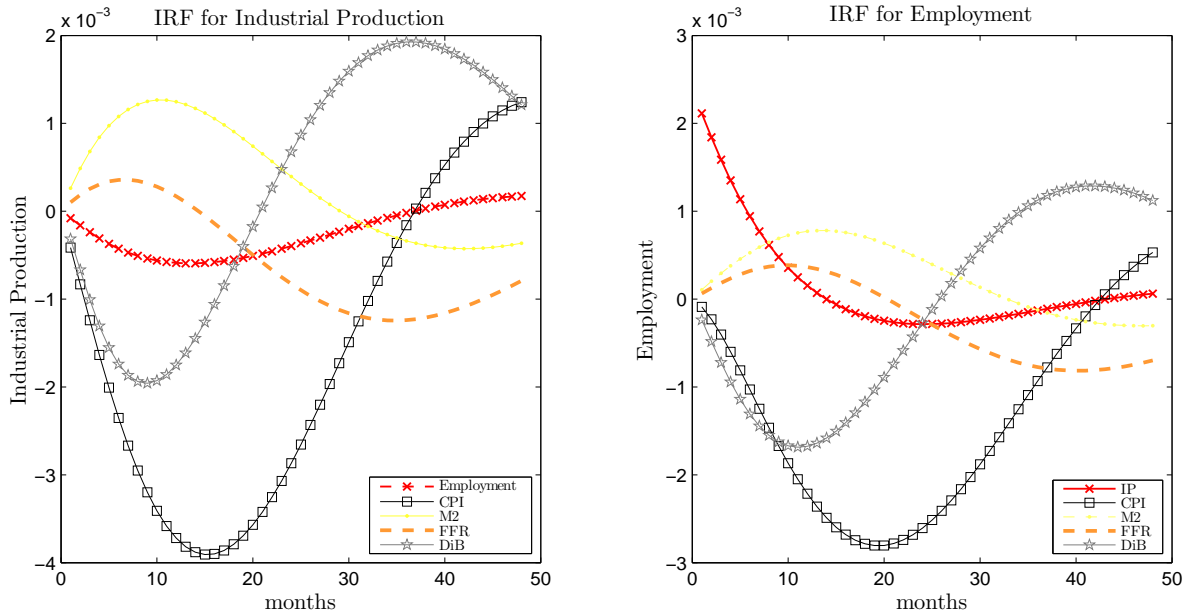


FIGURE 1.2: **Impulse Response Functions**

The left panel plots the impulse response function to a one standard deviation change in the corresponding variables with respect to industrial production. The right panel plots the same but for employment.

The focus of this thesis is not so much the macroeconomy as a whole but I aim at studying in more detail the implications of economic uncertainty on asset prices. Clearly, the current crisis gives a vivid example of how an increase in default risk is coupled with economic uncertainty. To see this, I plot in Figure 1.3, the uncertainty proxies together with their sector-wide asset swap spread, which is a measure of default risk in this industry.⁶

There seems to be a strong co-movement between the asset swap spread and the uncertainty proxies. Indeed, I find that the unconditional correlation between these series is around 0.69 with a p-value for testing the null hypothesis of zero correlation almost at zero. The implied volatility of near maturity options is said to be a good proxy of agent's

⁶I define the asset swap spread as the difference between the yield on the corporate bond and the LIBOR.

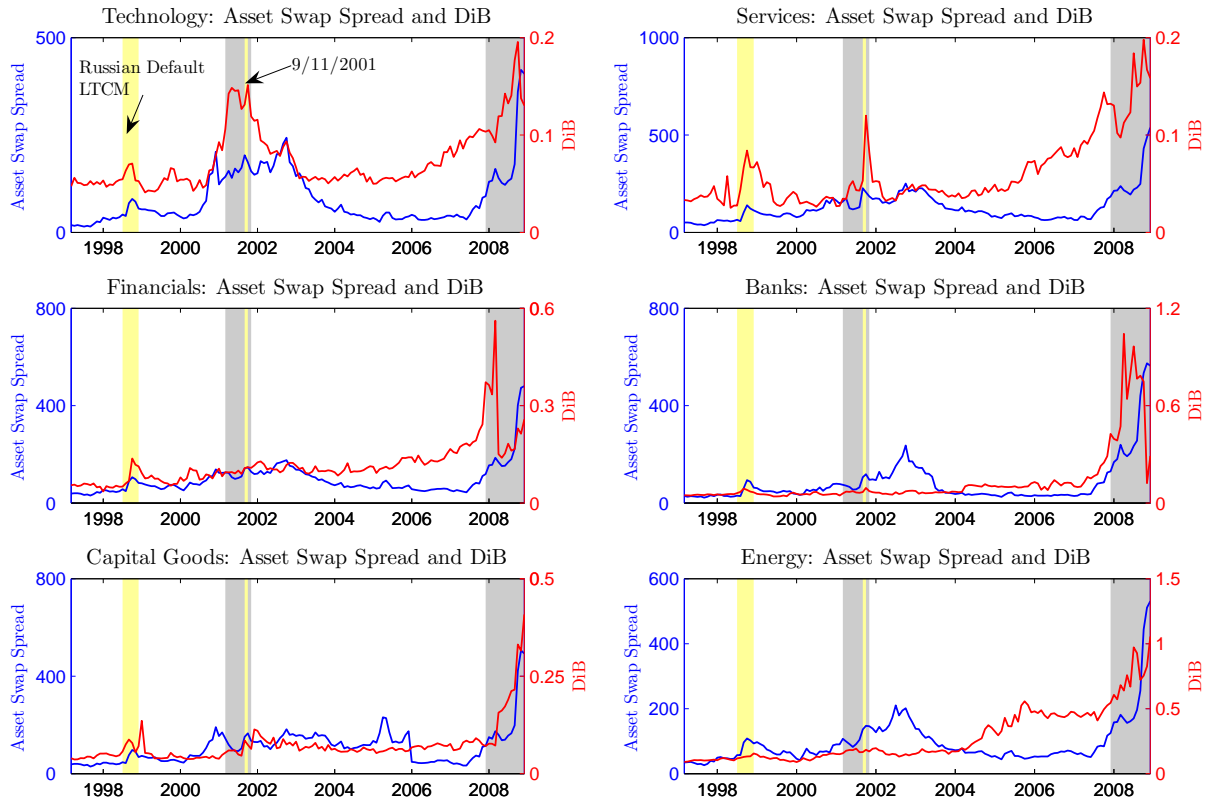


FIGURE 1.3: **Cross-Section of Uncertainty and Asset Swap Spreads**

I plot sector-wide uncertainty and asset swap spreads for six different sectors. Industries are defined according to the four-digit GICS code. The yellow shaded areas represent financial crises (LTCM debacle and terrorists' attacks), the gray shaded areas economic crisis as defined according to the NBER. Data is monthly and runs from March 1997 to December 2008.

future expectations and therefore a measure of economic uncertainty.⁷ However, as the measure itself is extracted from prices and prices are endogenously determined by potentially many variables, it is natural to assume that the implied volatility itself is driven by these variables. I find that economic uncertainty/disagreement not only impacts on default risk, but also on the implied volatility of individual and index options. I plot in Figure 1.4 the sector-wide uncertainty proxies together with a sector-wide implied volatility index which is calculated from near maturity call and put options. Again, we note a remarkably high correlation between the implied volatilities and the uncertainty proxies.

⁷As opposed to realized volatility itself which is said to be a backward looking measure of uncertainty.

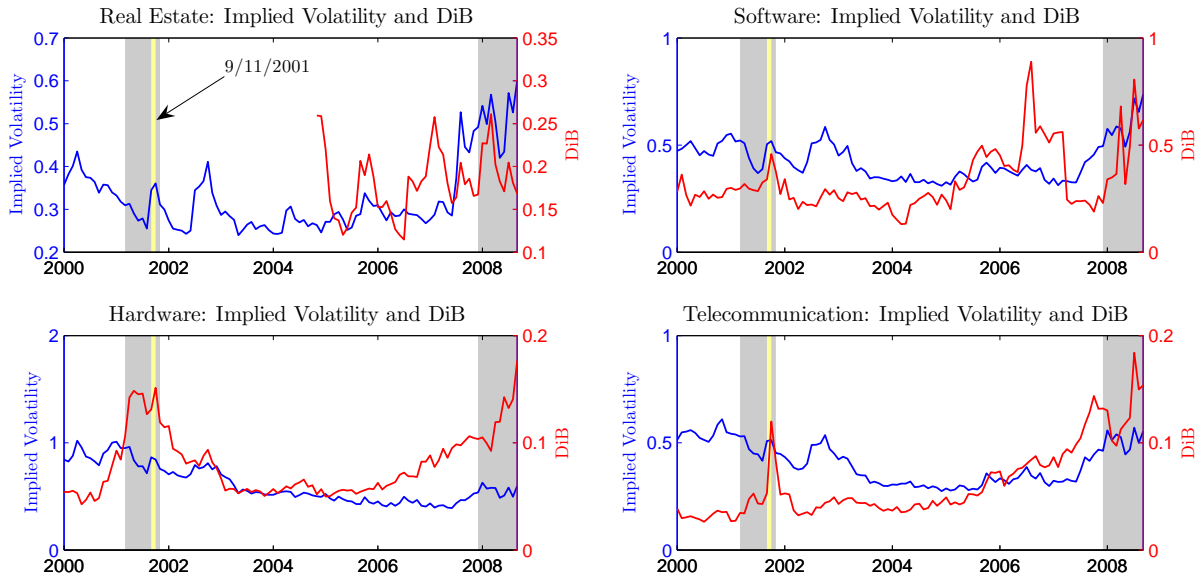


FIGURE 1.4: **Cross-Section of Uncertainty and Implied Volatility**

I plot sector-wide uncertainty and implied volatility for six different sectors. Industries are defined according to the four-digit GICS code. Implied volatility is defined as the average 30 day at-the-money implied volatility from put options. The yellow shaded areas represent the terrorists' attacks, the gray shaded areas economic crisis as defined according to the NBER. Data is monthly and runs from March 1997 to December 2008.

The fact of heightened uncertainty during crises periods suggests the existence of a systemic uncertainty component which is common to individual uncertainty proxies. In Figure 1.5, right panel, I plot such a common uncertainty proxy constructed from the cross-section of all firm-specific uncertainty proxies together with the default spread defined as the difference between the Moody's Baa and Aaa bond yields. Again, we note a remarkably high co-movement between these two series. Indeed, a simple linear regression from the default spread on the common uncertainty yields a t-statistic of more than 10 and the R^2 is 44%. On the left panel, we plot the common uncertainty together with the VIX which is an index of implied volatilities of options on the S&P 500. Running a regressions from the VIX on the common uncertainty yields a highly significant beta coefficient with a R^2 of almost 20%.

Starting from these observations, I seek to build a parsimonious model of partial information that entails salient features found in credit, equity, and option markets. Uncertainty is a central pillar in the economics and finance literature and the most common way to

model imperfect information is a Bayesian inference framework (see Kandel and Stambaugh, 1996, Lewellen and Shanken, 2001, and Pástor, 2000).⁸ The main feature of this modeling approach is that if a parameter is unknown, a prior distribution is assumed for this parameter and agents update their beliefs using Bayes' law. Another way of introducing model uncertainty would be to assume that agents are ambiguous about the data generating process itself. In these models of Knightian uncertainty, rational agents distinguish between situations where they know the probability distributions of random variables (known unknowns) and situations where they do not have this information (unknown unknowns).⁹ In this thesis, I abstract both from the important work on ambiguity aversion and incomplete information but homogeneous beliefs. In the latter class of work, the representative agent learns, by applying Bayes' law,¹⁰ about the hidden state of the economy which evolves over time as a Markov process. The early literature analyzes the portfolio problem of a investor who does not observe the true state of the economy (see e.g. Detemple, 1986, Gennotte, 1986, and Ghysels, 1986). Veronesi (1999) studies the impact on asset prices and shows that agents react more at times when uncertainty is high which generates higher asset price volatility. This induces risk-averse investors to demand higher returns for bearing this risk. David (1997) derives similar results in a production economy.

Surprisingly little effort has been undertaken to distinguish the learning uncertainty component in asset prices from the dispersion in beliefs both theoretically and empirically. Massa and Simonov (2005) empirically decompose the learning component into the learning uncertainty and dispersion in beliefs using forecasts errors from a state space model. They show that both components are priced risk factors and contribute different portions to the explanation of risk premia. Similarly, Ozoguz (2008) extracts dispersion of beliefs and Bayesian uncertainty from time series of state probabilities estimated from a regime switching model. The author finds in predictability regressions that both the learning uncertainty and disagreement predict first and second moments

⁸A very different way to consider economic uncertainty is found in the long-run risk models of Bansal and Yaron (2004) or Bekaert, Engstrom, and Xing (2008). In these models, economic uncertainty is modeled as the time-varying volatility of consumption and dividend growth together with recursive preferences.

⁹Dow and Werlang (1992), Epstein and Wang (1994, 1995), and Epstein and Miao (2003) use multi-prior expected utility preferences as in Gilboa and Schmeidler (1989). Anderson, Hansen, and Sargent (2000), and Uppal and Wang (2003) follow another line whereby in their models uncertainty is modeled by a set of priors and the investor's aversion to it is introduced via a penalty function. Recently, the literature has adopted a learning plus ambiguity approach. Hansen (2007) summarizes the literature.

¹⁰Brandt, Zeng, and Zhang (2004) also analyze alternative learning rules.

of the aggregate market return. While a deeper distinction in a learning uncertainty and dispersion component is highly desirable, I leave this for future research.

One of the most compelling features of heterogeneous beliefs models is that they induce trade among agents. When agents have different beliefs, optimal portfolios in equilibrium depend on the investors' relative wealth and their beliefs. My thesis seeks at extending this growing literature on models with differences in beliefs amongst agents regarding the fundamentals and some signal process in an incomplete information economy.

The literature on heterogeneous beliefs is vast and one line of the literature, e.g., Harris and Raviv (1993), Detemple and Murthy (1994, 1997), and Basak (2000), assumes that agents hold heterogeneous prior beliefs about unobservable economic variables. In these models, agents continue to disagree with each other even after they update their beliefs using identical information, but the difference in their beliefs deterministically converges to zero. Therefore, the belief dispersion in these models can only have a temporary effect. In another strand of the literature, e.g., Scheinkman and Xiong (2003), Dumas, Kurshev and Uppal (2008), Buraschi and Jiltsov (2006) and David (2008) heterogeneous beliefs arise from agents' different prior knowledge about the informativeness of signals and the dynamics of unobservable economic variables.¹¹ In the models of Scheinkman and Xiong (2003) and Hong, Scheinkman, and Xiong (2006), risk neutral investors subject to short-selling constraints update their beliefs based on their personal interpretations of incoming news. It follows that trade occurs whenever investors valuations "cross", i.e. whenever the more optimistic investor switches to being the more pessimistic. In a world in which investors interpret news differently, the greater news stimulus associated with an asset leads to a higher time-series variability of investors' relative valuations and larger trading volumes via the "crossing". At the same time, the short-sales constraint make some assets overpriced relative to others, so that their valuation reflects the value-weighted average opinion. Dumas, Kurshev, and Uppal (2008) study a similar economy as Scheinkman and Xing (2003) and refer to the speculative risk of unexpected movements in competitors' beliefs as sentiment risk. In these kind of models, behavioral biases such as overconfidence prevent the agents from learning the true parameters in the model. In David (2008) agents have different underlying models of the data generating process as opposed to differing initial priors. In his model, the difference in beliefs does

¹¹Another appealing way to prevent Bayes' law to die out, would be to assume that two groups of investors receive two different pieces of information opposed to receiving the same piece of information and processing it differently. This approach is studied in the work of Grossman and Stiglitz (1980), Admati (1985) and many others.

not converge asymptotically to zero because the drift of the underlying fundamentals is allowed to switch according to a two-state Markov process.

The distinction between an incomplete information rational expectations model and a model incorporating behavioral biases might not always be so trivial. For instance, Anderson, Ghysels, and Juergens (2005) simply sidestep the potential behavioral aspects of their framework, meaning whether the contribution of disagreement on asset prices could also be interpreted as an overreaction of irrational agents. Brav and Heaton (2002) demonstrate that from a mathematical point the rational expectations incomplete information economy and an economy with irrational agents are hard to distinguish. Irrationality-induced anomalies cannot survive the presence of rational arbitrageurs unless there are limits to arbitrage that prevent the effectiveness of rational bets against mispricing.¹² Unquestionably, behavioral biases are appealing,¹³ however, my motivation to remain in the class of rational beliefs is a practical one: The empirical quantification of these biases in beliefs or probability such as overconfidence or anchoring on a firm-specific level is difficult if not impossible.¹⁴ I choose to remain in the class of rational beliefs and economic agents update their beliefs based on the available information using Bayes' rule. The difference in their posteriors can arise either from a difference in agents' priors or a difference in subjective parameters of the dynamics of cash flows or signals. In particular, I assume that the volatility of the signal growth rate is perceived differently by the two agents in my economy. In this way, belief dispersion does not converge to zero even asymptotically. I aim at investigating these properties both theoretically and empirically in two different projects: First, in the context of an equilibrium structural model of credit risk and second, in option markets for individual and index options.

The first essay studies the potential relation between firm-specific and common economic uncertainty and credit risk in an economy with heterogeneous beliefs. Merton's (1974)

¹²Even if the rational agent's victory over the overconfident agent in the model of Dumas, Kurshev, and Uppal (2008) is only in the "fairly long run".

¹³I fully acknowledge that financial economics is a social science and we can learn much by studying how people effectively process data and take decisions. I cite Knight (1964): We perceive the world before we react to it, and we react not to what we perceive but always to what we infer.

¹⁴It would be important to distinguish between price or survey data based indices. Baker and Wurgler (2006) form a composite index of sentiment that is based on six known proxies for sentiment (closed-end fund discount, share turnover, dividend premium etc.). They show that sentiment indeed affects stock returns, but there are a size-, a volatility and some other effects which influence the impact of sentiment on prizes. www.sentix.de provides sentiment indices for all major stock indices, interest rates, and currencies from weekly web-based surveys. It would be interesting to see whether survey based measures of belief heterogeneity and these sentiment indicators have similar properties.

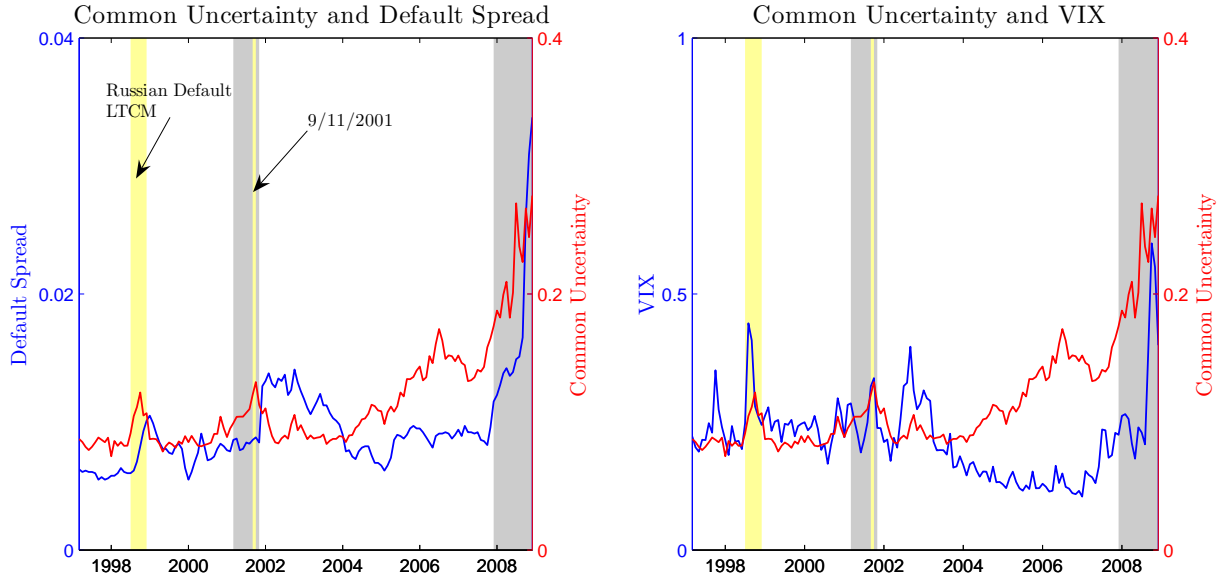


FIGURE 1.5: **Common Uncertainty, Default Spread and VIX**

I plot a common uncertainty proxy together with the default spread and the VIX. The default spread is defined as the difference between the yield on the Moody's Baa and Aaa bond. The yellow shaded areas represent financial crises (LTCM debacle and terrorists' attacks), the gray shaded areas economic crisis as defined according to the NBER. Data is monthly and runs from March 1997 to December 2008.

structural model of credit risk, and its extensions in, e.g., Black and Cox (1976), offer a framework for pricing corporate debt as a contingent claim on the value of the firm. An appealing property of structural models is that they motivate a very explicit link between credit spreads and a set of firm-specific variables. The empirical performance of standard structural models is, however, mixed, as these models tend to predict counterfactually low credit spreads, especially for short-term, high quality bonds. In the first essay, we aim at taking a different stance. We consider an economy in which the growth rate of a firm's cash flows is unknown and agents have different perceptions of the (unknown) volatility of expected future cash flows, i.e. the level of economic uncertainty. A higher (lower) volatility of expected cash flow growth in such an economy is directly linked to a higher (lower) economic uncertainty. Since expected cash flow growth is unobservable, agents estimate it from a common set of observable variables and agree to disagree about the firm's future cash flows and a common signal. In order to estimate a firm's expected growth rate of cash flows, they observe a firm-specific signal (firm's cash flows themselves) and a systematic (market wide) signal. The systematic signal is linked to the aggregate state of the economy and is correlated with firm's cash flows expected growth rate. This setting allows us to investigate the role played by both the systematic

and firm-specific belief disagreement components raised by economic uncertainty, and to study their effects on credit spreads both in the cross-section and the time series. The introduction of uncertainty and belief disagreement extends the classical Merton (1974) structural model to an incomplete information economy with heterogeneous beliefs. In this economy, changes in beliefs modify individual allocations into defaultable securities in order to balance ex-ante differences in expected marginal utilities across good and bad states of the economy. We solve the model and obtain testable empirical predictions for the impact of economic uncertainty and differences in beliefs on credit spreads and asset prices. We merge a dataset of firm-specific differences in beliefs and credit spreads and test these predictions empirically. We find five important sets of results that are new to the literature: (a) Counter-cyclical uncertainty is positively related to a common disagreement component about future earning opportunities by financial analysts; (b) The common component of the difference in beliefs in earning forecasts is the most significant variable in time-series regressions for credit spreads. At the same time, firm-specific difference in beliefs is the most significant component in the cross-section; (c) Uncertainty induces a significant co-movement between credit spreads and stock volatility; (d) During the 2008 Credit Crisis the link between uncertainty and credit spreads is even stronger than in previous crisis periods; (e) Uncertainty and belief heterogeneity have significant explanatory power for no-arbitrage violations implied by single factor models in credit markets

This essay is joint work with [Fabio Trojani](#) (University of Lugano) and [Andrea Buraschi](#) (Imperial College London).

The second essay studies the differential pricing of index options and single-stock options and their difference in volatility risk premia, despite the relative similarity of the underlying distributions (see e.g., Bakshi, Kapadia, and Madan, 2003 and Bollen and Whaley, 2004). In particular, single-stock options appear to be cheaper and have a volatility risk premium which is virtually zero, while the risk premium on the index options is potentially large. We extend the standard Lucas (1978) economy to two investors and multiple goods. The growth rates of the different firms are not observable and have to be estimated by the investors who agree to disagree about the firms' future dividends and a business cycle indicator. The wedge between the index and individual volatility risk premium is mainly driven by a correlation risk premium which emerges endogenously due to the following model features: In a full information economy with independent fundamentals, returns correlate solely due to the correlation of the individual stock with

the aggregate endowment (“diversification effect”). In our economy, stock return correlation is endogenously driven by idiosyncratic and systemic (business-cycle) disagreement (“risk-sharing effect”). We show that this effect dominates the diversification effect, moreover it is independent of the number of firms and a firm’s share in the aggregate market. In equilibrium, the skewness of the individual stocks and the index differ due to a correlation risk premium. Depending on the share of the firm in the aggregate market, and the size of the disagreement about the business cycle, the skewness of the index can be larger (in absolute values) or smaller than the one of individual stocks. As a consequence, the volatility risk premium of the index is larger or smaller than the individual. In equilibrium, this different exposure to disagreement risk is compensated in the cross-section of options and model-implied trading strategies exploiting differences in disagreement earn substantial excess returns. We test the model predictions in a set of panel regressions, by merging three datasets of firm-specific information on analysts earning forecasts, options data on S&P 100 index options, options on all constituents, and stock returns. Sorting stocks based on differences in beliefs, we find that volatility trading strategies exploiting different exposures to disagreement risk in the cross-section of options earn high Sharpe ratios. The results are robust to different standard control variables and transaction costs and are not subsumed by other theories explaining the volatility risk premia.

This essay is joint work with [Fabio Trojani](#) (University of Lugano) and [Andrea Buraschi](#) (Imperial College London).

All chapters are self-contained and can be read independently of each other.

Chapter 2

Economic Uncertainty, Disagreement and Credit Markets

joint with [Andrea Buraschi](#) and [Fabio Trojani](#)

Introduction

Structural models with additive preferences find it difficult to explain the joint behavior of corporate bond spreads and stock returns. The credit spread puzzle is determined by the difficulty to generate large average credit spreads of high-quality bonds, while keeping model-implied default and recovery rates at the empirically observed levels. It is intrinsically related to the broader challenge of standard theories to explain asset prices based on realistic assumptions about cash-flow innovations and discount rates. Notable well-known manifestations of this challenge for the US stock market are the equity premium and excess volatility puzzles.

Motivated by the need to understand these puzzling features of asset prices, a large literature has explored from different perspectives and with different approaches the general equilibrium link between uncertainty and asset returns. An important strand of the more recent literature has investigated in a variety of dynamic models the structural link between uncertainty, the properties of agents' subjective beliefs and asset prices. Three major directions have been followed. The first explores homogenous agents economies, in which incomplete information about the investment opportunity set is modeled with a

(single-prior) Bayesian belief formation mechanism that causes equilibrium asset prices deviating from their complete-information values (e.g., Veronesi, 1999 and 2000 and Pástor and Veronesi, 2003). A second strand of literature maintains the representative agent assumption and studies the asset pricing implications of partial information and learning when investors are averse to Knightian uncertainty. In these models, learning is linked to a whole set of beliefs. However, agents' concern to follow decisions robust to uncertainty leads them to focus on the most conservative (pessimistic) belief. This belief distortion can create even larger deviations of asset prices from their equilibrium values under complete information (e.g., Epstein and Schneider, 2008 and Leippold, Trojani, and Vanini, 2008). The third stream of literature investigates partial information and learning in heterogeneous agents economies where investors have diverse beliefs. In these models, agents agree to disagree and their equilibrium risk-sharing behavior creates interactions with non trivial asset pricing implications (e.g., Basak, 2000, Scheinkman and Xiong, 2003, Buraschi and Jiltsov, 2006).

The different aspects of uncertainty studied in these models can produce quite distinct effects on asset returns. For instance, in Veronesi (1999) a higher Bayesian uncertainty can increase expected returns because of an increased sensitivity of the marginal utility to news. Leippold, Trojani, and Vanini (2008) show that ambiguity aversion coupled with learning further increases the equilibrium equity premium when the elasticity of intertemporal substitution is larger than one. Uncertainty can lead to opposite implications for expected returns in models with diverse beliefs, depending on the tightness of trading constraints. Scheinkman and Xiong (2003) draw inspiration from Miller's (1977) hypothesis and show that, when risk-neutral agents face short-selling constraints and agree to disagree, equilibrium prices reflect the beliefs of overconfident investors. In this context, a more dispersed belief heterogeneity implies lower excess returns. Basak (2000), Buraschi and Jiltsov (2006) and Dumas, Krushev, and Uppal (2009), among others, study the risk-sharing mechanism that takes place in a frictionless economy with risk averse investors. Their results show that when disagreement is large the resulting stochastic discount factor effect yields larger excess returns in equilibrium. Credit markets provide a useful laboratory to produce sharp evidence on the direction of the effect of uncertainty and diversity of beliefs on asset prices. While most equities can be easily sold short, the search costs of an over-the-counter market like the corporate bond market can be significant, implying short-selling constraints that are likely more binding. Therefore, an empirical rejection of Miller's (1977) hypothesis for the corporate bond

and stock markets is a potentially strong evidence in favor of the risk-sharing-induced impact of belief heterogeneity on asset returns.

In order to understand the equilibrium implications of this risk-sharing effect on credit spreads and stock returns, we solve a frictionless structural Merton (1974)-type credit risk model, in which we derive explicit testable restrictions for the equilibrium link between belief heterogeneity, credit spreads and stock returns. In our model, agents have different perceptions of future firm cash flows and their degree of uncertainty, captured by the volatility of expected cash flows growth. They form individual forecasts of firm cash-flow growth using a common set of observable variables and agree to disagree based on these beliefs. The common observable variables include both a firm-level signal (firm's cash flows themselves) and a systematic (market-wide) signal correlated with firm's cash-flow growth. The specification of a common uncertainty component linked to market-wide economic growth is motivated by the empirical observation that proxies of difference in beliefs across firms share a large common component that surges during recessions and periods of financial crises, when there is less confidence about future macroeconomic conditions and firm's cash flows, which suggests that belief-driven risk-sharing effects are stronger in phases of market downturn. Kurz (1994) discusses this link and explains how higher uncertainty can lead economic agents to have more disperse beliefs. There is also evidence that credit spreads systematically widen in bad economic states: A simple analysis reveals that the default spread, given by the difference of Moody's Baa and Aaa bond yields has a monthly correlation of 60% with the first principal component of the dispersion of analyst forecasts.¹

The equilibrium stochastic discount factor in our economy is a direct function of the (stochastic) degree of belief heterogeneity. This feature gives rise to a stochastic firm value volatility and risk-neutral skewness. The equilibrium risk-sharing mechanism between optimistic and pessimistic agents implies that a higher uncertainty or belief heterogeneity produces at the same time a lower equilibrium firm value, a higher firm value volatility, and a more negative firm value risk-neutral skewness. Therefore, the market price of default risk stays in an unambiguously positive relation to the latent degree of uncertainty and diversity of beliefs. This general equilibrium structure has the following testable implications for the joint relation between belief-driven risk-sharing effects, credit spreads and stock returns.

¹This correlation is higher than the one of any other risk factor used in the literature. For instance, the correlation between the default spread and the VIX index (another proxy often used to measure uncertainty) around crisis times is only 28%.

First, higher uncertainty and belief heterogeneity are unambiguously linked to higher credit spreads. Since this feature arises mostly via the price of default risk, and less through the probability of default, we can find model parameter choices that imply together realistic credit spreads and default probabilities. Second, the sign of the relation between belief heterogeneity and stock returns is ambiguous and leverage-dependent: It is positive for high leverage firms, but it can turn negative for less levered firms. This finding is striking, because it arises in a frictionless economy.² It is due to the different skewness sensitivity of the default options embedded in the stock: For low (high) leverage companies, this option is far out-of-the-money (closer to be in-the-money) and its value is more (less) sensitive to changes in skewness. Third, in contrast to single-factor structural models of credit risk, the equilibrium joint distribution of credit spreads and stock returns implies an ambiguous leverage-dependent sign for their correlation. This feature gives rise to equilibrium violations of standard capital structure arbitrage relations implied by single factor models, which are less likely for highly levered firms.

We test empirically these three model predictions using a variety of panel and Logit regressions. We first construct measures of individual cash-flow disagreement for a large cross-section of 337 US firms, using earning forecast data from the Institutional Brokers Estimate System (I/B/E/S) in the period from 1996 to 2005. In a second step, we compute an empirical proxy for the systematic component of the disagreement and estimate by dynamic factor analysis a common component from the whole panel of individual disagreement proxies. We find that this common component is highly counter-cyclical and that it explains a large fraction of the variation in individual proxies, thus supporting the conjecture that cross-sectional firm's cash-flow uncertainty is largely systematic.

We support the first model prediction by showing that disagreement proxies have an unambiguously positive impact on credit spreads. A one standard deviation increase in firm-specific disagreement increases credit spreads by approximately 18 basis points, which is more than one third the credit spread sample standard deviation in our data. Similarly, a one standard deviation increase in systematic disagreement increases the

²This finding can potentially help reconcile the mixed sign of the empirical link between disagreement and stock returns found in the literature. Diether, Malloy, and Scherbina (2002) find a negative relation between dispersion of analysts' earnings forecasts and stock returns. Johnson (2004) argues that if dispersion proxies for idiosyncratic risk the return of a levered firm should decrease with dispersion. Qu, Starks, and Yan (2004) find a positive link between disagreement and returns using a different scaling variable. Anderson, Ghysels, and Juergens (2005) and Banerjee (2011) also estimate a positive relation. Avramov, Chordia, Jostova, and Philipov (2009) indicate that the negative relation between stock returns and dispersion could be a manifestation of financial distress.

average credit spread by about 10 basis points. These results are robust to the inclusion of various other predictors of credit spreads, the choice of the sample period, and different econometric methods. Our tests of the second model prediction find that disagreement has a strong explanatory power for stock returns, but the sign of this relation depends on leverage, a possibility that is included in the implications of the equilibrium risk-sharing dynamics in our model. We find that one third of the firms with lowest leverage features a negative relation, but the remaining two thirds of firms imply instead a positive relation. Third, disagreement significantly increases the conditional likelihood of an empirical violations of capital structure no-arbitrage restrictions implied by single-factor credit risk models. We find that Merton's and similar structural models' negative relation between credit spreads and stock prices is often empirically violated, with a frequency between 14% and 19% that tends to decrease with firm leverage.³ These patterns of arbitrage-free violations can be well reproduced within our calibrated model and empirical proxies of disagreement are the most significant variables in explaining the likelihood of such violations.

Several papers have studied structurally the impact of uncertainty in equity markets, but little is known for corporate bond markets. David (2008) extends the Merton (1974) model to a structural regime-switching economy, in which higher uncertainty implies higher corporate credit spreads via a convexity effect generated by the time-variation in the solvency ratio over the business cycle. Cremers and Yan (2010) extend the intuition in Pástor and Veronesi (2003) to study the effects of uncertainty about profitability in a single period model with both equity and corporate debt. They show that the convexity effect generated by a higher uncertainty increases stock valuations and typically dampens corporate bond prices, where this effect is stronger if firm's leverage is higher. Empirically, they support a positive association between stock valuation and uncertainty, for most but not all uncertainty measures. However, they do not support the positive association between uncertainty and credit spreads, particularly for uncertainty proxies related to firm age. Korteweg and Polson (2010) take a distinct Monte Carlo Markov Chain approach and directly add parameter uncertainty in a structural credit risk model with exogenous firm value dynamics. Their estimation results show that parameter uncertainty can explain up to 40% of the credit spread that is typically attributed to liquidity, taxes and jump risk, without significantly raising default probabilities. Finally, Guntay and Hackbarth (2010) find empirically a positive relation

³Recently, Kapadia and Pu (2008) give further support to our findings by showing that the frequency of violations in credit default swap markets can be as large as 35%.

between credit spreads and uncertainty, based on firm-specific measures of dispersion in analyst forecasts.

The contribution of our paper is different along several dimensions. First, we develop a structural equilibrium model with disagreeing investors in which the predictions of uncertainty and diversity of beliefs for the joint distribution of credit spreads and stock returns can be consistently studied. Second, we validate empirically the model predictions, while controlling for other potentially endogenous effects. Consistently with the findings in Güntay and Hackbarth (2010), we estimate a positive link between diversity of beliefs and credits spreads. In addition to their results, we first find that this link is not subsumed by firm-specific explanatory variables measuring volatility or jump risk, which are endogenously driven by uncertainty in our theoretical model. Second, our estimation results show that while belief disagreement systematically rises credit spreads, it generates an ambiguous correlation with stock returns that has strong explanatory power for the likelihood of an arbitrage-free violation in credit markets, as predicted by our theoretical model.

Our paper also borrows from the asset pricing literature studying the effects of difference in beliefs. Basak (2000) shows that the risk of an extraneous process uncorrelated with fundamentals can be priced in equilibrium. Dumas, Kurshev, and Uppal (2009) study the relation between overconfidence and the excess volatility of stock returns. Buraschi and Jiltsov (2006) analyze an economy in which belief disagreement makes options non redundant assets and study empirically the implications for the joint behavior of index option prices and trading volume. Our approach follows the earlier literature, but crucially departs from this research by introducing the default risk dimension. This feature complicates the solution of the equilibrium, as stocks and corporate bonds are derivatives on an endogenous firm value process with stochastic volatility and skewness, and gives rise to novel model predictions for this literature.⁴ Such predictions include the ambiguous leverage-dependent character of the correlation between credit spreads and stock returns and the direct dependence of the likelihood of a credit market arbitrage-free violation on the degree of diversity of opinions.

The remainder of the paper is organized as follows. Section 2.1 introduces our structural equilibrium model of credit risk with uncertainty. It then presents and discusses the equilibrium solutions for asset prices. Section 2.2 investigates in detail the empirical

⁴See also a related point raised in Longstaff and Wang (2008).

model predictions. Section 3.3 describes our data set and Section 3.4 presents the empirical findings. Section 2.5 provides robustness checks and Section 2.6 concludes. Proofs are collected in the Appendix.

2.1 The Economy with Uncertainty and Disagreement

We study asset prices in an economy where macro-economic uncertainty is linked to investors' heterogeneous beliefs on the growth opportunities of a firm financed by both equity and debt. In reality the growth rate of future earnings and cash flows is typically unknown to economic agents and financial analysts largely disagree on it. This additional element of uncertainty is not surprising since future cash flows depend on several variables that are difficult to predict, like future sales and costs, the regulatory environment, and general business conditions.

We borrow from the literature and consider a Bayesian inference framework to model agents' heterogeneous beliefs and learning. One line of the literature, e.g., Harris and Raviv (1993), Detemple and Murthy (1994), Morris (1996), Basak (2000) and Buraschi and Jiltsov (2006) assumes agents with heterogeneous prior beliefs about unobservable economic variables. In these models agents disagree even after they update their beliefs using identical information, but the difference in their beliefs typically converges to zero asymptotically. In a second strand of the literature, e.g. Scheinkman and Xiong (2003) and Dumas, Kurshev, and Uppal (2009), heterogeneous beliefs follow from agents' different perception on either the informativeness of signals or the parameters in the dynamics of some economic variable. We follow this route in order to allow for a truly stochastic steady-state dynamics of the disagreement process. This feature allows us to capture the relevant link between uncertainty and belief disagreement. In our economy investors observe the same piece of information. To circumvent the no-trade theorem and induce trading in equilibrium, we follow the standard assumption that heterogeneous priors are not motivated by private information, so that it is rational for agents to "agree to disagree".

2.1.1 The Model

We start from the structural model of Merton (1974) and take the dynamics of the asset cash flows of the firm as a primitive.⁵ The main departure from the Merton (1974) model is the introduction of heterogeneous investors who disagree about the growth rate of future cash flows. Investors are identical in all other aspects, such as preferences, endowments, and risk aversion. The firm has a simple capital structure consisting of debt and equity, where debt is the sum of two defaultable zero bonds with different seniority and identical maturity. Since cash flows are stochastic, the firm may default on its obligations. It follows that in equilibrium the value of the debt depends on the probability that the firm will be able to generate enough cash flows to cover its liabilities. Let the assets' cash flows $A(t)$ follow the process:

$$\begin{aligned} d \log A(t) &= \mu_A(t)dt + \sigma_A dW_A(t), \\ d\mu_A(t) &= (a_{0A} + a_{1A}\mu_A(t))dt + \sigma_{\mu_A} dW_{\mu_A}(t), \end{aligned}$$

where $\mu_A(t)$ is the cash flow's expected growth rate, $\sigma_A > 0$ its volatility, $a_{0A} \in \mathbb{R}$ the growth rate of expected cash flow growth, $a_{1A} < 0$ its mean-reversion parameter and $\sigma_{\mu_A} > 0$ the volatility. The vector composed of $W_A(t)$ and $W_{\mu_A}(t)$ is assumed to be a standard two-dimensional Brownian motion.

The cash flow process $A(t)$ is observable by all investors in the economy, but the expected growth rate $\mu_A(t)$ is unknown and must be estimated. The volatility parameter σ_{μ_A} measures the objective uncertainty of the expected future growth of firm cash flows. It is linked to the subjective degree of uncertainty among investors via their Bayesian updating rules and the observed realizations of $A(t)$. Denote by $m_A(t)$ the estimated value of the true growth rate $\mu_A(t)$. In practice, estimating future earnings or cash flows is the goal of financial analysts. These forecasts inherently display some degree of subjective uncertainty about firms' future earnings. Thus, agents may eventually disagree and have different values for $m_A^i(t)$, for $i = 1, 2$. This apparently innocuous departure from the basic Merton model conveys some important implications. First, whereas in Merton's model the value of assets can be taken as exogenous, in our model it depends on agents' relative demands, which are functions of their subjective beliefs. This simple

⁵Merton's (1974) model of credit risk assumes an exogenous firm value process with constant volatility. Even if we treat the firm value as endogenous, the predictions of our model for the case in which there is no disagreement are identical to those of Merton (1974).

feature makes the equilibrium value of the firm a function of the degree of uncertainty and belief disagreement in the economy. Second, due to market incompleteness, contingent claims on the firm value cannot be priced by standard replication arguments: A dynamic portfolio investing in the firm assets and the risk-free bond does not replicate the payoffs of equity and corporate bonds. Additional financial assets are needed to complete the market and their prices depend on equilibrium portfolio demands.

To reproduce a common business cycle component of belief disagreement, we assume that the expected growth of firm cash flows is linked to a market-wide risk factor related to the business cycle and the competitive landscape.⁶ We model this feature by a signal $z(t)$ that analysts can use to improve the estimation of the cash-flow growth rate. The dynamics of $z(t)$ follows a Gaussian process with a drift that is related to the drift of firm cash flows:

$$\begin{aligned} dz(t) &= (\alpha\mu_A(t) + \beta\mu_z(t))dt + \sigma_z dW_z(t), \\ d\mu_z(t) &= (a_{0z} + a_{1z}\mu_z(t))dt + \sigma_{\mu_z} dW_{\mu_z}(t), \end{aligned}$$

where $\sigma_z > 0$ is the volatility of the signal, $a_{0z} \in \mathbb{R}$ the long-term growth rate of expected signal growth, $a_{1z} < 0$ the mean-reversion parameter and $\sigma_{\mu_z} > 0$ the volatility. $(W_z(t), W_{\mu_z}(t))'$ is a standard two-dimensional Brownian motion independent of $(W_A(t), W_{\mu_A}(t))'$. Investors use observations of both $A(t)$ and $z(t)$ to make inferences about $\mu_A(t)$. When $\beta = 0$, the expected change in $z(t)$ is perfectly correlated with $\mu_A(t)$ and $z(t)$ contains pure information about the expected growth rate of firm's cash flows. When $\beta \neq 0$, the expected change in $z(t)$ is a mixture of $\mu_A(t)$ and the growth rate $\mu_z(t)$ of another systematic risk factor, which can be linked, e.g., to market-wide information about the state of the economy. In this way one can interpret $\mu_z(t)$ as the part of aggregate expected growth rate in the economy that is orthogonal to the firm

⁶Therefore, it is optimal for analysts to use information that goes beyond simple evidence on historical firm-specific cash flows (Buyd, Hu, and Jagannathan, 2005 and Beber and Brandt, 2006). Malmendier and Tate (2005) find that especially during the new economy boom in 2000 the accounting values of some companies were not very reliable. An imprecisely observed firm value is treated in Duffie and Lando (2001), who show that the quality of the firm's information disclosure can affect the term structure of corporate bond yields. Yu (2005) finds empirically that firms with higher disclosure rankings tend to have lower credit spreads. An imperfectly observed firm value is also modeled by Çetin, Jarrow, Protter, and Yildirim (2004), who assume that investors can access only a coarsened subset of the manager's information set. Giesecke (2004) develops a model with an imperfectly observed default boundary. Collin-Dufresne, Goldstein, and Helwege (2003) assume that firm values are observed with one time-lag. An industry implementation of these academic concepts is presented in CreditGradesTM, as described in the RiskMetrics (2002) technical document, which models the unobservable distance to default by a latent process explicitly.

specific expected growth rate $\mu_A(t)$. The volatility parameter σ_{μ_z} then measures the aggregate uncertainty associated with this market-wide component.

2.1.2 Modeling Uncertainty and Disagreement

We consider a simple specification for the uncertainty and disagreement in our economy. Economic agents update their beliefs based on the available information using Bayes' rule. The difference in their posteriors can arise either from a difference in agents' priors or a difference in some subjective parameter value of the dynamics of cash flows or signals. In the second case, a parsimonious model can be based on the realistic assumption that the uncertainty parameter σ_{μ_z} of the market-wide expected component $\mu_z(t)$ is agent dependent. We follow this assumption below when we derive a set of testable empirical predictions of our model.

Given that the model-implied state dynamics is conditionally Gaussian, the Bayesian prior updating rules of each agent follow with standard arguments, and the heterogeneity in beliefs can be completely summarized by the differences in means $m^i(t)$ and covariance matrices $\gamma^i(t)$ across agents.

Let $m^i(t) := (m_A^i(t), m_z^i(t))' := E^i((\mu_A(t), \mu_z(t))' | \mathcal{F}_t^Y)$, where $\mathcal{F}_t^Y := \mathcal{F}_t^{A,z}$ is the information generated by $A(t)$ and $z(t)$ up to time t , and $E^i(\cdot)$ denotes expectation relative to the subjective probability of investor $i = 1, 2$. Let also $Y(t) = (\log A(t), z(t))$, $b^1 = \text{diag}(\sigma_{\mu_A}^1, \sigma_{\mu_z}^1)$, $a_0 = (a_{0A}, a_{0z})'$, $a_1 = \text{diag}(a_{1A}, a_{1z})$, $B = \text{diag}(\sigma_A, \sigma_Z)$ and $A = \begin{pmatrix} 1 & 0 \\ \alpha & \beta \end{pmatrix}$. The (posterior) belief dynamics of agent one then follows by a standard application of the Kalman-Bucy filter:

$$dm^1(t) = (a_0 + a_1 m^1(t))dt + \gamma^1(t) A' B^{-1} dW_Y^1(t), \quad (2.1)$$

$$d\gamma^1(t)/dt = a_1 \gamma^1(t) + \gamma^1(t) a_1' + b^1 b^{1'} - \gamma^1(t) A' (B B')^{-1} A \gamma^1(t), \quad (2.2)$$

with initial conditions $m^1(0) = m_0^1$ and $\gamma^1(0) = \gamma_0^1$, where $dW_Y^1(t) := B^{-1} (dY(t) - A m^1(t) dt)$ is the innovation process induced by the first investor's belief and filtration.⁷ To completely specify the disagreement structure in the economy, we finally specify the disagreement process implied by the learning dynamics of agent two. This process is the

⁷A formal proof of this result can be found in Liptser and Shiryaev (2000); see also the technical Appendix.

key state variable driving all equilibrium quantities in our economy. It is defined by the two dimensional process:

$$\Psi(t) := \begin{pmatrix} \Psi_A(t) \\ \Psi_z(t) \end{pmatrix} = \begin{pmatrix} (m_A^1(t) - m_A^2(t))/\sigma_A \\ (m_z^1(t) - m_z^2(t))/\sigma_z \end{pmatrix}.$$

The first component of $\Psi(t)$ measures disagreement about the expected growth rate of future firm cash flows, while the second component measures the disagreement about the market-wide signal $z(t)$. Since the market wide uncertainty parameter σ_{μ_z} influences the subjective dynamics of each individual belief $m^i(t)$, it also has implications for the stochastic properties of the disagreement process itself. The dynamics for $\Psi(t)$ follows directly:

$$d\Psi(t) = B^{-1} (a_1 B + \gamma^2(t) A' B^{-1}) \Psi(t) dt + B^{-1} (\gamma^1(t) - \gamma^2(t)) A' B^{-1} dW_Y^1(t), \quad (2.3)$$

with initial conditions $\Psi(0) = (m_A^1(0) - m_A^2(0))/\sigma_A, (m_z^1(0) - m_z^2(0))/\sigma_z$, where $\gamma^2(t)$ satisfies the same differential equation as $\gamma^1(t)$, but with agent specific parameter b^1 replaced by b^2 . The dynamics of $m_1(t)$ and $\Psi(t)$ completely characterize the beliefs induced by investors' priors and filtrations. Heterogeneity in prior beliefs alone is sufficient to let investors disagree about $\mu_A(t)$ and $\mu_z(t)$ at all times, even when they agree on the dynamics of cash flows and signals (i.e. $b^1 = b^2$), but in this case heterogeneity in beliefs vanishes asymptotically.⁸ If prior variances are also identical ($\gamma^1(0) = \gamma^2(0)$) then the disagreement process is deterministic, as in the model solved by Buraschi and Jiltsov (2006). We consider a model with truly stochastic disagreement dynamics, in which heterogeneity in beliefs does not vanish asymptotically, by assuming that $b^1 \neq b^2$, i.e., we allow for the presence of heterogeneous subjective uncertainty across agents. Note that since both $\mu_A(t)$ and $\mu_z(t)$ are unobservable, the parameter b^i cannot be uniquely inferred by investors from the quadratic variation of these processes. Moreover, we emphasize that the average level and the heterogeneity of the subjective uncertainty parameters across agents both impact directly on the steady-state distribution of the disagreement process implied by dynamics (3.1). This feature offers a natural interesting link between economic uncertainty and the stochastic properties of the heterogeneity in beliefs, which motivates the use of belief disagreement proxies as indicators of economic uncertainty in our tests of the main empirical predictions of the model.

⁸Acemoglu, Chernozhukov, and Yildiz (2008) show that when agents are uncertain about a random variable and about the informativeness of a signal even an infinite sequence of signal observations does not lead agents' heterogeneous prior beliefs to converge.

Remark: Our way of modeling disagreement and the economy with heterogeneous beliefs is related to, but distinct from models that study the impact of behavioral biases on asset prices. In Scheinkman and Xiong (2003) and Hong, Scheinkman, and Xiong (2006), risk neutral investors subject to short-selling constraints update their beliefs based on their personal interpretations of incoming news. It follows that trading occurs whenever investors valuations “cross”, i.e., whenever the more optimistic investor switches to being the more pessimistic. In a world in which investors interpret news differently, the greater news stimulus associated with an asset leads to a higher time-series variability of investors’ relative valuations and larger trading volumes via the “crossing”. At the same time, the short-sales constraint makes some assets overpriced relative to others, so that their valuation reflects more the opinion of the optimist. In contrast, in an economy without constraints the stock price simply reflects the value-weighted average opinion.⁹ In our model, agents are rational, but risk averse, and short-selling is allowed. The local volatility of the growth rate of the underlying state variables is unknown and cannot be estimated from the quadratic variations of the drift, because the drift itself is unobservable. Thus, agents can disagree without imposing an axiomatic behavioral bias assumption. Even without short-selling constraints, uncertainty affects asset prices in our economy. This is consistent with the less important role of short-selling restrictions in bond markets; see, e.g., Longstaff, Mithal, and Neis (2005).

2.1.3 The Link Between Economic Uncertainty and Belief Disagreement

An important characteristic of the data is an apparent link between economic uncertainty and belief disagreement. This relationship is well motivated within our model setting because the disagreement dynamics (3.1) explicitly depend on the value of the subjective uncertainty parameter $\sigma_{\mu_z}^i$, $i = 1, 2$, of investors. Intuitively, a higher value of subjective uncertainty impacts more on the drift of $\Psi(t)$, while the difference in subjective uncertainty between agent 1 and 2 will mainly have an impact on the diffusion of $\Psi(t)$. Note that if the subjective uncertainty parameters of agent 1 and 2 are identical, i.e. $\sigma_{\mu_z}^1 = \sigma_{\mu_z}^2$, then the disagreement process is deterministic. On the other hand, we

⁹Duffie, Gârleanu, and Pedersen (2002) derive similar implications for a setting with heterogeneous beliefs in which short-selling is allowed but costly.

find that the higher the average subjective uncertainty (subjective uncertainty variability) across agents in the model, the higher is the average disagreement (disagreement variability).

The steady state distribution of $\Psi(t)$ in our economy is Gaussian and follows easily from equation (3.1):

$$\Psi(t)|\Psi(0) \sim \mathcal{N}\left(e^{tM}\Psi(0), te^{tM}B^{-1}(\gamma^1 - \gamma^2)A'(BB')^{-1}A(\gamma^1 - \gamma^2)B^{-1}e^{tM}\right),$$

where $M = B^{-1}(a_1B + \gamma^2A'B^{-1})$; the solution γ^i of the matrix Riccati equation

$$0 = a_1\gamma^i + \gamma^i a'_1 + b^i b^{i'} - \gamma^i A'(BB')^{-1}A\gamma^i; \quad i = 1, 2, \quad (2.4)$$

is the steady state variance covariance matrix of the learning dynamics of agent $i = 1, 2$ in the model. The closed-form expression for γ^i is given by:

$$\gamma^i = \left(F_i^{1/2} - (a'_1 + a_1)/2\right)^{-1} b^i b^{i'}, \quad (2.5)$$

where $F_i^{1/2}$ is the square root of matrix $F_i = b^i b^{i'} A'(BB')^{-1}A + a_1 a'_1$, and it makes explicit the dependence of the moments of the steady state distribution of $\Psi(t)$ on the subjective uncertainty parameters $\sigma_{\mu_z^1}$ and $\sigma_{\mu_z^2}$. Even if this dependence is highly nonlinear and dependent on all model parameters, we find that the first and second moments of the steady-state distribution of $\Psi(t)$ are both increasing in the average subjective uncertainty $\bar{\sigma}_{\mu_z} \equiv \frac{1}{2}(\sigma_{\mu_z^1} + \sigma_{\mu_z^2})$. Similarly, the variance of this distribution is increasing in the cross sectional uncertainty heterogeneity $\Delta\sigma_{\mu_z} \equiv \sigma_{\mu_z^1}^2 - \sigma_{\mu_z^2}^2$.

To illustrate concretely these model features, we depict in Figure 2.1 the comparative statics of the steady state mean and standard deviation of $\Psi_z(t)$ with respect to the average subjective uncertainty $\bar{\sigma}_{\mu_z}$ and the difference in uncertainty $\Delta\sigma_{\mu_z}$.

An increase in $\bar{\sigma}_{\mu_z}$ ($\Delta\sigma_{\mu_z}$) increases the expectation (variance) of the steady-state distribution of $\Psi_z(t)$. These findings give a theoretical support in our model for the positive link between uncertainty and disagreement in the data. $\Delta\sigma_{\mu_z}$ has virtually no effect on the expected value of $\Psi_z(t)$, so that average disagreement is indeed mostly driven by the the average subjective uncertainty in the economy. The variance of $\Psi_z(t)$, instead, is mostly driven by the heterogeneity in subjective uncertainty in the economy. When $\Delta\sigma_{\mu_z}$ is zero, the variance of the disagreement process vanishes. Average uncertainty

1st Moment of Steady–State Distribution of Ψ 2nd Moment of Steady–State Distribution of Ψ

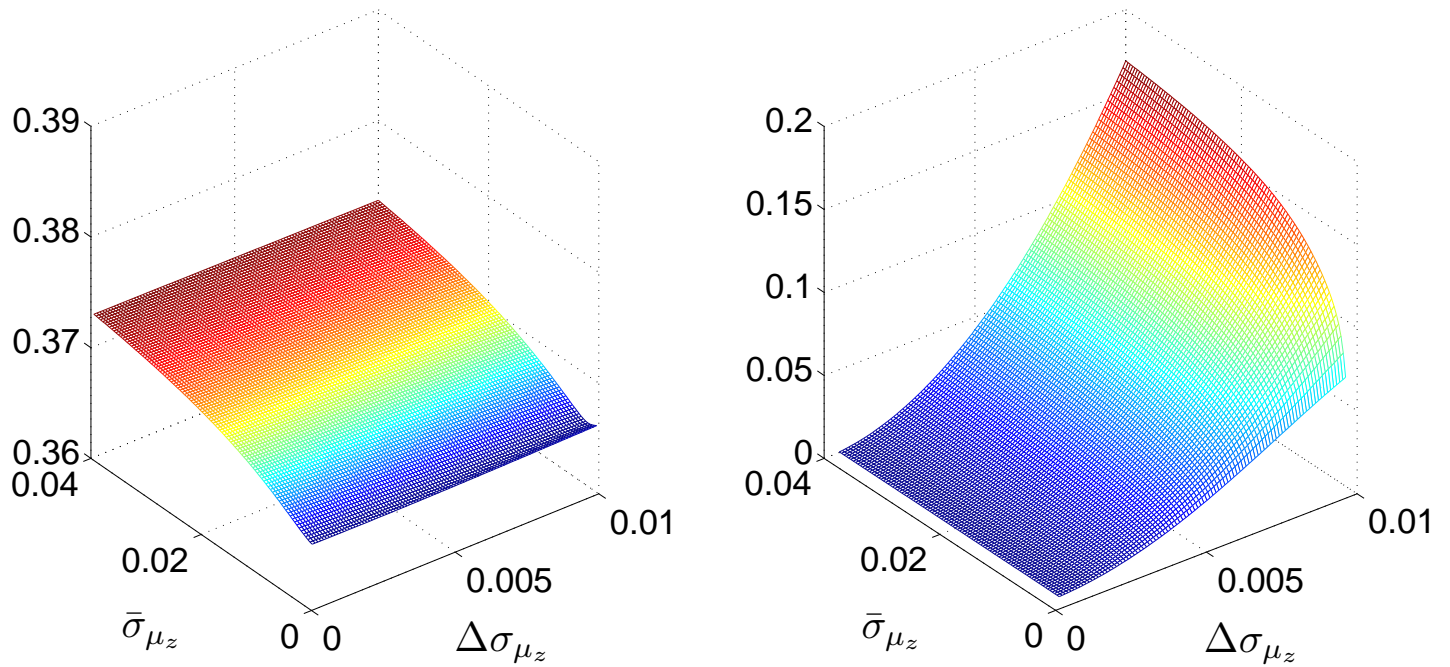


FIGURE 2.1: Disagreement as a Function of Average Uncertainty and Difference in Uncertainty

We plot the first and second moment of the steady state distribution of Ψ_z as a function of the average uncertainty, i.e. $\bar{\sigma}_{\mu_z} \equiv \frac{1}{2}(\sigma_{\mu_z}^1 + \sigma_{\mu_z}^2)$ and the agent perceived difference in uncertainty, i.e. $\Delta\sigma_{\mu_z} \equiv \sigma_{\mu_z}^1 - \sigma_{\mu_z}^2$.

$\bar{\sigma}_{\mu_z}$ also increases the variance of $\Psi_z(t)$. This last effect is more pronounced for settings of low average uncertainty and large uncertainty heterogeneity. These features suggest that periods of high average subjective uncertainty and high uncertainty heterogeneity might be those associated with the largest average disagreement and disagreement variability over time. Overall, our model suggests a clear link between uncertainty and belief disagreement, which motivates our interpretation of common and firm specific belief disagreement proxies as indicators of economic uncertainty.

Remark: The role of economic uncertainty has been studied from a different angle by Bansal and Yaron (2004), among others, who find that time varying uncertainty directly affects the risk premia of financial assets in a representative agent economy displaying recursive preferences. In a similar spirit, Lettau, Ludvigson, and Wachter (2008) study

macroeconomic risk and investigate a learning model where both first and second moments of consumption growth can switch according to a Markov Chain. In these models, objective economic uncertainty carries a risk premium if and only if agents have Epstein-Zin recursive utility. Bekaert, Engstrom, and Xing (2008) study an economy with a time-varying conditional variance of fundamentals and external habit utility. In this way, the authors disentangle the different implications of a time-varying risk aversion and economic uncertainty. In our model, subjective uncertainty influences the dynamic properties of the disagreement process and the optimal risk sharing among agents. This feature generates direct asset pricing implications also when investors have time-additive preferences. Economic uncertainty in our setting also has a distinct role in the growing literature based on (recursive) multiple-priors utility; see, for instance, Gilboa and Schmeidler (1989) and Epstein and Schneider (2008).¹⁰ In our model, individual investors are uncertain about future growth opportunities but do not display any ambiguity aversion as they make their decisions using a single prior. The difference in beliefs approach has the important empirical advantage that it can be more easily proxied from the data than individual-specific multiple priors.

2.1.4 Investors' Preferences, Financial Markets and Equilibrium

We work in a standard two-agent economy populated by investors with heterogeneous beliefs, but identical in all other aspects. Agents maximize life-time expected power utility subject to their budget constraint:¹¹

$$V^i = \sup_{c_i} E^i \left(\int_0^\infty e^{-\rho t} \frac{c_i(t)^{1-\gamma}}{1-\gamma} dt \mid \mathcal{F}_0^Y \right), \quad (2.6)$$

where $c_i(t)$ is the consumption of agent $i = 1, 2$ at time t , $\gamma > 0$ is the relative risk aversion coefficient, and $\rho \geq 0$ is the time preference rate. Investors finance their consumption plans by trading in financial assets. In contrast to Merton's (1974) structural

¹⁰There is also a number of important applications linked to multiple-priors utility in the context of robust optimal control problems (Hansen, Sargent, and Tallarini, 1999, Cagetti, Hansen, Sargent, and Williams, 2002, Maenhout, 2004, Leippold, Vanini, and Trojani, 2008). None of these papers studies the equilibrium interaction of heterogeneous agents. In most of these models, ambiguity averse investors typically act as if assuming a worst case scenario in terms of mean earnings.

¹¹In many standard models without disagreement, power utility implies low credit spreads and a low equity premium. In the attempt to solve for these tensions, Chen, Collin-Dufresne, and Goldstein (2008) and Bhamra, Kühn, and Strebulaev (2008) study representative agent economies with recursive preferences.

model, trading in the risk-less bond and the assets of the firm does not complete the market. Uncertainty influences the prices of all financial assets, which – from the perspective of agent one – can be written as functions of the two-dimensional filtered Brownian motion $W_Y^1(t)$ in equation (3.1). It follows that at least one additional financial asset is needed to determine a unique stochastic discount factor. We focus on an economy including corporate bonds and equity. We denote with $r(t)$ the interest rate on the risk-less bond, assumed to be in zero net supply, with $S(t)$ the price of the stock of the firm, in positive net supply, and with $B^s(t)$ ($B^j(t)$) the price of the senior (junior) bond, also in positive supply. $V(t)$ denotes the value of the single firm in our economy.

Definition 2.1 (Equilibrium). An equilibrium consists of a unique stochastic discount factor such that: (I) given equilibrium prices, all agents in the economy solve the optimization problem (3.2), subject to their budget constraint. (II) Good and financial markets clear.

The equilibrium is solved using the martingale approach, originally developed by Cox and Huang (1986). The extension to the case of heterogeneous beliefs has been studied, among others, by Cuoco and He (1994), Karatzas and Shreve (1998), and Basak and Cuoco (1998). In this extension, a stochastic weighting process $\lambda(t)$ captures the equilibrium impact of belief disagreement. Optimal consumption policies are of the form $c_i(t) = (y_i \xi^i(t))^{-1/\gamma}$, where y_i is the Lagrange multiplier associated with the static budget constraint of agent i and ξ^i is the state-price density of this agent. Under the given assumptions, closed-form expressions for $c_i(t)$ and $\xi^i(t)$ in terms of the exogenous state-variables follow with standard computations.

In equilibrium, the individual state price densities of agent one and two are:

$$\xi^1(t) = \frac{e^{-\rho t}}{y_1} A(t)^{-\gamma} \left(1 + \lambda(t)^{1/\gamma}\right)^\gamma, \quad \xi^2(t) = \frac{e^{-\rho t}}{y_2} A(t)^{-\gamma} \left(1 + \lambda(t)^{1/\gamma}\right)^\gamma \lambda(t)^{-1},$$

where the weighting process $\lambda(t) = y_1 \xi^1(t) / (y_2 \xi^2(t))$ follows the dynamics:

$$\frac{d\lambda(t)}{\lambda(t)} = -\Psi_A(t) dW_A^1(t) - \left(\alpha \Psi_A(t) \frac{\sigma_A}{\sigma_z} + \beta \Psi_z(t) \right) dW_z^1(t). \quad (2.7)$$

Individual state price densities are functions of the exogenous cash flow process $A(t)$ and the weighting process $\lambda(t)$. The dynamics of $\lambda(t)$ is completely described by the disagreement process $\Psi(t)$, which determines the volatility of $\lambda(t)$. Thus, in contrast to

the classical Merton (1974) model, the stochastic discount factor is a function of both $A(t)$ and $\Psi(t)$. When agents agree, i.e. $\Psi(t) = 0$ and $b^1 = b^2$, we obtain as a special case Merton's economy with identical agents, the relative weight $\lambda(t)$ is equal to one and optimal consumption plans across investors are proportional.¹² When agents have different subjective beliefs, i.e. $\Psi(t) \neq 0$, they implement different optimal consumption plans. Optimistic investors consume more in states associated with high aggregate cash flows, at a lower marginal utility of consumption, because they perceive these states as more likely. Similarly, pessimistic investors have a lower marginal utility in states of low aggregate cash flows. It follows that the relative consumption share of the optimist is higher (lower) in states of high (low) aggregate cash flows.¹³ This feature has direct implications for the equilibrium stochastic discount factor.

The impact of disagreement on the stochastic discount factor yields another important difference with respect to the classical Merton (1974) model for the shape of the risk premia.

Let θ_A^i and θ_z^i be the subjective market prices of risk for cash flow and signal shocks of agent i . Itô's Lemma applied to the state price density equilibrium expressions implies the following well-known result:

$$\begin{aligned} \theta_A^1(t) &= \gamma\sigma_A + \Psi_A(t) (1 + \lambda(t)^{1/\gamma})^{-1} \lambda(t)^{1/\gamma}, & \theta_z^1(t) &= (1 + \lambda(t)^{1/\gamma})^{-1} \lambda(t)^{1/\gamma} \left(\alpha\Psi_A(t)\frac{\sigma_A}{\sigma_z} + \beta\Psi_z(t) \right), \\ \theta_A^2(t) &= \gamma\sigma_A - \Psi_A(t) (1 + \lambda(t)^{1/\gamma})^{-1}, & \theta_z^2(t) &= - (1 + \lambda(t)^{1/\gamma})^{-1} \left(\alpha\Psi_A(t)\frac{\sigma_A}{\sigma_z} + \beta\Psi_z(t) \right). \end{aligned}$$

The market prices of firm cash flow shocks $\theta_A^i(t)$ is the sum of two terms. The first term is the standard price of risk that is also found in the homogeneous Merton (1974) economy. It is the product of relative risk aversion and cash flow risk. The second term is due to the belief heterogeneity. Since the stochastic discount factor depends on $\lambda(t)$, the market prices of risk contain a term related to the volatility of this process, which is proportional to the disagreement index $\Psi(t)$. This is due to the fact that belief-dependent state prices are necessary in order to support belief-dependent consumption shares. When $\Psi(t) \neq 0$, the market-wide signal is priced. When this signal is related to cash flow growth ($\alpha > 0$), agents use it to improve their beliefs on future firm cash

¹²To focus on the impact of disagreement, we always assume identical initial endowments of the total cash flow stream $A(t)$ across investors.

¹³The cyclical behavior of the consumption share is reflected in the dynamics of the stochastic weight $\lambda(t)$: If agent one (agent two) is optimistic about future cash flows, the stochastic weight is counter-cyclical (pro-cyclical) because the individual marginal utilities of consumption are proportional to the price of the underlying state.

flows. At the same time, higher signal volatility generates higher asset price volatility.¹⁴ The market prices of risk perceived by the optimist are greater than those perceived by the pessimist. The economic intuition is simple: In equilibrium, trading occurs between investors who have to finance their different optimal consumption plans. The pessimist buys financial protection against low cash flow states from the optimist. This excess demand lowers the price of securities with positive exposure to cash flow shocks, and the risk implied by bad cash flow states is transferred to the optimist. If a negative state occurs, the more optimistic agent is hit twice: First, because the aggregate endowment is lower; second, because her relative consumption share is lower due to the protection agreement. The size of this risk transfer is proportional to the size of the disagreement among agents $\Psi(t)$.

2.1.5 Pricing of Financial Assets

Given the expressions for the individual state price densities $\xi^1(t)$ and $\xi^2(t)$, we can price any contingent claim in the economy by computing expectations of its contingent payoffs weighted by state price densities.

We assume that the firm repayment structure satisfies a strict priority rule, in which payments to the junior bond holders are made only if the contractual payments to the senior bond holders have been made. As in the Merton (1974) model, default occurs only at maturity of the corporate bonds, if the value of the assets of the firm is less than the face value of these bonds. We assume zero-bankruptcy costs. Therefore, in the event of default, equity holders are left with a zero price of equity and the corporate bond holders share the residual firm value according to the pre-specified seniority rules. To focus on the implications of disagreement in credit risk, we keep the default structure as simple as possible and do not explore more general default rules or more flexible default and liquidation procedures.¹⁵ In this setting, the price of the senior bond is the sum of the prices of the zero-coupon bond and the price of a short put option on the firm value.

¹⁴Since the signal is not an explicit argument of consumer preferences, the economy generates dynamics that could be interpreted or labeled as having “excess volatility”. It is also interesting to notice that even if there would be no disagreement on the dynamics of the market-wide signal $z(t)$, but agents were to disagree on the firm’s factor loadings β , then the risk of unexpected innovations in the signal would still affect agents optimal behavior and equilibrium market prices.

¹⁵See, e.g., Black and Cox (1976) for a model with premature default. Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) model strategic debt service. Broadie, Chernov, and Sundaresan (2007) propose a setting that incorporates Chapter 7 and Chapter 11 issues.

Similarly, the price of equity is the firm value residual in excess of the price of the total corporate debt.

To compute the expectations in the relevant pricing expressions, the joint density of $A(t)$, $\lambda(t)$, and the contingent claim payoff is needed, since the equilibrium stochastic discount factor is a function of both $A(t)$ and $\lambda(t)$. The joint distribution of $A(t)$ and $\lambda(t)$ is typically unavailable in closed-form. However, we can calculate their joint Laplace transform in closed-form, which can be used, in a second step, to price more efficiently all securities using Fourier transform methods. In this way, we can avoid a pricing approach that relies exclusively on Monte Carlo simulation methods, which would be too computationally intensive in our setting, especially in the computation of corporate credit spreads and stock prices. For convenience, we compute the Laplace transform of $A(t)$ and $\lambda(t)$ in the case $\sigma_{\mu_z}^1 \neq \sigma_{\mu_z}^2$, which admits a steady-state distribution for the relevant state variables in our model. The following technical Lemma, which draws upon Dumas, Kurshev, and Uppal (2009), gives the required result.¹⁶

Lemma 2.2. *The joint Laplace transform of $A(t)$ and $\lambda(t)$ under the belief of agent one is given by:*

$$E_t^1 \left(\left(\frac{A(T)}{A(t)} \right)^\epsilon \left(\frac{\lambda(T)}{\lambda(t)} \right)^\chi \right) = F_{m_A^1}(m_A^1, t, T; \epsilon) F_{\Psi_A, \Psi_z}(\Psi_A, \Psi_z, t, T; \epsilon, \chi), \quad (2.8)$$

where

$$\begin{aligned} F_{m_A^1}(m_A^1, t, T; \epsilon) &= \exp \left(\frac{\epsilon}{a_{1A}} \left(-a_{0A}\tau + \left(\frac{a_{0A}}{a_{1A}} + m_A^1 \right) (e^{a_{1A}\tau} - 1) \right) + \frac{1}{2}\epsilon(\epsilon - 1)\sigma_A^2\tau \right. \\ &\quad \left. + \frac{\epsilon^2}{4a_{1A}^2} \left(\frac{\gamma_A^2}{\sigma_A} \right)^2 (3 - 4e^{a_{1A}\tau} + e^{2a_{1A}\tau} + 2a_{1A}\tau) \right. \\ &\quad \left. + \frac{\epsilon^2\gamma_A^2}{a_{1A}} \left(-\tau + \frac{1}{a_{1A}} (e^{a_{1A}\tau} - 1) \right) \right), \end{aligned} \quad (2.9)$$

with $\tau = T - t$ and

$$F_{\Psi_A, \Psi_z}(\Psi_A, \Psi_z, t, T; \epsilon, \chi) = e^{A_0(\tau) + B_1(\tau)\Psi_A + B_2(\tau)\Psi_z + C_1(\tau)\Psi_A^2 + C_2(\tau)\Psi_z^2 + D_0(\tau)\Psi_A\Psi_z}, \quad (2.10)$$

for functions A_0, B_1, B_2, C_1, C_2 and D_0 detailed in the proof in the Appendix.

¹⁶In contrast to Dumas, Kurshev, and Uppal (2009), however, we can reduce the system of ordinary differential equations for functions A_0, B_1, B_2, C_1, C_2 and D_0 to a system of matrix Riccati equations. Matrix Riccati equations can be linearized using Radon's Lemma. In this way, we obtain fully explicit expressions for the coefficients in the exponentially quadratic solution for the Laplace transform.

The Laplace transform in Lemma 2.2 depends on $m_A^1(t)$, $\Psi_A(t)$, and $\Psi_z(t)$. The dependence on $m_A^1(t)$ is exponentially affine. The dependence on $\Psi_A(t)$ and $\Psi_z(t)$ is exponentially quadratic. Using the closed-form characteristic function of $A(t)$ and $\lambda(t)$, we can now price the contingent claims in the economy by Fourier inversion and Monte Carlo methods. The spirit of the Fourier inversion approach is similar to the one used to price derivatives in stochastic volatility models, such as Heston (1993), Duffie, Pan, and Singleton (2000), and Carr, Geman, Madan, and Yor (2001), or in interest-rate models, such as Chacko and Das (2002).

The pricing expressions for the contingent claims in our economy are summarized in the next technical Lemma.

Lemma 2.3. *Let*

$$G(t, T, x; \Psi_A, \Psi_z) \equiv \int_0^\infty \left(\frac{1 + \lambda(T)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \right)^\gamma \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} \left(\frac{\lambda(T)}{\lambda(t)} \right)^{-i\chi} F_{\Psi_A, \Psi_z}(\Psi_A, \Psi_z, t, T; x, i\chi) d\chi \right] \frac{d\lambda(T)}{\lambda(T)}.$$

1. *The equilibrium firm value is:*

$$V(t) := V(A, m_A^1, \Psi_A, \Psi_z) = A(t) \int_t^\infty e^{-\rho(u-t)} F_{m_A^1}(m_A^1, t, u; 1 - \gamma) G(t, u, 1 - \gamma; \Psi_A, \Psi_z) du.$$

2. *The equilibrium price of the corporate zero-coupon bond is:*

$$B(t, T) := B(t, T; m_A^1, \Psi_A, \Psi_z) = e^{-\rho(T-t)} F_{m_A^1}(m_A^1, t, T; -\gamma) G(t, T, -\gamma; \Psi_A, \Psi_z).$$

3. *The equilibrium price of the senior defaultable bond is:*

$$\begin{aligned} B^s(t, T) &:= B^s(t, T; A, m_A^1, \Psi_A, \Psi_z), \\ &= B(t, T) - E_t^1 \left(e^{-\rho(T-t)} \left(\frac{A(t)}{A(T)} \frac{1 + \lambda(T)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \right)^\gamma (K_1 - V(T))^+ \right). \end{aligned}$$

4. *The equilibrium price of the junior defaultable bond is:*

$$\begin{aligned} B^j(t, T) &:= B^j(t, T; A, m_A^1, \Psi_A, \Psi_z), \\ &= E_t^1 \left(e^{-\rho(T-t)} \left(\frac{A(t)}{A(T)} \frac{1 + \lambda(T)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \right)^\gamma \left((V(T) - K_1)^+ - (V(T) - (K_1 + K_2))^+ \right) \right). \end{aligned}$$

5. *The equilibrium price of equity is:*

$$S(t) := S(t, T; A, m_A^1, \Psi_A, \Psi_z) = V(t) - B^s(t, T) - B^j(t, T).$$

Firm value and risk-less zero bond prices are in semi-closed form, up to a numerical integration. We note, however, that corporate bond and stock prices, which are options written on the firm value, require a Monte Carlo simulation step in their computation.

From the expressions in Lemma 2.3, we can study the dependence of corporate credit spreads and the price of equity on the degree of disagreement about cash flows and the market-wide signal. In this way, we can obtain testable empirical predictions of our model for the joint behavior of belief disagreement, credit spreads, and stock returns. We focus on the following main questions, which give rise to the null hypotheses tested in the empirical part of the paper:

- Q1.** What is the relation between uncertainty, belief disagreement, and credit spreads?
- Q2.** Does time variation in uncertainty create a structural link between stock returns volatility and credit spreads?
- Q3.** What is the link between stock returns, uncertainty, and belief disagreement?
- Q4.** Does heterogeneity in beliefs help to explain no arbitrage violations of single factor credit risk models?

2.2 Model Predictions

We use the solutions of the model to develop empirical predictions for the joint behavior of credit spreads, stock prices, and stock volatility, as a function of the difference in beliefs. We compute equilibrium quantities with respect to the steady-state distribution of beliefs, which is non-trivial, assuming that agents disagree about σ_{μ_z} . We then calibrate the model to the cash-flow dynamics of a representative firm in our sample. Table 3.1 summarizes the set of calibrated parameters. We assume a level of risk aversion equal to 2 and a firm cash flows volatility equal to 7% (Zhang, 2006).¹⁷ The calibrated values of the learning parameters are consistent with the estimates obtained in Xia (2001) and Brennan and Xia (2001). The median difference in beliefs of firms' future earnings in our I/B/E/S forecast data is 0.22. Consequently, in our comparative statics we consider disagreement $\Psi_A(t)$ and $\Psi_z(t)$ between zero and 0.2.

¹⁷Brennan and Xia (2001), Ait-Sahalia, Parker, and Yogo (2004) and Chen, Collin-Dufresne, and Goldstein (2008) use risk aversion parameters between six and ten. The main focus of these studies, however, is the equity premium puzzle.

TABLE 2.1: Choice of Parameter Values and Benchmark Values of State Variables

This table lists the parameter values used for all figures in the paper. We calibrate to the mean and volatility of the time-series average of operating cash flow for all firms present in our database. Operating cash flow is earnings before extraordinary items (Compustat item 18) minus total accruals, scaled by average total assets (Compustat item 6), where total accruals are equal to changes in current assets (Compustat item 4) minus changes in cash (Compustat item 1), changes in current liabilities (Compustat item 5), and depreciation expense (Compustat item 14) plus changes in short-term debt (Compustat item 34). The initial values for the conditional variances are set to their steady-state variances. Agent specific values are consistent with estimated values from Brennan and Xia (2001).

| Parameters for Cash Flow | | |
|--|------------|-------|
| Long-term growth rate of cash flow growth | a_{0A} | 0.01 |
| Mean-reversion parameter of cash flow growth | a_{1A} | -0.01 |
| Volatility of cash flow | σ_A | 0.07 |
| Initial level of cash flow | A | 1.00 |
| Initial level of cash flow growth | m_A^1 | 0.01 |
| Parameters for Signal | | |
| Long-term growth rate of signal | a_{0z} | 0.01 |
| Mean-reversion parameter of signal | a_{1z} | -0.03 |
| Volatility of signal | σ_z | 0.06 |
| Agent specific Parameters | | |
| Relative risk aversion for both agents | γ | 2.00 |
| Time Preference Parameter | ρ | 0.02 |

2.2.1 Credit Spreads and the Volatility of Stock Returns

It is well-known that the Merton (1974) model implies corporate credit spreads which are on average too low. Structural credit risk models fail to explain credit spreads especially for firms with high credit ratings; see, e.g., Eom, Helwege, and Huang (2004), Huang and Huang (2004) and David (2007) for an alternative explanation. This is known as the “credit spread puzzle”. We find that belief disagreement can help to explain this puzzle: An increase in the difference in beliefs reduces the firm value and increases the risk premium for the default event, by lowering the risk-neutral skewness of the firm. These features generate the higher credit spreads, even if the probability of default is held below 4% for low leverage firms. To illustrate this effect, the right panel of Figure

2.2 shows that as $\Psi_A(t)$ and $\Psi_z(t)$ increase from zero to 0.2, the senior credit spread increases by 29%, from 123 to 159 basis points, even for moderate levels of risk aversion. This increase is larger than the standard deviation of credit spreads for senior debt in our data set. For a larger relative risk aversion $\gamma = 3$, the increase in credit spreads is as large as 45 basis points.

These features give rise to a testable empirical prediction, which is directly related to the question Q1 raised at the end of Section 2.1:

- \mathcal{H}_1 : Uncertainty-DiB and credit spreads are positively related.

Another main implication of our model is that disagreement increases at the same time the volatility of equity.¹⁸ This feature is important because Campbell and Taksler (2003) have documented empirically a positive co-movement between credit spreads and the volatility of stock returns, which even exceeds the co-movement predicted by standard structural models.¹⁹ Our model offers a structural explanation for these findings, in which the co-movement between credit spreads and equity volatility follows endogenously from the time-variation of the difference in beliefs. The left panel of Figure 2.2 shows, for the same scenario as above, that the volatility of equity increases from 7% to 29% as disagreement increases; see the left panel of Figure 2.2. For firms with lower leverage, not shown in Figure 2.2, the volatility increases to 20%. These volatilities are consistent with the average volatility of stock returns in our data set.

These findings give rise to a second testable empirical prediction, related to question Q2 at the end of Section 2.1:

- \mathcal{H}_2 : As Uncertainty-DiB varies, credit spreads and stock volatility co-move positively.

2.2.1.1 Understanding the Link between Difference in Beliefs and the Price of Default

To understand the economic link between belief disagreement, stock volatility and the price of default risk, it is convenient to investigate the equilibrium properties of the firm

¹⁸The detailed specification of the equity volatility is derived in the technical Appendix.

¹⁹This finding has been documented also by Zhang, Zhou, and Zhu (2006), and Avramov, Jostova, and Philipov (2007), using reduced form models. Chen, Collin-Dufresne, and Goldstein (2008) and Bhamra, Kühn, and Strebulaev (2008) obtain similar effects from two consumption-based equilibrium models.

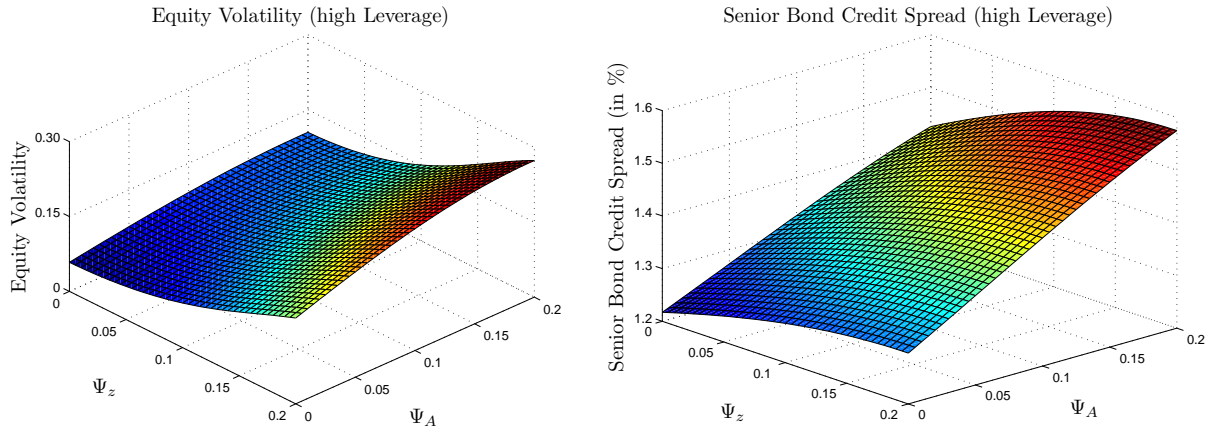


FIGURE 2.2: **Equity Volatility and Senior Bond Credit Spreads for High Leverage Ratio**

These figures plot equity volatility (left panel) and senior bond credit spreads (right panel) for a high leverage ratio as a function of difference in beliefs $\Psi_A(t)$ and the common disagreement, $\Psi_z(t)$. The parameter values used are given in Table 3.1.

value process in our economy. When $\Psi_A(t) = \Psi_z(t) = 0$ the model implications are those of Merton's (1974): At the calibrated parameters the firm value is 161 and the constant firm value volatility is 7%. Disagreement across investors affects the equilibrium stochastic discount factor and thus prices of all financial assets. Figure 2.3 (left panel) shows that the firm value decreases with disagreement about cash flows and the signal between agent one and two: An increase in $\Psi_A(t)$ and $\Psi_z(t)$ from zero to 0.2 reduces the equilibrium (asset) value of the firm by approximately 1.5%. Therefore, this effect has a negative impact on corporate bond prices, which contain a short put position on the firm value.

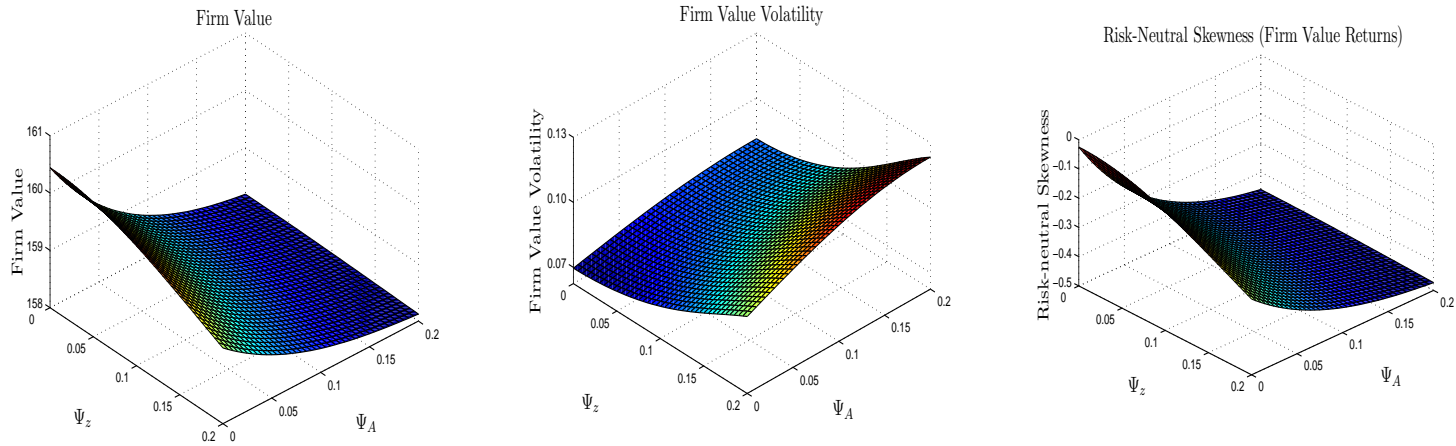


FIGURE 2.3: **Firm Value, Firm Value Volatility, and Risk-Neutral Skewness**

These figures plot the firm value, the firm value volatility, and the firm value risk-neutral skewness as a function of belief disagreement about cash flow $\Psi_A(t)$ and the common disagreement, $\Psi_z(t)$. The parameter values used are given in Table 3.1.

To understand this finding, it is helpful to compare the discount factor of the optimistic agent in our model with the stochastic discount factor in Merton's (1974) model. The stochastic discount factor of the optimist can be written as:

$$\xi_i(t) = \frac{1}{y_i} e^{-\rho t} A(t)^{-\gamma} s_i(t)^{-\gamma}, \quad (2.11)$$

where $s_i(t) = c_i(t)/A(t)$ is this investor's share of total consumption $A(t)$ and $\xi_i(t)$ is proportional to the marginal utility of a stochastic share $s_i(t)$ of total consumption $A(t)$. In the economy with homogeneous beliefs, this share is a constant that depends only on the tightness of the individual budget constraints (i.e., the ratio y_1/y_2). Thus, the discount factor is simply proportional to the marginal utility $A(t)^{-\gamma}$ of aggregate consumption. In the economy with disagreement, the random share $s_i(t)$ is greater (lower) when $A(t)$ is higher (lower). Therefore, in good (bad) cash flow states, the marginal utility of the optimist has a larger (lower) impact on the stochastic discount factor. Since in good (bad) states the marginal utility of the optimist (pessimist) is lower, the present value of future cash flows is lower, implying a lower equilibrium firm value than in the economy with homogeneous beliefs. To implement the optimal ex-ante consumption plan, the optimist (pessimist) buys financial assets that finance the higher future consumption share in good (bad) cash flow states. Therefore, in the competitive equilibrium the optimist sells financial protection against low cash flow states to the pessimist. It follows that the additional risk created by the stochastic consumption share in the economy is compensated to investors in an asymmetric way because the individual state prices are proportional to different marginal utilities. Since the optimist (pessimist) perceives a lower (higher) state price for large cash flow states, the market price of risk of cash flow shocks is higher (lower) for the optimist (pessimist).

An important feature of the stochastic discount factor (2.11) is that its volatility is stochastic when the consumption share is stochastic. When disagreement is zero, $s_i(t)$ is constant and the volatility of $d\xi_i(t)/\xi_i(t)$ is proportional to the volatility of firm cash flows: The firm value process features a constant volatility as in Merton's (1974) model. When beliefs are heterogeneous, the volatility of $s_i(t)$ is stochastic and proportional to $\Psi(t)$ because it depends on the volatility of the ratio of individual marginal utilities of optimal consumption. It follows that the volatility of the firm value is also stochastic and increasing with the degree of belief disagreement. A higher uncertainty about future state prices therefore increases the volatility of the discounted value of future cash flows. Figure 2.3 (middle panel) shows a plot of the firm value volatility (obtained from Itô's

Lemma). At the calibrated parameters an increase in $\Psi_A(t)$ and $\Psi_z(t)$ to 0.2 increases the equilibrium firm value volatility from 7% to approximately 12.5%. Since equity is a call option on the firm value, a higher volatility of the underlying in the economy with heterogeneous beliefs produces a higher stock volatility. At the same time, it further reduces corporate bond prices.

As disagreement varies over time, we obtain a negative co-movement between firm value and firm value volatility. This feature generates a moderate negative skewness of the physical distribution of the firm value, which yields a moderate increase of the physical probability of default from 3.2% (5.6%) to 4% (7.1%) for low (high) leverage firms.²⁰ It is well-known that negative skewness can also be obtained in partial equilibrium models with jumps.²¹ In our model, negative skewness arises endogenously, even if cash flows and security prices do not include a jump component. This follows from the form of the equilibrium stochastic discount factor. Using Itô's Lemma, the diffusion term of the dynamics of the individual stochastic discount factors is given by:

$$\begin{aligned} d\xi_i(t)/\xi_i(t) - E_t[d\xi_i(t)/\xi_i(t)] &= -(\gamma\sigma_A + (1 - s_i(t))\Psi_A(t)) dW_A^i(t) \\ &\quad - (1 - s_i(t)) \left(\alpha\Psi_A(t) \frac{\sigma_A}{\sigma_z} + \beta\Psi_z(t) \right) dW_z^i(t), \end{aligned} \quad (2.12)$$

where $s_i(t) = c_i(t)/A(t)$ is the share of agent i of total consumption. It follows that the volatility of the individual state prices is asymmetric and systematically related to the consumption share of agents in the economy: A positive cash flow or signal shock lowers the volatility of the individual stochastic discount factors, and vice versa. This feature is due to the decreasing marginal utility of consumption of the two investors and generates endogenously the negative skewness in the economy.

A second key point in the understanding of the behavior of the price of default-dependent securities in our economy is the link between Uncertainty-DiB and the firm value risk neutral skewness, which directly measures the price of likely default states. Consistent with the feature for the physical measure, the model also generates endogenously a pronounced negative risk-neutral skewness. Figure 2.3, right panel, illustrates the link between difference in beliefs and risk-neutral skewness: An increase in $\Psi_A(t)$ and $\Psi_z(t)$

²⁰For comparison, the average default probability of senior secured bonds between 1980 and 2006 is 4.3%, according to Moody's Corporate Default and Recovery Rates Report 2007. The positive relation between default probabilities and volatility is consistent with the recent evidence documented in Bharath and Shumway (2008). Further recent studies on default probabilities include Vassalou and Xing (2004), Duffie, Saita, and Wang (2006), and Campbell, Hilscher, and Szilagyi (2008).

²¹See Pan (2002) for an application in option pricing, and Zhang, Zhou, and Zhu (2006), Cremers, Driessen, and Maenhout (2008) and Tauchen and Zhou (2006), in credit risk

from zero to 0.2 decreases the risk-neutral skewness of the firm value from zero to -0.5.²² This feature is important because it generates a further effect that directly influences corporate bonds and equity prices, given that they are non-linear claims on firm's asset values. It implies a large disagreement-induced risk premium for the default event, which can have a substantial impact on the price of default-dependent securities, even if the physical probability of default at the calibrated parameters is less than 4%.

2.2.2 Price of Equity, Skewness of Stock Returns and Capital Structure Arbitrage

Credit risk models inspired by the structural (partial equilibrium) approach of Merton (1974) imply negative correlations between stock prices and credit spreads. In this approach, corporate debt and equity are different (non-linear but monotonic) contingent claims collateralized by the same balance sheet. Since firm's cash flows act as the single pricing factor, negative cash-flow shocks reduce the value of any claim in the capital structure. The correlation between corporate bond spreads and equity prices is restricted, by no-arbitrage, to be negative at all times and states. This link forms the basis of capital structure arbitrage strategies, which are based on relative value analysis and implemented using different parts of the capital structure, as well as over-the-counter contracts such as CDS.

In our economy, the volatility and risk-neutral skewness of the firm value are stochastic. For firms with different degrees of leverage, we find that the price of equity can either increase or decrease with disagreement. This feature has important implications for the sign of the co-movement of credit spreads and stock prices, which is crucial for the successful implementation of capital structure arbitrage strategies.

Figure 2.4 illustrates the different effects of an increase in the disagreement $\Psi_A(t)$ and $\Psi_z(t)$, conditional on a low or a high firm leverage.

For high leverage firms, an increase in $\Psi_A(t)$ and $\Psi_z(t)$ to 0.2 lowers the price of equity by 1 percent. For low leverage firms, the price increases by 3.1 percent. From the results in the previous sections, it follows that in the first case the price and the volatility of

²²To compute the risk-neutral skewness, we follow Bakshi and Madan (2000), who show that any payoff function can be spanned by a continuum of out-of-the-money calls and puts: When the risk-neutral distribution is left skewed, the combined cost of the positioning in puts is larger than the one of the combined positions in calls. The expression for the risk-neutral skewness of the firm value returns is provided in the technical Appendix.

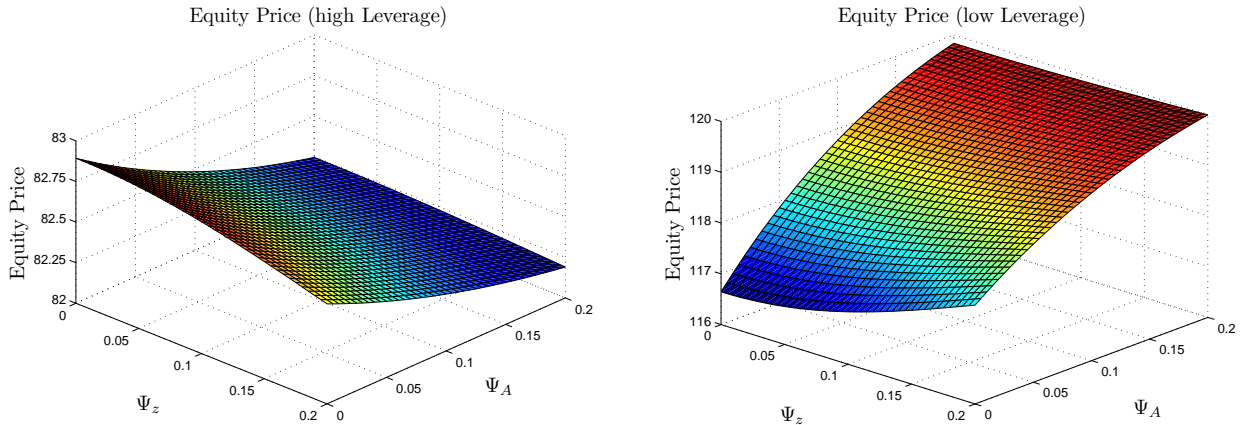


FIGURE 2.4: **Firm Equity Price for High and Low Leverage Ratio**

The price of equity with high leverage ratio (left panel) and low leverage ratio (right panel) is plotted as a function of the difference in beliefs $\Psi_A(t)$ and the common disagreement, $\Psi_z(t)$. The parameter values used are given in Table 3.1.

equity co-move negatively. A positive co-movement arises in the second case. This is an important departure from Merton's (1974) model – with immediate implications for capital structure arbitrage strategies – because it implies that the standard hedge ratio might even change sign.

To understand why these features exist, note that the price of equity can be represented as a portfolio consisting of a long position in the firm value $V(t)$, a short position in K_1 risk-less zero bonds with price $ZCB(t)$, and a long position in an out-of-the-money put on the firm value, with strike K_1 and price $P(t)$:²³

$$S(t) = V(t) - K_1 \cdot ZCB(t) + P(t, K_1) .$$

The first term, $V(t)$, is independent of leverage and is decreasing in disagreement. The price of the zero coupon bond can be shown to be decreasing in disagreement for a relative risk aversion parameter greater than one. Thus, the effects of the first two components of the price of equity tend to offset each other, with the second component increasing proportionally to firm leverage. The last term – i.e. the price of the put option $P(t, K_1)$ – has a positive impact on the price of equity, but the size of the effect depends significantly and in a non monotonic way on firm's leverage. For some regions of leverage, we find that this effect can be large enough to reverse the negative impact

²³We consider for brevity of exposition a firm without junior debt.

of the change in the value of the firm:

$$\frac{dS}{d\Psi} = \frac{dV}{d\Psi} - K_1 \cdot \frac{dZCB}{d\Psi} + \left[\overbrace{\frac{dP}{dV} \cdot \frac{dV}{d\Psi}}^{\text{Delta: +}} + \overbrace{\frac{dP}{d\sigma_V} \cdot \frac{d\sigma_V}{d\Psi}}^{\text{Vega: +}} + \overbrace{\frac{dP}{dSk_V} \cdot \frac{dSk_V}{d\Psi}}^{\text{Skewness: +}} \right]. \quad (2.13)$$

When leverage is high, the dominating effect on the price of equity comes from the first two terms in (2.13), as the Delta, Vega, and Skewness effects on the put price are all small in relative terms. For very low leverage values, the values of the put option and the position in the zero bond are a small fraction of firm value. Therefore, the price of equity is dominated by the first term in (2.13). It follows that for high and very low leverage the value of equity is decreasing with disagreement at the calibrated model parameters. For the intermediate leverage region, however, the price of the embedded out-of-the-money put option can be a non-negligible fraction of the firm value, and its sensitivity to increases in negative skewness (the last term in square brackets) is high. We find that the last effect can be high enough to compensate the negative change of the firm value and make the price of equity increase. Figure 2.5 illustrates the trade-off between these effects as $\Psi_A(t)$ changes from 0 to 0.20, dependent on firm leverage.

For levels of leverage between approximately 0.01 and 0.03 the effect of the higher negative skewness is large enough to increase the price of equity as beliefs dispersion increases. The leverage region in which disagreement and stock price have a positive co-movement depends on the calibrated parameters in the model. For instance, for a relative risk aversion parameter $\gamma = 4$ this region is broader and contains leverage ratios between 0.01 and 0.06.

Overall, the following additional empirical predictions, which are the direct counterparts to question Q3 and Q4 in Section 2.1, arise from our analysis:

- \mathcal{H}_3 : The relation between stock returns and heterogeneity in beliefs is negative for highly levered firms, but it can switch sign for firms that are moderately levered.
- \mathcal{H}_4 : The co-movement of credit spreads and stock returns tends to be positive for highly levered firms but is more likely to turn negative for firms that are less levered when Uncertainty-DiB is higher.

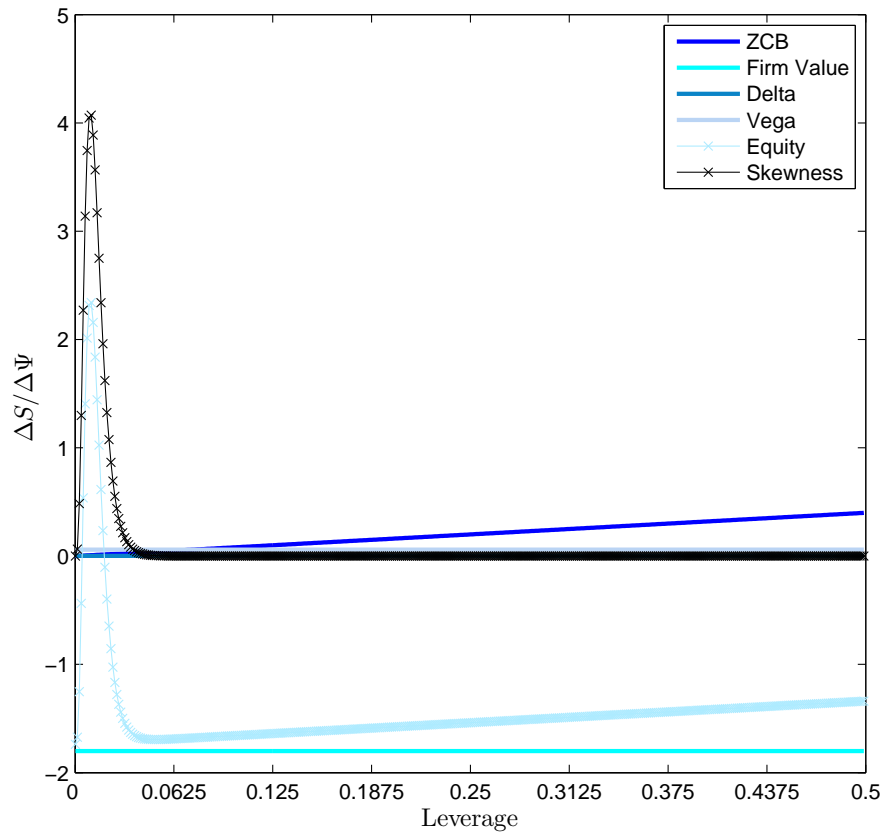


FIGURE 2.5: **Change in the Equity Price**

This figure plots the change in the equity price as a function of the bonds face values. We split up the total variation of equity, into four main effects: The Delta effect, which is due to a change in the underlying, the Vega effect, which is due to a change in the firm value volatility, a Skew effect, which is due to a change in the risk-neutral skewness, and a bond effect.

Remark: Similar to the findings for the firm value, the endogenous stochastic co-movement between the price and the volatility of equity generates an asymmetric physical stock price density. However, in contrast to the unambiguously negative sign of the skewness of firm value, the skewness of stock returns can be both positive and negative in our model: The positive (negative) co-movement between the price and the volatility of equity tends to generate stock returns that are positively (negatively) skewed. This is interesting because skewness has been found by several authors to be a key determinant of stock returns; see, among others, Kraus and Litzenberger (1976), Harvey and Siddique

(2000) and Dittmar (2002).²⁴ Moreover, these features have important implications for the co-movement between stock and corporate bond prices and for the relation between disagreement and firm-specific measures of distance to default. As disagreement increases, the price of the corporate bond decreases, but the price of equity can increase if the firm is moderately levered. It follows that in our model the higher credit spreads do not have to be coupled with, for example, a lower distance to default.

2.3 The Data Sets

To empirically test the main implications of our model, we merge four data sets and match, for each firm, information on professional earnings forecasts, balance-sheet data, corporate bond spreads, stock returns, and stock option prices. The merged data set contains monthly information on 337 firms for the period 1996 – 2005.

2.3.1 Bond Data

The bond data is obtained from the Fixed Income Securities Database (FISD) on corporate bond characteristics and the National Association of Insurance Commissioners (NAIC) database on bond transactions. The FISD database contains issue and issuer-specific information for all U.S. corporate bonds. The NAIC data set contains all transactions on these bonds by life insurance, property and casualty insurance, and health maintenance companies, as distributed by Warga (2000). This database is an alternative to the no longer available database used by Duffee (1998), Elton, Gruber, Agrawal, and Mann (2001), and Collin-Dufresne, Goldstein, and Martin (2001). U.S. regulations stipulate that insurance companies must report all changes in their fixed income portfolios, including prices at which fixed income instruments were bought and sold. Insurance companies are major investors in the fixed income market and, according to Campbell

²⁴Kraus and Litzenberger (1976) study a CAPM with investors that have preferences for skewness. Harvey and Siddique (2000) show that stocks with increasing prices when volatility spikes up have a positive skewness. Moreover, investors with preferences for skewness bid up the prices of assets with positive (co)skewness. Dittmar (2002) studies the impact of a non-linear pricing kernel in an economy, in which agents are averse to kurtosis and prefer positive skewness. Barberis and Huang (2007) study the impact of a preference for skewness in a Prospect Theory-type model with exogenously distorted beliefs. Brunnermeier, Gollier, and Parker (2007) analyze preferences for skewness in general equilibrium and find that positively skewed assets have lower expected returns. Recently, Conrad, Dittmar, and Ghysels (2007) link the higher prices of assets with positive skewness to the existence of stock market bubbles.

and Taksler (2003), they hold about one-third of outstanding corporate bonds. These data represent actual transaction data and not trader quotes or matrix prices.²⁵

Initially, we eliminate all bonds with embedded optionalities, such as callable, puttable, exchangeable, convertible securities, bonds with sinking fund provisions, non-fixed coupon bonds, and asset-backed issues. The data set contains information on the seniority level of the bonds. We are thus able to divide our data sample into senior secured and junior subordinated bonds. We manually delete all data entry errors. Moreover, to control for the possibility of residual errors, we winsorize our database at the 1% and 99% level. We are then left with a final database of 337 firms with senior secured bonds and junior subordinated bonds. Finally, to compute corporate bond credit spreads, we use zero-coupon yields available from the Center for Research in Security Prices (CRSP).

2.3.2 Uncertainty-DiB Proxy

To obtain a proxy of belief disagreement, we use analyst forecasts of earnings per share, from the Institutional Brokers Estimate System (I/B/E/S) database. This database contains individual analyst's forecasts organized by forecast date and the last date the forecast was revised and confirmed as accurate. To circumvent the problem of using stock-split adjusted data, as described in Diether, Malloy, and Scherbina (2002), we use unadjusted data. In an initial step, we match analysts forecast data with the bond data. We extend each forecast date to its revision date.²⁶ If an analyst makes more than one forecast per month, we take the last forecast that was confirmed.

Firm-Specific Disagreement:

In our model, belief disagreement of firms future earnings is defined as $\frac{m_A^1 - m_A^2}{\sigma_A}$, i.e., the difference between the subjective expected growth of a firm's cash flows scaled by the volatility of cash flows. We proxy disagreement about future cash flows by computing for each firm the mean absolute difference in the available earnings forecasts, scaled by an indicator of earnings uncertainty. Since the data on subjective earnings uncertainty of analyst forecasts is not available in our data set, we proxy earnings uncertainty by the standard deviation of earnings forecasts. Therefore, our measure of disagreement is the

²⁵Earlier data sets offered only indirect information about actual market prices since values of non-traded bonds were estimated based on matrix algorithms.

²⁶E.g., if a forecast is made in July and last confirmed in September, then we use this information for the months July, August, and September.

ratio of the mean absolute difference and the standard deviation of earnings forecasts.²⁷ The average firm-specific disagreement is 0.22. Firm-specific disagreement is highly volatile and exhibits a substantial cross-sectional variation. The minimum disagreement is 0.01 and the maximum 0.82, with an average time-series standard deviation of 0.14.

Common Disagreement:

The common disagreement is defined as $\frac{m_z^1 - m_z^2}{\sigma_z}$, i.e., the difference between the subjective expected growth of the signal, scaled by the signal volatility. To empirically form a common Uncertainty-DiB measure, we extract the first principal component from the cross-section of individual uncertainty proxies, where each individual uncertainty proxy is weighted by the market capitalization of the firm.²⁸

2.3.3 Option Data

The option data is taken from OptionMetrics, LLC. This database covers all exchange listed call and put options on U.S. equities. With each trade, OptionMetrics reports the option's implied volatility. Implied volatilities are calculated using LIBOR and Eurodollar rates, taking into account European and American exercise styles. We apply the following data filters to eliminate possible data errors. First, we exclude options which mature in the given month, since it is known (see, e.g., Bondarenko, 2003) that these suffer from illiquidity. Second, we eliminate all observations for which the ask is lower than the bid, for which the bid price is equal to zero, or for which the bid ask spread is lower than the minimum ticksize.²⁹ In a first step, we take implied volatilities of single-stock options which are closest to at-the-money since these are known to be the most liquid ones. The implied volatility skew is calculated as the difference between the implied volatility of a put option with moneyness 0.92 and the implied volatility of an at-the-money put, scaled by the difference $0.92 - 1$ in strike to spot ratios.

²⁷We checked the robustness of our disagreement proxy with respect to other measures and found our measure to dominate. To save space, we omit these results here. However, they are available upon request.

²⁸As a robustness check, we also applied an equally weighted scheme. However, the results do not change quantitatively.

²⁹The minimum ticksize equals USD 0.05 for options trading below USD 3 and USD 0.1 in any other case.

2.3.4 Control Variables

A large empirical literature has studied the determining factors of credit spreads and stock returns. A first obvious control variable in all our regressions is firm leverage, which is defined as total debt divided by the sum of total debt and the book value of shareholders' equity. In the regression for credit spreads, we additionally control for firm size, defined as the log total book value of assets. Leverage and firm size data are retrieved from the COMPUSTAT database. We also add to our regressions earnings volatility, defined as the time-series sample standard deviation of quarterly earnings per share over the last eight quarters, scaled by lagged stock price. A second set of natural control variables captures business-cycle and term structure effects, as well as further systematic pricing factors. To this end, we include in the regressions for credit spreads and stock returns a NBER dummy, the monthly S&P500 returns and non-farm payroll. The NBER dummy is taking the value 1 in expansion periods, as defined by the NBER. A third set of control variables are the Fama and French factors. Fama and French show that a zero-cost factor mimicking portfolio exposed to the size and value premia can explain a considerable component of the cross-section of equity returns. Although corporate bonds and equity are different non-linear contingent claims, they are written on the same firm's balance sheet. Thus, one might expect their dynamics to be driven by the same value/risk drivers and the Fama and French factors to be significant in explaining the cross-section of corporate bonds. Fourth, the recent literature studying determinants of corporate credit spreads emphasizes the role of liquidity risk. This literature is large and one of the debates is about the most appropriate measure of liquidity for corporate bond and equity pricing; see Mahanti, Nashikkar, Subrahmanyam, Chacko, and Mallik, 2008 for a discussion.³⁰ We apply two different proxies of liquidity. For the corporate bond spreads, we follow Fontaine and Garcia (2011) and use their measure of aggregate liquidity, which is extracted from U.S. Treasuries.³¹ For equity returns, we apply the Pástor and Stambaugh (2003) liquidity measure.³²

³⁰Chacko (2006) and Downing, Underwood, and Xing (2007) study the pricing of liquidity risk in credit markets by constructing direct measures from the bond market. These studies focus on the impact of the liquidity level per se. de Jong and Driessen (2006) build liquidity risk factors, which represent systematic liquidity shocks in equity and government bond markets, similar to Acharya and Pedersen (2005), and show that corporate bond prices carry a substantial liquidity risk premium. Fontaine and Garcia (2011) extract an aggregate liquidity measure from the cross-section of on-the-run premia and show that this measure has a significant impact on risk premia for on-the-run and off-the-run bonds, LIBOR loans, swap contracts, and corporate bonds.

³¹We thank René Garcia for providing us their data.

³²Pástor and Stambaugh (2003) find that during months with low liquidity, the correlation between their liquidity proxy and the return on corporate bonds is found to be as large as -27%. However, in our

2.4 Empirical Analysis

2.4.1 *The Dynamics of Disagreement*

We investigate the cross-sectional and time-series properties of our proxies of disagreement. A first result is striking: A single dynamic common component explains more than 87% of the time-series variation in the cross-section of individual firm disagreement proxies. This is indeed remarkable given that we are considering a cross-section of more than 337 firms, thus suggesting a large systematic component driving difference in beliefs on firms' growth opportunities. The common disagreement component is highly time varying and exhibits a counter-cyclical behavior. It has an average of 0.38, with a minimum (maximum) of 0.12 (0.78), and a standard deviation of 0.13. It typically rises in the six months before crisis periods and decreases shortly afterwards: The average increase in the six months before a crisis period is 41% and the average decrease six months after the crisis is around 34%. The average firm-specific disagreement component is 0.22. The firm-specific disagreement is also highly volatile and it exhibits a substantial cross-sectional dispersion: The minimum (maximum) firm-specific disagreement is 0.01 (0.82), and its average time-series standard deviation is 0.14.

In what follows, we study systematically the empirical links between disagreement and asset prices suggested by our theoretical model. First, we use panel regression techniques to investigate the impact of individual and common disagreement proxies on corporate credit spreads and stock returns, after controlling for other explanatory factors. Second, we study the joint testable implications for capital structure arbitrage strategies.

2.4.2 *The Dynamics of Credit Spreads*

In our model, belief disagreement unambiguously increases credit spreads. We investigate the relevance of disagreement proxies with respect to several empirical reduced-form models in the literature and we consider the following empirical specifications, which embody specific sets of control variables previously studied in this literature: (1) Implied volatility and option-related factors; (2) Macro-financial factors; (3) Firm-specific

data, we find a very weak relationship between their measure and corporate credit spreads. We therefore prefer the liquidity measure by Fontaine and Garcia (2008), which they show has a significant impact on corporate credit spreads, using the same data as we do. Vice versa, their measure has a marginal impact on the equity market. We therefore rely on the Pástor and Stambaugh (2003) measure in this case.

factors, (4) Fama and French, liquidity, and volatility factors. Model (5) includes all determinants, while model (6) includes only statistically significant regressors.

In model (1), we control for the equity implied volatility and other option-related variables. This aspect is important as Cremers, Driessen, Maenhout, and Weinbaum (2006) and Cao, Yu, and Zhong (2010) find that the option-implied volatility and skewness are powerful explanatory factors of corporate credit spreads. They interpret these variables as proxies for a volatility and jump risk premium component in credit spreads. In our model, the option implied volatility and skewness are indeed important and are correlated with credit spreads. However, both option implied volatility and credit spreads are endogenously driven by heterogeneity in beliefs, so that the information contained in option-implied volatility should be subsumed by heterogeneity in beliefs. Table 2.2 summarizes the results for model (1). Indeed we find that, after controlling for the difference in beliefs, the option-implied volatility, which is significant in univariate regressions, is no longer statistically significant. This result suggests that option-implied (risk neutral) volatility is highly correlated with uncertainty, rather than being a pure proxy for measurable and uncertainty-free jump risk. The two disagreement proxies and the option-related variables explain 58% of the variation in corporate credit spreads. Both the individual uncertainty and common uncertainty proxies are highly statistically significant and the estimated coefficient has the expected positive sign. We also find that the marginal impact of changes in disagreement on credit spreads is economically important. Given that the cross-firm average of the standard deviation of a monthly change in belief disagreement is 0.18, the slope parameter estimates imply that a one-standard deviation change in the firm-specific belief disagreement leads to an increase of approximately 18 basis points in corporate credit spreads. The effect of the common disagreement is slightly smaller. The standard deviation of the common disagreement is 0.13, hence, a one standard deviation change in the common disagreement leads to an increase of approximately 10 basis points. The adjusted R^2 of the regression is 0.54. This means that the disagreement proxies together with the option implied variables explain half of the time-series and cross-sectional variation of corporate credit spreads.

In model (2), we include macro-financial variables as controls. Controlling for these variables is relevant for three reasons. First, Collin-Dufresne, Goldstein, and Martin (2001) show that macro-financial variables such as the risk-free rate level and the slope of the yield curve are correlated with credit spreads. Thus, in addition to these variables, we

also consider a NBER dummy. Second, Huang and Kong (2007) show that macroeconomic announcements of business-cycle related variables have a significant effect on corporate credit spreads. Among many possible announcement variables, we use non-farm pay-roll as a further control for the state of the economy. These variables are clearly potentially important, as macroeconomic conditions are related to both the probability of default and the recovery rate. Non-farm pay-roll is labeled by Andersen and Bollerslev (1998) the “king” of announcements. Moreover, Beber and Brandt (2009) document that it is the most influential macro announcement variable. Third, given the relative importance of a single market-wide factor in explaining the dynamics of differences in beliefs, it is interesting to measure the extent to which the role of the differences in beliefs is subsumed by the realizations of macro-financial and macro-economics variables. Does disagreement play an independent role? The results in Table 2.2 (second column) show that, in addition to the firm-specific and common uncertainty proxy, the risk-free rate and non-farm pay-roll are highly significant. The NBER dummy is significant and has an expected negative sign. The sign of the coefficient for the other macro-financial variables is consistent with the findings in Collin-Dufresne, Goldstein, and Martin (2001). We find that, even after controlling for macroeconomic factors, the disagreement measures are both statistically and economic significant. Interestingly, however, when we control for these factors the size of the slope coefficient on the common component of the difference in beliefs declines from 0.718 to 0.476, supporting the conjecture that market-wide disagreement interacts with macroeconomic announcements (non-farm payroll and NBER recessions).

Model (3) controls for firm-specific features. Leverage is an obvious control variable as it is positively related to the probability of default in structural models of credit risk, and it is found to be an important explanatory variable of credit spreads, as documented – among others – by Avramov, Jostova, and Philipov (2007). We also control for firm size, to proxy for the higher sensitivity of smaller firms to business-cycle factors (Fama and French, 1993). In the third column of Table 2.2, the regression results show that both the individual and common disagreement proxies are highly significant, despite the significance of leverage and firm size. Consistent with economic intuition, these variables have a positive estimated coefficient. Moreover, their significance is robust to the inclusion of additional control variables in model (6). If we compare their coefficients with the size of the disagreement slope coefficients, we find that given a standard deviation of 0.08, a one standard deviation change in leverage leads to a 3 basis points

increase in corporate credit spreads, which is six times smaller than the contribution of the systematic disagreement, for instance. Firm size contributes to approximately 4 basis points.

In model (4), we control for the Fama and French, liquidity, and volatility factors. Schaefer and Strebulaev (2008) find that corporate bond prices are significantly influenced by two Fama-French factors and the VIX implied volatility index. In our regressions, we use implied volatilities from individual stock options, rather than the aggregate index option implied volatility, to improve the granularity of the information provided by the risk neutral implied volatility. Longstaff, Mithal, and Neis (2005) find that a large fraction of the non-default component of corporate credit spreads can be related to illiquidity, and Fontaine and Garcia (2008) show that their aggregate measure of liquidity extracted from on-the-run premia is a good predictor of the default spread. The fourth column of Table 2.2 shows that even after controlling for these factors, both the individual and common disagreement measures are highly significant with t-statistics well above 5. Some of the additional explanatory variables, which are significant in univariate regressions, such as the market and Fama-French factors, are no longer significant after controlling for the disagreement measures. There are three important exceptions: earning volatility,³³ SMB factor, and aggregate liquidity. Earnings volatility and the size factor have the expected positive sign and the liquidity factor has the expected negative sign. The result on the earning volatility is interesting since this variable plays a key role in any Merton-type structural model of credit spreads. Our regressions confirm this link but add a potentially interesting element to our understanding of it. When we regress credit spreads on both earning volatility and differences in beliefs, we find that a one standard deviation change in earnings volatility accounts for an approximate increase of 4 basis points in credit spreads. On the other hand, the effect of disagreement is statistically more significant and economically seven times larger.³⁴

In model (5), we run a regression including all explanatory factors. The statistical significance of the individual and common disagreement proxies remain remarkably high. Quite surprisingly, the economic significance remains stable even when we include all other variables. In terms of adjusted R^2 , we find that all determinants together explain

³³We use a one-year moving average to calculate the volatility. Moreover, the earning volatility is scaled so that it has the same time-series standard deviation as the corresponding average implied volatility.

³⁴Avramov, Jostova, and Philipov (2007) also find that Fama and French factors lose their significance for credit spreads when combined with other control variables.

approximately 80% of the time series and cross-sectional variation of corporate credit spreads.

In model (6), finally, we run the regression with only the explanatory factors that were found significant in the previous specifications (1)-(5). The main result do not change. Overall, our results show that the explanatory power of both the individual and common disagreement for credit spreads is high and robust with respect to several common control variables. These findings are remarkably robust also with respect to a stratification of the sample with respect to firm leverage.

2.4.3 Capital Structure No-Arbitrage Violations

Capital structure arbitrage has become increasingly popular among long/short, multi-strategy, event driven hedge funds. The success of these strategies depends on the empirical realism of the key assumptions on the joint behavior of the value of debt and equity, such as the sensitivities of corporate bond prices to changes in the price of equity. Anecdotal evidence suggests that this relationship often fails. However, although it is very relevant from a practical point of view, we know very little about this link both at an empirical and theoretical level. Nonetheless, the joint behavior of the value of debt and equity offers powerful additional information about alternative models of credit spreads.

While much of the literature has focussed on predicting corporate bond price levels, second moment predictions like hedge ratios have so far been mostly neglected. The hedge ratio of standard single-factor models takes the form $(1/\Delta_S - 1)S/D$, where Δ_S is the sensitivity of the price of equity to the underlying price of the corporate bond and S/D is the inverse leverage ratio. In contrast to single-factor models such as the Merton model, our theory implies that the sign of Δ_S might be both positive or negative, depending on the leverage of the firm, and that disagreement could play a role in explaining some of the violations.³⁵ To simplify our discussion, we first specify the

³⁵A prominent example of a failure of capital structure arbitrage occurred in May 2005, when General Motors (GM) and Ford got downgraded to junk status. Before May 2005, many hedge funds sold CDS on GM and hedged their exposure by shorting the equity (or by creating a long volatility position in GM options). The rationale for this strategy was that, consistent with the structural Merton model, wider credit spreads would be accompanied by a drop in the share price (or an increase in the option implied volatility). After the downgrade of GM to junk status by Standard & Poors, credit spreads on a 10 year CDS increased by almost 200 basis points in one month. The share price, however, rose almost 25% to 32.75 USD, and the implied volatility of short-term at-the-money options on GM increased by 50% to reach 62.73%. Many widely known hedge funds engaging in capital structure arbitrage posted large losses and the state of the hedge fund industry obtained center stage in the financial press.

TABLE 2.2: OLS Panel Regression Results for Credit Spreads

Using data running from January 1996 to December 2004, we regress credit spreads on corporate bonds on a set of variables listed below. * denotes significance at the 10% level, ** denotes significance at the 5% level and *** denotes significance at the 1% level. Model (1) corresponds to the regression model with option-implied determinants (see Cremers, Driessen, and Maenhout (2008)). Model (2) corresponds to macro determinants. Model (3) corresponds to firm specific variables, Model (4) corresponds to the regression model with systematic risk factors and Model (5) includes all determinants. Model (6) is the regression with significant values only. All estimations use autocorrelation and heteroscedasticity-consistent t-statistics.

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---------------------|--------------------|----------------------|--------------------|---------------------|----------------------|----------------------|
| Constant | 0.756*** [8.45] | 0.613*** [6.38] | 0.594*** [7.39] | 0.420*** [5.29] | 0.639*** [6.38] | 0.508*** [5.99] |
| Individual DiB | 0.976*** [8.56] | 0.837** [8.93] | 0.837*** [8.31] | 0.880*** [7.73] | 0.820*** [7.27] | 0.735*** [7.11] |
| Common DiB | 0.718*** [3.85] | 0.476*** [4.09] | 0.521*** [5.10] | 0.469*** [5.28] | 0.560*** [5.83] | 0.489*** [5.00] |
| Implied Volatility | 0.713 [1.43] | | | | 0.672* [1.77] | 0.523 [1.43] |
| IV Skew | -0.256* [-1.65] | | | | -0.231 [-1.38] | |
| Slope of TS | | -0.412* [-1.71] | | | -1.746* [-1.67] | -1.652* [-1.87] |
| Risk-free Rate | | -0.532*** [-4.35] | | | -0.429*** [-3.43] | -0.493*** [-3.84] |
| Non-Farm Payroll | | -0.632*** [-4.33] | | | -0.561*** [-3.56] | -0.461*** [-3.98] |
| NBER Dummy | | -2.545* [-1.73] | | | -1.939 [-1.63] | |
| Leverage | | | 0.375*** [4.13] | | 0.564*** [3.34] | 0.488*** [3.43] |
| Firm Size | | | 0.045*** [3.40] | | 0.035** [2.45] | 0.027** [2.31] |
| Liquidity | | | | -0.124** [-2.51] | -0.102** [-1.99] | -0.120** [-2.01] |
| $R_m - R_f$ | | | | -0.632 [-1.02] | -0.541 [-1.01] | |
| SMB | | | | 0.427*** [3.22] | 0.321** [1.99] | 0.308* [1.76] |
| HML | | | | 0.321* [1.77] | 0.491* [1.67] | 0.410* [1.64] |
| Earnings Volatility | | | | 0.400** [2.47] | 0.391** [2.30] | 0.418** [2.34] |
| Adjusted R^2 | 0.54 | 0.68 | 0.70 | 0.68 | 0.80 | 0.80 |

main object of our analysis.

Definition 2.4. We define a violation event when $\Delta CS \cdot \Delta S > 0$, where ΔCS is the change in the credit spread and ΔS the change in the corresponding individual stock price.

The leverage-dependent co-movement of stock returns and credit spreads in our model suggests that a violation according to Definition 2.4 is more likely for low leverage firms and when disagreement is large. This gives rise to the following testable implication related to question Q2:

- Arbitrage violations according to Definition 2.4 are more likely to arise for moderately levered firms and when disagreement is high.

In the sequel, we first compare the empirical and model-implied frequencies at which arbitrage violations of single-factor models occur. Then, we use panel Logit techniques to learn additional conditional properties of the data.

2.4.3.1 Unconditional Violations and Disagreement

We use the calibrated parameters in Table 3.1 to simulate our model and calculate the occurrence frequencies of a credit violation. The results are collected in Table 2.3, Panel A.

The unconditional average frequency of a violation in the data is approximately 13.9% (see Table 2.3, Panel B). This is a large number, considerably higher than other no-arbitrage restriction violations studied in index option markets. For instance, Bakshi, Cao, and Chen (2000) study violations of the one factor Black and Scholes (1973) model in index option markets and find that the probability that the delta of a call (put) is negative (positive) is between 1% and 4%, depending on the moneyness level. The frequency of occurrence of capital structure violations is larger by a factor of almost 5.

When we simulate our model with disagreement using the calibrated parameters in Table 3.1, we find that the model generates capital structure violations: The model-implied frequency is 14.2%, which is quite close to what is observed empirically. After we stratify the sample with respect to firm's leverage, we find that the empirical violation frequency

TABLE 2.3: Simulated and Empirical Occurrence of Violations

This table shows the violation frequencies implied by the Monte Carlo simulation of our model (Panel A). The reported numbers are the simulated fractions of the violation occurrence across 10,000 simulation trials of the model. Panel B summarizes the empirical frequency arbitrage violations as a percentage of total observations at a given leverage. Low, Average, and High refer to the leverage ratios.

| Panel A: Simulated Violations | | | |
|--------------------------------------|------------|----------------|-------------|
| | Low | Average | High |
| | 15.3 | 14.2 | 12.2 |
| Panel B: Empirical Violations | | | |
| | Low | Average | High |
| | 18.9 | 15.4 | 14.3 |

is substantially higher (18.9%) for firms with low leverage than for firms with high leverage (14.3%). This result is consistent with the results obtained from the simulation of the model, where we find that the violations are 15.3% for low leverage firms and 12.2% for high leverage firms. Thus, the model helps to explain another feature of the data that traditional models cannot explain. This finding is relevant since it implies a co-movement of different parts of the capital structure that is consistent with the role of disagreement, after taking into account cross-sectional differences in leverage.

2.4.3.2 Conditional Violations and Disagreement

The previous unconditional results do not necessarily imply that disagreement is the only factor able to explain no-arbitrage violations, since they might be generated, e.g., by a misspecification of the Merton (1974) model or by market frictions. For instance, Acharya, Schaefer, and Zhang (2008) study in much detail the GM and Ford downgrade of 2005 and find empirical evidence that institutional frictions and liquidity are responsible for a segmentation of equity and credit markets at this time. Capital structure

arbitrage strategies can be set up as a convergence trade and illiquidity of a market increases transaction costs as well as risks of these trades.³⁶ More generally, the liquidity of the underlying securities impacts arbitrage activities in these markets. Even though the results in Acharya, Schaefer, and Zhang (2008) focus on two particular firms, their evidence is suggestive of the potential importance of a joint study of liquidity factors and disagreement.

We consider two different liquidity factors in our Logit regressions: The Fontaine and Garcia (2008) liquidity measure and the Pàstor and Stambaugh (2003) measure. As capital structure arbitrage requires trading in the corporate bond and the stock, both the liquidity in the credit and equity market might be relevant.³⁷ We also add as controls the Fama and French size and book-to-market factors to our regressions, as they turn out to be significantly related to the corporate bond spreads, and the VIX index.

We study how belief disagreement is linked to the conditional probability of a violation in credit markets. To this end, we estimate a set of panel Logit regressions, in which the binary variable $y(it)$, denoting the occurrence of a violation at time t for firm i , is regressed onto a set of variables that include both disagreement proxies. The probability that a violation occurs at time t for firm i is specified as:

$$P(y(it) = 1) = F(\beta_0 + \beta_1 \log \Psi_i(t) + \beta_2 \log \Psi_z(t) + \sum_{j=1}^2 \delta_j F_{ij}(t) + \sum_{k=1}^7 \gamma_k T_k(t)),$$

where F is the cumulative distribution function of a logistic distribution, β_1 and β_2 are the loadings on the individual and common disagreement proxies, δ_j the loading of implied volatility and leverage, and γ_k the loading of the VIX, the equity market liquidity, the credit market liquidity, market, size, book-to-market mimicking factors, and earnings volatility. The model is estimated by Maximum-Likelihood. The results are given in Table 2.4.

We find that individual and common disagreement increase the conditional probability of a violation, with an estimated coefficient that is highly significant and stable across the

³⁶Convergence trading strategies are popular among hedge funds. A typical convergence trading strategy is to bet that the price difference between two assets with similar characteristics will narrow in the future. The collapse of Long-Term Capital Management (LTCM) is often cited as a chatoyant example illustrating the interplay of convergence trades and a deterioration of liquidity, together with its impact on asset prices and volatility (see Xiong, 2001).

³⁷The unconditional correlation between the two liquidity proxies is fairly low: In the period from December 1985 to December 2007 the unconditional correlation is only -10%.

TABLE 2.4: Logit Regression of Arbitrage Violations on Credit Markets

This table summarizes the Logit regression results for the violation frequency in credit markets. The probability that a violation event occurs is specified as:

$$P(y(it) = 1) = F(\beta_0 + \beta_1 \log \Psi_i(t) + \beta_2 \log \Psi_z(t) + \sum_{j=1}^2 \delta_j F_{ij}(t) + \sum_{k=1}^7 \gamma_k T_k(t)),$$

where Ψ_i is the individual disagreement, Ψ is the common disagreement, F_j the implied volatility and leverage, and T indicate the VIX, Pastor and Stambaugh Liquidity proxy (PS Liquidity), Fontaine and Garcia Liquidity (FG Liquidity) proxy, Fama and French factors (market, size, and book-to-market), and earnings volatility. t-values are in brackets. * denotes significance at the 10% level, ** denotes significant at the 5% level and *** denotes significance at the 1% level, respectively.

| Dependant | Leverage | | |
|---------------------|--------------------|--------------------|--------------------|
| | Low | Average | High |
| Constant | -0.21** [-2.39] | -0.32** [-2.19] | -0.37** [-2.43] |
| Individual DiB | 0.17*** [3.32] | 0.18*** [3.83] | 0.23*** [4.27] |
| Common DiB | 0.12** [2.40] | 0.18** [2.36] | 0.13** [2.43] |
| Implied Volatility | 0.19* [1.68] | 0.26* [1.75] | 0.18* [1.90] |
| VIX | 0.05* [1.67] | 0.04* [1.66] | 0.02 [1.38] |
| Leverage | 0.13** [2.02] | 0.18** [2.30] | 0.21* [1.71] |
| PS Liquidity | -0.16 [-1.22] | -0.12 [-1.58] | -0.11* [-1.73] |
| FG Liquidity | -0.17* [-1.88] | -0.20** [-2.01] | -0.21** [-2.29] |
| $R_m - R_f$ | 0.05 [1.32] | 0.10 [1.57] | 0.12 [1.63] |
| SMB | 0.24* [1.90] | 0.17 [1.43] | 1.28* [1.91] |
| HML | 0.18 [1.47] | 0.23 [1.62] | 0.48** [2.52] |
| Earnings Volatility | 0.40* [1.69] | 0.49* [1.91] | 0.42* [1.75] |
| Pseudo R^2 | 0.19 | 0.19 | 0.22 |

different leverage levels. Leverage positively impacts the conditional probability of a violation and the estimated coefficients are significant. The implied volatility of individual

options increases the probability of an arbitrage violation in credit markets. However, the estimated coefficients are significant at the 10% level only. The estimated coefficients for the equity market liquidity proxy (PS liquidity) are negative and marginally significant for the highest leverage bin. They are not distinguishable from zero for the other leverage ratios. In contrast, the estimated coefficients for the credit market liquidity proxy (FG liquidity) are significant across all leverage bins. The coefficients are negative and the size increases in absolute terms with leverage, implying (i) that a higher liquidity reduces the probability of an arbitrage violation and (ii) that liquidity becomes more important in explaining these violations for firms in more distress. This finding is similar to Kapadia and Pu (2008), who find that liquidity of the credit market is significant in linking equity and credit markets, while equity market liquidity has no effect.

Overall, we obtain corroborating evidence that belief disagreement is a missing factor in structural models, explaining part of the empirical no-arbitrage violations of single-factor models in credit markets.

2.4.4 Stock Returns

As shown by the calibrated structural model a positive (negative) relation between disagreement and future equity returns is more likely for higher (lower) levels of leverage, since the skewness effect of the firm value tends to vanish for increasing leverage levels. Thus, we investigate in further details this testable implication of the model.

We stratify by leverage by introducing a dummy variables according to three different, equally weighted, leverage bins: The low/medium/high bins corresponds to the first/second/third leverage terciles.³⁸ Table 2.5 summarizes the results.

We find two regularities. First, in the credit spread regression the significance and the size of the estimated coefficients for disagreement are monotone cross-sectionally, with a sign independent of leverage. Second, in equity return regressions (last column), the estimated coefficient of disagreement is statistically significant across all leverage bins, but the sign of the estimated coefficient for the low leverage firms is negative. This finding is interesting for two reasons. First, it provides further evidence of the

³⁸The three terciles correspond to leverage ratios below 0.045, between 0.045 and 0.35, and above 0.35, respectively.

TABLE 2.5: OLS Panel Regressions with Dummies

Using data running from January 1996 to December 2004, we regress credit spreads on corporate bonds and firm stock returns on a set of variables listed below. The coefficients for Dispersion (HL), Dispersion (AL), and Dispersion (LL) are obtained by multiplying the coefficient with a dummy variable that takes the value 1 if the firm is in the high, average, and low leverage bin and zero otherwise. The same applies to the variables Implied Volatility, Implied Volatility Skewness, and Leverage. * denotes significance at the 10% level, ** denotes significance at the 5% level and *** denotes significance at the 1% level. All estimations use autocorrelation and heteroskedasticity-consistent t-statistics.

| | Credit Spreads | Stock Returns |
|------------------------------|----------------------|---------------------|
| Constant | 0.653*** [3.83] | 0.001*** [5.35] |
| Individual DiB (LL) | 0.756*** [6.82] | -0.012** [-2.31] |
| Individual DiB (AL) | 0.786** [2.48] | 0.001* [1.83] |
| Individual DiB (HL) | 0.859** [2.39] | 0.002* [2.42] |
| Common DiB | 0.721** [2.22] | 0.007** [2.31] |
| Implied Volatility (LL) | 0.625* [1.93] | -0.001 [-1.11] |
| Implied Volatility (AL) | 0.520 [1.09] | -0.001 [-1.01] |
| Implied Volatility (HL) | 0.492 [1.00] | -0.002 [-1.00] |
| Implied Volatility Skew (LL) | -0.261 [-1.38] | -0.026 [-1.10] |
| Implied Volatility Skew (AL) | -0.324 [-1.44] | -0.017 [-1.31] |
| Implied Volatility Skew (HL) | -0.529 [-1.30] | -0.015 [-1.00] |
| Slope of Term Structure | -0.377 [-1.34] | |
| Risk-free Rate | -0.506*** [-3.10] | |
| Non-Farm Payroll | -0.597** [-2.42] | 0.013 [1.38] |
| NBER Dummy | -1.657 [-1.09] | -0.002 [-1.07] |
| Leverage (LL) | 0.427** [3.45] | 0.017* [1.73] |
| Leverage (AL) | 0.287*** [3.01] | 0.015 [0.83] |
| Leverage (HL) | 0.265*** [3.00] | 0.013 [0.93] |
| Liquidity | -0.129** [-2.28] | 0.029* [1.74] |
| $R_m - R_f$ | -0.532 [-1.03] | 0.001*** [4.39] |
| SMB | 0.417** [2.37] | 0.002*** [4.94] |
| HML | 0.313* [1.73] | 0.001*** [2.99] |
| Earnings Volatility | 0.373* [1.71] | -0.002* [-1.68] |
| Adjusted R^2 | 0.75 | 0.05 |

importance of credit risk in the context of a structural model of a levered firm, in which skewness effects generated by an increases in the difference in beliefs tend to vanish for high leverage. Second, it can help to explain the contradicting result found by previous empirical reduced-form equity return literature.³⁹

³⁹Diether, Malloy, and Scherbina (2002) find a negative coefficient and interpret the result as supporting evidence for the behavioral style model; Anderson, Ghysels, and Juergens (2005) run a similar

2.5 Robustness

In this section, we assess the robustness of our results by studying (i) the extent to which our disagreement proxies capture other sources of risk and (ii) a setting with time dummies and firm fixed effects, in order to explore to what extent our disagreement proxies measure cross-sectional versus time-series variation.

2.5.1 Idiosyncratic Volatility

Idiosyncratic volatility is a potentially important risk factor for stock returns and credit spreads. Johnson (2004), for example, studies a model, in which stock returns of a levered firm are decreasing in the asset's idiosyncratic risk. Chen, Collin-Dufresne, and Goldstein (2008) emphasize that the credit spread puzzle is closely related to the ratio of idiosyncratic and total volatility of stock returns. Campbell and Taksler (2003) empirically document that idiosyncratic stock return volatility is an important explanatory factor for the cross-section of credit spreads. Even if firm-level belief disagreement is not itself a measure of pure idiosyncratic risk, it is a natural robustness check to investigate the extent to which it proxies for idiosyncratic volatility. To this end, we calculate the time-series standard deviation of daily market adjusted stock returns 180 days preceding each observation, and use it as a further explanatory variable in our regressions, leading to the results in Table 2.6.

Confirming the results in Campbell and Taksler (2003), idiosyncratic risk significantly and positively affects corporate credit spreads. However, it does not have an impact on either the economic or statistical significance of our belief disagreement proxy, which is the most significant variable in the model. The adjusted R^2 of the regressions with idiosyncratic risk without disagreement are slightly lower than those of our benchmark regressions without idiosyncratic risk. The estimated coefficient for disagreement in the aggregated regression for stock returns is highly significant. The one of idiosyncratic volatility is negative and significant, but only at the 10 percent significance level.⁴⁰ Interestingly, in the regression for stock returns with dummy variables for leverage, our

regression using a different dataset of firms and find a positive slope coefficient. They interpret the result as supporting evidence of a neoclassical model.

⁴⁰This last finding is consistent with the results in Ang, Hodrick, Xing, and Zhang (2006) and Guo and Savickas (2006).

TABLE 2.6: OLS Panel Regressions with Idiosyncratic Volatility

Using data running from January 1996 to December 2004, we regress credit spreads on corporate bonds and firm stock returns on a set of variables listed below. The variable Idiosyncratic Volatility is the time-series sample standard deviation of daily stock returns 180 days preceding the observation. We standardize the variable to have it the same standard deviation as the option implied volatility, to make it comparable. * denotes significance at the 10% level, ** denotes significant at the 5% level and *** denotes significance at the 1% level. All estimations use autocorrelation and heteroskedasticity-consistent t-statistics.

| | Credit Spreads | Stock Returns |
|--------------------------|---------------------|--------------------|
| Constant | 0.283*** [4.82] | 0.001*** [3.29] |
| Individual DiB | 0.762*** [7.48] | 0.018*** [5.56] |
| Common DiB | 0.432** [2.34] | 0.001** [1.99] |
| Implied Volatility | 0.329* [1.78] | -0.001 [-1.00] |
| Implied Volatility Skew | -0.233 [-1.47] | -0.019 [-1.28] |
| Idiosyncratic Volatility | 0.332** [2.27] | -0.001* [-1.74] |
| Slope of Term Structure | -0.210 [-1.29] | |
| Risk-free Rate | -0.220* [-1.95] | |
| Non-Farm Payroll | -0.492** [-2.01] | -0.004 [-1.17] |
| Leverage | 0.372*** [3.93] | 0.038* [1.95] |
| Liquidity | -0.113** [-2.18] | 0.017* [1.94] |
| $R_m - R_f$ | -0.219 [-1.00] | 0.001** [1.98] |
| SMB | 0.126 [1.58] | 0.001** [1.90] |
| HML | 0.124* [1.66] | 0.001** [2.34] |
| Adjusted R^2 | 0.75 | 0.04 |

proxy for disagreement consistently maintains a significant explanatory power, but the idiosyncratic volatility does not.

2.5.2 Time and Firm Fixed Effects

To prevent potential biases due to a spurious time series correlation, we add time dummies to our baseline regressions. The results are summarized in Table 2.7. We note that the time dummies have very little effect on the coefficients of disagreement. For the idiosyncratic disagreement, the coefficients remain similar as in the regressions without time effects and the t-statistics remain essentially the same. The coefficient of the common disagreement proxy is lower and significant at the 10% level. This is not surprising, since the systematic disagreement picks up time variation only. Similar findings apply to the other time-series variables. For example, non-farm payroll and the slope of the risk-free rate lose almost all their explanatory power. The adjusted R^2 remains almost unchanged, which means that our determinants already account for low frequency time variation. To gauge more intuition about the cross-sectional variation, we run a firm fixed effects regression. The coefficients of the idiosyncratic disagreement remain almost unchanged and their estimates remain highly statistically significant, with t-statistics of 7.4 for credit spreads and 5.2 for expected stock returns. The largest effect can be observed for the coefficient of leverage. For the credit spreads regression, the coefficient drops from a 1% significance level to a 5% significance level. This means that leverage is more closely related to individual cross-sectional variation in credit spreads, and less related to time-series variation.

2.6 Conclusion

In a frictionless Merton (1974) type credit risk model, we derive equilibrium implications of risk-sharing between heterogeneous agents and we study its impact on credit spreads and stock returns. In our model, investors have different perceptions of future cash flows and their degree of uncertainty, captured by the volatility of expected cash flow growth. They agree to disagree and form forecasts of cash flow growth using a common set of observable variables. The intertemporal risk-sharing gives rise to three testable implications. First, higher disagreement unambiguously implies higher credit spreads and volatility. Second, it leads to a higher frequency of capital structure violations. Third, it can reduce expected equity returns of low leverage firms, but it increases expected equity returns for high levered firms.

TABLE 2.7: OLS Panel Regressions with Year Dummies and Fixed Effects

Using data running from January 1996 to December 2004, we regress credit spreads and stock returns on a set of variables listed below including either year dummies or firm fixed effects. Their coefficients are not reported. * denotes significance at the 10% level, ** denotes significant at the 5% level and *** denotes significance at the 1% level. All estimations use autocorrelation and heteroskedasticity-consistent t-statistics.

| | Credit Spreads | | Stock Returns | |
|-------------------------|---------------------|---------------------|--------------------|--------------------|
| Constant | 0.582*** [3.28] | 0.529*** [4.49] | 0.001*** [4.02] | 0.001*** [4.09] |
| Individual DiB | 0.739*** [6.99] | 0.787** [7.43] | 0.010*** [5.10] | 0.011*** [5.22] |
| Common DiB | 0.323** [1.99] | 0.312** [2.01] | 0.003* [1.76] | 0.002* [1.84] |
| Implied Volatility | 0.626* [1.75] | 0.642* [1.86] | -0.001 [1.10] | -0.001 [1.02] |
| Implied Volatility Skew | -0.213 [-1.47] | -0.218 [-1.56] | -0.021 [-1.39] | -0.029 [-1.48] |
| Slope of Term Structure | -0.134 [-1.00] | -0.198 [-1.03] | | |
| Risk-free Rate | -0.313** [-1.99] | -0.372** [-2.01] | | |
| Non-Farm Payroll | -0.355* [-1.85] | -0.365* [-1.78] | -0.003 [-1.20] | -0.004 [-1.17] |
| Leverage | 0.426*** [3.31] | 0.389** [2.03] | 0.034* [1.83] | 0.038* [1.95] |
| Liquidity | -0.183** [-2.19] | -0.121** [-2.18] | 0.027* [1.91] | 0.017* [1.83] |
| $R_m - R_f$ | -0.212 [-1.00] | -0.272 [-1.04] | 0.001** [1.87] | 0.001** [1.93] |
| SMB | 0.124* [1.69] | 0.120* [1.94] | 0.001** [1.84] | 0.001** [1.90] |
| HML | 0.124* [1.63] | 0.120* [1.71] | 0.001** [1.98] | 0.001** [2.34] |
| Earnings Volatility | 0.203* [1.67] | 0.201* [1.68] | -0.002 [-1.21] | -0.001 [-0.99] |
| Adjusted R^2 | 0.69 | 0.69 | 0.03 | 0.04 |
| Year Dummies | ✓ | ✓ | ✓ | ✓ |
| Firm Dummies | | ✓ | | ✓ |

Using a merged data-set of individual firm earning forecasts, credit spreads, and stock returns, we test the model predictions using a set of panel and Logit regressions. Our empirical study produces a number of results. First, disagreement increases corporate

credit spreads through the risk-sharing mechanism in our model. This result is significant, both economically and statistically, and robust to the inclusion of a variety of commonly used control variables.

Second, disagreement helps to explain the large frequencies of violations of capital structure no-arbitrage restrictions of single-factor credit risk models. These models imply a negative relation between credit spreads and the price of equity. In our model, this monotonic relation can be violated for some regions of leverage, due to a time-varying risk-neutral skewness driven by the degree of heterogeneity in beliefs. The percentage of no-arbitrage violations in our calibrated model is substantial and is comparable to the one in the data. We investigate the extent to which belief disagreement explains the conditional probability of these violations in a set of Logit regressions. In all regressions, we find that the slope coefficient of both individual and common disagreement is positive and highly significant.

Third, the empirical analysis identifies a significant positive relation between stock returns and disagreement using aggregated data, but this relation is reversed and significant for low leverage firms. This result is consistent with the predictions of our structural model when leverage is introduced. The recent empirical literature has debated on the sign of the relation between stock returns and divergence of opinions. Our analysis offers a structural explanation for these mixed results in a frictionless economy where asset prices reflect the risk-sharing behavior of disagreeing investors.

Our results rise some potentially interesting questions for future research. A first issue is, e.g., the link between heterogeneity in beliefs and liquidity. When corporate credit spreads began to surge in mid 2007, funding liquidity fell quite substantially; See, e.g., Brunnermeier (2009). Such increases in illiquidity were not confined to the 2008 credit crisis.⁴¹ Routledge and Zin (2009) propose a model to study the impact of Knightian uncertainty on liquidity risk in times of economic crises, and it is natural to expect that disagreement and liquidity can interact in interesting ways in a setting with heterogeneous beliefs and market frictions. A surge in uncertainty might rise the incentives to trade for symmetrically informed investors, but it might also make it more difficult to distinguish informed from uninformed traders, and thus reduce the incentive of uninformed agents to trade. Empirically, the interaction of disagreement and liquidity is

⁴¹For instance, Acharya and Pedersen (2005) find that their proxy of illiquidity was particularly high during the Iraqi invasion in 1990, the Asian crisis in 1997, and the Russian default in 1998. Bao, Pan, and Wang (2011) obtain similar findings with a different illiquidity proxy.

supported by the data. In preliminary results, we find that disagreement and illiquidity proxies can predict a large portion of the variation of the asset swap spread across different sectors. These findings indicate an interesting empirical link that could constitute an important topic of future work.

Chapter 3

When Uncertainty Blows in the Orchard: Comovement and Volatility Risk Premia

joint with [Andrea Buraschi](#) and [Fabio Trojani](#)

Introduction

The pricing of index and individual stock options differs in a number of striking empirical dimensions, which represent important challenges to modern asset pricing models. First, as noted in Bakshi, Kapadia, and Madan (2003), the index option implied volatility smile is on average steeper and more negatively skewed than the smile of single stock options. Second, the average volatility risk premium of index options is typically larger than the one of individual stock options. For instance, while the average at-the-money implied volatility of S&P 100 index options has been about 19.2% on a yearly basis, the realized volatility has been only about 16.7%, which implies an index volatility risk premium of 2.5%. In contrast, the average volatility risk premium of single-name S&P 100 stocks has been only 0.9% in the same sample period. Third, such significant cross-sectional differences in compensation for volatility risk result in large risk adjusted returns for a number of option strategies. For instance, we find that dispersion portfolios being short index volatility and long individual stock volatility can achieve an annualized Sharpe

ratio of about 1.5 without an apparent exposure to standard risk factors.¹ Similar findings are obtained for spread portfolios of straddles, which are sorted with respect to the differences of implied and realized volatilities.

Given this evidence, it may not appear surprising that variance and correlation have emerged as new asset classes, for which trading strategies have been standardized to create a novel generation of financial products, thus highlighting their potential economic importance for economic risk-sharing. The recent financial crisis has shown even more dramatically the need for a good understanding of the sources and the pricing peculiarities of volatility and correlation. The Greek debt crisis and fears of contagion caused investors to sell risky assets in the second quarter of 2010 which contributed to a jump in not only market volatility (the VIX tripled between the first and second quarter of 2010) but also realized and implied correlation: “The decision to reopen our dispersion trading desk after shutting it down last year [2009] has largely been driven by the macro environment. The disparity between implied and realized correlation has really attracted a lot of attention, with correlation on the Dow Jones Eurostoxx 50 index realizing on average 20 points below implied. Investors have looked to profit from the carry, and there has been a lot of opportunity in this space,” says Cyrille Walter, head of European equity trading at Morgan Stanley. For example, Och-Ziff Capital Management placed USD 12 bn in a dispersion trade in May 2011, which documents how profitable variance and correlation trading is regarded in the industry.²

Whereas these empirical facts are well documented, little is known about their potential explanation within a structural economic model. For example, while it is often argued that the option implied volatility provides useful information about investors’ perception of future uncertainty, the potential equilibrium mechanisms linking uncertainty and volatility or correlation risk premia are still not fully understood. In this paper, we motivate theoretically and explore empirically a distinct equilibrium channel for these links, generated by the optimal risk sharing of investors having diverse opinions about the degree of future uncertainty in the economy.

In order to produce a structural explanation for these links, we posit a two-trees Lucas (1978) economy, in which heterogeneity in beliefs is a priced equilibrium risk factor. Our

¹A dispersion portfolio involves a short position in the index volatility and a long position in the constituents volatility, which can be based on either straddles or strangles. Since at-the-money straddles have a delta exposure close to zero, a dispersion portfolio short an index straddle and long the constituents straddles is well hedged against market movements.

²See [Bloomberg News](#).

model allows us to motivate a number of testable predictions for the relation between difference in opinions and (i) the differential pricing of index and individual options, (ii) the variance and correlation risk premia embedded in index and individual options and (iii) the risk-return profile of option strategies creating exposure to volatility and correlation risk. We make two key assumptions. First, the growth rates of firms' dividend streams are unknown. Second, agents have heterogeneous perceptions about the level of economic uncertainty. Because of these assumptions, agents process information differently and eventually disagree, leading to a stochastic disagreement with a non-degenerate asymptotic distribution. Agents' optimal consumption share depends on the level of disagreement, which becomes a priced state variable, and investors engage in active risk sharing, in order to finance their optimal consumption plans.

In equilibrium, stock volatilities and correlations co-move with the degree of diversity of investors' opinions. Volatility is priced and volatility risk premia reflect both the perceived level of investors' uncertainty and the heterogeneity in beliefs. Moreover, the volatility risk premium on index options is larger, because of an endogenous correlation risk premium generated by investors' optimal risk sharing.

Using the model solution, we derive a number of testable predictions. First, we show that the difference in beliefs creates a wedge between risk-neutral and realized volatilities. The larger the difference in beliefs, the larger the volatility risk premium of index options relative to the volatility risk premium of individual options, because of the larger ex-ante compensation required by the optimistic investor to hedge the pessimist in aggregate bad states. Second, the difference between index and individual volatility risk premia is driven by the correlation premium. The correlation risk premium depends on the degree to which market-wide information is used by investors to form their beliefs on dividends. During periods of market stress, it has been argued that agents are affected by limited attention, thus increasing their reliance on market-wide signals to conduct their inference and limiting attention to firm specific information. We model this element by introducing a market-wide signal correlated with the aggregate dividend process. When subjective economic uncertainty is greater and agents rely more on the common signal, the correlation of beliefs as well as the correlation of disagreement over different firms is larger. This implies greater correlation of asset prices and eventually a larger correlation risk premium. Third, we find that a higher disagreement about future dividends induces a larger negative difference in the skewness of index and individual options. This effect is larger when agents disagree on the market-wide signal and market-wide uncertainty

is high. Fourth, we study the risk-return features of model-implied option portfolio strategies. We calibrate our economy and investigate the model-implied features of the returns of these portfolios. We find that exposure to common disagreement components can explain a large fraction of the excess returns of both spread straddle portfolios and dispersion portfolios. At the same time, other standard risk factors, like, e.g., the market return, have no apparent explanatory power. This is important since it suggests a possible rational explanation for the failure of traditional models to explain the data.

We test the empirical predictions of our model and use data on S&P 100 index options and single-stock options for all index constituents, in the period January 1996 to June 2007. We merge this dataset with analysts earning forecasts from the Institutional Brokers Estimate System (I/B/E/S) and stock return data from CRSP. We first compute a belief disagreement proxy for each individual firm and then apply dynamic factor analysis to construct a common factor that proxies for the common component of belief disagreement across firms.

Our analysis produces a number of novel results. First, in line with our model we find that individual and common disagreement proxies increase the difference of index and individual option skews. This effect is both economically and statistically significant. For instance, a one standard deviation change in individual disagreement implies about a half standard deviation change in the difference of index and individual option slopes of the smile. Second, consistent with the model predictions we find that diversity in beliefs also significantly increases the difference of volatility risk premia of index and individual stock options, as well as the correlation risk premium, in a way that is remarkably robust to the inclusion of several other control variables. For instance, a one standard deviation change in the individual disagreement proxy implies almost a one standard deviation change in the difference of index and individual volatility risk premia. Similarly, common disagreement is the most significant variable for the correlation risk premium, even after controlling for demand pressure and sentiment effects, which have been studied in the previous literature. Third, we study the risk-return tradeoff of dispersion and spread straddle portfolios. A standard dispersion trade shorts index straddles and buys straddles of individual stocks that compose the index. Since we investigate the effect of dispersion in beliefs, we sort straddles by the difference in beliefs of the underlying firms. Consistent with the model predictions, we find that spread straddle and dispersion portfolios, in which the long leg is based on stocks with low individual difference in beliefs, both generate attractive expected excess returns, without apparent exposure to

standard risk factors typically used to explain cross-sectional differences of stock returns. For example, the most profitable dispersion portfolio yields an annualized Sharpe ratio of 1.5, which is 20% higher than the Sharpe ratio derived from investing all wealth in a short index put. Goyal and Saretto (2009) find similar high returns and Sharpe ratios for spread straddle portfolios, when sorting individual options according to the difference of implied and realized volatility.

These findings are inconsistent with a single factor option-pricing setting, such as the Black and Scholes (1973) and Merton (1973) model, but can be reproduced in our economy because of the additional systematic risk factor linked to disagreement risk. We find that both spread straddle and dispersion portfolios feature an economically and statistically significant exposure to the dynamic common component in the difference of beliefs across firms. Finally, we formally investigate whether dispersion in beliefs is a priced risk factor that helps to explain the cross-section of spread straddle and dispersion option portfolio returns, as our model implies. We adopt the two-pass Fama and French (1992) approach, with Shanken and Zhou (2007) correction for misspecification robust standard errors. We first find that the price of risk for the common disagreement factor is strongly economically and statistically significant: The point estimate of the factor risk premium for the common disagreement is about 28% on an annual basis. Second, we find that a large fraction of spread straddle and dispersion option portfolio returns is explained by exposure to the common disagreement factor, while the risk premia estimated for other risk factors (such as the market, momentum, and HML factors) result insignificant. Third, we find some evidence of a potential nonlinear relation between option returns and common disagreement risk, indicating that infrequent but large increases in disagreement risk can harm the return of option portfolios with exposure to it. This finding follows from a negative estimated risk premium of the quadratic term in a two-pass Fama and French (1992) model allowing for a quadratic link between option returns and the common disagreement factor. Overall, these results confirm the hypothesis that a good fraction of the cross-section of option returns can be explained by an exposure to priced common disagreement risk.

Related Literature. Our paper is related to different strands of the literature. The first strand studies equilibrium models with multiple assets. Menzly, Santos, and Veronesi (2004) and Santos and Veronesi (2010) study a multi-asset economy with external habit, in which dividend shares are mean-reverting. Pavlova and Rigobon (2007) investigate a two country two good economy with demand shocks and log-linear preferences.

Cochrane, Longstaff, and Santa-Clara (2008) analyze a two-trees Lucas (1978) economy and the implications for stock returns, correlations, and the equity risk premium. Martin (2011) extends this framework to a collection of Lucas trees, so called Lucas orchards, with rare disasters. Our contribution departs from these papers along several dimensions. First, our model not only generates comovement in asset prices, but also a correlation risk premium. Second, we link the price of this risk to a quantity that is related to the formation of expectations, as opposed to pure dividend shocks and the relative size of the trees.

Our work is closely related to the literature that investigates the characteristics of the volatility risk premium. The early literature on volatility risk premia is large and deals mostly with index options. Fleming, Ostdiek, and Whaley (1995), Jackwerth and Rubinstein (1996), and Christensen and Prabhala (1998) observe that average realized index volatilities tend to be substantially lower than implied volatilities of index options. Buraschi and Jackwerth (2001), Coval and Shumway (2001), and Bakshi and Kapadia (2003a) find evidence that option returns are not spanned by dynamic delta hedging strategies and that volatility risk is priced at the index level. These findings have ignited a broader literature that studies also single-name options. There is empirical consensus that volatility risk premia for single-name options are substantially smaller than for index options (see Bakshi and Kapadia, 2003b, Duarte and Jones, 2007, and Carr and Wu, 2009). Driessen, Maenhout, and Vilkov (2009) find insignificant differences between implied and realized volatilities of single stocks based on average model-free volatility measures. They interpret the significant difference between index and single name options risk premia as evidence of a large correlation risk premium. They argue that a simple option-based dispersion strategy earns large Sharpe ratios. An important economic question is therefore related to the nature of these excess returns. We contribute to this literature by finding that a single common disagreement factor helps to understand the dynamics of both volatility and correlation risk premia, which reconciles the findings in the literature.

Another important stream of the literature examines the cross-section of option returns. Goyal and Saretto (2009), among others, find that long-short portfolios of straddles sorted with respect to the individual differences of implied and realized volatilities can produce attractive returns, with no apparent exposure to standard risk factors. They interpret the result as a challenge to rational expectation models and evidence, instead, of behavioral biases in the option market. We address this challenge by examining the

role played by exposure to the common difference in beliefs factor in the context of a Fama-MacBeth analysis. Our results indicate that a significant fraction of the cross-sectional differences in excess returns can be explained by exposure to this common factor. The evidence is consistent with intertemporal and state-dependent risk sharing.

Finally, we are not the first to study the impact of belief disagreement on the implied volatility of options. Buraschi and Jiltsov (2006) demonstrate in a single-tree Lucas economy with heterogeneous beliefs that belief disagreement increases the implied volatility smile of index options. This paper differs from their work in several dimensions. In an economy with several stocks and diversity of opinions, we provide a structural explanation for the difference in the volatility risk premia of index and single-stock options, the different cross-sectional slopes of the smile, the correlation risk premium embedded in index options, and the risk-return tradeoff of well-known option trading strategies. Empirically, we employ a more structural proxy for the common disagreement, computed as a dynamic common component out of the cross-section of individual differences in firm earning forecasts, and show that the model predictions are broadly consistent with the empirical evidence.

The rest of the paper is organized as follows. Section 3.1 provides the model setup. Section 3.2 derives the main theoretical model predictions. Section 3.3 describes our panel data set and Section 3.4 presents the results of our empirical study. Section 3.5 concludes.

3.1 The Economy with Uncertainty and Heterogeneous Beliefs

We borrow from the single-firm model in Dumas, Kurshev, and Uppal (2009) and extend it to a two-tree Lucas economy with heterogeneous beliefs, in which belief heterogeneity is related to dividend shocks of both firms and to independent news of aggregate economic growth. In this setting, we obtain a number of testable empirical predictions for (i) the differential pricing of index and individual options and (ii) the link with index and individual option volatility risk premia.

3.1.1 State Dynamics

Uncertainty is driven by a Brownian motion $W = (W_{D_i}, W_z, W_{\mu_{D_i}}, W_{\mu_z})'_{i=1,2}$ on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$. Two firms, indexed by $i = 1, 2$, produce their perishable good and pay a dividend process with dynamics $d \log D_i(t) = \mu_{D_i}(t)dt + \sigma_{D_i}dW_{D_i}(t)$, with $\sigma_{D_i} > 0$. The expected growth rates of dividends $\mu_{D_i}(t)$ are unobservable and follow the dynamics:

$$d\mu_{D_i}(t) = (a_{0D_i} + a_{1D_i}\mu_{D_i}(t)) dt + \sigma_{\mu_{D_i}}dW_{\mu_{D_i}}(t) .$$

By definition, parameter $\sigma_{\mu_{D_i}} > 0$ is a measure of economic uncertainty about firm i individual growth rate.

In reality, investors are exposed to many potential sources of news, which can be relevant to form their beliefs about future prospects of a firm. Some of this information can be more firm-specific, focusing, e.g., on individual firm characteristics, while some other news can have a more systematic character, producing information about the growth rate of the economy as a whole. Since the degree of common information used has important implications for the co-movement of beliefs about firms future prospects, we embed this feature into our model by a signal $dz(t)$ potentially related to the dividend dynamics of both firms:

$$\begin{aligned} dz(t) &= (\alpha_{D_1}\mu_{D_1}(t) + \alpha_{D_2}\mu_{D_2}(t) + \beta\mu_z(t)) dt + \sigma_zdW_z(t), \\ d\mu_z(t) &= (a_{0z} + a_{1z}\mu_z(t)) dt + \sigma_{\mu_z}dW_{\mu_z}(t), \end{aligned}$$

where $\sigma_z > 0$. By definition, $\sigma_{\mu_z} > 0$ measures the uncertainty linked with the expected value of signal component $dz(t)$.

If $\alpha_{D_1} = \alpha_{D_2} = 0$, dividends and signals are independent and investors use exclusively dividend information to build their beliefs. If $\beta = 0$ and $\alpha_{D_1} = \alpha_{D_2}$, $dz(t)$ is a signal related to aggregate dividend growth, which is optimally used by rational investors to forecast future dividends. Therefore, an heterogeneity in uncertainty parameter σ_{μ_z} between agents implies a diverse weight attributed by investors to news generated from aggregate dividend growth signals. Similarly, parameters β and α_{D_i} determine the strength of the relation between dividends and signals: As α_{D_i}/β increases, the signal is more informative for cash flows growth and agents use such information more and more to build their dividend forecasts.

Remark: When $\alpha_{D_1} = \alpha_{D_2}$, the relation between β and α_{D_i} can be interpreted as a parsimonious parametrization of rational inattention features, in which a lower ratio β/α_{D_i} parametrizes a higher degree of inattention. Rational inattention has been studied by a large literature in psychology and finance, and has been shown to be linked to agents' reaction to market-wide news. Peng and Xiong (2006) and Peng, Xiong, and Bollerslev (2007) show that after a macroeconomic shock, investors mainly focus on information about aggregate market indicators, while Gilbert, Kogan, Lochstoer, and Ozyildirim (2011) find that the market response of investors focusing on summary statistics can impact stock prices, volatility, and trading volume.

3.1.2 Specification of Disagreement

Two investors, indexed by $n = A, B$, update their beliefs following Bayes' rule and a standard Kalman-Bucy filter. To write the model dynamics in vector form, denote investor specific parameters by $b^n = \text{diag}(\sigma_{\mu_{D_1}}, \sigma_{\mu_{D_2}}, \sigma_{\mu_z}^n)$ and parameters that are common across investors by $a_0 = (a_{0D_1}, a_{0D_2}, a_{0z})'$, $a_1 = \text{diag}(a_{1D_1}, a_{1D_2}, a_{1z})$, $B = \text{diag}(\sigma_{D_1}, \sigma_{D_2}, \sigma_z)$, and

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \alpha_{D_1} & \alpha_{D_2} & \beta \end{pmatrix}.$$

$Y(t) := (\log D_1(t), \log D_2(t), z(t))'$ is the relevant state vector, \mathcal{F}_t^Y the information generated up to time t and $E^n(\cdot)$ the expectation operator under the subjective probability of investor n .

The posterior belief for state $\mu(t) := (\mu_{D_1}(t), \mu_{D_2}(t), \mu_z(t))'$ is $m^n(t) := E^n(\mu(t)|\mathcal{F}_t^Y)$. Its posterior covariance matrix solves a matrix Riccati differential equation, with solution dependent on the initial prior. We consider the steady state solution, which is independent on agent's prior. The posterior belief dynamics of agent A then follows as: $dm^A(t) = (a_0 + a_1 m^A(t))dt + \gamma^A A' B^{-1} dW_Y^A(t)$, with initial conditions $m^A(0) = m_0^A$, where $dW_Y^A(t)$ is the innovation process from the perspective of investor A and γ^A the corresponding steady state posterior covariance matrix.³

³A formal proof of this result can be found in Liptser and Shiryaev (2000)

In our model, disagreement can be summarized by the vector $\Psi(t) := (\Psi_{D_1}(t), \Psi_{D_2}(t), \Psi_z(t))' := B^{-1}(m^A(t) - m^B(t))$. The first two components of $\Psi(t)$ capture disagreement about dividend growth rates, the third one disagreement related to signal $dz(t)$. Using Itô's Lemma, the dynamics of $\Psi(t)$ are given by:

$$d\Psi(t) = B^{-1}(a_1 B + \gamma^B A' B^{-1}) \Psi(t) dt + B^{-1}(\gamma^A - \gamma^B) A' B^{-1} dW_Y^A(t), \quad (3.1)$$

where γ^B is investor's B steady state posterior covariance matrix. For interpretations purposes, we often refer to the case $\alpha_{D_1} = \alpha_{D_2}$, so that $dz(t)$ is effectively a signal of economy-wide dividend growth and $\Psi_z(t) \propto \frac{1}{\sigma_z}(\sigma_{D_1} \Psi_{D_1}(t) + \sigma_{D_2} \Psi_{D_2}(t))$. Even if $\Psi_{D_1}(t)$, $\Psi_{D_2}(t)$, $\Psi_z(t)$ are linearly related, signal news have economically distinct implications from dividend news: From equation (3.1), we see that dividend shocks load exclusively on the individual dynamics of $d\Psi_{D_i}(t)$. They therefore generate idiosyncratic dividend disagreement risk. In contrast, signal news load on both $d\Psi_{D_1}(t)$ and $d\Psi_{D_2}(t)$ and give rise to a common systematic risk factor in dividend disagreement. To this end, the covariance of the dividend disagreements, $Cov(d\Psi_{D_1}(t), d\Psi_{D_2}(t))$, directly depends on how signal shocks are assumed to affect the beliefs dynamics: Even if dividends are weakly dependent or independent, a correlation is generated whenever $dz(t)$ is perceived to produce information on aggregate dividend growth ($\alpha_{D_1}, \alpha_{D_2} \neq 0$). This feature further motivates the belief-driven channel for a commonality of stock returns in our economy.

3.1.3 Co-movement of the Diversity in Beliefs

A co-movement of $\Psi_{D_1}(t)$ and $\Psi_{D_2}(t)$ arises when signal shock $dz(t)$ is perceived as a useful predictor for dividend growth. Figure 3.1 presents the comparative statics of the model-implied correlation between $d\Psi_{D_1}$ and $d\Psi_{D_2}$ with respect to parameters α_{D_1} and α_{D_2} .⁴

According to intuition, $\rho(d\Psi_{D_1}(t), d\Psi_{D_2}(t))$ is an increasing function of $\alpha_{D_1}, \alpha_{D_2}$, which attains a maximum for $\alpha_{D_1} = \alpha_{D_2}$, i.e., when signal shocks $dz(t)$ imply large belief revisions about aggregate dividend growth. At the calibrated parameters, such excess co-movement can be large: For $\alpha_{D_1} = \alpha_{D_2} = 0.4$ it can be as high as 30%, depending on

⁴For simplicity, we consider a symmetric economy with identically distributed dividends across firms.

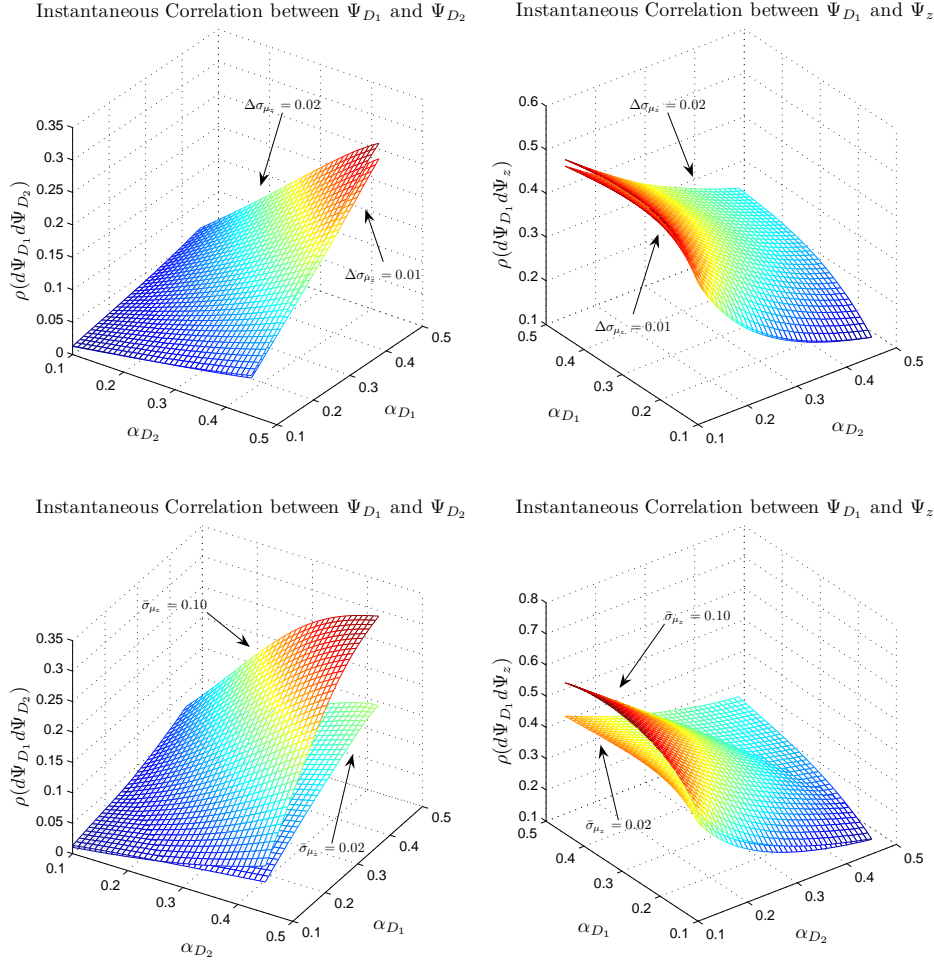


FIGURE 3.1: Uncertainty Correlation

The upper left panel plots the instantaneous correlation between the disagreement about firm 1, Ψ_{D_1} , and firm 2, Ψ_{D_2} , as a function of the weights α_{D_1} and α_{D_2} for different levels of difference in subjective uncertainty ($\sigma_{\mu_z^1} - \sigma_{\mu_z^2} \equiv \Delta\sigma_{\mu_z}$). The upper right panel plots the instantaneous correlation between the disagreement about firm 1, Ψ_{D_1} , and the signal, Ψ_z , as a function of the weights α_{D_1} and α_{D_2} for different levels of difference in subjective uncertainty ($\sigma_{\mu_z^1} - \sigma_{\mu_z^2} \equiv \Delta\sigma_{\mu_z}$). The lower panels plot the same correlations but for different levels of average economic uncertainty, ($\bar{\sigma}_{\mu_z}$) and $\Delta\sigma_{\mu_z}$ fixed to 0.01. The parameters chosen are summarized in Table 3.1.

parameters $\sigma_{\mu_z}^A, \sigma_{\mu_z}^B$. $\rho(d\Psi_{D_1}(t), d\Psi_{D_2}(t))$ is also linked to the structure of the individual uncertainty parameters $\sigma_{\mu_z}^n$ across agents: As either the average uncertainty $\bar{\sigma}_{\mu_z}$ or the uncertainty heterogeneity $\Delta\sigma_{\mu_z} \equiv \sigma_{\mu_z}^A - \sigma_{\mu_z}^B$ increase, the correlation increases, because signal shocks imply larger revisions of dividend beliefs for all agents.

Given that empirically rational inattention behavior appears to be more likely during crisis periods, when agents focus more on aggregate information, the model offers a

structural channel for the counter-cyclical co-movement of disagreement proxies across firms. If the average perceived uncertainty or the uncertainty heterogeneity across investors increase in crisis times, these counter-cyclical co-movement features are even stronger.

3.1.4 Equilibrium

We derive testable implications for the relation between belief disagreement and index and individual options volatility risk premia using a simple two-trees exchange economy with two consumption goods, in which investors maximize their life-time expected utility subject to the budget constraint:

$$V^n = \sup_{c_{D_1}^n, c_{D_2}^n} E^n \left(\int_0^\infty e^{-\delta t} \left(\frac{c_{D_1}^n(t)^{1-\gamma}}{1-\gamma} + \frac{c_{D_2}^n(t)^{1-\gamma}}{1-\gamma} \right) dt \mid \mathcal{F}_0^Y \right), \quad (3.2)$$

where $c_{D_i}^n(t)$ is the consumption of agent n of good i and $\gamma, \delta > 0$ are the common relative risk aversion and time preference parameters.

We shut down the distinction between risk aversion and elasticity of inter-temporal substitution using a time-separable utility, in order to simplify the computation of the equilibrium and the interpretation of some of the asset pricing implications of the heterogeneity in beliefs. By allowing each tree to produce its own fruit, we also introduce an imperfect substitution rate between goods. Representative-agent models studying multiple trees with a single fruit include, among others, Menzly, Santos, and Veronesi (2004), Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2011). In these economies, co-movement of returns can arise through direct market clearing effects, which are however weak in our setting because of the imperfect substitution between goods. In our model, return co-movement arises solely due to the equilibrium risk-sharing policies of investors with diverse beliefs.

To identify a unique equilibrium stochastic discount factor, we assume that agents can trade in a risk-free bond, in zero net supply, shares of the two firms, in positive supply, and equity and index options, all in zero net supply. The equilibrium is solved using standard martingale methods and produces closed-form expressions for the stochastic discount factor of agent n , denoted by $\xi^n(t)$, which is a function of a stochastic weighting

process $\lambda(t)$:⁵

$$\xi^A(t) = \frac{e^{-\delta t}}{y_A} D_1(t)^{-\gamma} \left(1 + \lambda(t)^{1/\gamma}\right)^\gamma, \quad \xi^B(t) = \frac{e^{-\delta t}}{y_B} D_1(t)^{-\gamma} \left(1 + \lambda(t)^{1/\gamma}\right)^\gamma \lambda(t)^{-1}, \quad (3.3)$$

where y_A and y_B are Lagrange multipliers in the (static) budget constraint of agent A and B , respectively, and $\lambda(t) := y_A \xi^A(t) / (y_B \xi^B(t))$ follows the dynamics:

$$\frac{d\lambda(t)}{\lambda(t)} = - \left(\sum_{i=1}^2 \Psi_{D_i}(t) dW_{D_i}^A(t) + \left(\sum_{i=1}^2 \alpha_{D_i} \Psi_{D_i}(t) \frac{\sigma_{D_i}}{\sigma_z} + \beta \Psi_z(t) \right) dW_z^A(t) \right). \quad (3.4)$$

The dynamics of $\lambda(t)$ depends on perceived shocks $W_{D_1}^A(t)$, $W_{D_2}^A(t)$ and $W_z^A(t)$, which load on $\lambda(t)$ proportionally to $\Psi_{D_1}(t)$, $\Psi_{D_2}(t)$, and $\Psi_z(t)$. The dependence of state prices on signal shocks $dW_z^A(t)$ increases when the relative precision of signals and dividends increases (σ_{D_i}/σ_z is large) or when the signal is more strongly related to expected dividend growth ($\alpha_{D_1}, \alpha_{D_2}$ increase).

State prices feature an endogenous stochastic volatility, which is increasing with respect to all disagreement proxies. Thus, option prices and volatility risk premia also directly depend on the belief disagreement. The intuition is simple. State prices reflect the desire of pessimistic investors to ensure a satisfactory consumption level in bad states. Since investors' marginal utility must equate in each state, the consumption share of the pessimistic investor is counter-cyclical and the protection seller (the optimist) must be compensated ex-ante by a premium, which is state dependent because it is a function of the degree of heterogeneity of beliefs.

Financial protection can be bought through individual stock or index put options. The first contract gives protection against a low dividend of a single firm. The second contract, against a low aggregate dividend. If dividends are perceived as more correlated, low aggregate consumption states are more likely and protection against bad aggregate dividend states becomes relatively more expensive. Thus, the protection premium embedded in out-of-the-money put index options gets larger than the one implied by individual put option prices. Through this mechanism, the model can create a wedge

⁵See Cox and Huang (1989). The extension to the case with heterogenous beliefs is due, among others, to Cuoco and He (1994), Karatzas and Shreve (1998), and Basak and Cuoco (1998). In this extension, the utility function of the representative agent is a weighted average of the utility functions of the individual agents. In contrast to the standard setting, a stochastic relative weight $\lambda(t)$ captures the equilibrium effect of the heterogeneity in beliefs across agents.

between the pricing of index and individual stock options, which can help to explain the observed difference in volatility risk premia of index and individual option markets.

3.1.5 Pricing

Given the equilibrium state price densities, we can price any contingent claim in our economy.⁶ State prices are functions of $D_1(t)$, $D_2(t)$ and $\lambda(t)$. Therefore, the joint density of these variables is needed for pricing purposes. This density is not analytically tractable, but its Laplace transform can be computed in closed form. Borrowing from the solution techniques in Dumas, Kurshev, and Uppal (2009), we can characterize this Laplace transform. Equilibrium stock and index prices are in semi-closed form, up to a numerical integration. This feature largely reduces the computational costs of the equilibrium. Option prices, which are derivatives written on equilibrium stock and index values, require a Monte Carlo simulation step in their computation.

3.2 Model Predictions

We can now study the impact of belief disagreement on the volatility risk premia of individual stocks and the index. To this end, we calibrate the model to the dividend dynamics of the S&P 500 and assume for simplicity of exposition a symmetric economy. The parameters are summarized in Table 3.1. Risk aversion is set to 2 and the dividend volatility of firm 1 and 2 is 4%. We study comparative statistics for changes in belief disagreement Ψ_{D_1} and Ψ_z between zero and 0.3.

3.2.1 The Link Between Stock Return Correlation and Common Disagreement

Assets returns in our economy can be correlated even if dividends are only weakly linked. In contrast to representative agent economies, such as Cochrane, Longstaff, and Santa-Clara (2008) and Martin (2011), market clearing effects are not the key driver of an endogenous stock co-movement. In our economy, the interdependence of market-wide informational effects with the optimal risk-sharing among agents produces the

⁶For convenience, we compute all pricing expressions from the perspective of agent A , using the first good as a numéraire.

TABLE 3.1: Choice of Parameter Values and Benchmark Values of State Variables

This table lists the parameter values used for all figures in the paper. We calibrate the model to the mean and volatility of the dividends on the S&P 500. The average growth rate for the period 1996-2006 is 5.93% and the volatility is 3.52%. The initial values for the conditional variances are set to their steady-state variances.

| Parameters for Fundamentals | | |
|---|----------------|-------|
| Long-term growth rate of dividend growth | a_{0D_i} | 0.01 |
| Mean-reversion parameter of dividend growth | a_{1D_i} | -0.01 |
| Volatility of dividend | σ_{D_i} | 0.07 |
| Initial level of dividend | D_i | 1.00 |
| Initial level of dividend growth | $m_{D_i}^A$ | 0.01 |
| Parameters for Signal | | |
| Long-term growth rate of signal | a_{0z} | 0.01 |
| Mean-reversion parameter of signal | a_{1z} | -0.03 |
| Volatility of signal | σ_z | 0.06 |
| Agent specific Parameters | | |
| Relative risk aversion for both agents | γ | 2.00 |
| Time Preference Parameter | ρ | 0.02 |

main channel for a co-movement of (i) belief disagreement across stocks and (ii) stock returns. The first channel is potentially completely belief-driven: Rational inattention behavior or a higher economic uncertainty produce a larger co-movement in beliefs, which increases the aggregate cash-flow risk perceived by investors. The second channel works through the endogenous risk-sharing across agents. In equilibrium, the more pessimistic agent consumes a higher fraction of aggregate consumption in bad consumption states. Therefore, she tries to reduce the exposure to stocks with a dividend process correlated with aggregate consumption. To reach market clearing, those stocks are sold to the more optimistic investor at a discount. When market-wide signals increase agents' perception of the degree of dividend correlation, the price adjustment needed for an ex-ante optimal risk-sharing across agents implies a higher discount for both stocks. This risk-sharing induced channel of co-movement is absent in Lucas economies with homogeneous agents.

We can now solve for the conditional covariance between stock 1 and 2. Figure 3.2 presents the comparative statics of the stock return correlation.⁷

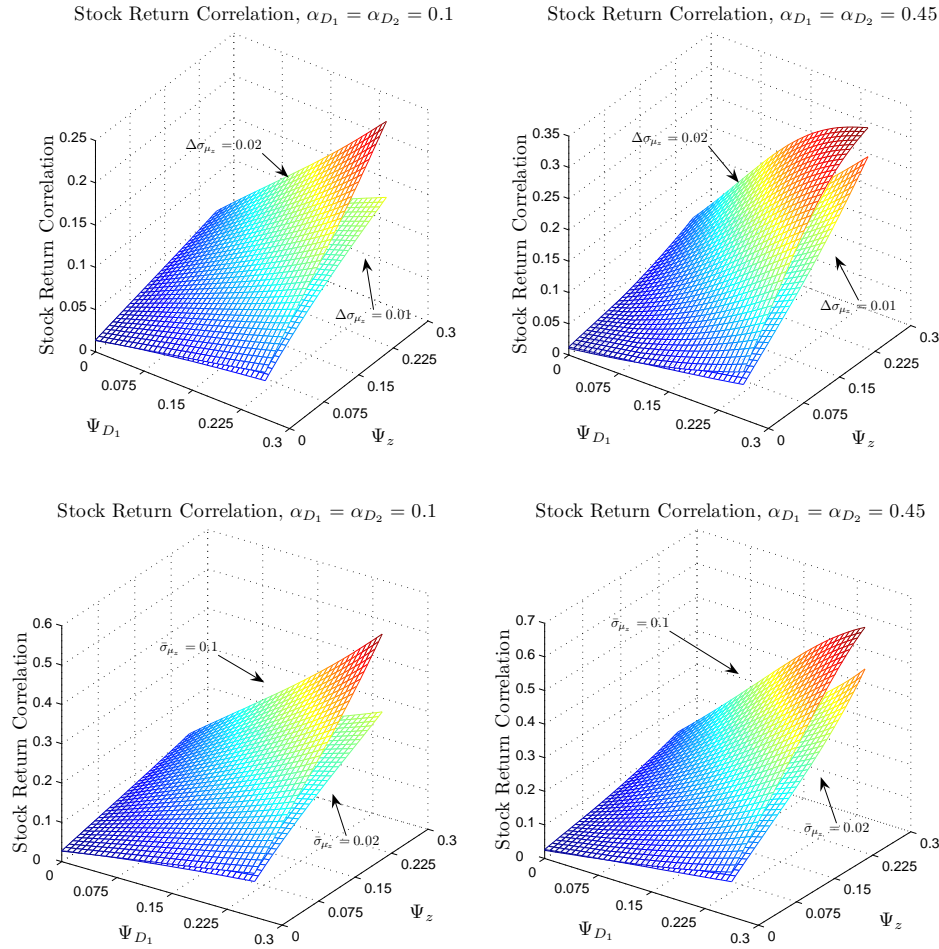


FIGURE 3.2: **Stock Return Correlation**

This figure plots the return correlation of stock 1 and stock 2 as a function of belief disagreement Ψ_{D_1} and Ψ_z . The parameters chosen are summarized in Table 3.1.

Consistent with intuition, the stock correlation is increasing in $\Psi_{D_1}(t)$ and $\Psi_z(t)$. In absence of disagreement, the correlation is small, confirming the minor role of market clearing-driven comovement effects in our economy. As expected, when (i) rational inattention behavior is stronger ($\alpha_{D_1}, \alpha_{D_2}$ increase) or (ii) market-wide uncertainty or uncertainty heterogeneity rise ($\bar{\sigma}_{\mu_z}$ or $\Delta\sigma_{\mu_z}$ increase), the stock correlation is higher.

⁷We set $\Psi_{D_2}(t) = 0$ for simplicity, which is a conservative assumption for the implied correlation level.

3.2.2 The Differential Pricing of Index and Individual Option Smiles

A second key feature of the model is that it allows for an endogenously steeper smile for index options than for individual stock options. Bakshi, Kapadia, and Madan (2003) link the differential pricing of index and individual options to their different risk-neutral skewness. In our model, risk-neutral skewness is generated by the asymmetrically increased equilibrium state price of low relative to high dividend states: Since the more optimistic agent sells financial protection at a premium, the price of low dividend states increases with the degree of investors' disagreement. Additivity of investors' utility function implies that asymmetric state price adjustments are amplified when pricing aggregate dividend states: The risk-neutral skewness of aggregate dividends is more negative than the one of individual firm dividends. As a consequence, index returns feature an even more negative risk-neutral skewness than individual stock returns. Moreover, when dividends are perceived as more correlated under agents' beliefs, the higher likelihood of aggregate low dividend states further increases the difference between index and single stock risk-neutral skewness. This endogenously generates a steeper smile for index options. Figure 3.3 illustrates this intuition.

First, we see that the difference of index and single stock negative risk neutral skewness is larger when $\Psi_{D_1}(t)$, $\Psi_z(t)$ increase, and it is more sensitive to market-wide than dividend disagreement shocks. Second, this difference is larger in presence of rational inattention (larger parameters $\alpha_{D_1}, \alpha_{D_2}$) and a higher average uncertainty or uncertainty heterogeneity (larger parameters $\bar{\sigma}_{\mu_z}$ or $\Delta\sigma_{\mu_z}$). The more pronounced negative risk-neutral skewness of index returns is directly mapped into a stronger skew of index option relative to individual option smiles. Figure 3.4 shows that, at the calibrated model parameters, index option smiles are steeper than individual stock smiles, as disagreement proxies $\Psi_{D_1}(t)$ or $\Psi_z(t)$ increase, where the difference in steepness is stronger either in presence of more pronounced rational inattention features or under a higher market-wide uncertainty. We summarize these relations between option-implied volatility smiles and proxies of investors' disagreement as follows.

Model prediction I: *A larger belief heterogeneity increases the (negative) difference of index and individual slopes of the option smile, where the difference is larger in presence of a larger common component in the cross section of individual disagreement proxies about firm cash-flows.*

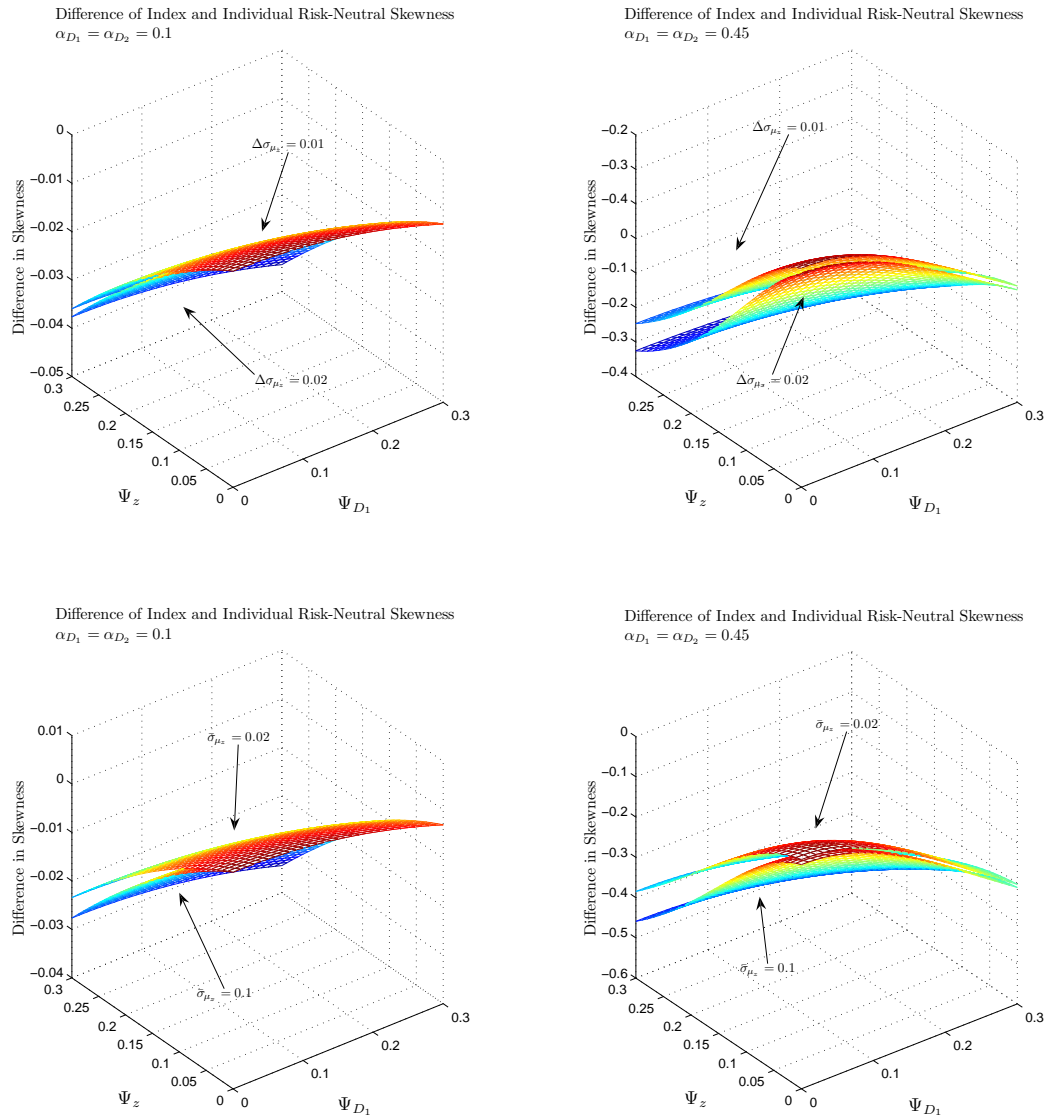


FIGURE 3.3: **Risk-Neutral Skewness for Individual Stock and Index Options**

These figures plot the difference in risk-neutral skewness of the returns of stock 1 (left panel) and the index (right panel). The parameters chosen are summarized in Table 3.1.

In our empirical analysis, we take model prediction I systematically to the data, by testing which fraction of the difference between index and individual risk-neutral skewness can be explained by common and individual disagreement proxies extracted from I/B/E/S data on professional earning forecasts.

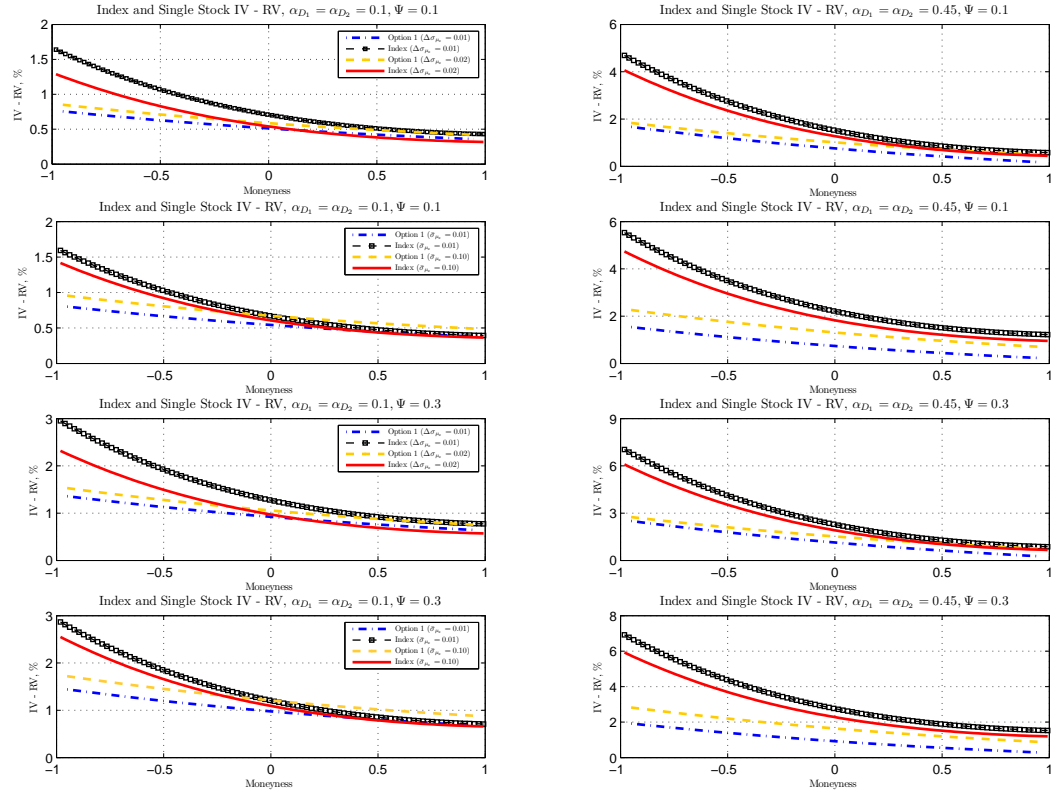


FIGURE 3.4: **Index and Individual IV - RV**

This figure plots the difference between the implied (IV) and realized (RV) volatility for the individual stock and the index, for different levels of disagreement, average and difference in subjective uncertainty. The volatility risk premium is defined as the difference between the implied volatility and the square root of the integrated variance $E_t^A \left(\int_t^T \sigma^2(s) ds \right)$. The parameters chosen are summarized in Table 3.1. Moneyness is defined as $\ln\left(\frac{S}{K}\right)$.

3.2.3 Variance and Correlation Risk Premia

The third key feature of the model is that correlation risk is priced in equilibrium. Figure 3.4 indicates (i) that the difference of expected and at-the-money option-implied variance is more negative for index than for individual options and (ii) that this pattern is more apparent as the heterogeneity in beliefs, rational inattention behavior or market-wide uncertainty increase. Is it well-known that the difference of expected and implied volatility is naturally linked to the volatility risk premium of the option's underlying

(see, e.g., Carr and Wu, 2009). Similarly, the more negative volatility risk premium of index options can be rationalized by a correlation risk premium embedded in the index constituents.⁸ In this way, our model motivates a potential tight link between commonality in dividend disagreement across firms, the more negative volatility risk premium of index options and the implied correlation risk premium of the index basket.

Remark: The natural way to measure the correlation risk premium is using correlation swaps. A swap buyer pays the implied correlation $SC_{t,T}$ at maturity T and receives the average realized correlation $RC_{t,T}$ in a basket of stocks, where $SC_{t,T} = E_t^{\mathbb{Q}}[RC_{t,T}]$, with \mathbb{Q} the relevant risk-neutral probability. The payoff of a (normalized) long correlation swap is $CR_{t,T} = RC_{t,T} - SC_{t,T}$ and its expected value is the correlation risk premium: $CRP_{t,T} = E_t^{\mathbb{P}}[RC_{t,T}] - E_t^{\mathbb{Q}}[RC_{t,T}]$.

Absence of arbitrage implies that the correlation risk premium is given by:

$$CRP_{t,T} = -Cov\left(\frac{\xi(T)}{\xi(t)}, RC_{t,T}\right), \quad (3.5)$$

where ζ is, without loss of generality, the risk-neutral density process of agent A in the economy: A negative correlation risk premium arises because large correlation states are more likely under the risk-neutral distribution, as disagreement increases.⁹ Observable proxies for $SC_{t,T}$ can be inferred from the cross-section of index and individual variance swaps. A long variance swap pays implied variance of a given asset at maturity T , i.e., the variance swap rate $SV_{t,T}$, and receives the average realized variance $RV_{t,T}$. The payoff of a (normalized) long variance swap is $VR_{t,T} = RV_{t,T} - E_t^{\mathbb{Q}}[RV_{t,T}]$ and its expected value measures the variance risk premium: $E_t^{\mathbb{P}}[RV_{t,T}] - SV_{t,T}$.¹⁰ We synthesize the implied correlation $IC_{t,T}$ using the relation: $IC_{t,T} \approx \frac{SV_{t,T}^I - \sum_{i=1}^m w_i^2 SV_{t,T}^i}{\sum_{i \neq j} w_i w_j \sqrt{SV_{t,T}^i SV_{t,T}^j}}$, where $SV_{t,T}^I$ and $SV_{t,T}^i$, $i = 1, \dots, n$ are index and individual variance swap rates, respectively, and w_i is the market capitalization of stock i at time t . Index and individual variance swap rates

⁸If one can only observe stock correlations (as opposed to the correlation of the differences in beliefs across stocks), the average stock return correlation will emerge as a risk factor explaining (in reduced-form regressions) the spread between the volatility risk premium of index and individual options. A related reduced-form argument is used by Driessen, Maenhout, and Vilkov (2009) to explain the difference of index and individual volatility risk premia by the presence of a large (negative) correlation risk premium.

⁹It is the equilibrium risk-sharing that increases stock correlations when disagreement among investors rises, because low aggregate consumption states require a larger equilibrium reallocation of financial protection.

¹⁰Volatility risk premia are simply defined as $VOLRP_{t,T} = \sqrt{E_t^{\mathbb{P}}[RV_{t,T}]} - \sqrt{SV_{t,T}}$, both for individual stocks and the index. Variance swaps are exposed to disagreement risk and pay a negative risk premium, because realized variances and disagreement are in a positive relation in our economy.

are synthesized from (listed) plain vanilla option prices.¹¹ Figure 3.5 summarizes the main predictions of our model for the relation between volatility risk premia, correlation risk premia and belief heterogeneity.

Individual volatility risk premia are larger (the correlation risk premium is larger), in absolute value, as disagreement, market-wide uncertainty, or rational inattention features are more pronounced; see the left (right) Panels of Figure 3.5. For instance, the correlation risk premium increases from nearly zero to about 1% ($\alpha_{D_i} = 0.1$) and 1.6% ($\alpha_{D_i} = 0.45$) as market-wide and individual firm disagreement rise: The largest increases are caused by shocks in the market-wide disagreement component, which tend to be linked more strongly to stock correlation shocks. In summary, we obtain the following model prediction.

Model prediction II: *A higher heterogeneity in beliefs implies a more negative difference between index and individual volatility risk premia (a more negative correlation risk premium). This feature is stronger under a larger co-movement in the heterogeneity of beliefs across firms.*

In the empirical part, we study model prediction II more systematically, by estimating the part of (i) the correlation risk premium and (ii) the difference in index and individual variance risk premia, which can be explained by empirical proxies of common and individual disagreement.

3.2.4 Simulated Option Trading Strategies

Since the model suggests the existence of a risk premium related to disagreement, it generates novel implications for expected excess returns on options strategies. We therefore investigate the extent to which the abnormal excess returns documented in previous empirical literature are explained by these risk premia. One of the most studied option (volatility) strategies is related to straddles, which involve a long position in a call and

¹¹Under the assumption of no arbitrage and a continuous swap rate process, the following relation is exact (see, e.g., Carr and Madan, 1998, Britten-Jones and Neuberger, 2000 and Carr and Wu, 2009):

$$SV_{t,T} = E_t^{\mathbb{Q}}(RV_{t,T}) = \frac{2}{(T-t)B(t,T)} \int_0^{\infty} \frac{P(K,T)}{K^2} dK, \quad (3.6)$$

where $B(t,T)$ is the price of a zero coupon bond with maturity T and $P(K,T)$ the price of an out-of-the-money put option with strike K and maturity T . We define the 30-days realized variance as: $RV_{t,t+30} = \frac{365}{30} \sum_{i=1}^{30} R_{t_n}^2$ where $t = t_0 < t_1 < \dots < t_N = T$ and $R_{t_n} = \log(S_{t_n}/S_{t_{n-1}})$.

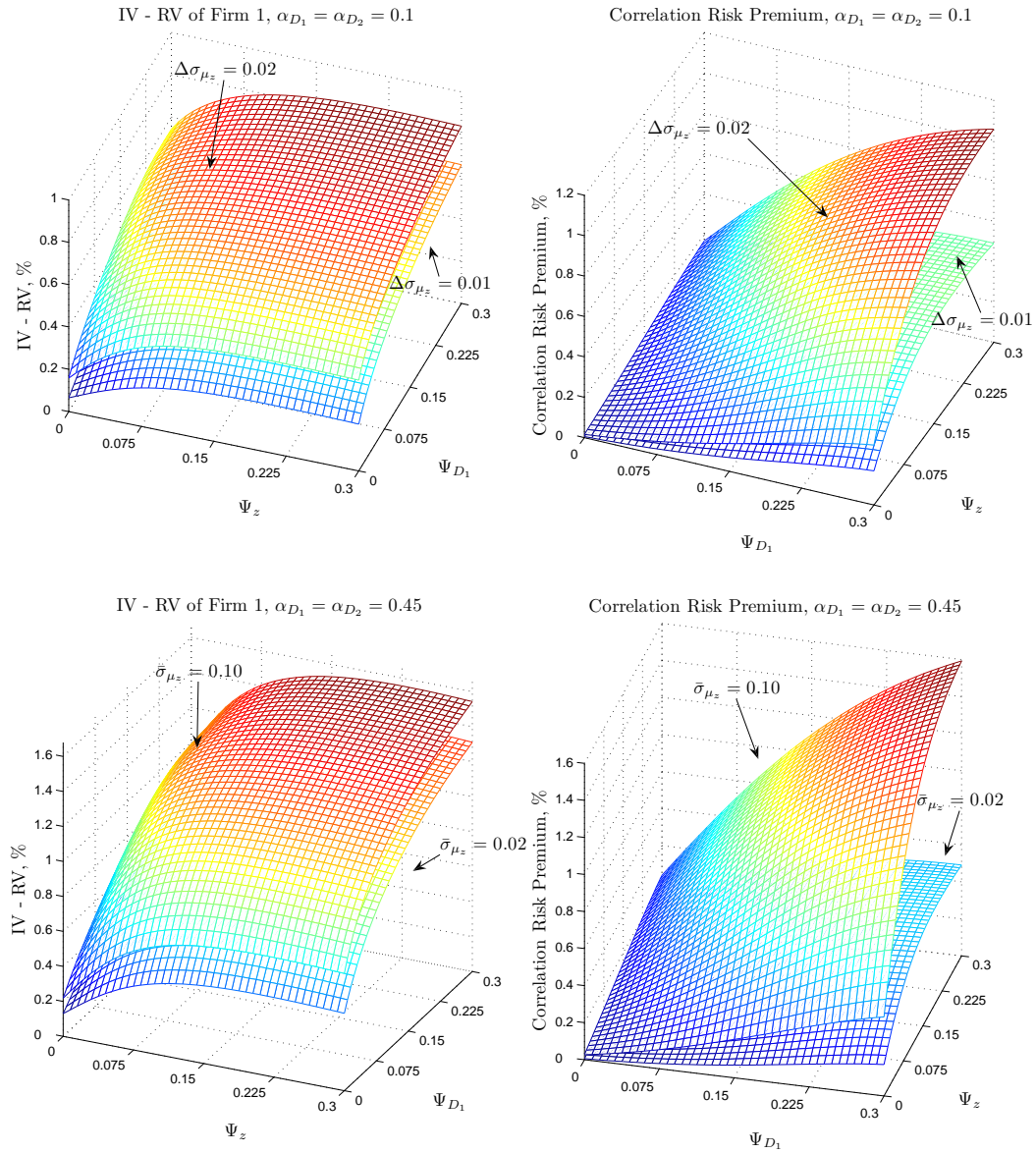


FIGURE 3.5: Volatility and Correlation Risk Premia

The left panels plot the volatility risk premium for firm 1 as a function of the disagreement about the growth rate of firm 1, Ψ_{D_1} , and the disagreement about the signal, Ψ_z . The volatility risk premium is calculated as the difference between the volatility swap rate and the 30 day realized volatility. The 30 day realized volatility is calculated from running 10,000 simulations and averaging. The right panel plot the correlation risk premium which are synthesized using correlation swaps as in equation (3.5). The parameters chosen are summarized in Table 3.1.

a put, with identical moneyness and underlying. Straddle portfolios have been analyzed extensively in the literature; see, among others, Coval and Shumway (2001) and Driessen

and Maenhout (2007). A straddle generates positive (negative) expected excess returns if the variance risk premium is positive (negative). A second well-known (correlation) option strategy is a dispersion portfolio, i.e., a combined position in (i) a short straddle of index options and (ii) a long straddle portfolio of individual stock options. The short index straddle earns on average the index volatility risk premium. The long individual straddles pay an average of the individual volatility risk premia. Since the index volatility risk premium is more negative than the average individual volatility risk premium, a dispersion portfolio has a positive expected excess return and earns the negative correlation premium embedded in short index straddles. Instead of purchasing the whole basket of constituent straddles, a dispersion strategy can be enhanced by selecting a subset of individual options with low implied volatility relative to the expected volatility. Since expected volatility is unobservable, correlation option traders often proxy it by the current realized volatility, because volatilities feature a substantial degree of autocorrelation.

Previous literature has often highlighted that it is difficult to reconcile the excess returns of these strategies with traditional risk factors (see Goyal and Saretto, 2009). Our model suggest a simple alternative explanation. The cheapest options are those of stocks with the lowest degree of dividend disagreement: When belief disagreement is large, the negative volatility risk premium of that particular firm tends to be large as well, because investors want to be compensated for holding disagreement risk. Therefore, if our model predictions are correct, dispersion portfolios sorted with respect to the degree of disagreement of stocks in the option straddles of their long side should yield sorted expected excess returns. Similarly, the spread between sorted dispersion portfolios with respect to high and low disagreement should yield positive expected excess returns. Finally, the excess returns of these option strategies should depend on risk factors related to (common) disagreement components. We verify these predictions within our calibrated model.

To be broadly consistent with the main features of our data, we simulate return time series of 11 years and 5 months (2,877 days). At the beginning of each month, at-the-money index straddles are sold and at-the-money straddles on one of the two individual stocks are bought, in a way that makes the portfolio vega neutral with respect to the individual stock volatility. To construct individual straddles and the sorted dispersion portfolios, we consider both the stock with the lowest and the highest dividend disagreement at the beginning of each month. The position is held for one month and is

dynamically delta hedged, by buying a corresponding amount of the individual stock and investing the remainder in the risk-free bond. We obtain simulated time series of 137 monthly returns of sorted dispersion trades. We also study the returns of a short index put strategy, which is known to generate high returns and Sharpe ratios in the data. This strategy is a natural benchmark for our sorted dispersion portfolios. Table 3.2 summarizes our findings.

Dispersion portfolios sorted with respect to the lowest stock disagreement yield the most attractive risk-return (mean-variance) profile. Their annualized Sharpe ratio is approximately 1.9 (1.65) in the economy with low (high) rational inattention parameters $\alpha_{D_1}, \alpha_{D_2}$: The economy with more pronounced rational inattention implies a slightly higher expected return, but also a clearly higher volatility, leading to the lower Sharpe ratio. All dispersion portfolios provide a larger Sharpe ratio than both short index put and sorted straddle portfolios of individual stocks. Compared to short index puts, they also feature a lower kurtosis and negative skewness, indicating a slightly lower tail risk. In contrast to dispersion strategies, sorted straddle returns feature no pronounced tail risk exposure and, in particular, a positive skewness. In our model, each contingent claim is priced at its fair price and excess returns arise because of exposure to systematic risk factors: Disagreement risk is priced and can motivate the emergence of non-zero excess returns in equilibrium. We study whether part of the excess return of the option strategies in Table 3.2 is linked to (common) disagreement risk exposure. We proxy common-disagreement risk by the weighted average of the dividend disagreements $\Psi_{D_1}(t)$ and $\Psi_{D_2}(t)$ of the two firms. We then perform regressions analysis of excess returns on risk factors, controlling for (i) the market return, (ii) the monthly change in the common disagreement proxy and (iii) a nonlinear common disagreement effect, modeled by the squared change in the common disagreement proxy. Estimated coefficients for factor regressions are summarized in the last four rows of Table 3.2.

We find that all sorted dispersion and straddle strategies have a zero market beta. The short index put is the only strategy with a positive market exposure. This is consistent with the well known (delta) exposure of short put returns to large markets drops (see Broadie, Chernov, and Johannes (2007)). All strategies have a positive beta with respect to the common disagreement factor. Strategies with the highest disagreement beta are, respectively, the dispersion portfolios sorted with respect to low and high disagreement, the short index put and the sorted portfolio of individual straddles. These betas are highest in the economy with more pronounced rational inattention behavior

TABLE 3.2: Moments of Simulated Option Strategies

This table reports summary statistics of the simulated strategy returns mean, standard deviation, skewness, kurtosis and Sharpe ratio. The DiB sorted straddle portfolio is long the stock with the lower disagreement and short the stock with the higher disagreement. The DiB sorted dispersion portfolio is formed by investing 100% of the wealth in shorting index straddles and investing a fraction of wealth into the options of the firm with either the lowest or highest belief disagreement such that the portfolio is vega neutral. The remainder is invested in the individual stock of this particular firm such that the portfolio is delta neutral. The index consists of two equally weighted stocks. We simulate 2,877 trading days and 1,000 simulation runs. All options are at-the-money with a maturity of 28 trading days. We run the following regression: $exret_i(t) = \alpha_i + \beta_i^M MRKT(t) + \beta_i^D \epsilon^D(t) + \beta_i^{(\epsilon^D)^2} (\epsilon^D(t))^2 + u_i(t)$, where $exret_i(t)$ is the return in excess of the one month risk-free rate, $MRKT(t)$ is the index excess return, $\epsilon^D(t)$ is the monthly change in the common disagreement factor, and $(\epsilon^D(t))^2$ is the monthly change in the common disagreement squared. The market excess return is defined as the return on the index (i.e., the equal weighted sum of both stocks) and the common disagreement is a weighted average of the firm-specific disagreement proxies.

| | Less rational inattention: $\alpha_{D_i} = 0.1$ | | | | More rational inattention: $\alpha_{D_i} = 0.45$ | | | | INDEX |
|------------------|---|-------------------|-------------------|-------------------|--|-------------------|-------------------|-------------------|--------|
| | DiB STRDL | Low DiB DISP | HIGH DiB DISP | SHORT PUT | DiB STRDL | Low DiB DISP | HIGH DiB DISP | SHORT PUT | |
| Mean | 0.081 | 0.127 | 0.087 | 0.117 | 0.079 | 0.137 | 0.098 | 0.114 | 0.010 |
| StDev | 0.273 | 0.231 | 0.242 | 0.653 | 0.269 | 0.288 | 0.250 | 0.598 | 0.038 |
| Kurtosis | 6.938 | 7.532 | 7.146 | 8.477 | 5.487 | 7.814 | 7.417 | 7.879 | 5.819 |
| Skewness | 1.102 | -3.127 | -4.154 | -8.239 | 1.144 | -3.130 | -4.014 | -8.743 | -7.423 |
| Sharpe Ratio | 1.03 | 1.90 | 1.25 | 0.62 | 1.02 | 1.65 | 1.36 | 0.66 | 0.91 |
| Alpha | 0.081 (1.77) | 0.079 (1.89) | 0.065 (1.87) | 0.091 (2.02) | 0.071 (1.71) | 0.074 (2.04) | 0.061 (1.87) | 0.094 (1.98) | |
| MRKT | 1.942 (1.25) | -0.975 (-0.99) | -0.889 (-1.04) | 4.290 (2.00) | 1.647 (1.04) | -0.984 (-1.04) | -0.841 (-1.46) | 3.478 (2.40) | |
| ϵ^D | 2.784 (2.67) | 4.105 (3.04) | 3.475 (2.48) | 3.144 (2.07) | 2.414 (2.55) | 5.017 (3.05) | 4.071 (3.12) | 3.020 (2.20) | |
| $(\epsilon^D)^2$ | -0.081 (-1.88) | -0.104 (-1.61) | -0.214 (-1.55) | -0.115 (-1.64) | -0.150 (-1.70) | -0.142 (-1.72) | -0.211 (-1.67) | -0.514 (-1.44) | |

($\alpha_{D_1} = \alpha_{D_2} = 0.45$). We find weak evidence of a potentially negative dependence on squared common disagreement shocks, highlighting a potential nonlinear dependence of option expected returns on disagreement risk. In summary, these findings give rise to the following model prediction.

Model prediction III: *The returns of sorted straddle and dispersion portfolios feature a zero beta to market risk, but a positive beta with respect to the common proxy of stock disagreement.*

Empirically, we can test model prediction III by studying whether the excess returns of straddle and dispersion portfolios feature a systematic exposure to empirical proxies of common disagreement in the time series or in the cross-section.

3.3 Data

In order to test empirically the main predictions of our model, we build a monthly merged panel data set, consisting of option and stock prices on all constituents of the S&P 100 index, the time-series of index spot and option prices, and analysts' forecasts of future earnings from the I/B/E/S database. The sample period of our study goes from January 1996 to June 2007.

Options Data: We use option information from the OptionMetrics Ivy DB database. Our option sample contains trades and quotes of S&P 100 index options and all individual options on all constituents of the S&P 100, traded on the Chicago Board Options Exchange (CBOE). The S&P 100 is a capitalization-weighted index with quarterly rebalancing. Options on the index are European style and expire on the third Friday of the contract month. Individual stock options are American style and usually expire on the Saturday following the third Friday of the contract month. We apply a number of data filters to our option data, which are explained in a supplemental data Appendix.

Stock Returns Data: In order to calculate the realized volatility of the index and individual stocks, we use daily returns from CRSP for single-stocks and from OptionMetrics for the index. We calculate the realized volatility over 21-day windows, requiring that the stock has at least 15 non-zero return observations within each window.

Uncertainty and Differences in Beliefs Proxies: We use standard measures of belief disagreement using forecast data from the Institutional Brokers Estimate System (I/B/E/S) database (see e.g. Yu, 2011). For each firm i in each month t , we calculate the cross-sectional standard deviation and consensus estimate of analyst forecasts of earnings per share from the unadjusted I/B/E/S detail database. Firm-specific disagreement is then defined as the ratio between the cross-sectional standard deviation and the consensus estimate. Finally, common disagreement is defined as the market capitalization weighted average of all firms. Market capitalization data is taken from the COMPUSTAT database.¹²

¹²An alternative to use market capitalization would be to use an equally weighted average. We note however that the unconditional correlation is almost 1.

While in our regressions we use the level of disagreement, we use innovations in common disagreement in our portfolio sorting exercise. Innovations in common disagreement are constructed as first differences.¹³

Other Control Variables: It is well-known that cross-sectional differences in option implied volatilities are related to firm-specific CAPM Betas (see e.g. Chang, Christoffersen, Jacobs, and Vainberg, 2011). In order to control for such effects, we compute monthly conditional betas, using historical returns over a window of 180 days, and include them as explanatory variables in our regressions.

We control for liquidity effects using two different proxies. To control for option liquidity effects, we calculate the ratio of option trading volume and the number of shares in the underlying outstanding. In order to account for equity market liquidity effects, we use the Pástor and Stambaugh (2003) measure of liquidity.

In volatility risk premium regressions, we control for jump risk by explicitly proxying for differences in risk neutral skewness across stocks. As a proxy for risk neutral skewness, we use the difference between the implied volatility of a put with 0.92 strike-to-spot ratio (or the closest available) and the implied volatility of an at-the-money put, divided by the absolute difference in strike-to-spot ratios.

An alternative explanation for the emergence of volatility risk premia of single-stock and index options is the demand-based argument in Bollen and Whaley (2004). We therefore consider buying pressure as an additional explanatory variable in our regressions. Following Bollen and Whaley (2004), we define net buying pressure as the number of contracts traded during the month at prices higher than the prevailing bid/ask quote midpoint, minus the number of contracts traded during the month at prices below the prevailing bid/ask quote midpoint, multiplied by the absolute value of the option's delta and scaled by the total trading volume across all option series.

An additional alternative hypothesis for time-varying endogenous correlation is suggested by Shiller (1984) and Shleifer and Summers (1990). They argue that the dynamic interaction of noise traders and rational arbitrageurs can lead to price formation processes in which arbitrage activity may not fully absorb correlated demand shocks. In this context, investors' sentiment may induce endogenous (realized) return correlation.

¹³As a robustness check, we also considered a model with shocks in the common disagreement proxy defined by the residuals of a linear AR(2) dynamics. Results do not change quantitatively.

We control for this effect and quantify the different roles played by differences in beliefs versus investors sentiment. To this end, we follow Baker and Wurgler (2007) and proxy sentiment by the first principal component of trading volume (as measured by NYSE turnover), the dividend premium, the closed-end fund discount, the number and first-day returns on IPOs and the equity share in new issues; see Baker and Wurgler (2007).¹⁴

Finally, we control for other standard risk factors in the literature using the two Fama and French (1993) and the momentum factors. Business-cycle effects are captured by a number of other variables, including the market price-earning ratio, industrial production, housing start number, the producer price index, and non-farm employment.¹⁵ Our macro proxy is then the first principal component estimated from these variables.

3.4 Empirical Analysis

We test empirically the main predictions of our model and start by focusing on an initial set of predictions using panel regression methods. First, we study the extent to which the individual and common belief disagreement components help to explain (i) the difference between index and individual stock volatility risk premia and (ii) the difference in the slope of the smile of index and individual stock options (risk neutral skewness). Second, we investigate whether the common disagreement component helps to explain correlation risk premia.

A second set of predictions is related to the fact that belief disagreement is priced. We tackle this question both in the time-series and the cross-section: In the time series, we estimate the beta of excess returns of option-based trading strategies (dispersion and straddle portfolios) with respect to unexpected variations in common disagreement. Then, in the cross-section, we sort the returns of these portfolios with respect to their beta on differences in beliefs and investigate the extent to which cross-sectional differences in risk adjusted returns are explained by their exposure to disagreement. Finally, we quantify the price of disagreement risk in option returns using a Fama-MacBeth (1973) approach.

¹⁴These data are available on [Jeffrey Wurgler's webpage](#).

¹⁵Since the price-earning ratio for the S&P 100 does not exist, we use price-earning data from the S&P 500. We retrieve S&P 500 price-earnings data from the S&P webpage, while all other macro variables from [FRED](#).

3.4.1 Disagreement and Variance Risk Premium

Our model predicts that the difference between index and individual volatility risk premia is increasing in the degree of heterogeneity in beliefs. We denote by $IV_i(t) - RV_i(t+1) \equiv VRP_i(t)$ the difference between the implied and realized volatility, at time t and $t+1$, respectively, for firm i .¹⁶ Similarly, $IV(t) - RV(t+1) \equiv VRP(t)$ is the corresponding difference for the index. Note that $E_t(RV(t+1) - IV(t))$ and $E_t(RV_i(t+1) - IV_i(t))$ are proxies for the volatility risk premium on index and individual stock stock, respectively. Therefore, a regression with $VRP(t) - VRP_i(t)$ as an endogenous variable should imply a positive regression parameter with respect to individual and common proxies of heterogeneity in beliefs.¹⁷ We consider the following panel regression model:

$$\begin{aligned}
 VRP(t) - VRP_i(t) = & \beta_0 + \beta_{i1}DIB_i(t) + \beta_2\overline{DIB}(t) + \sum_{j=3}^5 \beta_{ij}\text{Control}_{ij}(t) \\
 & + \sum_{k=1}^3 \gamma_k \text{Control}_k(t) + \epsilon_i(t),
 \end{aligned} \tag{3.7}$$

where $DIB_i(t)$ is the proxy of belief disagreement of each individual firm i at time t , $\overline{DIB}(t)$ is the common disagreement proxy at time t , $\text{Control}_i(t)$ is a vector of control variables for firm i at time t , and $\text{Control}(t)$ is a vector of time-series determinants at time t , such as market volatility, the macro factor, and the sentiment index. Table 3.3 (first three columns) summarizes the results.

Consistent with the model predictions, parameter estimates in column 1 of Table 3.3 for individual and common disagreement proxies are positive and significant, both economically and statistically. For instance, a one standard deviation change in the individual DiB proxy implies almost a 3/4 standard deviation change in the difference between the index and individual volatility risk premia. In order to control for potential biases due to spurious time-series correlation, we also add time dummies to the baseline regression (column 2 in Table 3.3) and find that they have a negligible effect on the estimated coefficients for the individual disagreement proxy. The estimated coefficient for the common disagreement proxy drops by approximately 20% and remains significant at the 10% level. This is consistent with the fact that the common disagreement proxy picks up

¹⁶To simplify notation, we denote $IV_i(t, t+1)$ by $IV_i(t)$ and $RV_i(t, t+1)$ by $RV_i(t+1)$.

¹⁷In a supplemental Appendix, we present also panel regression results using as endogenous variable the difference of $[IV(t) - RV(t)]$ and $[IV_i(t) - RV_i(t)]$. Because of the high autocorrelation of the monthly realized volatility, $[RV(t) - IV(t, t+1)]$ and $[RV_i(t) - IV_i(t)]$ are natural alternative proxies of index and individual volatility risk premia. However, the results for the impact of disagreement on the cross-sectional differences in index and individual volatility risk premia do not change.

time variation effects only. Similar findings apply to the other time-series variables, like market volatility or the macro factor. To learn more about the degree of cross-sectional variation explained, we finally run a fixed effects panel regression (column 3 in Table 3.3) and find that the coefficient estimate of the individual DiB proxy is virtually unchanged and still significant. This finding indicates that individual DiB likely captures a time-varying cross sectional heterogeneity in the difference of index and individual volatility risk premia, rather than a pure time-independent cross sectional effect.

3.4.1.1 Disagreement and the Skew of Index and Individual Options

Our model also predicts that cross sectional differences in the slope of the smile of index and individual options are explained by the heterogeneity in beliefs: A larger degree of disagreement is linked *coeteris paribus* to a more negative difference between the slopes of index and individual option smiles. We compute a standardized measure of slope of the smile, both for index and individual options, as the difference between the implied volatility of a put with 0.92 strike-to-spot ratio (or the closest available) and the implied volatility of an at-the-money put, divided by the absolute difference in strike-to-spot ratios. According to this metric, a larger negative skewness is equivalent to a larger positive slope of the smile. Therefore, a larger disagreement is expected to increase the difference between the index and individual proxies for the slope of the smile in the following panel regressions:

$$\underbrace{Slope(t)}_{\text{Slope Index}} - \underbrace{Slope_i(t)}_{\text{Slope Individual}} = \beta_0 + \beta_{i1}DIB_i(t) + \beta_2\overline{DIB}(t) + \sum_{j=3}^5 \beta_{ij}\text{Control}_{ij}(t) + \sum_{k=1}^3 \gamma_k \text{Control}_k(t) + \epsilon_i(t), \quad (3.8)$$

where $Slope(t) - Slope_i(t)$ is the difference between index and individual slopes. Further explanatory and control variables are identical to those in model (3.7) with, in addition, the index and individual at-the-money implied volatilities, which control for the direct impact of the volatility level on the skew of the smile. Table 3.3 (columns 4 to 6) reports the results.

We find a positive relation between disagreement and the difference of index and individual option slopes (column 4 of Table 3.3). Individual and common DiB proxies are both economically and statistically significant. For instance, a one standard deviation change in the individual DiB proxies implies about a half standard deviation change in the difference between index and individual option slopes. Regressions including time and

fixed effects (column 5 and 6 of Table 3.3) show that the coefficient estimate of individual DiB is not driven by spurious time-series correlations or by simple time-independent cross sectional features.

3.4.1.2 Disagreement and Correlation Risk Premium

Our model predicts that the correlation risk premium embedded in the cross-section of index and individual options is explained by the common component in the heterogeneity of beliefs. We denote by $IC(t) - RC(t + 1)$ the difference between the option implied and the average realized correlation at time t and $t + 1$, respectively.¹⁸ According to the model predictions, a linear regression model:

$$IC(t) - RC(t + 1) = \beta_0 + \beta_1 \overline{DIB}(t) + \sum_{k=2}^5 \beta_k \text{Control}_k(t) + \epsilon(t) , \quad (3.9)$$

should imply a positive parameter for the common disagreement proxy. Table 3.4 summarizes the results.

We find that common disagreement components have a significant positive impact on the (negative) correlation risk premium (column 1 of Table 3.4). Common disagreement is the most significant variable, also after controlling for demand pressure (column 2 of table 3.4) and sentiment (column 3 of Table 3.4) proxies, which are natural control variables, because (i) buying pressure is known to be related to the implied volatility of index versus individual options (Bollen and Whaley, 2004 and Gârleanu, Pedersen, and Poteshman, 2009) and (ii) sentiment is a known systematic factor driving correlation shocks. While we find that demand pressure has little impact on correlation risk premia, we find that sentiment is positively related to the negative correlation risk premium.

3.4.2 Option Strategy Returns

We now take the previous results a step further by studying the excess returns of volatility and correlation strategies. If systematic disagreement risk is priced and helps to explain volatility (correlation) then Low-minus-High straddle (dispersion) portfolios sorted with respect to difference in beliefs will generate positive excess returns. This should

¹⁸To simplify notation, we denote $IC(t, t + 1)$ by $IC(t)$ and $RC(t, t + 1)$ by $RC(t + 1)$.

occur both in the time-series and in the cross-section. The properties of these excess returns are the focus of the following analysis.

3.4.2.1 Disagreement and the Time Series of Straddle Portfolio Returns

Goyal and Saretto (2009) study option trading strategies that take advantage of cross-sectional differences in stock volatility: They show that portfolios long (short) the options with the largest (smallest) difference between realized and implied volatility yield economically significant returns, with no apparent exposure to standard systematic risk factors. Our model suggests a novel potential explanation for these findings: If the common factor driving stock disagreement is correlated with states of active risk sharing among agents, straddle returns might reflect a compensation for bearing systematic disagreement risk.

To test empirically this hypothesis, we run a similar exercise as in Goyal and Saretto (2009) and construct quintile sorted straddle portfolios, in dependence of the level of individual disagreement: Each month, quintile 1 (5) consists of firms with the lowest (highest) disagreement proxy. For each stock in each quintile, we compute individual straddle portfolio returns. Finally, we average the straddle returns within each quintile and obtain equally weighted straddle portfolio returns sorted with respect to individual stock disagreement.¹⁹ Then, we regress the corresponding excess returns on our common disagreement proxy, $\epsilon^D(t)$, after controlling for traditional risk factors:

$$\begin{aligned} exret_i(t) = & \alpha_i + \beta_i^M MRKT(t) + \beta_i^S SMB(t) + \beta_i^{BM} HML(t) + \beta_i^{MOM} MOM(t) \\ & + \beta_i^V \epsilon^V(t) + \beta_i^L \epsilon^L(t) + \beta_i^S \epsilon^S(t) + \beta_i^C \epsilon^C(t) + \beta_i^D \epsilon^D(t) + u_i(t), \end{aligned} \quad (3.10)$$

where $exret_i(t)$ is the monthly excess return of quintile straddle portfolio i , $i = 1, 5, 1-5$, $MRKT(t)$ is the market excess return, and $SMB(t)$, $HML(t)$ and $MOM(t)$ are the size, book-to-market and momentum factors, respectively. Table 3.5 summarizes the results of straddle returns in quintiles 1 (low) and 5 (high), together with those of a spread straddle portfolio (LmH), consisting of a long position in quintile 1 straddles and a short position in quintile 5 straddles.

¹⁹In a supplemental Appendix, we repeat this procedure using as sorting criterion the difference of realized and implied volatility, as in Goyal and Saretto (2009), and obtain very similar results.

First, we find that the straddle average excess return decreases from 7.2% for quintile 1 to -8.7% for quintile 5, implying a return of 15.9% and a Sharpe ratio of about 1.32 for the spread straddle portfolio. Second, none of the traditional risk factors is significant at standard confidence levels (see the third column of Table 3.5). The most significant explanatory variable for the dynamics of spread straddle returns is $\epsilon^D(t)$, the common disagreement proxy, which has a t-statistics equal to 1.98 and a positive slope coefficient. Third, we consider the impact of additional proxies of potentially relevant systematic risks that might affect straddle returns. We use monthly changes in the VIX index $\epsilon^V(t)$ (realized stock correlation $\epsilon^C(t)$) to capture potential time-varying volatility (correlation) risk effects, while we use monthly changes $\epsilon^L(t)$ and $\epsilon^S(t)$ in liquidity and sentiment proxies to capture unexpected shocks in overall liquidity and sentiment conditions. As the third column of Table 3.5 shows, these risk factors are also largely insignificant in explaining the spread straddle portfolio returns. The only exception is the slope coefficient on realized correlation which is significant at the 10% probability level.

This empirical evidence is consistent with the main predictions of the calibrated model-implied straddle returns in Table 3.2. Overall, we cannot reject the hypothesis that spread straddle returns contain a systematic return component that is positively linked to unexpected changes in systematic disagreement risk.

3.4.2.2 Disagreement and the Time Series of Dispersion Portfolio Returns

By construction, dispersion strategies generate a risk exposure to changes in correlation. Our model predicts an economic link between dispersion returns and their exposure to common disagreement shocks, which we address in this section. Our empirical tests start from the simple practical consideration that trading a whole book of constituent volatilities can either generate unattractive transaction costs or be difficult due to illiquidity issues: It is more appropriate to trade a sub basket of individual stock volatilities, by means of so-called proxy hedging.²⁰ We adopt a number of different criteria to sort into quintiles the individual options for the long position in the dispersion portfolio. First, we sort based on the degree of individual firm disagreement (DiB sorting). Second, we

²⁰The art of such a trade is to find criteria to select the cheapest options to form the basket. A successful choice of cheap options allows the dispersion trader to partly disentangle excess return components for average correlation risk from those for average individual volatility risk, while at the same time producing attractive excess returns.

sort according to the CAPM beta (Beta sorting) of each underlying stock, since the results in Table 3.3 indicate a positive link between CAPM beta and individual volatility risk premia. Third, we sort based on the degree of illiquidity of the underlying stock (Liquidity sorting), since more liquid stocks are likely associated with cheaper options. Fourth, we sort based on the difference between implied and realized volatility (VRP sorting), as suggested in Goyal and Saretto (2009). Finally, we select the stocks in each bottom quintile to build the long side of the dispersion portfolio in our empirical tests. For comparison purposes, for the DiB sorting, we present the evidence for two cases: Dispersion returns for a portfolio with a long side consisting of options of stocks in the top and bottom quintile.²¹ Table 3.5 (columns 4 and 5) summarizes the properties of dispersion returns.

We find that the DiB sorted low quintile dispersion portfolio features the most attractive average return (16.54%) and unconditional Sharpe ratio (1.51). Estimated regressions of factor model (3.10) for dispersion returns highlight additional interesting features. First, dispersion portfolios have no apparent exposure to standard risk factors, such as the market return or the size, book-to-market and momentum factors, and to liquidity shocks. Second, they feature a slightly significant negative (positive) exposure to aggregate implied volatility (sentiment) shocks. Third, the most significant positive (negative) exposure arises with respect to shocks in common disagreement components (average realized stock correlation). Fourth, the economic impact of common disagreement shocks on the average return of a dispersion portfolio is economically large: For instance, a one standard deviation shock implies an increase of the average return of DiB sorted dispersion portfolios of about 3.5%. Consistently with the model predictions, we conclude that (i) dispersion excess returns are not easily explained by traditional economic sources of risk and that (ii) they feature a distinct and economically important exposure to common disagreement shocks.

²¹We fix the portfolio weights of the individual options by hedging away individual volatility risks using a vega-neutral position. We then determine delta-neutral portfolio weights for the individual stocks and invest the residual wealth in the risk-free asset. Thus, each dispersion portfolio shorts an (index) straddle and invests the residual wealth in (individual) straddles, individual stocks and the risk-free bond. We circumvent microstructure biases and initiate all option strategies on Tuesdays, as opposed to the first trading day (Monday). Returns are computed using as a reference initial price the average of the closing bid and ask quotes on the previous day. As a reference final price for all options in the portfolio, we compute the realized value of the terminal payoff at the option's expiration date. After expiration of each option, a new option is selected and a new monthly return is calculated following the above procedure. We use equally weighted monthly returns on calls and puts. Such a procedure is repeated for each month.

3.4.2.3 Double Sorted Dispersion Portfolios

According to the empirical evidence in Table 3.5, different exposures to common disagreement shocks in the time series might be linked to systematically different excess returns in the cross-section of dispersion returns. For instance, dispersion returns for the bottom DiB quintile have both a larger exposure to disagreement shocks and a higher excess return than dispersion returns for the top DiB quintile. Are such cross-sectional differences significant, after controlling for other common risk factors, such as, e.g., liquidity risk? We address this question with a simple nonparametric approach: In each month, we construct double sorted straddle portfolios for the long side of the dispersion trades. The first sort is based on the degree of exposure to common disagreement risk. The second sort is with respect to the degree of exposure to some of the other systematic risk factors in regression model (3.10). In order to keep the double sorted portfolios sufficiently well populated, we double sort options into terciles, rather than quintiles, along each sorting dimension.

Each column in Panel A of Table 3.6 reports aggregated average returns (and related t -statistics) for double sorted dispersion portfolios: Average returns in each row of Panel A are obtained by aggregating the corresponding double sorted portfolio returns along the Market, Size, Book-to-Market, Momentum, Liquidity and Volatility risk premium sorting dimension.

First, we find that differences in dispersion returns between bottom and top terciles of common DiB exposure are significant and economically relevant, irrespective of the second sorting criterion: Differences in returns range from about 1.9% to 5.6%. Second, while in each tercile of common DiB exposure there is some return variability along terciles of the second sorting dimension, this variability is typically lower than the one observed along the DiB exposure dimension. Panel B and C of Table 3.6 summarize these findings for the double sorted portfolios with respect to the market (Panel B) and liquidity (Panel C) risk exposure: While the average return of the high-minus-low portfolio with respect to common DiB exposure is 5.4% (4.3%) in the double sorting with respect to market (liquidity) exposure, the aggregate high-minus-low portfolio with respect to market (liquidity) exposure has an average return of only 1.3% (1.4%).²² Overall, this

²²Intuitively, the very low return differences for the market sorting reflect the fact that dispersion trades try to achieve a market neutral position by delta hedging market risk.

analysis shows that an economically relevant fraction of cross sectional variation in dispersion returns is driven by a conditional exposure to systematic disagreement risk.

3.4.2.4 The Economic Relevance of Disagreement Risk

Section IV.3.4.2.3 and Table 3.5 document an increasing cross-sectional pattern in the average return of quintile dispersion portfolios, which is unrelated to cross-sectional differences in standard risk factor exposures and positively linked to an increasing beta with respect to common disagreement shocks. A natural question is whether the cross-section of dispersion returns is linked to other nonlinear sources of systematic risk, e.g., downside risk, which could provide a rationale for the different cross-sectional dispersion returns as a compensation for hidden downside risk. A potentially important downside risk component in dispersion returns is motivated, e.g., by the nonlinear structure of option straddle payoffs and the pronounced negative skewness of dispersion returns in the data.

We investigate the relation between disagreement risk and downside risk in dispersion returns, borrowing from the Koenker and Basset (1978) quantile regression approach, and specify each conditional quantile of dispersion returns as a linear function of the relevant risk factors.²³ This methodology allows us to estimate the potentially different effect of systematic disagreement components on distinct quantiles of the conditional distribution of dispersion returns. Table 3.7 presents estimated quantile regression coefficients, together with the corresponding lower and upper 10% and 90% confidence bounds.

For brevity, we focus on regression estimates for the 0.25, the 0.5, and the 0.75 quantiles and present results only for regressors that are significant in explaining at least one of these quantiles.²⁴

²³Quantile regression extends standard least squares estimation of the conditional mean function to a collection of linear regression models for each conditional quantile of the endogenous variable. By combining the quantile regression results for different quantiles, we thus obtain a more complete description of the impact of disagreement on the conditional distribution of dispersion returns.

²⁴Standard errors are computed by a (x, y) -pair bootstrap procedure (see Koenker and Hallock, 2001). By construction, the regression quantile point estimates for the 0.5 quantile are identical to the OLS point estimates presented in Table 3.5.

TABLE 3.4: Correlation Risk Premium Regressions

Using data from January 1996 to June 2007, we run regressions from the correlation risk premium on a number of determinants (see equation (13)). The correlation risk premium is approximated by the difference between the index volatility risk premium and a weighted average of the constituents volatility risk premia. Common DiB is our proxy for the common component in the difference in beliefs for the market, Market Vola is the 21 day realized volatility of the index, Demand Pressure is defined as the ratio of the open interest for the out-of-the-money index put options to the open interest for the near and at-the-money index options. Sentiment is the first principal component from trading volume as measured by NYSE turnover, the dividend premium, the closed-end fund discount, the number and first-day returns on IPOs and the equity share in new issues. * denotes significance at the 10% level, ** denotes significance at the 5% level and *** denotes significance at the 1% level. All estimations use autocorrelation and heteroskedasticity-consistent t-statistics reported in parenthesis below the estimated coefficient.

| | (1) | (2) | (3) |
|-----------------|--------------------|--------------------|--------------------|
| Constant | 0.002*** (3.45) | 0.002*** (3.47) | 0.001*** (3.78) |
| Common DiB | 0.104*** (2.71) | 0.112*** (2.65) | 0.108*** (2.74) |
| Market Vola | 0.057** (2.45) | 0.068** (2.21) | 0.055** (2.01) |
| Macro Factor | -0.079 (-1.49) | -0.065 (-1.41) | -0.065 (-1.12) |
| Demand Pressure | | 0.875 (0.89) | |
| Sentiment | | | 0.888** (2.46) |
| Adj. R^2 | 0.10 | 0.09 | 0.13 |
| Observations | 138 | 138 | 138 |

TABLE 3.5: Returns on Straddles and Dispersion Sorted on Disagreement

This table reports the monthly mean and annualized Sharpe ratio of the straddle and dispersion portfolio returns. Portfolios are sorted according to the size of each firm's individual DiB. Quintile 1 (5) consists of stocks with the lowest (largest) DiB. Option returns of single-stocks are sampled between January 1996 and June 2007. The alpha and beta coefficients are estimated from the following least-squares regression:

$$\begin{aligned} exret_i(t) = & \alpha_i + \beta_i^M MRKT(t) + \beta_i^S SMB(t) + \beta_i^{BM} HML(t) + \beta_i^{MOM} MOM(t) \\ & + \beta_i^V \epsilon^V(t) + \beta_i^L \epsilon^L(t) + \beta_i^S \epsilon^S(t) + \beta_i^D \epsilon^D(t) + \beta_i^C \epsilon^C(t) + u_i(t), \end{aligned}$$

where $exret_i(t)$ is the strategy return in excess of the one month Libor, $MRKT$ is the value-weighted excess return on all NYSE, AMEX, and NASDAQ stocks, SMB is the size factor, HML is the book-to-market factor and MOM is the momentum factor. ϵ^V is the monthly change of the VIX, ϵ^L the monthly change of the aggregate liquidity measure, ϵ^D the change of the common disagreement factor, and ϵ^C the change in the realized correlation.

| | STRADDLES | | | DISPERSION | |
|--------------|--------------------|--------------------|-------------------|----------------------|----------------------|
| | Low | High | LmH | Low | High |
| Mean | 0.0719 | -0.0872 | 0.1591 | 0.1654 | 0.1038 |
| Sharpe Ratio | 0.6072 | -1.5635 | 1.3249 | 1.5097 | 1.1940 |
| Alpha | 0.083** (2.01) | -0.104* (-1.75) | 0.186* (1.92) | 0.094 (2.58) | 0.067 (2.04) |
| MRKT | 0.928 (1.02) | -1.192 (-1.24) | 2.120 (1.04) | -0.060 (-0.42) | -0.103 (-0.33) |
| SMB | -1.293* (-1.93) | 1.823 (1.02) | -3.116 (-0.14) | 1.621 (1.20) | 1.384 (1.39) |
| HML | -0.193 (-0.33) | -0.299 (-0.92) | 0.106 (0.94) | -0.639 (-0.50) | -0.552 (-0.99) |
| MOM | 0.102 (0.41) | -0.373* (-1.84) | 0.475 (1.38) | -1.301 (-0.39) | -1.208 (-1.08) |
| ϵ^D | 0.934** (2.12) | 0.680* (1.67) | 0.252** (1.98) | 4.841*** (2.58) | 4.142** (2.14) |
| ϵ^L | 1.001 (0.83) | 0.098 (0.97) | 0.903 (1.02) | -0.202 (-0.22) | -0.372 (-0.28) |
| ϵ^V | -0.195 (-1.04) | 1.235 (1.03) | -1.430 (-0.55) | -2.184* (-1.84) | -2.842* (-1.73) |
| ϵ^S | 0.287* (1.78) | 0.302* (1.93) | -0.015 (-1.03) | 3.833* (1.90) | 2.377** (2.01) |
| ϵ^C | -0.183* (-1.82) | -0.199* (-1.77) | 0.016* (1.69) | -0.005*** (-3.01) | -0.011*** (-2.45) |
| Adj. R^2 | 0.08 | 0.09 | 0.08 | 0.05 | 0.06 |
| Observations | 132 | 132 | 132 | 132 | 132 |

TABLE 3.6: Returns on Double-Sorted Option Strategies

This table reports average monthly returns of double sorted dispersion portfolio returns. Dispersion portfolios are formed by investing 100% of the wealth in shorting index straddles and investing a fraction of wealth into double-sorted stocks. Each month, we first sort according to exposure to Common DiB. Then, within each tercile, we sort stocks into terciles based on the exposure to the market return, the size, book-to-market, momentum, and aggregate liquidity factor in Panel A. The returns on the double sorted dispersion portfolios are then averaged over each of the six stock characteristic portfolios. Panels B and C report the break-downs of average returns for the Market and Liquidity double sort, respectively. The t-statistics reported in parenthesis are based on Newey-West adjusted standard errors using 12 lags. The returns are sampled between January 1996 and June 2007.

| Control | PANEL A: AVERAGE RETURNS | | | |
|---------------------|--------------------------|-----------------|-----------------|-----------------|
| | Low | MED | HIGH | HML |
| MRKT | 0.099 (3.30) | 0.135 (2.92) | 0.155 (2.23) | 0.056 (4.30) |
| SMB | 0.111 (1.94) | 0.135 (2.01) | 0.155 (1.80) | 0.044 (2.17) |
| HML | 0.114 (3.40) | 0.134 (4.30) | 0.162 (3.20) | 0.048 (4.04) |
| MOM | 0.093 (2.67) | 0.117 (2.31) | 0.129 (1.76) | 0.036 (2.70) |
| ϵ^L | 0.124 (4.11) | 0.148 (3.74) | 0.161 (2.50) | 0.037 (3.87) |
| ϵ^V | 0.145 (5.30) | 0.146 (4.11) | 0.164 (4.20) | 0.019 (5.11) |
| PANEL B1: MARKET | | | | |
| Low | 0.094 (2.50) | 0.124 (2.85) | 0.145 (3.02) | 0.051 (2.41) |
| Med | 0.100 (2.74) | 0.135 (3.01) | 0.154 (2.44) | 0.054 (2.85) |
| High | 0.102 (3.22) | 0.140 (2.41) | 0.161 (3.45) | 0.059 (2.47) |
| HmL | 0.008 (2.50) | 0.016 (2.44) | 0.016 (2.48) | |
| PANEL B2: LIQUIDITY | | | | |
| Low | 0.112 (2.44) | 0.135 (2.48) | 0.160 (2.35) | 0.048 (2.61) |
| Med | 0.122 (1.98) | 0.147 (2.11) | 0.166 (2.50) | 0.044 (2.38) |
| High | 0.130 (2.19) | 0.151 (2.57) | 0.169 (2.14) | 0.039 (2.22) |
| HmL | 0.018 (1.88) | 0.016 (2.04) | 0.009 (2.81) | |

TABLE 3.7: Quantile Regressions on Dispersion Portfolio Returns

This table presents estimated coefficients from running quantile regressions of the returns of disagreement sorted dispersion portfolios on changes in common DiB, ϵ^D , changes in the VIX, ϵ^V , changes in sentiment, ϵ^S , and finally changes in realized correlation, ϵ^C . The upper (lower) row for each determinant consists of the 10% (90%) confidence bound. Bold numbers indicate estimated coefficients.

| DiB Bin | 1 | | | 2 | | | 3 | | | 4 | | | 5 | | |
|--------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|---------------|
| Quantile | 0.25 | 0.5 | 0.75 | 0.25 | 0.5 | 0.75 | 0.25 | 0.5 | 0.75 | 0.25 | 0.5 | 0.75 | 0.25 | 0.5 | 0.75 |
| ϵ^D | 1.633 | 8.487 | 10.420 | 2.028 | 7.943 | 9.858 | 2.133 | 7.698 | 9.726 | 2.066 | 7.759 | 9.503 | 2.132 | 7.425 | 7.840 |
| | -2.013 | 4.841 | 6.774 | -1.255 | 4.660 | 6.575 | -1.150 | 4.415 | 6.443 | -1.217 | 4.476 | 6.220 | -1.151 | 4.142 | 4.557 |
| ϵ^V | -5.659 | 1.195 | 3.128 | -4.901 | 1.014 | 2.929 | -4.796 | 0.769 | 2.797 | -4.863 | 0.830 | 2.574 | -4.434 | 0.859 | 1.274 |
| | 0.318 | 0.142 | 0.075 | 0.125 | 0.075 | 0.004 | 0.039 | -0.153 | -0.225 | -0.332 | -0.351 | -0.407 | -0.802 | -0.898 | 0.237 |
| ϵ^S | -4.334 | -4.510 | -4.577 | -4.527 | -4.577 | -4.648 | -4.613 | -4.805 | -4.877 | -4.984 | -5.003 | -5.059 | -4.722 | -4.818 | -4.943 |
| | 5.205 | 7.791 | 8.439 | 5.501 | 7.430 | 8.357 | 5.505 | 7.003 | 7.292 | 5.443 | 6.853 | 7.079 | 3.999 | 4.695 | 5.320 |
| ϵ^C | 1.251 | 3.837 | 4.485 | 1.547 | 3.476 | 4.403 | 1.551 | 3.049 | 3.338 | 1.489 | 2.899 | 3.125 | 1.681 | 2.377 | 3.002 |
| | -2.703 | -0.117 | 0.531 | -2.407 | -0.478 | 0.449 | -2.403 | -0.905 | -0.616 | -0.829 | 0.581 | 0.807 | -0.637 | 0.059 | 0.684 |
| ϵ^C | -0.006 | -0.001 | 0.000 | -0.004 | -0.003 | -0.002 | -0.006 | -0.003 | -0.004 | -0.008 | -0.005 | -0.006 | -0.003 | -0.002 | 0.001 |
| | -0.009 | -0.004 | -0.003 | -0.007 | -0.006 | -0.005 | -0.009 | -0.006 | -0.007 | -0.011 | -0.008 | -0.009 | -0.012 | -0.011 | -0.008 |
| | -0.012 | -0.007 | -0.006 | -0.010 | -0.009 | -0.008 | -0.012 | -0.009 | -0.010 | -0.014 | -0.011 | -0.012 | -0.021 | -0.020 | -0.017 |

We find that shocks in the VIX, the sentiment proxy, and the realized correlation proxy all have a similar estimated coefficient, with identical sign across the different quantiles of dispersion portfolios: While the point estimates for ϵ^V and ϵ^C are all negative, those for ϵ^S are all positive. Shocks to realized correlation have a significant impact on each quantile of dispersion portfolios. Shocks to sentiment (the VIX index) have some significant explanatory power only for the 75% quantiles of dispersion portfolios (the quantiles of the fourth and fifth quintile portfolios).

Shocks in the common disagreement proxy ϵ^D , on the other hand, have an economically and statistically significant positive impact on the upper (0.75) and central (0.5) quantiles, but they tend to imply a negative impact on the bottom (0.25) return quantile of all dispersion portfolios. An increase in disagreement increases expected returns in average and good states (0.50 and 0.75 quantiles), while in bad states (bottom 0.25 quantile) stocks associated with largest increase in disagreement generate losses. This evidence suggests the existence of a potentially infrequent downside risk linked to systematic disagreement shocks. According to these findings, the positive effect of disagreement risk on the top 50% dispersion returns might be interpreted as a compensation for its negative impact on rare, but potentially large, dispersion losses.

3.4.3 The Price of Disagreement Risk

The basic intuition of our model for a cross-sectional differential pricing of index and individual options is based on the presence of priced disagreement components, generated by the optimal equilibrium risk sharing among disagreeing agents, which explains a fraction of the excess returns of option volatility and correlation strategies based on, e.g., straddle and dispersion portfolios. We conclude our empirical analysis by testing whether disagreement risk is a priced risk factor in the cross-section of these option strategy returns.

Similar to Breeden, Gibbons, and Litzenberger (1989) and Ang, Hodrick, Xing, and Zhang (2006), we build a factor-mimicking portfolio of innovations in common disagreement. By having a traded factor at hand, we are able to study the factor price of risk in a natural way. To obtain the factor mimicking portfolio (CDiB), we regress innovations in the common disagreement on the excess returns of the dispersion portfolio: $\epsilon^D(t+1) = a + b' \mathbf{rx}(t+1) + u(t+1)$, where $\mathbf{rx}(t+1)$ is the vector of excess

returns on the dispersion portfolios. The factor mimicking factor is then given by $CDiB(t+1) = \hat{b}'\mathbf{r}\mathbf{x}(t+1)$.

We estimate the price of disagreement risk components using a two-pass Fama-MacBeth (1973) regression approach, in which we consider the return of a factor mimicking portfolio for common disagreement risk. Following Fama and French (1992), the factor mimicking portfolio is long (short) the quintile of stocks having the highest (lowest) beta with respect to our common DiB (CDiB) proxy. In the first stage of the two-pass methodology, we regress the time-series of monthly excess returns of straddle option quintile portfolios in Table 3.5 on a number of potential pricing factors, in order to estimate the corresponding factor betas. We follow the same procedure for the cross-section of dispersion quintile portfolios, in which the long side simply consists of the five quintile straddle portfolios in Table 3.5, respectively. In addition to standard risk factors like, e.g., the market return, in these regressions we additionally include as explanatory variables the return and the squared return of the factor-mimicking portfolio for common DiB, in order to account for potential nonlinearities of option strategy returns with respect to common DiB shocks. In the second stage of the two-pass methodology, we regress each month the cross-section of excess returns of straddle and dispersion portfolios on their estimated factor betas from the first step. In this way, we obtain monthly estimates of the price of risk for each factor. Finally, we average monthly point estimates of prices of risk over time to obtain unconditional estimates of factor risk premia.

For each time point $t = 1 \dots, T$, in the second step of the two-pass methodology we estimate the following cross-sectional regressions, in the full model specification:

$$\begin{aligned} exret_i(t) = & \alpha_i + \beta_i^{MRKT} \lambda_{MRKT} + \beta_i^{SML} \lambda_{SML} + \beta_i^{HML} \lambda_{HML} \\ & + \beta_i^{MOM} \lambda_{MOM} + \beta_i^{CDiB} \lambda_{CDiB} + \beta_i^{CDiB^2} \lambda_{CDiB^2} + u_i(t), \end{aligned} \quad (3.11)$$

where $exret_i(t)$ denotes the excess returns of option straddle or dispersion portfolio $i = 1, \dots, N$ and Table 3.8 summarizes our findings.²⁵

Panel A reports estimated prices of risk from the second step of the procedure, both for (i) the full factor specification (3.11) and (ii) a CAPM-type factor specification, extended only by the betas for the return and the squared return of the factor-mimicking portfolio

²⁵We adopt standard refinements of the two-pass methodology by using a Shanken (1992) correction in order to obtain errors-in-variables robust standard errors. We also compute Shanken and Zhou (2007) correction, in order to obtain standard errors robust against misspecification, however, the results remain quantitatively the same.

TABLE 3.8: The Price of Disagreement Risk

Panel A reports the Fama-MacBeth (1973) factor premia of sorted straddle and dispersion portfolios. *MRKT* is the excess return on the market portfolio, *SMB* and *HML* are the Fama and French (1993) size and value factors, *MOM* is the momentum factor, and *CDiB* is the mimicking factor for common disagreement. *CDiB* is constructed as a zero cost portfolio which is long the quintile of stocks with highest exposure to Common DiB and short the quintile with lowest exposure to Common DiB. In Panel B, we report factor loadings from the time-series regression on the *MRKT* factor, *CDiB* and *CDiB*². The t-statistics are corrected for estimation error in the first stage as proposed by Shanken (1992). The data covers January 1996 until June 2007.

| PANEL A: FAMA AND MACBETH (1973) FACTOR PREMIA | | | | | | | |
|--|-----------------|-----------------|-----------------|-----------------|-------------------|-----------------|-------------------|
| | Alpha | MRKT | SMB | HML | MOM | CDiB | CDiB ² |
| A.I. | 0.025 (0.45) | 0.371 (1.47) | | | | 0.311 (3.41) | -0.045 (-1.65) |
| A.II | 0.041 (3.14) | 0.414 (1.77) | 0.387 (2.47) | 0.141 (0.47) | -0.530 (-1.01) | | |
| A.III | 0.030 (1.47) | 0.410 (0.88) | 0.224 (1.45) | 0.317 (0.33) | -0.458 (-0.56) | 0.222 (2.58) | -0.014 (-1.15) |

| PANEL B: FACTOR EXPOSURES STRADDLES | | | | | | |
|-------------------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| | 1 | 2 | 3 | 4 | 5 | 1-5 |
| β_{MRKT} | 0.971 (1.22) | 0.210 (1.47) | -0.410 (-1.18) | -1.147 (-1.47) | -1.251 (-1.66) | 2.222 (1.47) |
| β_{CDiB} | 0.501 (5.14) | 0.314 (2.48) | -0.001 (3.01) | -0.142 (4.27) | -0.155 (3.87) | 0.656 (3.56) |
| β_{CDiB^2} | -0.014 (-1.45) | -0.018 (-1.48) | -0.005 (-1.66) | 0.005 (1.25) | 0.017 (1.47) | -0.031 (-1.44) |

| PANEL B: FACTOR EXPOSURES DISPERSION | | | | | | |
|--------------------------------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| | 1 | 2 | 3 | 4 | 5 | 1-5 |
| β_{MRKT} | -0.075 (-0.40) | -0.017 (-0.12) | -0.055 (-0.47) | -0.159 (-0.82) | -0.167 (-0.11) | 0.092 (-0.45) |
| β_{CDiB} | 0.298 (3.45) | 0.235 (2.45) | 0.014 (2.74) | -0.144 (-1.97) | -0.149 (-2.01) | 0.447 (2.85) |
| β_{CDiB^2} | -0.146 (-1.11) | -0.455 (-1.45) | -0.211 (-0.55) | -0.044 (-0.26) | 0.001 (-0.33) | -0.147 (-1.10) |

for common DiB. Overall, we find that exposure to the return of the CDiB factor-mimicking portfolio is the only one implying a strongly significant positive price of risk: When the CDiB mimicking portfolio return is included in the regression, exposure to market risk and all other standard risk factors has no significant impact on the cross-section of option portfolio returns. The estimated market price for CDiB risk of 26% is instead strongly significant, both economically and statistically. In the CAPM-type factor specification (row A.I), the results also produce some (weak) evidence of

a negative market price of risk for the squared returns of the CDiB factor-mimicking portfolio, indicating a potential nonlinearity in the relation between some option returns and disagreement risk.

Panel B reports for brevity the cross-section of estimated betas for the CAPM-type extended specification. We find that the CDiB beta of straddle (dispersion) quintile portfolios ranges from 0.501 to -0.155 (0.398 to -0.149). Thus, cross-sectional variations in factor betas for common DiB are responsible for differences in average excess returns of sorted straddle (dispersion) portfolios of about $(0.501 - 0.155) \cdot (0.26) = 8.99\%$ ($(0.298 - 0.149) \cdot (0.26) = 3.87\%$). This amounts to about two third (half) of the total cross-sectional variation of 15% (6%) in the excess returns of straddle (dispersion) portfolios.

In summary, we conclude that the cross-section of option portfolio returns contains an economically relevant component generated by different cross-sectional exposures to systematic disagreement risk. When neglecting exposures to common disagreement components, average alphas of dispersion and straddle returns are significantly different from zero. However, this finding seems more related to the presence of an uncovered systematic disagreement risk factor than to an evident option mispricing.

3.5 Conclusion

The interaction of incomplete information and disagreement give rise to a number of novel implications for the dynamics and cross-section of volatility and correlation risk premia. In this paper, we depart from the previous literature by investigating a two-trees Lucas economy in which the growth rates of firms' dividend streams are unknown and investors have different perceptions of future market-wide uncertainty. Agents agree to disagree and equilibrium consumption plans depend on the degree of disagreement among investors, who engage in active risk sharing to finance their optimal policies. In this setting, investors' disagreement about a market-wide economic signal can generate substantial stock return co-movement, even when underlying fundamentals are weakly related. This generally occurs in presence of greater subjective uncertainty. Even more important, volatility and correlation risk are priced and their risk premia are driven by the degree of heterogeneity in beliefs among investors and affected by the level of perceived market-wide uncertainty. The equilibrium volatility risk premium on *index* options is larger, because of an endogenous correlation risk premium generated by investors'

optimal risk sharing, and model-implied *index* option smiles are steeper than individual option smiles. These features produce option volatility and correlation strategies that can generate large risk-adjusted excess returns, as a consequence of their exposure to unexpected changes in the degree of diversity in beliefs in the economy. While these strategies have no apparent exposure to standard risk factors, they produce an economically and statistically significant exposure to proxies of systematic disagreement risk. This is important since it has the potential to explain some puzzles highlighted by the extant empirical literature.

Using analyst forecast data on future earnings to construct a proxy of disagreement, we find that disagreement has strong predictive power in explaining (a) the spread between index and individual volatility risk premia, (b) the skewness of implied volatility smiles and (c) option-implied correlation risk premia. Option trading strategies that exploit these risk premia yield very attractive Sharpe ratios. At the same time, these excess returns are explained by a significant exposure to unexpected changes in the common disagreement component, while exposure to market risk and a number of other traditional systematic risk factors are not statistically significant. After we control for common disagreement, Fama and MacBeth (1973) regression alphas are no longer statistically different from zero.

The results in this paper support the notion that beliefs heterogeneity add an economically important dimension to neoclassical asset pricing and to our understanding of the dynamics and cross-section of volatility and correlation risk premia.

Appendix A

Appendix A

A.1 Beliefs Disagreement Dynamics and Learning

Given the parametric structure of the dynamics for dividends and signals in our model, it is possible to solve the filtering problem of a Bayesian investor that believes in this dynamics. Since the relevant state variables define a multivariate Gaussian process, this problem is standard. In particular, with Gaussian initial conditions the implied conditional beliefs are Gaussian; see Rogers and Williams (1987) Sec. 6.9, and Liptser and Shiryaev (2000), Theorem 12.7.

Lemma A.1. *Let $m(t) = E(\mu(t)|\mathcal{F}_t^Y)$ and $\gamma(t) = E((\mu(t) - m(t))(\mu(t) - m(t))'|\mathcal{F}_t^Y)$. It then follows that $m(t)$ and $\gamma(t)$ are continuous \mathcal{F}_t^Y -measurable for any t solutions of the system of equations:*

$$dm(t) = (a_0 + a_1 m(t))dt + \gamma(t)A'(BB')^{-1}(dY(t) - Am(t)dt), \quad (\text{A.1})$$

$$d\gamma(t)/dt = a_1 \gamma(t) + \gamma(t)a_1' + bb' - \gamma(t)A'(BB')^{-1}A\gamma(t), \quad (\text{A.2})$$

subject to the initial conditions $m(0) = E(\mu_0|\mathcal{F}_0^Y)$ and $\gamma(0) = E((\mu_0 - m(0))(\mu_0 - m(0))'|\mathcal{F}_0^Y)$. Moreover, if the matrix $\gamma(0)$ is positive definite, then the matrices $\gamma(t)$, $0 \leq t \leq T$, have the same property.

Closed form solutions of the matrix Riccati equation¹ for $\gamma(t)$ in Lemma A.1 are obtained, via Radon's Lemma, by linearizing the flow of the differential equation (A.2).

¹For a review of Riccati equations, see Freiling (2002).

Lemma A.2. *Let*

$$\begin{pmatrix} g_{11}(t) & g_{12}(t) \\ g_{21}(t) & g_{22}(t) \end{pmatrix} = \exp \left(t \begin{pmatrix} a_1 & A'(BB')^{-1}A \\ bb' & -a'_1 \end{pmatrix} \right).$$

Then the solution of equation (A.2) can be written as:

$$\gamma(t) = (\gamma(0)g_{12}(t) + g_{22}(t))^{-1} (\gamma(0)g_{11}(t) + g_{21}(t)).$$

Proof: Let $\gamma(t) = F(t)^{-1}G(t)$, for two differentiable functions $F(t)$ and $G(t)$ such that $G(t)$ is invertible and $F(0) = id_{2 \times 2}$. It then follows:

$$\frac{d}{dt} [F(t)\gamma(t)] - \frac{d}{dt} [F(t)] \gamma(t) = F(t) \frac{d}{dt} \gamma(t),$$

and

$$\frac{d}{dt} G(t) - \frac{d}{dt} [F(t)] \gamma(t) = bb'F(t) + G(t)a_1 + (F(t)a'_1 + G(t)A'(BB')^{-1}A) \gamma(t). \quad (\text{A.3})$$

The last ordinary differential equation leads to the following system of linear equations:

$$\begin{aligned} \frac{d}{dt} G(t) &= F(t)bb' + G(t)a_1, \\ \frac{d}{dt} F(t) &= -F(t)a_1 + G(t)A'(BB')^{-1}A. \end{aligned}$$

The solution of this system of differential equations is:

$$\begin{aligned} (G(t) \quad F(t)) &= (G(0) \quad F(0)) \exp \left(t \begin{pmatrix} a_1 & A'(BB')^{-1}A \\ bb' & -a'_1 \end{pmatrix} \right), \\ &= (\gamma(0)g_{11}(t) + g_{21}(t) \quad \gamma(0)g_{12}(t) + g_{22}(t)). \end{aligned}$$

It then follows that the closed form solution of the matrix Riccati equation (A.2) is:

$$\gamma(t) = (\gamma(0)g_{12}(t) + g_{22}(t))^{-1} (\gamma(0)g_{11}(t) + g_{21}(t)).$$

If $\gamma(t)$ is at the steady-state, then the matrix Riccati becomes an algebraic Riccati equation (ARE) of the form:

$$0 = a_1\gamma + \gamma a'_1 + bb' - \gamma A'(BB')^{-1}A\gamma, \quad (\text{A.4})$$

which is equivalent to the following system of equations:

$$\begin{aligned} 0 &= 2a_{1A}\gamma_A + \sigma_{\mu_A}^2 - \frac{\gamma_A^2}{\sigma_A^2} - \frac{(\alpha\gamma_A + \beta\gamma_{Az})^2}{\sigma_z^2}, \\ 0 &= 2a_{1z}\gamma_z + \sigma_{\mu_z}^2 - \frac{\gamma_{Az}^2}{\sigma_A^2} - \frac{(\alpha\gamma_{Az} + \beta\gamma_z)^2}{\sigma_z^2}, \\ 0 &= (a_{1A} + a_{1z})\gamma_{Az} - \frac{\gamma_{Az}\gamma_A}{\sigma_A^2} - \frac{\alpha\gamma_{Az}(\alpha\gamma_A + \beta\gamma_{Az}) + \beta\gamma_z(\alpha\gamma_A + \beta\gamma_{Az})}{\sigma_z^2}. \end{aligned}$$

Lemma A.3. *Let*

$$F = bb'A'(BB')^{-1}A + a_1a_1', \quad (\text{A.5})$$

then the solution to equation (A.4) is given by:

$$\gamma = \left(F^{1/2} - (a_1' + a_1)/2 \right)^{-1} bb'. \quad (\text{A.6})$$

Proof: Analytical solutions to ARE are in general not available, however, under the condition that bb' is non-singular and $bb'a_1$ is a symmetric matrix, Incertis (1981) derives analytical solutions for equation (A.4). Applying the steps in the paper, we find that the closed-form solution of γ is given by (A.6), where $F^{1/2}$ is the square root of the matrix F , which is defined in (A.5). In the special case $\beta = 0$, the solution takes the particularly simple form $\gamma_{Az} = 0$ and:

$$\begin{aligned} \gamma_A &= \frac{a_{1A} + \sqrt{a_{1A}^2 + \sigma_{\mu_A}^2 \left(\frac{\sigma_z^2 + \alpha^2 \sigma_A^2}{\sigma_A^2 \sigma_z^2} \right)}}{\left(\frac{\sigma_z^2 + \alpha^2 \sigma_A^2}{\sigma_A^2 \sigma_z} \right)}, \\ \gamma_z &= -\frac{\sigma_{\mu_z}^2}{2a_{1z}}. \end{aligned}$$

From Lemma A.1, the disagreement dynamics in our economy follows in a straightforward way, by applying the above results individually to the filtering problems of the two investors in our economy. The dynamics of the individual beliefs are:

$$\begin{aligned} dm^1(t) &= (a_0 + a_1 m^1(t))dt + \gamma^1(t)A'B^{-1}dW_Y^1(t), \\ dm^2(t) &= (a_0 + a_1 m^2(t))dt + \gamma^2(t)A'(BB')^{-1}(m^1(t) - m^2(t))dt + \gamma^2(t)A'B^{-1}dW_Y^1(t), \end{aligned}$$

using the symmetry of B . The dynamics of $\Psi(t) = B^{-1}(m^1(t) - m^2(t))$ follows, as:

$$\begin{aligned} d\Psi(t) &= B^{-1} (a_1 + \gamma^2(t)A'(BB')^{-1}) (m^1(t) - m^2(t))dt + B^{-1} (\gamma^1(t) - \gamma^2(t)) A'B^{-1}dW_Y^1(t) \\ &= B^{-1} (a_1B + \gamma^2(t)A'B^{-1}) \Psi(t)dt + B^{-1} (\gamma^1(t) - \gamma^2(t)) A'B^{-1}dW_Y^1(t), \end{aligned}$$

with initial condition $\Psi(0) = B^{-1}(m^1(0) - m^2(0))$. The solution of this stochastic differential condition is:

$$\Psi(t) = \exp \left\{ \int_0^t M(s)ds \right\} \Psi(0) + \int_0^t \exp \left\{ \int_0^s M(u)du \right\} B^{-1} (\gamma^1(s) - \gamma^2(s)) A'B^{-1}dW_Y^1(s) \quad (\text{A.7})$$

where $M(s) = B^{-1} (a_1B + \gamma^2(s)A'B^{-1})$. It follows that $\Psi(t)$ is normally distributed as:

$$\Psi(t) \sim \mathcal{N} \left(e^{\int_0^t M(s)ds} \Psi(0), \int_0^t e^{\int_0^s M(u)du} B^{-1} (\gamma^1(s) - \gamma^2(s)) A'(BB')^{-1} A (\gamma^1(s) - \gamma^2(s))' B^{-1} e^{\int_0^s M(u)du} ds \right).$$

The parameter $b = \text{diag}(\sigma_{\mu_A}, \sigma_{\mu_z})$ in the dynamics for $\gamma(t)$ in Lemma A.1 impacts the distribution of $m(t)$ only indirectly, by influencing the Riccati differential equation for $\gamma(t)$. Therefore, when we assume that this parameter is perceived identically by all investors, we can model a setting of rational Bayesian investors that can disagree because of different priors at time zero. If we assume that this parameter is perceived differently by some investor, we can model parsimoniously an economy with overconfidence, in which, e.g., some investor perceives a lower variance for the expected consumption growth or the expected change in the signal. To this end, we just need to use an investor-dependent parameter $b^i = \text{diag}(\sigma_{\mu_A}^i, \sigma_{\mu_z}^i)$ in the matrix Riccati differential equation for $\gamma^i(t)$, where $i = 1, 2$.

For $i = 1, 2$, define:

$$\gamma^i(t) = \begin{pmatrix} \gamma_A^i(t) & \gamma_{Az}^i(t) \\ \gamma_{Az}^i(t) & \gamma_z^i(t) \end{pmatrix}. \quad (\text{A.8})$$

From the vector dynamics for $\Psi(t)$, the dynamics for $\Psi_A(t)$ and $\Psi_z(t)$ read explicitly:

$$\begin{aligned} d\Psi_A(t) &= \left(\left(a_{1A} + \frac{\gamma_A^2(t)}{\sigma_A^2} \right) \Psi_A(t) + \frac{\alpha\gamma_A^2(t) + \beta\gamma_{Az}^2(t)}{\sigma_A\sigma_z} \Psi_z(t) \right) dt \\ &\quad + \frac{\gamma_A^1(t) - \gamma_A^2(t)}{\sigma_A^2} dW_A^1(t) + \frac{\alpha(\gamma_A^1(t) - \gamma_A^2(t)) + \beta(\gamma_{Az}^1(t) - \gamma_{Az}^2(t))}{\sigma_A\sigma_z} dW_z^1(t), \end{aligned}$$

and

$$d\Psi_z(t) = \left(\frac{\gamma_{Az}^2(t)}{\sigma_A \sigma_z} \Psi_A(t) + \left(a_{1z} + \frac{\alpha \gamma_{Az}^2(t) + \beta \gamma_z^2(t)}{\sigma_z^2} \right) \Psi_z(t) \right) dt + \frac{\gamma_{Az}^1(t) - \gamma_{Az}^2(t)}{\sigma_A \sigma_z} dW_A^1(t) + \frac{\alpha(\gamma_{Az}^1(t) - \gamma_{Az}^2(t)) + \beta(\gamma_z^1(t) - \gamma_z^2(t))}{\sigma_z^2} dW_z^1(t) .$$

Similarly, the dynamics for $m_A^1(t)$ and $m_z^1(t)$ read explicitly:

$$dm_A^1(t) = (a_{0A} + a_{1A} m_A^1(t)) dt + \frac{\gamma_A^1(t)}{\sigma_A} dW_A^1(t) + \frac{\alpha \gamma_A^1(t) + \beta \gamma_{Az}^1(t)}{\sigma_z} dW_z^1(t) ,$$

and

$$dm_z^1(t) = (a_{0z} + a_{1z} m_z^1(t)) dt + \frac{\gamma_{Az}^1(t)}{\sigma_A} dW_A^1(t) + \frac{\alpha \gamma_{Az}^1(t) + \beta \gamma_z^1(t)}{\sigma_z} dW_z^1(t) .$$

This concludes the discussion of the disagreement dynamics in our model.

□

A.2 Equilibrium Quantities

For completeness, we derive all equilibrium quantities in this Appendix. The proofs follow grossly Basak (2005).

(i) Dynamics of the stochastic weighting process λ : Itô's Lemma applied to $\eta(t) = \xi^1(t)/\xi^2(t)$ gives:

$$d\eta(t) = \frac{d\xi^1(t)}{\xi^2(t)} - \frac{\xi^1(t)}{(\xi^1(t))^2} d\xi^2(t) + \frac{1}{2} \frac{2\xi^1(t)}{(\xi^2(t))^3} (d\xi^2(t))^2 - \frac{1}{(\xi^2(t))^2} d\xi^2(t) d\xi^1(t) .$$

Since markets are complete, there exists a unique stochastic discount factor for each agent. Absence of arbitrage implies for $i = 1, 2$:

$$\frac{d\xi^i(t)}{\xi^i(t)} = -r(t)dt - \theta^i(A(t), z(t))' dW_Y^i ,$$

where $\theta^i = (\theta_A^i(t), \theta_z^i(t))'$ is the vector of market prices of risk perceived by agent i . It then follows,

$$\begin{aligned} d\eta(t) &= \frac{\xi^1(t)}{\xi^2(t)} \frac{d\xi^1(t)}{\xi^1(t)} - \frac{\xi^1(t)}{\xi^2(t)} \frac{d\xi^2(t)}{\xi^2(t)} + \frac{\xi^1(t)}{\xi^2(t)} \left(\frac{d\xi^2(t)}{\xi^2(t)} \right)^2 - \frac{1}{(\xi^2(t))^2} d\xi^2(t) d\xi^1(t), \\ &= \eta(t) \left(-r(t)dt - \theta_A^1(t)dW_A(t) - \theta_z^1(t)dW_z(t) - (-r(t)dt - \theta_A^2(t)dW_A(t) - \theta_z^2(t)dW_z(t)) \right. \\ &\quad \left. + \left((\theta_A^2(t))^2 + (\theta_z^2(t))^2 - \theta_A^1(t)\theta_A^2(t) - \theta_z^1(t)\theta_z^2(t) \right) dt \right). \end{aligned} \quad (\text{A.9})$$

The prices of the stock and the senior bond in our economy follow the dynamics:

$$dS(t) = S(t) (\mu_S(t)dt + \sigma_{SA}dW_A(t) + \sigma_{S_z}dW_z(t)), \quad (\text{A.10})$$

$$dB^S(t) = B^S(t) (\mu_{B^S}(t)dt + \sigma_{B^S A}dW_A(t) + \sigma_{B^S z}dW_z(t)), \quad (\text{A.11})$$

where $S(t)$ is the price of equity and $B^s(t)$ the price of the senior bond, and the expected growth rates $\mu_S(t)$ and $\mu_{B^s}(t)$ and the volatility coefficients $\sigma_{SA}, \sigma_{B^S A}, \sigma_{S_z}$ and $\sigma_{B^S z}$ are determined in equilibrium. It is easily shown that the difference in the perceived rates of return have to satisfy the consistency condition:

$$\mu_n^1(t) - \mu_n^2(t) = \sigma_n \left(\Psi_A(t), \quad \alpha \Psi_A(t) \frac{\sigma_A}{\sigma_z} + \beta \Psi_z(t) \right)',$$

where n denotes security n . The definition of market price of risk yields:

$$\sigma_{nA}\theta_A^i(t) + \sigma_{nz}\theta_z^i(t) = \mu_n^i(t) - r(t).$$

After some simple algebra, we obtain:

$$\sigma_{nA}(t) (\theta_A^1(t) - \theta_A^2(t)) + \sigma_{nz}(t) (\theta_z^1(t) - \theta_z^2(t)) = \sigma_{nA}(t)\Psi_A(t) + \sigma_{nz}(t) \left(\alpha \Psi_A(t) \frac{\sigma_A}{\sigma_z} + \beta \Psi_z(t) \right).$$

Since this equation has to hold for any $\sigma_{nA}(t)$ and $\sigma_{nz}(t)$, it follows:

$$\begin{aligned} \theta_A^1(t) - \theta_A^2(t) &= \Psi_A(t), \\ \theta_z^1(t) - \theta_z^2(t) &= \left(\alpha \Psi_A(t) \frac{\sigma_A}{\sigma_z} + \beta \Psi_z(t) \right). \end{aligned}$$

By construction, we also have:

$$dW_A(t) = \frac{m_A^i(t) - \mu_A(t)}{\sigma_A} dt + dW_A^i(t), \quad dW_z(t) = \left(\alpha \frac{m_z^i(t) - \mu_z(t)}{\sigma_z} + \beta \frac{m_z^i(t) - \mu_z(t)}{\sigma_z} + dW_z^i(t) \right).$$

Therefore, after substituting in equation (A.9), we get:

$$\begin{aligned} \frac{d\eta(t)}{\eta(t)} &= -dW_A^1(t)\Psi_A(t) - \theta_z^1(t)dW_z^1(t) + \theta_z^2(t) \left(dW_z^1(t) + \alpha\Psi_A(t)\frac{\sigma_A}{\sigma_z} + \beta\Psi_z(t) \right) \\ &\quad + \left((\theta_A^1(t) - \Psi_A(t))^2 + \theta_z^2(t) (\theta_z^2(t) - \theta_z^1(t)) - \theta_A^1(t) (\theta_A^1(t) - \Psi_A(t)) \right) dt, \\ &= -dW_A^1(t)\Psi_A(t) - dW_z^1(t) \left(\alpha\Psi_A(t)\frac{\sigma_A}{\sigma_z} + \beta\Psi_z(t) \right). \end{aligned}$$

(ii) Representative investor optimization and optimal consumption policies:

The representative agent in the economy faces the following optimization problem:

$$\sup_{c_1(t)+c_2(t)=A(t)} U(c_1(t), c_2(t), \lambda(t)) = \frac{c_1(t)^{1-\gamma}}{1-\gamma} + \lambda(t) \frac{c_2(t)^{1-\gamma}}{1-\gamma}, \quad (\text{A.12})$$

where $\lambda(t) > 0$. Optimality of individual consumption plans implies that the stochastic weight takes the following form:

$$\lambda(t) = u'(c_1(t))/u'(c_2(t)) = y_1\xi^1(t)/y_2\xi^2(t),$$

where $u'(c(t)) = c(t)^{-1/\gamma}$ is the marginal utility function, which is assumed identical across agents. The first order condition for agent one is:

$$e^{-\rho t} c_1(t)^{-\gamma} = y_1\xi^1(t).$$

The first order condition for agent two is:

$$\eta(t)e^{-\rho t} c_2(t)^{-\gamma} = y_2\xi^1(t).$$

The aggregate resource constraint can now be easily derived as:

$$\left(\frac{y_2\xi^1(t)e^{\rho t}}{\eta(t)} \right)^{-1/\gamma} + (y_1\xi^1(t)e^{\rho t})^{-1/\gamma} = A(t).$$

Thus, the solutions for the individual state price densities are:

$$\xi^1(t) = e^{-\rho t} \frac{1}{y_1} A(t)^{-\gamma} \left(1 + \lambda(t)^{1/\gamma}\right)^\gamma, \quad \xi^2(t) = e^{-\rho t} \frac{1}{y_2} A(t)^{-\gamma} \left(1 + \lambda(t)^{1/\gamma}\right)^\gamma \lambda(t)^{-1}.$$

To solve for the optimal consumption policy of each agent, we plug in the functional forms for the individual state price densities:

$$c_1(t) = (y_1 \xi^1(t) e^{\rho t})^{-1/\gamma} = A(t) \left(1 + \lambda(t)^{1/\gamma}\right)^{-1}.$$

Good's market clearing, finally implies:

$$c_2(t) = A(t) - c_1(t) = A(t) \lambda(t)^{1/\gamma} \left(1 + \lambda(t)^{1/\gamma}\right)^{-1}.$$

This concludes the proof. □

Remark A.4 (Equilibrium risk-less interest rate). The expression for the equilibrium interest rate can be now easily obtained from the above results, using the same arguments as in Basak (2005):

$$\begin{aligned} r(t) = & \rho + \gamma \left(\frac{1}{1 + \lambda(t)^{1/\gamma}} \mu_A^1(t) + \frac{\lambda(t)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \mu_A^2(t) \right) - \frac{1}{2} (1 + \gamma) \gamma \sigma_A^2 \\ & + \frac{\gamma - 1}{2\gamma} \frac{\lambda(t)^{1/\gamma}}{(1 + \lambda(t)^{1/\gamma})^2} \left((\alpha \Psi_A(t) \frac{\sigma_A}{\sigma_z} + \beta \Psi_z(t))^2 + \Psi_A(t)^2 \right). \end{aligned} \quad (\text{A.13})$$

In particular, one can see that for a relative risk aversion $\gamma > 1$ the contribution of disagreement to the equilibrium interest rate is positive.

A.3 Market Prices of Risk

The optimal consumption policy of agent i is:

$$c_i(t) = I(y_i \xi^i(t)),$$

where $I(h) = h^{-1/\gamma}$ is the inverse marginal utility function of consumption, which is identical across agents. Applying Itô's Lemma to this equation yields:

$$\begin{aligned} dc_i(t) &= \frac{\partial I(y_i \xi^i(t))}{\partial \xi^i(t)} d\xi^i(t) + \frac{1}{2} \frac{\partial^2 I(y_i \xi^i(t))}{\partial (\xi^i(t))^2} (d\xi^i(t))^2, \\ &= \mu_{c_i}(t) dt + \sigma_{c_i A}(t) dW_A(t) + \sigma_{c_i z}(t) dW_z(t). \end{aligned} \quad (\text{A.14})$$

Good market clearing and equation (A.14) imply:

$$\frac{c_1(t)}{\gamma} \theta_A^1(t) + \frac{c_2(t)}{\gamma} \theta_A^2(t) = \sigma_A A(t).$$

and

$$\theta_A^1(t) = \left(\frac{c_1(t)}{\gamma} + \frac{c_2(t)}{\gamma} \right)^{-1} \left(\sigma_A A(t) + \Psi_A(t) \frac{c_2(t)}{\gamma} \right) = \gamma \sigma_A + \frac{c_2(t)}{A(t)} \Psi_A(t).$$

The market price of risk of cash flow for the second agent follows easily as:

$$\theta_A^2(t) = \gamma \sigma_A - \frac{c_1(t)}{A(t)} \Psi_A(t).$$

By inserting in these formulas the optimal consumption policies, the individual market prices of risk for cash flow risk are:

$$\theta_A^1(t) = \gamma \sigma_A + \Psi_A(t) \left(1 + \lambda(t)^{1/\gamma} \right)^{-1} \lambda(t)^{1/\gamma}, \quad \theta_A^2(t) = \gamma \sigma_A - \Psi_A(t) \left(1 + \lambda(t)^{1/\gamma} \right)^{-1}.$$

For the market prices of signal risk we first obtain, using good market clearing:

$$\frac{c_1(t)}{\gamma} \theta_z^1(t) + \frac{c_2(t)}{\gamma} \theta_z^2(t) = 0.$$

Since, by construction:

$$\theta_z^1(t) - \theta_z^2(t) = \left(\alpha \Psi_A(t) \frac{\sigma_A}{\sigma_z} + \beta \Psi_z(t) \right),$$

we immediately obtain:

$$\theta_z^1(t) = \frac{c_2(t)}{A(t)} \left(\alpha \Psi_A(t) \frac{\sigma_A}{\sigma_z} + \beta \Psi_z(t) \right), \quad \theta_z^2(t) = -\frac{c_1(t)}{A(t)} \left(\alpha \Psi_A(t) \frac{\sigma_A}{\sigma_z} + \beta \Psi_z(t) \right).$$

By inserting in these formulas the form of the optimal consumption policies, the market prices of signal risk follow in closed form:

$$\begin{aligned}\theta_z^1(t) &= \left(1 + \lambda(t)^{1/\gamma}\right)^{-1} \lambda(t)^{1/\gamma} \left(\alpha \Psi_A(t) \frac{\sigma_A}{\sigma_z} + \beta \Psi_z(t)\right), \\ \theta_z^2(t) &= - \left(1 + \lambda(t)^{1/\gamma}\right)^{-1} \left(\alpha \Psi_A(t) \frac{\sigma_A}{\sigma_z} + \beta \Psi_z(t)\right).\end{aligned}$$

This concludes the discussion.

□

A.4 Proof of Lemma 2.2 (Joint Laplace Transform of $A(t)$ and $\eta(t)$)

We want to compute the following moment-generating function

$$F(A, \eta, m_A^1, \Psi_A, \Psi_z, t, u; \epsilon, \chi) = E_{A, \eta, m_A^1, \Psi_A, \Psi_z} (A(u)^\epsilon \eta(u)^\chi).$$

This function satisfies the following partial differential equation (PDE):

$$0 \equiv \mathcal{D}F(A, \eta, m_A^1, \Psi_A, \Psi_z, t, u; \epsilon, \chi) + \frac{\partial F}{\partial t}(A, \eta, m_A^1, \Psi_A, \Psi_z, t, u; \epsilon, \chi), \quad (\text{A.15})$$

with the initial condition $F(A, \eta, m_A^1, \Psi_A, \Psi_z, t, t; \epsilon, \chi) = A^\epsilon \eta^\chi$, and where \mathcal{D} is the differential generator of the multivariate process $(A(t), \eta(t), m_A^1(t), \Psi_A(t), \Psi_z(t))$ under

the probability measure of agent 1. Spelling out Feynman-Kač (A.16), we get

$$\begin{aligned}
0 = & \frac{\partial F}{\partial A} A m_A^1 + \frac{\partial F}{\partial m_A^1} (a_{0A} + a_{1A} m_A^1) + \frac{\partial F}{\partial \Psi_A} \left(\left(a_{1A} + \frac{\gamma_A^2}{\sigma_A^2} \right) \Psi_A + \left(\frac{\alpha \gamma_A^2 + \beta \gamma_{Az}^2}{\sigma_A \sigma_z} \right) \Psi_z \right) \\
& + \frac{\partial F}{\partial \Psi_z} \left(\left(a_{1z} + \frac{\alpha \gamma_{Az}^2 + \beta \gamma_z^2}{\sigma_z^2} \right) \Psi_z + \frac{\gamma_{Az}^2}{\sigma_A \sigma_z} \Psi_A \right) + \frac{1}{2} \frac{\partial^2 F}{\partial A^2} (A \sigma_A)^2 \\
& + \frac{1}{2} \frac{\partial^2 F}{\partial (m_A^1)^2} \left(\left(\frac{\gamma_A^1}{\sigma_A} \right)^2 + \left(\frac{\alpha \gamma_A^1 + \beta \gamma_{Az}^1}{\sigma_z} \right)^2 \right) \\
& + \frac{1}{2} \frac{\partial^2 F}{(\partial \Psi_A)^2} \left(\left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A^2} \right)^2 + \left(\frac{\alpha (\gamma_A^1 - \gamma_A^2) + \beta (\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A \sigma_z} \right)^2 \right) \\
& + \frac{1}{2} \frac{\partial^2 F}{(\partial \Psi_z)^2} \left(\left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_A \sigma_z} \right)^2 + \left(\frac{\alpha (\gamma_{Az}^1 - \gamma_{Az}^2) + \beta (\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right)^2 \right) \\
& + \frac{1}{2} \frac{\partial^2 F}{(\partial \eta)^2} \eta^2 \left(\Psi_A^2 + \left(\alpha \frac{\sigma_A}{\sigma_z} \Psi_A + \beta \Psi_z \right)^2 \right) \\
& + \frac{\partial^2 F}{\partial A \partial m_A^1} \gamma_A^1 A + \frac{\partial^2 F}{\partial A \partial \Psi_A} \left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A} \right) A + \frac{\partial^2 F}{\partial A \partial \Psi_z} \left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_z} \right) A - \frac{\partial^2 F}{\partial A \partial \eta} A \eta \Psi_A \sigma_A \\
& + \frac{\partial^2 F}{\partial m_A^1 \partial \Psi_A} \left(\frac{\gamma_A^1 (\gamma_A^1 - \gamma_A^2)}{\sigma_A^3} + \left(\frac{\alpha \gamma_A^1 + \beta \gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha (\gamma_A^1 - \gamma_A^2) + \beta (\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A \sigma_z} \right) \right) \\
& + \frac{\partial^2 F}{\partial m_A^1 \partial \Psi_z} \left(\frac{\gamma_A^1 (\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A^2 \sigma_z} + \left(\frac{\alpha \gamma_A^1 + \beta \gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha (\gamma_{Az}^1 - \gamma_{Az}^2) + \beta (\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right) \right) \\
& - \frac{\partial^2 F}{\partial m_A^1 \partial \eta} \eta \left(\Psi_A \frac{\gamma_A^1}{\sigma_A} + \left(\frac{\alpha \gamma_A^1 + \beta \gamma_{Az}^1}{\sigma_z} \right) \left(\alpha \Psi_A \frac{\sigma_A}{\sigma_z} + \beta \Psi_z \right) \right) \\
& - \frac{\partial^2 F}{\partial \Psi_A \partial \eta} \eta \left(\Psi_A \frac{\gamma_A^1 - \gamma_A^2}{\sigma_A^2} + \left(\frac{\alpha (\gamma_A^1 - \gamma_A^2) + \beta (\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A \sigma_z} \right) \left(\alpha \Psi_A \frac{\sigma_A}{\sigma_z} + \beta \Psi_z \right) \right) \\
& - \frac{\partial^2 F}{\partial \Psi_z \partial \eta} \eta \left(\Psi_A \frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_z \sigma_A} + \left(\frac{\alpha (\gamma_{Az}^1 - \gamma_{Az}^2) + \beta (\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right) \left(\alpha \Psi_A \frac{\sigma_A}{\sigma_z} + \beta \Psi_z \right) \right) \\
& + \frac{\partial^2 F}{\partial \Psi_A \partial \Psi_z} \left(\left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A^2} \right) \left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_A \sigma_z} \right) \right) \\
& + \left(\frac{\alpha (\gamma_A^1 - \gamma_A^2) + \beta (\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A \sigma_z} \right) \left(\frac{\alpha (\gamma_{Az}^1 - \gamma_{Az}^2) + \beta (\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right) + \frac{\partial F}{\partial t}.
\end{aligned}$$

The solution to this PDE takes the functional form

$$\begin{aligned}
F(A, \eta, m_A^1, \Psi_A, \Psi_z, t, u; \epsilon, \chi) &= A^\epsilon \eta^\chi F_{m_A^1} (m_A^1, t, u; \epsilon) F_{\Psi_A, \Psi_z} (\Psi_A, \Psi_z, t, u; \epsilon, \chi), \\
&=: A^\epsilon \eta^\chi \tilde{F} (m_A^1, \Psi_A, \Psi_z, t, u; \epsilon, \chi).
\end{aligned}$$

Plugging in this expression into equation (A.16), yields:

$$\begin{aligned}
0 = & \tilde{F}\epsilon m_A^1 + \frac{\partial \tilde{F}}{\partial m_A^1} (a_{0A} + a_{1A}m_A^1) + \frac{\partial \tilde{F}}{\partial \Psi_A} \left(\left(a_{1A} + \frac{\gamma_A^2}{\sigma_A^2} \right) \Psi_A + \left(\frac{\alpha\gamma_A^2 + \beta\gamma_{Az}^2}{\sigma_A\sigma_z} \right) \Psi_z \right) \\
& + \frac{\partial \tilde{F}}{\partial \Psi_z} \left(\left(a_{1z} + \frac{\alpha\gamma_{Az}^2 + \beta\gamma_z^2}{\sigma_z^2} \right) \Psi_z + \frac{\gamma_{Az}^2}{\sigma_A\sigma_z} \Psi_A \right) + \frac{1}{2}\epsilon(\epsilon - 1)\tilde{F}\sigma_A^2 \\
& + \frac{1}{2} \frac{\partial^2 \tilde{F}}{\partial (m_A^1)^2} \left(\left(\frac{\gamma_A^1}{\sigma_A} \right)^2 + \left(\frac{\alpha\gamma_A^1 + \beta\gamma_{Az}^1}{\sigma_z} \right)^2 \right) \\
& + \frac{1}{2} \frac{\partial^2 \tilde{F}}{(\partial \Psi_A)^2} \left(\left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A^2} \right)^2 + \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A\sigma_z} \right)^2 \right) \\
& + \frac{1}{2} \frac{\partial^2 \tilde{F}}{(\partial \Psi_z)^2} \left(\left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_A\sigma_z} \right)^2 + \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right)^2 \right) \\
& + \frac{1}{2}\chi(\chi - 1)\tilde{F} \left(\Psi_A^2 + \left(\alpha \frac{\sigma_A}{\sigma_z} \Psi_A + \beta \Psi_z \right)^2 \right) \\
& + \frac{\partial \tilde{F}}{\partial m_A^1} \epsilon \gamma_A^1 + \frac{\partial \tilde{F}}{\partial \Psi_A} \left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A} \right) \epsilon + \frac{\partial \tilde{F}}{\partial \Psi_z} \left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_z} \right) \epsilon - \epsilon \chi \tilde{F} \Psi_A \sigma_A \\
& + \frac{\partial^2 \tilde{F}}{\partial m_A^1 \partial \Psi_A} \left(\frac{\gamma_A^1 (\gamma_A^1 - \gamma_A^2)}{\sigma_A^3} + \left(\frac{\alpha\gamma_A^1 + \beta\gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A\sigma_z} \right) \right) \\
& + \frac{\partial^2 \tilde{F}}{\partial m_A^1 \partial \Psi_z} \left(\frac{\gamma_A^1 (\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A^2 \sigma_z} + \left(\frac{\alpha\gamma_A^1 + \beta\gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right) \right) \\
& - \frac{\partial \tilde{F}}{\partial m_A^1} \chi \left(\Psi_A \frac{\gamma_A^1}{\sigma_A} + \left(\frac{\alpha\gamma_A^1 + \beta\gamma_{Az}^1}{\sigma_z} \right) \left(\alpha \Psi_A \frac{\sigma_A}{\sigma_z} + \beta \Psi_z \right) \right) \\
& - \frac{\partial \tilde{F}}{\partial \Psi_A} \chi \left(\Psi_A \frac{\gamma_A^1 - \gamma_A^2}{\sigma_A^2} + \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A\sigma_z} \right) \left(\alpha \Psi_A \frac{\sigma_A}{\sigma_z} + \beta \Psi_z \right) \right) \\
& - \frac{\partial \tilde{F}}{\partial \Psi_z} \chi \left(\Psi_A \frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_z \sigma_A} + \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right) \left(\alpha \Psi_A \frac{\sigma_A}{\sigma_z} + \beta \Psi_z \right) \right) \\
& + \frac{\partial^2 \tilde{F}}{\partial \Psi_A \partial \Psi_z} \left(\left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A^2} \right) \left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_A\sigma_z} \right) \right. \\
& \left. + \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A\sigma_z} \right) \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right) \right) + \frac{\partial \tilde{F}}{\partial t}.
\end{aligned}$$

We can first factor out the expressions that do not involve η and Ψ (given in blue) and solve for $F_{m_A^1}$ by direct integration. To this end, we guess the following functional form:

$$F_{m_A^1}(m_A^1, t, u, \epsilon) = \exp(A(\epsilon, u - t)m_A^1 + C(\epsilon, u - t)),$$

with the explicit solutions for $A(\epsilon, \tau)$ and $C(\epsilon, \tau)$ given by:

$$\begin{aligned} A(\epsilon, u-t) &= \frac{\epsilon \left(e^{a_{1A}(u-t)} - 1 \right)}{a_{1A}}, \\ C(\epsilon, u-t) &= \frac{1}{2} \epsilon (\epsilon - 1) \sigma_A (u-t) + \frac{1}{a_{1A}} (a_{0A} + \epsilon \gamma_A^1) \left(e^{-a_{1A}(u-t)} + u-t \right) \\ &\quad + \frac{1}{a_{1A}} \left(\left(\frac{\gamma_A^1}{\sigma_A} \right)^2 + \left(\frac{\alpha \gamma_A^1 + \beta \gamma_{Az}^1}{\sigma_z} \right)^2 \right) \left(\frac{3}{2} e^{a_{1A}(u-t)} - a_{1A} (u-t) \right). \end{aligned}$$

Next, we guess the following functional form:

$$\begin{aligned} F_{\Psi_A, \Psi_z}(\Psi_A, \Psi_z, t, \epsilon, \chi, u) &= \exp \left(A_0(\epsilon, \chi, u-t) + B_1(\epsilon, \chi, u-t) \Psi_A + B_2(\epsilon, \chi, u-t) \Psi_z \right. \\ &\quad \left. + C_1(\epsilon, \chi, u-t) \Psi_A^2 + C_2(\epsilon, \chi, u-t) \Psi_z^2 + D_0(\epsilon, \chi, u-t) \Psi_A(t) \Psi_z \right). \end{aligned}$$

From this guess, we obtain the derivatives:

$$\begin{aligned} \frac{\partial \tilde{F}}{\partial \Psi_A} &= \tilde{F} (B_1(u-t) + 2C_1(u-t) \Psi_A + D_0(u-t) \Psi_z), \\ \frac{\partial^2 \tilde{F}}{\partial \Psi_A^2} &= \tilde{F} \left((B_1(u-t) + 2C_1(u-t) \Psi_A + D_0(u-t) \Psi_z)^2 + 2C_1(u-t) \right), \\ \frac{\partial \tilde{F}}{\partial \Psi_z} &= \tilde{F} (B_2(u-t) + 2C_2(u-t) \Psi_z + D_0(u-t) \Psi_A), \\ \frac{\partial^2 \tilde{F}}{\partial \Psi_z^2} &= \tilde{F} \left((B_2(u-t) + 2C_2(u-t) \Psi_z + D_0(u-t) \Psi_A)^2 + 2C_2(u-t) \right), \\ \frac{\partial \tilde{F}}{\partial \Psi_A \partial \Psi_z} &= \tilde{F} \left((B_1(u-t) + 2C_1(u-t) \Psi_A + D_0(u-t) \Psi_z) \left(B_2(u-t) + 2C_2(u-t) \Psi_z \right. \right. \\ &\quad \left. \left. + D_0(u-t) \Psi_A \right) + D_0(u-t) \right), \\ \frac{\partial \tilde{F}}{\partial t} &= -\tilde{F} \left(A'_0(u-t) + B'_1(u-t) \Psi_A + B'_2(u-t) \Psi_z + C'_1(u-t) \Psi_A^2 + C'_2(u-t) \Psi_z^2 \right. \\ &\quad \left. + D'_0(u-t) \Psi_A \Psi_z \right), \end{aligned}$$

which, plugged-in into the initial differential equation imply:

$$\begin{aligned}
0 = & (B_1(\tau) + 2C_1(\tau)\Psi_A + D_0(\tau)\Psi_z) \left(\left(a_{1A} + \frac{\gamma_A^2}{\sigma_A^2} \right) \Psi_A + \left(\frac{\alpha\gamma_A^2 + \beta\gamma_{Az}^2}{\sigma_A\sigma_z} \right) \Psi_z \right) \\
& + (B_2(\tau) + 2C_2(\tau)\Psi_z + D_0(\tau)\Psi_A) \left(\left(a_{1z} + \frac{\alpha\gamma_{Az}^2 + \beta\gamma_z^2}{\sigma_z^2} \right) \Psi_z + \frac{\gamma_{Az}^2}{\sigma_A\sigma_z} \Psi_A \right) + \\
& + \frac{1}{2} \left((B_1(\tau) + 2C_1(\tau)\Psi_A + D_0(\tau)\Psi_z)^2 + 2C_1(\tau) \right) \\
& \times \left(\left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A^2} \right)^2 + \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A\sigma_z} \right)^2 \right) \\
& + \frac{1}{2} \left((B_2(\tau) + 2C_2(\tau)\Psi_z + D_0(\tau)\Psi_A)^2 + 2C_2(\tau) \right) \\
& \times \left(\left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_A\sigma_z} \right)^2 + \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right)^2 \right) \\
& + \frac{1}{2}\chi(\chi - 1) \left(\Psi_A^2 + \left(\alpha \frac{\sigma_A}{\sigma_z} \Psi_A + \beta \Psi_z \right)^2 \right) + (B_1(\tau) + 2C_1(\tau)\Psi_A + D_0(\tau)\Psi_z) \left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A} \right) \epsilon \\
& + (B_2(\tau) + 2C_2(\tau)\Psi_z + D_0(\tau)\Psi_A) \left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_z} \right) \epsilon - \epsilon\chi\Psi_A\sigma_A \\
& + A(\epsilon, \tau) (B_1(\tau) + 2C_1(\tau)\Psi_A + D_0(\tau)\Psi_z) \\
& \times \left(\frac{\gamma_A^1(\gamma_A^1 - \gamma_A^2)}{\sigma_A^3} + \left(\frac{\alpha\gamma_A^1 + \beta\gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A\sigma_z} \right) \right) \\
& + A(\epsilon, \tau) (B_2(\tau) + 2C_2(\tau)\Psi_z + D_0(\tau)\Psi_A) \\
& \times \left(\frac{\gamma_A^1(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A^2\sigma_z} + \left(\frac{\alpha\gamma_A^1 + \beta\gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right) \right) \\
& - A(\epsilon, \tau)\chi \left(\Psi_A \frac{\gamma_A^1}{\sigma_A} + \left(\frac{\alpha\gamma_A^1 + \beta\gamma_{Az}^1}{\sigma_z} \right) \left(\alpha\Psi_A \frac{\sigma_A}{\sigma_z} + \beta\Psi_z \right) \right) \\
& - (B_1(\tau) + 2C_1(\tau)\Psi_A + D_0(\tau)\Psi_z) \chi \\
& \times \left(\Psi_A \frac{\gamma_A^1 - \gamma_A^2}{\sigma_A^2} + \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A\sigma_z} \right) \left(\alpha\Psi_A \frac{\sigma_A}{\sigma_z} + \beta\Psi_z \right) \right) \\
& - (B_2(\tau) + 2C_2(\tau)\Psi_z + D_0(\tau)\Psi_A) \chi \\
& \times \left(\Psi_A \frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_z\sigma_A} + \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right) \left(\alpha\Psi_A \frac{\sigma_A}{\sigma_z} + \beta\Psi_z \right) \right) \\
& + ((B_1(\tau) + 2C_1(\tau)\Psi_A + D_0(\tau)\Psi_z) (B_2(\tau) + 2C_2(\tau)\Psi_z + D_0(\tau)\Psi_A) + D_0(\tau)) \\
& \times \left(\left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A^2} \right) \left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_A\sigma_z} \right) + \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A\sigma_z} \right) \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right) \right) \\
& - (A'_0(\tau) + B'_1(\tau)\Psi_A + B'_2(\tau)\Psi_z + C'_1(\tau)\Psi_A^2 + C'_2(\tau)\Psi_z^2 + D'_0(\tau)\Psi_A\Psi_z) .
\end{aligned}$$

It follows that functions A_0, B_1, B_2, C_1, C_2 and D_0 must solve the following system of ODEs.

$$C_1'(\tau) = 2a_1 C_1^2(\tau) + 2b_1 C_1(\tau) + \frac{1}{2} \tilde{a}_1 D_0^2(\tau) + \tilde{b}_1 D_0(\tau) + 2c_1 C_1(\tau) D_0(\tau) + d_1, \quad (\text{A.16})$$

$$C_2'(\tau) = 2\tilde{a}_1 C_2^2(\tau) + 2b_2 C_2(\tau) + \frac{1}{2} a_1 D_0^2(\tau) + \tilde{b}_2 D_0(\tau) + 2c_1 C_2(\tau) D_0(\tau) + d_2, \quad (\text{A.17})$$

$$D_0'(\tau) = c_1 D_0^2(\tau) + \frac{1}{2} (b_1 + b_2) D_0(\tau) + 2\tilde{b}_1 C_2(\tau) + 2\tilde{b}_2 C_1(\tau) + 2a_1 C_1(\tau) D_0(\tau) + 2\tilde{a}_1 C_2(\tau) D_0(\tau) + 4c_1 C_1(\tau) C_2(\tau) + d_5, \quad (\text{A.18})$$

$$B_1'(\tau) = b_1 B_1(\tau) + \tilde{b}_1 B_2(\tau) + a_1 B_1(\tau) C_1(\tau) + \tilde{a}_1 B_2 D_0(\tau) + c_1 B_1(\tau) D_0(\tau) + 2c_1 C_1(\tau) B_2(\tau) + d_3 - e^{a_1 \Lambda \tau} \tilde{d}_3 + C_1(\tau) (\tilde{b}_3 e^{a_1 \Lambda \tau} - b_3) + D_0(\tau) (\tilde{c}_3 e^{a_1 \Lambda \tau} - c_3) + d_3, \quad (\text{A.19})$$

$$B_2'(\tau) = b_2 B_2(\tau) + b_1 B_1(\tau) + a_1 B_1(\tau) D_0(\tau) + \tilde{a}_1 B_2 C_2(\tau) + (2B_1(\tau) C_2(\tau) + B_2(\tau) D_0(\tau)) c_1 + C_2(\tau) (\tilde{b}_3 e^{a_1 \Lambda \tau} - b_3) + D_0(\tau) (\tilde{c}_3 e^{a_1 \Lambda \tau} - c_3) + d_4 (e^{a_1 \Lambda \tau} - 1), \quad (\text{A.20})$$

$$A_0'(\tau) = \frac{1}{2} a_1 B_1^2(\tau) + \frac{1}{2} \tilde{a}_1 B_2^2(\tau) + \tilde{a}_1 C_2(\tau) + b_3 B_1(\tau) + \frac{1}{2} c_3 B_2(\tau) + c_1 B_1(\tau) B_2(\tau), \quad (\text{A.21})$$

subject to the initial condition:

$$C_1(0) = C_2(0) = B_1(0) = B_2(0) = D_0(0) = A_0(0) = 0.$$

In these equations, the coefficients are given explicitly by:

$$\begin{aligned}
a_1 &= \left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A^2} \right)^2 + \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A \sigma_z} \right)^2, \\
b_1 &= a_{1A} + \frac{\gamma_A^2}{\sigma_A^2} - \chi \left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A} + \frac{\alpha \sigma_A}{\sigma_z} \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A \sigma_z} \right) \right), \\
\tilde{a}_1 &= \left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_A \sigma_z} \right)^2 + \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right)^2, \\
\tilde{b}_1 &= \frac{\gamma_{Az}^2}{\sigma_A \sigma_z} - \chi \left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_A \sigma_z} + \left(\frac{\alpha \sigma_A}{\sigma_z} \right) \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right) \right), \\
c_1 &= \left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A^2} \right) \left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_A \sigma_z} \right) + \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A \sigma_z} \right) \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right), \\
d_1 &= \frac{1}{2} \chi (\chi - 1) \left(1 + \left(\frac{\alpha \sigma_A}{\sigma_z} \right)^2 \right), \\
b_2 &= a_{1z} + \frac{\alpha \gamma_{Az}^2 - \chi \beta \gamma_{Az}^2}{\sigma_z^2} + \beta \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right), \\
\tilde{b}_2 &= \frac{\alpha \gamma_A^2 + \beta \gamma_{Az}^2}{\sigma_A \sigma_z} - \chi \beta \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A \sigma_z} \right), \\
d_2 &= \frac{1}{2} \chi (\chi - 1) \beta^2, \\
b_3 &= \epsilon \left(\frac{\gamma_A^1 - \gamma_A^2}{\sigma_A} \right) - \frac{\epsilon}{a_{1A}} \left(\frac{\gamma_A^1 (\gamma_A^1 - \gamma_A^2)}{\gamma_A^3} + \left(\frac{\alpha \gamma_A^1 + \beta \gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A \sigma_z} \right) \right), \\
\tilde{b}_3 &= \frac{\epsilon}{a_{1A}} \left(\frac{\gamma_A^1 (\gamma_A^1 - \gamma_A^2)}{\sigma_A^3} + \left(\frac{\alpha \gamma_A^1 + \beta \gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha(\gamma_A^1 - \gamma_A^2) + \beta(\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A \sigma_z} \right) \right), \\
c_3 &= 2\epsilon \left(\frac{\gamma_{Az}^1 - \gamma_{Az}^2}{\sigma_z} \right) - \frac{2\epsilon}{a_{1A}} \left(\frac{\gamma_A^1 (\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A^2 \sigma_z} + \left(\frac{\alpha \gamma_A^1 + \beta \gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z^2} \right) \right), \\
\tilde{c}_3 &= \frac{2\epsilon}{a_{1A}} \left(\frac{\gamma_A^1 (\gamma_{Az}^1 - \gamma_{Az}^2)}{\sigma_A^2 \sigma_z} + \left(\frac{\alpha \gamma_A^1 + \beta \gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha(\gamma_{Az}^1 - \gamma_{Az}^2) + \beta(\gamma_z^1 - \gamma_z^2)}{\sigma_z} \right) \right), \\
d_3 &= \epsilon \chi \left(\sigma_A - \frac{1}{a_{1A}} \left(\frac{\gamma_A^1}{\sigma_A} + \frac{\alpha \gamma_A^1 + \beta \gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha \sigma_A}{\sigma_z} \right) \right), \\
\tilde{d}_3 &= -\frac{\epsilon \chi}{a_{1A}} \left(\frac{\gamma_A^1}{\sigma_A} + \left(\frac{\alpha \gamma_A^1 + \beta \gamma_{Az}^1}{\sigma_z} \right) \left(\frac{\alpha \sigma_A}{\sigma_z} \right) \right), \\
d_5 &= \frac{\alpha \sigma_A}{\sigma_z} \chi (\chi - 1).
\end{aligned}$$

To solve the system of equations (A.16)-(A.21), we first solve equations (A.16)-(A.18). First, we observe that the system of differential equations (A.16)-(A.18) can be written as the following matrix Riccati equation:

$$\frac{dA}{d\tau} = AM'MA + AP + P'A + D \quad (\text{A.22})$$

with coefficient matrices defined by:

$$A = \begin{pmatrix} C_1 & D_0 \\ D_0 & C_2 \end{pmatrix}, \quad M = \begin{pmatrix} 2\sqrt{a_1} & 0 \\ \frac{c_1}{\sqrt{a_1}} & \sqrt{2\tilde{a}_1 - \frac{c_1^2}{a_1}} \end{pmatrix}, \quad P = \begin{pmatrix} 2b_1 & \tilde{b}_2 \\ \tilde{b}_1 & 2b_2 \end{pmatrix} \quad D = \begin{pmatrix} d_1 & d_5 \\ d_5 & d_1 \end{pmatrix}.$$

The matrix Riccati equation (A.22) can be solved in closed form by transforming it into a locally equivalent linear system of ordinary differential equations by a homogenization procedure (Radon's Lemma). Let

$$\begin{pmatrix} C_{11}(t) & C_{12}(t) \\ C_{21}(t) & C_{22}(t) \end{pmatrix} = \exp \left(t \begin{pmatrix} P & M'M \\ D & -P' \end{pmatrix} \right).$$

Then, the solution of differential equation (A.22) is:

$$A(t) = C_{22}(t)^{-1}C_{21}(t), \quad (\text{A.23})$$

using the fact that $A(0) = 0$. The solutions for B_1 , B_2 , and A_0 follow by direct integration. This concludes the proof. □

A.5 Proof of Lemma 2.3 (Security Prices)

By definition, the risk-less zero coupon bond price is given by:

$$B(t, T) = \frac{1}{\xi^1(t)} E_t^1 \left(e^{-\rho(T-t)} \xi^1(T) \right).$$

Using the expression for $\xi^1(t)$, we get:

$$B(t, T) = E_t^1 \left(e^{-\rho(T-t)} \left(\frac{A(T)}{A(t)} \right)^{-\gamma} \left(\frac{1 + \lambda(T)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \right)^\gamma \right). \quad (\text{A.24})$$

Let

$$G(t, T, x; \Psi_A, \Psi_z) \equiv \int_0^\infty \left(\frac{1 + \lambda(T)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \right)^\gamma \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} \left(\frac{\lambda(T)}{\lambda(t)} \right)^{-ix} F_{\Psi_A, \Psi_z}(\Psi_A, \Psi_z, t, T; -\gamma, ix) d\chi \right] \frac{d\lambda(T)}{\lambda(T)}.$$

By Fourier inversion, it then follows:

$$B(t, T) = e^{-\rho(T-t)} F_{m_A^1}(m_A^1, t, T; -\gamma) G(t, T, -\gamma; \Psi_A, \Psi_z).$$

In a similar way, the firm value is:

$$\begin{aligned} V(t) &= E_t^1 \left(\int_t^\infty e^{-\rho(u-t)} \frac{\xi^1(u)}{\xi^1(t)} A(u) du \right), \\ &= A(t) E_t^1 \left(\int_t^\infty e^{-\rho(u-t)} \left(\frac{1 + \lambda(t)^{1/\gamma}}{1 + \lambda(u)^{1/\gamma}} \right)^\gamma \left(\frac{A(u)}{A(t)} \right)^{1-\gamma} du \right), \\ &= A(t) \int_t^\infty \left(e^{-\rho(u-t)} F_{m_A^1}(m_A^1, t, u; 1-\gamma) G(u, T, 1-\gamma; \Psi_A, \Psi_z) \right) du. \end{aligned}$$

The price of the senior bond is:

$$\begin{aligned} B^s(t, T) &= K_1 B(t, T) - E_t^1 \left(e^{-\rho(T-t)} \frac{\xi^1(T)}{\xi^1(t)} (K_1 - V(T))^+ \right), \\ &= K_1 B(t, T) - E_t^1 \left(e^{-\rho(T-t)} \left(\frac{A(T)}{A(t)} \right)^{-\gamma} \left(\frac{1 + \lambda(T)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \right)^\gamma (K_1 - V(T))^+ \right), \\ &= K_1 B(t, T) - P(t, T, K_1), \end{aligned}$$

where $P(t, T, K_1)$ is the price of the put option on the firm value. The price of the junior bond is:

$$\begin{aligned} B^j(t, T) &= E_t^1 \left(e^{-\rho(T-t)} \frac{\xi^1(T)}{\xi^1(t)} (V(T) - K_1)^+ \right) - E_t^1 \left(e^{-\rho(T-t)} \frac{\xi^1(T)}{\xi^1(t)} (V(T) - (K_1 + K_2))^+ \right), \\ &= C(t, T, K_1) - C(t, T, K_1 + K_2), \end{aligned}$$

where $C(t, T, K_1)$ and $C(t, T, K_1 + K_2)$ are call options on the firm value with strikes K_1 and $K_1 + K_2$, respectively. Equity in our economy is a call option on the firm value with strike price $K_1 + K_2$. Therefore:

$$S(t) = E_t^1 \left(e^{-\rho(T-t)} \frac{\xi^1(T)}{\xi^1(t)} (V(T) - (K_1 + K_2))^+ \right) = C(t, T, K_1 + K_2).$$

A European call option on the equity value is derived in the following way:

$$O(t, T) = E_t^1 \left(e^{-\rho(T-t)} \frac{\xi^1(T)}{\xi^1(t)} (S(T) - K_e)^+ \right).$$

This concludes the proof.

□

A.6 Risk-neutral Skewness and Volatility Formulas

It follows from Bakshi and Madan (2000) that the entire collection of twice-continuously differentiable payoff functions with bounded expectation can be spanned algebraically. Applying this result to the firm value $V(t)$ (or equivalently to the stock value $S(t)$) yields:

$$G(V) = G(\tilde{V}) + (V - \tilde{V})G_V(\tilde{V}) + \int_{\tilde{V}}^{\infty} G_{VV}(K)(V - K)^+ dK + \int_0^{\tilde{V}} G_{VV}(K)(K - V)^+ dK,$$

where G_V is the partial derivative of the payoff function $G(V)$ with respect to V and G_{VV} the corresponding second-order partial derivative. By setting $\tilde{V} = V(t)$ we obtain the final formula for the firm value risk-neutral skewness, after applying the same steps as in Bakshi, Kapadia, and Madan (2003) (Theorem 1, p. 137).

Proposition A.5. *Let $v(t, T) = \ln(V(t+T)) - \ln(V(t))$ be the firm value return between time t and T . The risk-neutral skewness of $v(t, T)$ is given by:*

$$skew(t, T) = \frac{E_t \left((v(t, T) - E_t(v(t, T)))^3 \right)}{\left(E_t(v(t, T) - E_t(v(t, T)))^2 \right)^{3/2}} = \frac{e^{rT} W(t, T) - 2\mu(t, T)e^{rT} R(t, T) + 2\mu(t, T)^3}{(e^{rT} R(t, T) - \mu(t, T)^2)^{3/2}},$$

where

$$\begin{aligned}
R(t, T) &= \int_{V(t)}^{\infty} \frac{2 \left(1 - \ln \left(\frac{K}{V(t)}\right)\right)}{K^2} (V(T) - K)^+ dK + \int_0^{V(t)} \frac{2 \left(1 + \ln \left(\frac{V(t)}{K}\right)\right)}{K^2} (K - V(T))^+ dK, \\
W(t, T) &= \int_{V(t)}^{\infty} \frac{6 \ln \left(\frac{K}{V(t)}\right) - 3 \left(\ln \left(\frac{K}{V(t)}\right)\right)^2}{K^2} (V(T) - K)^+ dK \\
&\quad - \int_0^{V(t)} \frac{6 \ln \left(\frac{V(t)}{K}\right) - 3 \left(\ln \left(\frac{V(t)}{K}\right)\right)^2}{K^2} (K - V(T))^+ dK,
\end{aligned}$$

and

$$\begin{aligned}
X(t, T) &= \int_{V(t)}^{\infty} \frac{12 \left(\ln \left(\frac{K}{V(t)}\right)\right)^2 - 4 \left(\ln \left(\frac{K}{V(t)}\right)\right)^3}{K^2} (V(T) - K)^+ dK \\
&\quad - \int_0^{V(t)} \frac{12 \left(\ln \left(\frac{V(t)}{K}\right)\right)^2 - 4 \left(\ln \left(\frac{V(t)}{K}\right)\right)^3}{K^2} (K - V(T))^+ dK, \\
\mu(t, T) &= E_t \left(\ln \left(\frac{V(t+T)}{V(t)} \right) \right) \approx e^{rT} - 1 - \frac{e^{rT}}{2} R(t, T) - \frac{e^{rT}}{6} W(t, T) - \frac{e^{rT}}{24} X(t, T).
\end{aligned}$$

The expression for the stock returns volatility in the paper follows from a straight-forward application of Itô's Lemma.² The price of the stock, defined in equation (A.10), satisfies a diffusion process given by:

$$\frac{dS}{S} = \mu_S^1(t)dt + \sigma_{SA}(t)dW_A^1(t) + \sigma_{Sz}(t)dW_z^1(t),$$

Therefore, the diffusion term in this dynamics is characterized by:

$$\begin{aligned}
dS(t) - S(t)\mu_S^1(t)dt &= \frac{\partial S}{\partial A}(dA(t) - E_t^1(dA(t))) + \frac{\partial S}{\partial m_A^1}(dm_A^1(t) - E_t^1(dm_A^1(t))) \\
&\quad + \frac{\partial S}{\partial \Psi_A}(d\Psi_A(t) - E_t^1(d\Psi_A(t))) + \frac{\partial S}{\partial \Psi_z}(d\Psi_z(t) - E_t^1(d\Psi_z(t))) \\
&= \frac{\partial S}{\partial A}A(t)\sigma_A dW_A^1(t) + \frac{\partial S}{\partial m_A^1} \left(\frac{\gamma_A^1(t)}{\sigma_A} dW_A^1(t) + \left(\frac{\alpha\gamma_A^1(t) + \beta\gamma_{Az}^1(t)}{\sigma_z} \right) dW_z^1(t) \right) \\
&\quad + \frac{\partial S}{\partial \Psi_A} \left(\frac{\gamma_A^1(t) - \gamma_A^2(t)}{\sigma_A^2} dW_A^1(t) + \left(\frac{\alpha(\gamma_A^1(t) - \gamma_A^2(t)) + \beta(\gamma_{Az}^1(t) - \gamma_{Az}^2(t))}{\sigma_A\sigma_z} \right) dW_z^1(t) \right) \\
&\quad + \frac{\partial S}{\partial \Psi_z} \left(\frac{\gamma_{Az}^1(t) - \gamma_{Az}^2(t)}{\sigma_A\sigma_z} dW_A^1(t) + \left(\frac{\alpha(\gamma_{Az}^1(t) - \gamma_{Az}^2(t)) + \beta(\gamma_z^1(t) - \gamma_z^2(t))}{\sigma_z^2} \right) dW_z^1(t) \right).
\end{aligned}$$

²With the same procedure one also obtains the volatility of firm value returns.

By matching coefficients, we obtain:

$$\sigma_{SA}(t) = \frac{1}{S(t)} \left(\frac{\partial S}{\partial A} A(t) \sigma_A + \frac{\partial S}{\partial m_A^1} \frac{\gamma_A^1(t)}{\sigma_A} + \frac{\partial S}{\partial \Psi_A} \frac{\gamma_A^1(t) - \gamma_A^2(t)}{\sigma_A^2} + \frac{\partial S}{\partial \Psi_z} \frac{\gamma_{Az}^1(t) - \gamma_{Az}^2(t)}{\sigma_A \sigma_z} \right)$$

and

$$\begin{aligned} \sigma_{S_z}(t) = & \frac{1}{S(t)} \left(\frac{\partial S}{\partial m_A^1} \frac{\alpha \gamma_A^1(t) + \beta \gamma_{Az}^1(t)}{\sigma_z} + \frac{\partial S}{\partial \Psi_A} \frac{\alpha(\gamma_A^1(t) - \gamma_A^2(t)) + \beta(\gamma_{Az}^1(t) - \gamma_{Az}^2(t))}{\sigma_A \sigma_z} \right) \\ & + \frac{1}{S(t)} \frac{\partial S}{\partial \Psi_z} \frac{\alpha(\gamma_{Az}^1(t) - \gamma_{Az}^2(t)) + \beta(\gamma_z^1(t) - \gamma_z^2(t))}{\sigma_z^2} \end{aligned}$$

Thus, the volatility of stock returns at time t is $(\sigma_{SA}^2(t) + \sigma_{S_z}^2(t))^{1/2}$, with σ_{SA} and σ_{S_z} given above. For the special case, in which $\beta = 0$, the second of these expressions simplifies:

$$\sigma_{S_z}(t) = \frac{1}{S(t)} \left(\frac{\partial S}{\partial m_A^1} \frac{\alpha \gamma_A^1(t)}{\sigma_z} + \frac{\partial S}{\partial \Psi_A} \frac{\alpha(\gamma_A^1(t) - \gamma_A^2(t))}{\sigma_A \sigma_z} + \frac{\partial S}{\partial \Psi_z} \frac{\alpha(\gamma_{Az}^1(t) - \gamma_{Az}^2(t))}{\sigma_z^2} \right).$$

This concludes the discussion of the risk-neutral skewness and the volatility of stock returns in our model.

Appendix B

Appendix B

B.1 Technical Proofs

To save space, details of most calculations are presented in a technical Appendix to this paper, which is available on the authors' webpage.

Lemma B.1. *Let*

$$G(t, T, x_{D_1}, x_{D_2}; \Psi) \equiv \int_0^\infty \left(\frac{1 + \lambda(T)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \right)^\gamma \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} \left(\frac{\lambda(T)}{\lambda(t)} \right)^{-i\chi} F_\Psi(\Psi, t, T; x, i\chi) d\chi \right] \frac{d\lambda(T)}{\lambda(T)}.$$

1. *The equilibrium price of stock 1 is:*

$$\begin{aligned} S_1(t) &:= S_1(D_1, m^A, \Psi), \\ &= D_1(t) \int_t^\infty e^{-\delta(u-t)} F_{m^A}(m^A, t, u; 1 - \gamma, 0) G(t, u, 1 - \gamma, 0; \Psi) du. \end{aligned}$$

2. *The equilibrium price of stock 2 is:*

$$\begin{aligned} S_2(t) &:= S_2(D_1, D_2, m^A, \Psi), \\ &= D_2(t) \int_t^\infty e^{-\delta(u-t)} F_{m^A}(m^A, t, u; -2\gamma, 1 + \gamma) G(t, u, -2\gamma, 1 + \gamma; \Psi) du. \end{aligned}$$

3. The equilibrium price of the index is:

$$ID(t) := ID(D_1, D_2, m^A, \Psi) = \omega_1 S_1(t) + \omega_2 S_2(t).$$

4. The equilibrium price of the European option on stock 1 is:

$$\begin{aligned} O_1(t) &:= O_1(D_1, m^A, \Psi), \\ &= E_t^A \left(e^{-\delta(T-t)} \left(\frac{D_1(t)}{D_1(T)} \frac{1 + \lambda(T)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \right)^\gamma (S_1(T) - K_1)^+ \right). \end{aligned}$$

The formula for the option on stock 2 is identical with the corresponding replacements, and with $S_2(T)$ and K_2 replacing $S_1(T)$ and K_1 , respectively.

5. The equilibrium price of the European option on the index is:

$$\begin{aligned} I(t) &:= I(D_1, D_2, m^A, \Psi), \\ &= E_t^A \left(e^{-\delta(T-t)} \left(\frac{D_1(t)}{D_1(T)} \frac{1 + \lambda(T)^{1/\gamma}}{1 + \lambda(t)^{1/\gamma}} \right)^\gamma (ID(T) - K_{ID})^+ \right). \end{aligned}$$

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